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Analytical Solution of Linear, Quadratic and Cubic Model of PTT Fluid

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Abstract

An attempt is made for the first time to solve the quadratic and cubic model of magneto hydrodynamic Poiseuille flow of Phan-Thein-Tanner (PTT). A series solution of magneto hydrodynamic (MHD) flow is developed by using homotopy perturbation method (HPM). The results are presented graphically and the effects of non-dimensional parameters on the flow field are analyzed. The results reveal many interesting behaviors that warrant further study on the equations related to non-Newtonian fluid phenomena.

Keywords: Phan-Thein-Tanner (PTT) model; homotopy perturbation method; Nonlinear.

1. Introduction

Navier-Stokes equations explain the movement of viscous fluids. Convective terms play a vital role in the occurrence of nonlinearity in these equations and for non-Newtonian fluids it happens because of constitutive relations. Due to constitutive relations, higher-order derivative appears in the momentum equation and makes it difficult to solve compared to Newtonian fluids. In real life, all fluidic structures can be expressed in terms of Newtonian and non-Newtonian fluid models. Therefore, it is not possible to describe their mechanical behavior by a single comprehensive equation. As a result, a wide variety of constitutive equations have been proposed [1-29]. Of these, the constitutive equation proposed by Phan-Thein Tanner (PTT) [8] has attracted considerable attention in recent years.

The PTT model is one of the most widely used rheological models and can properly describe the common characteristics of viscoelastic non-Newtonian fluids as it is well regarded and derivable from molecular considerations. For this reason, many researchers are interested in studying PTT flow from different points of view and in different geometries including expansion and contraction entry flows which have been investigated by Quinznani et al., Quinznani et al. and Baloch et al. [9, 10]. In another study, Tichy et al. [11] examined the fluid in a thin confined space in spherical coordinated. A number of precise solutions of PTT flow have also been reported in the literature [12]. Similarly, there exist analytical solutions of PTT flow in the literature [13]. Likewise, there are many more studies of PTT flow [14-15].

In a remarkable study of PTT fluid, Hou Lei developed and solved the nonlinear problem of pressure and stress induced non-Newtonian fluid [16]. Hou Lei et al [17] studied the boundary layer approach in the contact interface by use of standard mean variables in material sciences. Furthermore, he investigated [18] resistance to the extensional and simple shear rate and presented the best estimate of the stress over-shoot for the elongating element over-stretch known as non-slip impact hardening in addition to the simple shear. He also studied other different PTT models [19-21].

There is huge literature about linear and exponential model of PTT flow. Similarly, the literature mentioned above is based on linear and exponential model. Therefore, the aim of the present work is to investigate quadratic and cubic model of PTT flow through homotopy perturbation method. There are different methods in the literature such as the variational iteration method [22], the Laplace decomposition method [23], and the homotopy perturbation transform method [24] to deal with such problems. However, the homotopy perturbation method [25] is the one which deforms continuously to a simple problem. The significant advantage of this method which does not require a small parameter in an equation is that it provides analytical approximate solutions to a wide range of nonlinear problems arising in applied sciences [26-28].

In this paper, we study the behavior of flow in viscoelastic quadratic and cubic Phan-Thein–Tanner (PTT) fluid flows in a channel. No external electric field but applied magnetic field is taken into account with low Reynolds number. To the best of our knowledge, no previous attempts have been made to study the above problem even in the steady-state case. The effects of the dimensionless number β = 2εN, De (where De is Deborah number) and magnetic parameter N on the flow field are investigated.

