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Analytical bending solution of fully clamped orthotropic rectangular plates resting on elastic foundations by the finite integral transform method

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Abstract

This study presents exact bending solution of fully clamped orthotropic rectangular plates subjected to arbitrary loads resting on elastic foundations, based on the finite integral transform method. In this method, it is not necessary to determine the deformation function because the basic governing equations of the classical plate theory for orthotropic plates have been used. A detailed parametric study is conducted to elucidate the influences of stiffness of elastic medium, plate length, flexural rigidities and distributed transverse load on the deflections. The applicability of the method is extensive since it can solve any plates with different loadings. Numerical results are presented to demonstrate the validity and accuracy of the approach, as it is totally in agreement with the other studies.

Keywords: Analytical solution, Finite integral transform method, Foundation plate, Orthotropic rectangular plate.

1. Introduction

Studying orthotropic structures such as rectangular plates is important, since orthotropic material analysis is used in composite structure analysis. Therefore, the deflections of these plates have been examined widely by different methods both analytically [1-6] and numerically [7-13]. These methods, which are mostly numerical, can satisfy neither the dominant equation nor the boundary conditions. Although, it is well-known that explicit analytical bending solutions of isotropic and orthotropic rectangular thin plates have been available for the cases with two opposite edges simply supported (i.e., Navier's solution and Lévy's solution). Otherwise, there are some problematic methods, semi-inverse and superposition, for instance. The semi-inverse method needs a predetermined function such as deflection or stress and in the superposition method [1], a problem must be divided into small ones which might pose difficulty being solved if not broken into manageable ones.

In this paper, an analytical finite integral transform method is adopted to acquire the exact bending solution. To the best of the authors' knowledge, there have been no reports of the present method on the analysis of clamped orthotropic rectangular plates resting on elastic foundations under arbitrary loadings. However, integral transform method is used to obtain exact solution for specific partial differential equations in elasticity theory [14, 15]. The main advantages of the method that are different from available analytical ones are: the trial functions are not required, which ensures the better accuracy and the broad generality; and it offers a systematic framework to explore the bending problems of plates with different boundary conditions such as cantilever plates by Rui Li [16] or free plates by Rui Li [17]. To solve this, an appropriate double finite sine integral transform is defined, and four infinite systems of linear simultaneous equations are achieved by applying the double integral transform on equation and considering boundary conditions. To verify the accuracy of the approach, the presented numerical results have been compared with the traditional semi-inverse approaches to the analysis of plates using classical plate theory by Timoshenko [1] and a symplectic superposition method [3]. Good agreement has been achieved, which confirms the accuracy and applicability of the approach.

2. Dominant equation and the boundary conditions of fully clamped orthotropic rectangular plates resting on elastic foundations

Consider a fully clamped orthotropic rectangular thin plate resting on an elastic Winkler foundation as shown in Fig. 1.

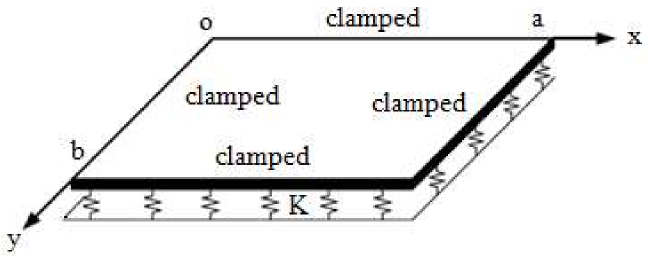

Fig. 1. A clamped rectangular thin plate resting on an elastic foundation.

The governing partial differential equation for bending of the plate for which the principal directions of orthotropic coincide with the x and y axes is as [1]:

$$D_x \frac{\partial^4 W}{\partial x^4} + 2H \frac{\partial^4 W}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 W}{\partial y^4} = q - KW \tag{1}$$

where D_x and D_y are the flexural rigidities about y and x axes, respectively, w refer to transverse deflection of middle surface of the plate, $H = D_1 + 2D_{xy}$ is the effective torsional rigidity in which $D_1 = \nu_2 D_x = \nu_1 D_y$ and ν_1 and ν_2 are Poisson's ratios, q is the distributed transverse load and K denotes the foundation module. In Fig. 1, a and b are length and width, respectively, where $0 \le x \le a$ and $0 \le y \le b$.