2. Constitutive Equations

The PTT fluid is characterized by the following equation:

$$\mathrm{T} = -p\mathrm{I} + \boldsymbol{\tau}, \tag{1}$$

$$f\left(tr(\boldsymbol{\tau})\right)\boldsymbol{\tau} + \lambda \overset{\nabla}{\boldsymbol{\tau}} = 2\mu \mathrm{A}_1 \tag{2}$$

$$\overset{\nabla}{\boldsymbol{\tau}} = \frac{d\boldsymbol{\tau}}{dt} - \boldsymbol{\tau}.\mathrm{L}^* - \mathrm{L}.\boldsymbol{\tau}, \tag{3}$$

whence

$$\mathrm{L} = \mathrm{grad}\mathrm{V}. \tag{4}$$

In the above equations, A_1 is the deformation-rate tensor, λ the relaxation time, τ the extra stress tensor, μ is dynamic viscosity, p the pressure, I the identity tensor, and $\overset{\nabla}{\tau}$ denotes Oldroyd's upper-convected derivative, d/dt the material derivative and tr the trace. In this analysis, we extended the analysis of Akylidiz [29] to the quadratic and cubic form of PTT model. The function f in quadratic and cubic forms are, respectively, given by

$$f\left(tr(\boldsymbol{\tau})\right) = 1 + \frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau}) \tag{5}$$

$$f\left(tr(\boldsymbol{\tau})\right) = 1 + \frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau}) + \frac{\delta_1}{2}\left(\frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau})\right)^2 \tag{6}$$

$$f\left(tr(\boldsymbol{\tau})\right) = 1 + \frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau}) + \frac{\delta_1}{2}\left(\frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau})\right)^2 + \frac{\delta_2}{6}\left(\frac{\varepsilon\lambda}{\mu} tr(\boldsymbol{\tau})\right)^3 \tag{6a}$$

In both forms, ε is a parameter related to the elongation behavior of the model. Also, if the trace of the stress tensor is small and ε vanishes, both forms reduce to the well-known upper convected Maxwell (UCM) model. In the case of unidirectional magnetohydrodynamic flow, the field equation takes the form

$$\rho \frac{d\mathrm{V}}{dt} = \nabla.\mathrm{T} + J \times B, \tag{7}$$

where ρ denotes the density of the fluid, d/dt the material time differentiation, J the electric current density, and B the magnetic flux.

The continuity and momentum equation for steady flow can be written in component form as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{8}$$

$$\rho\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)u+\frac{\partial p}{\partial x}=\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y} \tag{9}$$

$$\rho\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)v+\frac{\partial p}{\partial y}=\frac{\partial \tau_{yy}}{\partial x}+\frac{\partial \tau_{xy}}{\partial y} \tag{10}$$

where from eqs. (1) to (5), we get

$$f\,\tau_{xx}+\lambda\left(u\frac{\partial \tau_{xx}}{\partial x}+v\frac{\partial \tau_{xx}}{\partial y}-2\frac{\partial u}{\partial x}\tau_{xx}-2\frac{\partial u}{\partial y}\tau_{xy}\right)=2\mu\frac{\partial u}{\partial x} \tag{11}$$

$$f\,\tau_{yy}+\lambda\left(u\frac{\partial \tau_{yy}}{\partial x}+v\frac{\partial \tau_{yy}}{\partial y}-2\frac{\partial v}{\partial x}\tau_{yx}-2\frac{\partial v}{\partial y}\tau_{yy}\right)=2\mu\frac{\partial v}{\partial y} \tag{12}$$

$$f\,\tau_{xy}+\lambda\left(u\frac{\partial \tau_{xy}}{\partial x}+v\frac{\partial \tau_{xy}}{\partial y}-\frac{\partial v}{\partial x}\tau_{xx}-\frac{\partial v}{\partial y}\tau_{xy}-\frac{\partial u}{\partial x}\tau_{xy}-\frac{\partial u}{\partial y}\tau_{yy}\right)=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) \tag{13}$$