The internal forces of the plate are:

$$M_x = -(D_x \frac{\partial^2 W}{\partial x^2} + D_1 \frac{\partial^2 W}{\partial y^2}) \tag{2}$$

$$M_y = -(D_y \frac{\partial^2 W}{\partial y^2} + D_1 \frac{\partial^2 W}{\partial x^2}) \tag{3}$$

$$M_{xy} = -2D_{xy} \frac{\partial^2 W}{\partial x \partial y} \tag{4}$$

$$Q_x = -\frac{\partial}{\partial x}(D_x \frac{\partial^2 W}{\partial x^2} + H \frac{\partial^2 W}{\partial y^2}) \tag{5}$$

$$Q_y = -\frac{\partial}{\partial y}(D_y \frac{\partial^2 W}{\partial y^2} + H \frac{\partial^2 W}{\partial x^2}) \tag{6}$$

$$V_x = Q_x + \frac{\partial M_{xy}}{\partial y} \tag{7}$$

$$V_y = Q_y + \frac{\partial M_{xy}}{\partial x} \tag{8}$$

where M_x and M_y are the bending moments, M_{xy} is the torsional moment, Q_x and Q_y are the shear forces, V_x and V_y are the effective shear forces, and $D_{xy} = Gh^3/12$ is the torsional rigidity, where G is the shear modulus.

The boundary conditions of the plate are:

$$W|_{x=0,a} = 0 \quad , \quad W|_{y=0,b} = 0 \tag{9}$$

$$\frac{\partial W}{\partial x}|_{x=0,a} = 0 \quad , \quad \frac{\partial W}{\partial y}|_{y=0,b} = 0 \tag{10}$$

The above equations can be reduced to an isotropic plate with substituting $\nu_1 = \nu_2 = \nu$, $D_x = D_y = H = D$, $D_1 = \nu D$

and $D_{xy} = D(1-\nu)/2$, where D is the flexural rigidity and ν is Poisson's ratio.

3. Integral transform method solution

To solve Eq. (1) by double integral transform method, since $W(x,y)$ is defined within a rectangular domain $0 \le x \le a$ and $0 \le y \le b$, a double finite sine integral transform is defined as:

$$W_{mn} = \int_0^b\int_0^a W(x,y)\,sin(\alpha_m x)sin(\beta_n y)dxdy \qquad (m=1,2,3,\ldots),\ (n=1,2,3,\ldots) \tag{11}$$

that inversion formula can be represented as:

$$W(x,y) = \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\,sin(\alpha_m x)sin(\beta_n y) \qquad (m=1,2,3,\ldots),\ (n=1,2,3,\ldots) \tag{12}$$

where $\alpha_m = m\pi/a$ and $\beta_n = n\pi/b$.

In Eq. (1) with integration of higher-order partial derivatives of W and applying four boundary conditions, i.e. Eq. (9) the following results will be achieved.

$$\int_0^b\int_0^a \frac{\partial^4 W}{\partial x^4} sin(\alpha_m x)sin(\beta_n y)dxdy = \alpha_m^3\int_0^b\left[(-1)^m W|_{x=a} - W|_{x=0}\right] sin(\beta_n y)dy - \alpha_m\int_0^b\left[(-1)^m \frac{\partial^2 W}{\partial x^2}|_{x=a} - \frac{\partial^2 W}{\partial x^2}|_{x=0}\right] sin(\beta_n y)dy + \alpha_m^4 W_{mn} =$$
$$-\alpha_m\int_0^b\left[(-1)^m \frac{\partial^2 W}{\partial x^2}|_{x=a} - \frac{\partial^2 W}{\partial x^2}|_{x=0}\right] sin(\beta_n y)dy + \alpha_m^4 W_{mn} \tag{13}$$