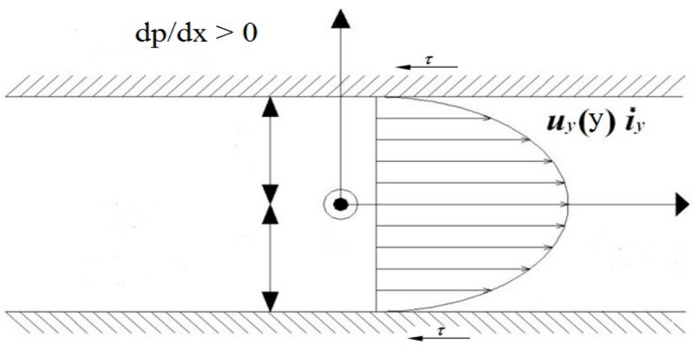

Fig. 1. Schematic diagram of the flow

For the flow under consideration, the velocity field and stress tensor take the form

$$V=u(y)i,\ \tau=\tau(y)i \tag{14}$$

where i and $u(y)$ are the unit vector and the velocity in the x-direction, respectively. According to the above assumptions, i.e. without external electric field and low Reynolds number [23], the MHD body force takes the following form:

$$J\times B=-\sigma B_0^2 v \tag{15}$$

where B_0 and σ are the constant flux density and electrical conductivity, respectively. Assumptions in equation (14) automatically satisfy the continuity equation. The equation of motion (7) yields the following scalar equations:

$$\frac{\partial p}{\partial x}=-\sigma B_0^2 u+\frac{\partial \tau_{xy}}{\partial y} \tag{16}$$

$$\frac{\partial p}{\partial x}=\frac{\partial \tau_{yy}}{\partial y} \tag{17}$$

By substituting (14) into (2), we obtain:

$$f\left(tr\left(\tau\right)\right)\tau_{xx}-2\lambda\tau_{xy}\frac{\partial u}{\partial y}=0 \tag{18}$$

$$f\left(tr\left(\tau\right)\right)\tau_{xy}-\lambda\tau_{yy}\frac{\partial u}{\partial y}=\mu\frac{\partial u}{\partial y}, \tag{19}$$

Upon solving the two above equations, we get the following expression:

$$\tau_{xx} = 2\frac{\lambda}{\mu}\tau_{xy}^2 \tag{20}$$

Defining non-dimensional variables [29]

$$y^* = \frac{y}{H}, u^* = \frac{u}{U}, \tau^* = \frac{\tau H}{\mu U}, p^* = \frac{pH}{\mu U}, N^* = \frac{NH^2}{\mu} \tag{21}$$

and substituting (21) into (16) by using the non-dimensional variables, we obtain (after dropping the asterisks):

$$0 = \frac{\partial p}{\partial x} + \frac{d\tau_{xy}}{dy} - Nu, \tag{22}$$

By using eq. (20), (5), (6) and (6a), we obtain:

$$\left(1 + 2\varepsilon D_e^2 \tau_{xy}^2\right)\tau_{xy} = \frac{du}{dy} \quad \text{Linear PTT} \tag{23}$$

$$\left(1 + 2\varepsilon D_e^2 \tau_{xy}^2 + 2\delta_1 \varepsilon D_e^3 \tau_{xy}^4\right)\tau_{xy} = \frac{du}{dy} \quad \text{Quadratic PTT} \tag{24}$$

$$\left(1 + 2\varepsilon D_e^2 \tau_{xy}^2 + 2\delta_1 \varepsilon D_e^3 \tau_{xy}^4 + \frac{4}{3}\delta_2 \varepsilon D_e^4 \tau_{xy}^6\right)\tau_{xy} = \frac{du}{dy} \quad \text{Cubic PTT} \tag{25}$$

We can generalize it for readers and they can use eq. (26) to carry out further investigation:

$$\tau_{xy} + \sum_{n=1}^{\infty} \frac{2^n \delta_{n-1} \varepsilon D_e^{n+1} \tau_{xy}^{2n+1}}{n!} = \frac{du}{dy} \tag{26}$$

where $D_e = \lambda U / L$ is the Deborah number and $N = \sigma B_0^2 > 0$ is the magnetic parameter and $\delta_0 = 1$. By using eqs. (22), we get:

$$\frac{du}{dy} = \frac{1}{N}\frac{d^2\tau_{xy}}{dy^2} \tag{27}$$

Substituting eq. (27) into (23) to (26), after simplification we get:

$$\left(N\tau_{xy} + \beta D_e^2 \tau_{xy}^3\right) = \frac{d^2\tau_{xy}}{dy^2} \tag{28}$$