$$\int_0^b\int_0^a \frac{\partial^4 W}{\partial y^4} sin(\alpha_m x)sin(\beta_n y)dxdy = \beta_n^3\int_0^a\left[(-1)^n W|_{y=b} - W|_{y=0}\right] sin(\alpha_m x)dx - \beta_n\int_0^a\left[(-1)^n \frac{\partial^2 W}{\partial y^2}|_{y=b} - \frac{\partial^2 W}{\partial y^2}|_{y=0}\right] sin(\alpha_m x)dx + \beta_n^4 W_{mn} =$$
$$-\beta_n\int_0^a\left[(-1)^n \frac{\partial^2 W}{\partial y^2}|_{y=b} - \frac{\partial^2 W}{\partial y^2}|_{y=0}\right] sin(\alpha_m x)dx + \beta_n^4 W_{mn} \tag{14}$$

$$\int_0^b\int_0^a \frac{\partial^4 W}{\partial x^2\partial y^2} sin(\alpha_m x)sin(\beta_n y)dxdy = \alpha_m^2\beta_n\int_0^a\left[(-1)^n W|_{y=b} - W|_{y=0}\right] sin(\alpha_m x)dx - \alpha_m\int_0^b\left[(-1)^m \frac{\partial^2 W}{\partial y^2}|_{x=a} - \frac{\partial^2 W}{\partial y^2}|_{x=0}\right] sin(\beta_n y)dy + \alpha_m^2\beta_n^2 W_{mn} = \alpha_m^2\beta_n^2 W_{mn} \tag{15}$$

The transform of the load function $q(x,y)$ is:

$$q_{mn} = \int_0^b\int_0^a q(x,y)\,sin(\alpha_m x)sin(\beta_n y)dxdy \tag{16}$$

By applying the double integral transform on Eq. (1) and the substitution of equations (13) - (16), one gets:

$$\left(D_x\alpha_m^4 + D_y\beta_n^4 + 2H\alpha_m^2\beta_n^2 + K\right)W_{mn} - \alpha_m D_x\int_0^b\left[(-1)^m \frac{\partial^2 W}{\partial x^2}|_{x=a} - \frac{\partial^2 W}{\partial x^2}|_{x=0}\right] sin(\beta_n y)dy$$
$$-\beta_n D_y\int_0^a\left[(-1)^n \frac{\partial^2 W}{\partial y^2}|_{y=b} - \frac{\partial^2 W}{\partial y^2}|_{y=0}\right] sin(\alpha_m x)dx = q_{mn} \tag{17}$$

Consider the following definitions as [2]:

$$I_m = \int_0^a \frac{\partial^2 W}{\partial y^2}|_{y=b}\, sin(\alpha_m x)dx \tag{18a}$$

$$J_m = \int_0^a \frac{\partial^2 W}{\partial y^2}|_{y=0}\, sin(\alpha_m x)dx \tag{18b}$$

$$K_n = \int_0^b \frac{\partial^2 W}{\partial x^2}|_{x=a}\, \sin(\beta_n y)dy \tag{18c}$$

$$L_n = \int_0^b \frac{\partial^2 W}{\partial x^2}|_{x=0}\, \sin(\beta_n y)dy \tag{18d}$$

Therefore, Eq. (17) is simplified in terms of unknown constants I_m, J_m , K_n and L_n as follows:

$$W_{mn} = \frac{1}{K + D_x\alpha_m^4 + D_y\beta_n^4 + 2H\alpha_m^2\beta_n^2}\left\{q_{mn} + \beta_n D_y\left[(-1)^n I_m - J_m\right] + \alpha_m D_x\left[(-1)^m K_n - L_n\right]\right\} \tag{19}$$

By substituting Eq. (19) into Eq. (12), one can get the expression for $W(x,y)$ with I_m, J_m, K_n and L_n for $m=1,2,3,\ldots$ and $n=1,2,3,\ldots$.