$$\left(N\tau_{xy} + \beta D_e^2 \tau_{xy}^3 + \delta_1 \beta D_e^3 \tau_{xy}^5\right) = \frac{d^2\tau_{xy}}{dy^2} \tag{29}$$

$$\left(N\tau_{xy} + \beta D_e^2 \tau_{xy}^3 + \delta_1 \beta D_e^3 \tau_{xy}^5 + \frac{2}{3}\delta_2 \beta D_e^4 \tau_{xy}^7\right) = \frac{d^2\tau_{xy}}{dy^2} \tag{30}$$

$$\tau_{xy} N + \sum_{n=1}^{\infty} \frac{2^{n-1} \delta_{n-1} \beta D_e^{n+1} \tau_{xy}^{2n+1}}{n!} = \frac{d^2\tau_{xy}}{dy^2} \tag{31}$$

where $\beta = 2\varepsilon N$ and appropriate no-slip boundary conditions for the problem are [29]:

$$u(0) = 0, u(1) = 0 \tag{32}$$

By using eq. (22), the above boundary conditions can be written as:

$$\frac{d}{dy}\tau_{xy}(0) = P, \frac{d}{dy}\tau_{xy}(1) = P, \tag{33}$$

Eqs. (28) to (31) can be solved by using HPM subject to the boundary conditions given in (33).

3. The HPM solution

For the HPM solution, we select:

$$\tau_{xy\,(0)} = \frac{P\left(-1+e^{-N}\right)e^{N\eta}}{N\,(e^{-N}-e^{N})} + \frac{P\left(-1+e^{N}\right)e^{-N\eta}}{N\,(e^{-N}-e^{N})} \tag{34}$$

as an initial approximation of τ_{xy}. We further choose the auxiliary linear operator as follows:

$$L = \frac{\partial^2}{\partial\eta^2}\tau_{xy} - N^2\tau_{xy} \tag{35}$$

which satisfy

$$L\left(C_1 e^{N\eta} + C_2 e^{-N\eta}\right) = 0 \tag{36}$$

In view of the basic idea of the HPM [19-22], Eqs. (28) to (30) can be expressed as:

$$\left(1-q\right)L\left(\tau_{xy}-\tau_{xy\,(0)}\right)+q\left(\left(N\,\tau_{xy}+\beta D_e^2\tau_{xy}^3\right)-\frac{d^2\tau_{xy}}{dy^2}\right)=0 \tag{36}$$

$$\left(1-q\right)L\left(\tau_{xy}-\tau_{xy\,(0)}\right)+q\left(\left(N\,\tau_{xy}+\beta D_e^2\tau_{xy}^2+\delta_1\beta D_e^3\tau_{xy}^4\right)-\frac{d^2\tau_{xy}}{dy^2}\right)=0 \tag{37}$$

$$\left(1-q\right)L\left(\tau_{xy}-\tau_{xy\,(0)}\right)+q\left(\left(N\,\tau_{xy}+\beta D_e^2\tau_{xy}^2+\delta_1\beta D_e^3\tau_{xy}^4+\frac{2}{3}\delta_2\beta D_e^4\tau_{xy}^6\right)-\frac{d^2\tau_{xy}}{dy^2}\right)=0 \tag{38}$$

After solving Eqs. (36-38) based on a computer algebra system, such as MATHEMATICA, MAPLE or MATLAB, we can approximate the behavior of the flow. If we substitute $\delta_2 = 0$, eq. (38) reduces to (37) and similarly, if we put $\delta_1 = 0$, eq. (37) will further reduce to eq. (36) which is the linear form of PTT fluid and identical with the linear form presented by Akyildiz [29].

4. Results and discussion

In this study our main focus is on investigating the effects of linear, quadratic and cubic PTT model on the velocity profile.