By applying remaining boundary conditions represented by Eq. (10), one gets:

$$\frac{\partial W}{\partial x}|_{x=0}=0 \Rightarrow \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4}{ab}\alpha_m W_{mn} sin(\beta_n y)=0 \tag{20a}$$

$$\frac{\partial W}{\partial x}|_{x=a}=0 \Rightarrow \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4}{ab}(-1)^m\alpha_m W_{mn} sin(\beta_n y)=0 \tag{20b}$$

$$\frac{\partial W}{\partial y}|_{y=0}=0 \Rightarrow \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4}{ab}\beta_n W_{mn}\sin(\alpha_m x)=0 \tag{20c}$$

$$\frac{\partial W}{\partial y}|_{y=b}=0 \Rightarrow \sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{4}{ab}(-1)^n\beta_n W_{mn}\sin(\alpha_m x)=0 \tag{20d}$$

By multiplying Eqs. (20a) and (20b) by $sin(\beta_n y)dy$ and integration from 0 to b and multiplying Eqs. (20c) and (20d) by $\sin(\alpha_m x)dx$ and integration from 0 to a the following results will be obtained:

$$\sum_{m=1}^{\infty}\alpha_m W_{mn}=0 \qquad (n=1,2,3,\ldots) \tag{21a}$$

$$\sum_{m=1}^{\infty}(-1)^m\alpha_m W_{mn}=0 \quad (n=1,2,3,\ldots) \tag{21b}$$

$$\sum_{n=1}^{\infty}\beta_n W_{mn}=0 \qquad (m=1,2,3,\ldots) \tag{21c}$$

$$\sum_{n=1}^{\infty}(-1)^n\beta_n W_{mn}=0 \quad (m=1,2,3,\ldots) \tag{21d}$$

By substituting Eq. (19) into Eq. (21), four infinite systems of linear simultaneous equations with respect to unknown constants I_m, J_m, K_n and L_n will be achieved as the followings:

$$\sum_{m=1}^{\infty}\frac{\alpha_m}{K+D_x\alpha_m^{\,4}+D_y\beta_n^{\,4}+2H\alpha_m^{\,2}\beta_n^{\,2}}\left\{q_{mn}+\beta_n D_y\left[(-1)^n I_m-J_m\right]+\alpha_m D_x\left[(-1)^m K_n-L_n\right]\right\}=0 \quad (n=1,2,3,\ldots) \tag{22a}$$

$$\sum_{m=1}^{\infty}\frac{(-1)^m\alpha_m}{K+D_x\alpha_m^{\,4}+D_y\beta_n^{\,4}+2H\alpha_m^{\,2}\beta_n^{\,2}}\left\{q_{mn}+\beta_n D_y\left[(-1)^n I_m-J_m\right]+\alpha_m D_x\left[(-1)^m K_n-L_n\right]\right\}=0 \quad (n=1,2,3,\ldots) \tag{22b}$$

$$\sum_{m=1}^{\infty}\frac{\beta_n}{K+D_x\alpha_m^{\,4}+D_y\beta_n^{\,4}+2H\alpha_m^{\,2}\beta_n^{\,2}}\left\{q_{mn}+\beta_n D_y\left[(-1)^n I_m-J_m\right]+\alpha_m D_x\left[(-1)^m K_n-L_n\right]\right\}=0 \quad (m=1,2,3,\ldots) \tag{22c}$$

$$\sum_{m=1}^{\infty}\frac{(-1)^n\beta_n}{K+D_x\alpha_m^{\,4}+D_y\beta_n^{\,4}+2H\alpha_m^{\,2}\beta_n^{\,2}}\left\{q_{mn}+\beta_n D_y\left[(-1)^n I_m-J_m\right]+\alpha_m D_x\left[(-1)^m K_n-L_n\right]\right\}=0 \quad (m=1,2,3,\ldots) \tag{22d}$$

In addition, the bending moments along the clamped edges can be obtained using Eq. (2) and Eq. (3) with following evident boundary conditions: $\partial^2 W/\partial y^2\big|_{x=0}=0,\ \partial^2 W/\partial y^2\big|_{x=a}=0,\ \partial^2 W/\partial x^2\big|_{y=0}=0,\ \partial^2 W/\partial x^2\big|_{y=b}=0$ So the bending moments will be:

$$M_x|_{x=0}=-\left(D_x\frac{\partial^2 W}{\partial x^2}+D_1\frac{\partial^2 W}{\partial y^2}\right)|_{x=0}=-\left(D_x\frac{\partial^2 W}{\partial x^2}\right)|_{x=0}=-D_x\left(\frac{2}{b}\sum_{n=1}^{\infty}L_n sin(\beta_n y)\right) \tag{23a}$$

$$M_x|_{x=a}=-\left(D_x\frac{\partial^2 W}{\partial x^2}+D_1\frac{\partial^2 W}{\partial y^2}\right)|_{x=a}=-\left(D_x\frac{\partial^2 W}{\partial x^2}\right)|_{x=a}=-D_x\left(\frac{2}{b}\sum_{n=1}^{\infty}K_n \sin(\beta_n y)\right) \tag{23b}$$

$$M_y|_{y=0}=-\left(D_y\frac{\partial^2 W}{\partial y^2}+D_1\frac{\partial^2 W}{\partial x^2}\right)|_{y=0}=-\left(D_y\frac{\partial^2 W}{\partial y^2}\right)|_{y=0}=-D_y\left(\frac{2}{a}\sum_{n=1}^{\infty}J_m \sin(\alpha_m x)\right) \tag{23c}$$

$$M_y|_{y=b}=-\left(D_y\frac{\partial^2 W}{\partial y^2}+D_1\frac{\partial^2 W}{\partial x^2}\right)|_{y=b}=-\left(D_y\frac{\partial^2 W}{\partial y^2}\right)|_{y=b}=-D_y\left(\frac{2}{a}\sum_{n=1}^{\infty}I_m \sin(\alpha_m x)\right) \tag{23d}$$

By solving four infinite systems of linear simultaneous equations (22a-d) for a finite number of terms in each set of equations, unknown constants I_m, J_m, K_n and L_n will be achieved. Then, by substituting the obtained constants into Eq. (19) and Eq. (12) respectively, the bending solutions of a fully clamped orthotropic rectangular thin plate resting on elastic foundations will be obtained. The results are theoretically exact solutions when m and $n \to \infty$, while in practice one can obtain the desired accuracy by taking an appropriate number of terms.

4. Numerical examples

To validate the proposed formulation, a fully clamped rectangular plate under the following conditions is examined:

(I) Deflections $W(qa^4/D)$ for a fully clamped isotropic rectangular plate without elastic foundations under uniform load of intensity q.

(II) Deflections $W(Pa^2/D)$ for a fully clamped isotropic rectangular plate without elastic foundations under central concentrated load P.

(III) Deflections $W(Pa^2/D_x)$ for a fully clamped orthotropic rectangular plate without elastic foundations subjected to central concentrated load P with $D_y = 4D_x$, $D_{xy} = 0.85D_x$, $\nu_1 = 0.075$, $\nu_2 = 0.3$

(IV) Deflections $W(Pa^2/D)$ for a fully clamped isotropic rectangular square plate resting on an elastic foundation with $Ka^4/D = 10^2$ subjected to central concentrated load P.

(V) Deflections $W(Pa^2/D_x)$ for a fully clamped orthotropic rectangular square plate resting on an elastic foundation with $Ka^4/D = 10^2$ subjected to central concentrated load P with $D_y = 4D_x$, $D_{xy} = 0.85D_x$, $\nu_1 = 0.075$, $\nu_2 = 0.3$.

Results are compared with the results of Timoshenko and Woinowsky-Krieger [1] and Pana et al. [3] at specific locations for the above cases in "Tables 1-5", respectively, which show good agreement. In this study, the first 40 terms of I_m, J_m, K_n and L_n in the calculation are considered for sufficient accuracy of the solutions.

Table 1. Deflections for a fully clamped isotropic rectangular Plate without elastic foundations under uniform load.

b/a	$W_{x=a/2,y=b/2}(qa^4/D)$		Difference (%)
	Ref.[1]	Present	
1.0	0.00126	0.00121	3.96825
1.1	0.00150	0.00145	3.33333
1.2	0.00172	0.00165	4.06977
1.3	0.00191	0.00183	4.18848
1.4	0.00207	0.00198	4.34783
1.5	0.00220	0.00211	4.09091
1.6	0.00230	0.00221	3.91304
1.7	0.00238	0.00229	3.78151
1.8	0.00245	0.00235	4.08163
1.9	0.00249	0.00240	3.61446
2.0	0.00254	0.00243	4.33071

Table 2. Deflections for a fully clamped isotropic rectangular plate without elastic foundations under central concentrated load.

b/a	$W_{x=a/2,y=b/2}(Pa^2/D)$		Difference (%)
	Ref.[1]	Present	
1.0	0.00560	0.00550	1.78571
1.2	0.00647	0.00634	2.00927
1.4	0.00691	0.00677	2.02605
1.6	0.00712	0.00697	2.10674
1.8	0.00720	0.00704	2.22222
2.0	0.00722	0.00707	2.07756

Table 3. Deflections for a fully clamped orthotropic rectangular plate without elastic foundations subjected to central concentrated load ($D_y = 4D_x$, $D_{xy} = 0.85D_x$, $\nu_1 = 0.075$, $\nu_2 = 0.3$).

b/a	m	n	$W_{x=a/2,y=b/2}(Pa^2/D_x)$
1.0	40	40	0.002402
1.1	40	40	0.002808
1.2	40	40	0.003192
1.3	40	40	0.003540
1.4	40	40	0.003844
1.5	40	40	0.004102
1.6	40	40	0.004314
1.7	40	40	0.004485
1.8	40	40	0.004620
1.9	40	40	0.004724
2.0	40	40	0.004803

Table 4. Deflections $W(Pa^2/D)$ for a fully clamped isotropic rectangular square plate resting on elastic foundation subjected to central concentrated load.

y		x	$a/8$	$a/4$	$3a/8$	$a/2$
		present	0.0000934	0.000345	0.000604	0.000714
$a/8$	n=m=40	Ref.[3]	0.0000964	0.000344	0.000597	0.000706
		Difference(%)	3.1120	0.2907	1.1725	1.1331
		present	0.000345	0.00120	0.00200	0.00240
$a/4$	n=m=40	Ref.[3]	0.000344	0.00112	0.00192	0.00228
		Difference(%)	0.2907	7.1429	4.1667	5.2632
		present	0.000604	0.00200	0.00350	0.00430
$3a/8$	n=m=40	Ref.[3]	0.000597	0.00192	0.00338	0.00410
		Difference(%)	1.1725	4.1667	3.5503	4.878
		present	0.000714	0.00240	0.00430	0.00550
$a/2$	n=m=40	Ref.[3]	0.000706	0.00228	0.00410	0.00527
		Difference(%)	1.1331	5.2632	4.878	4.3643

Table 5. Deflections $W(Pa^2/D_x)$ for a fully clamped orthotropic rectangular square plate resting on an elastic foundation subjected to central concentrated load ($D_y = 4D_x$, $D_{xy} = 0.85D_x$, $\nu_1 = 0.075$, $\nu_2 = 0.3$).

y	x	$a/8$	$a/4$	$3a/8$	$a/2$
$a/8$	n=m=40	0.0000340	0.000136	0.000263	0.000327
$a/4$	n=m=40	0.000124	0.000453	0.000866	0.00109
$3a/8$	n=m=40	0.0002140	0.000759	0.00148	0.00192
$a/2$	n=m=40	0.0002508	0.0008849	0.00175	0.00240

5. Conclusions

In this study, an exact bending solution of a fully clamped orthotropic rectangular thin plate resting on elastic foundations is obtained by the double finite sine integral transform method. Moreover, the present method can be used for different boundary conditions and does not need any predetermined function such as deflection or stress, so obviously it is simpler than other methods which are based on semi-inverse method.

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