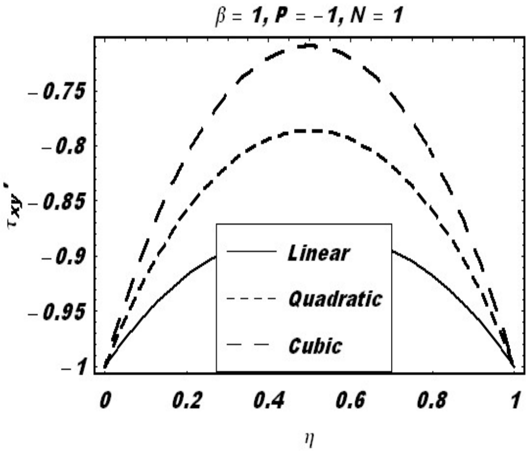

Fig. 2. Effect of small β and N for different models of PTT fluid

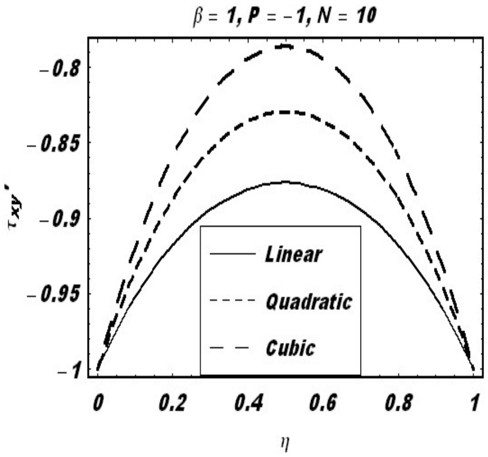

Fig. 3. Effect of small β and large N for different models of PTT fluid

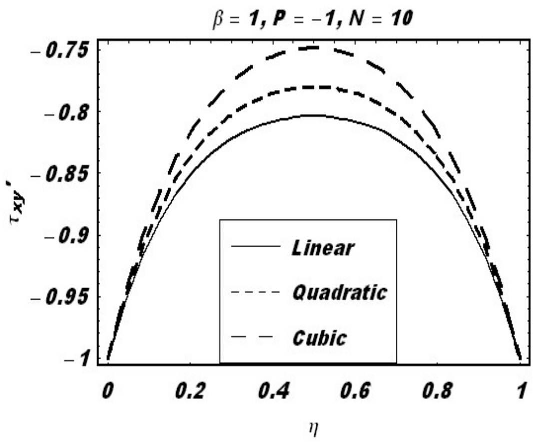

Fig. 4. Effect of large β and small N for different models of PTT fluid

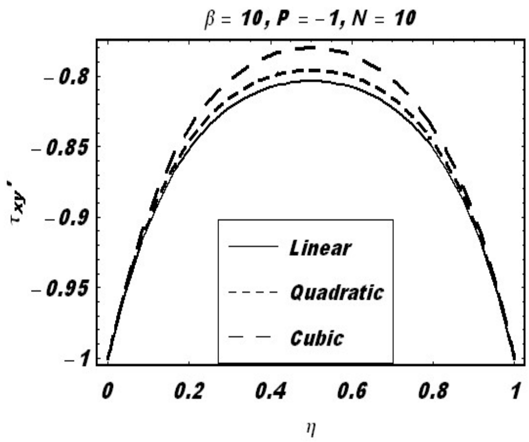

Fig. 5. Effect of large β and large N for different models of PTT fluid

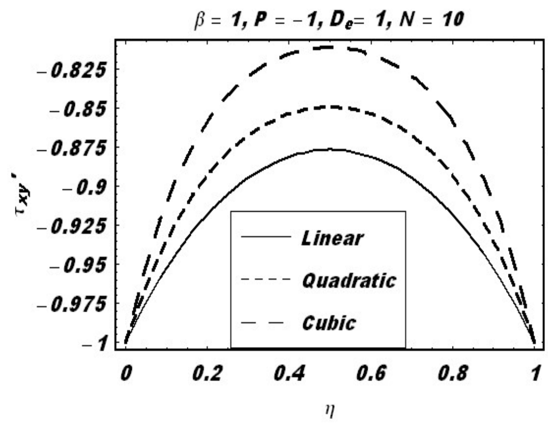

Fig. 6. Effect of small D_e for different models of PTT fluid

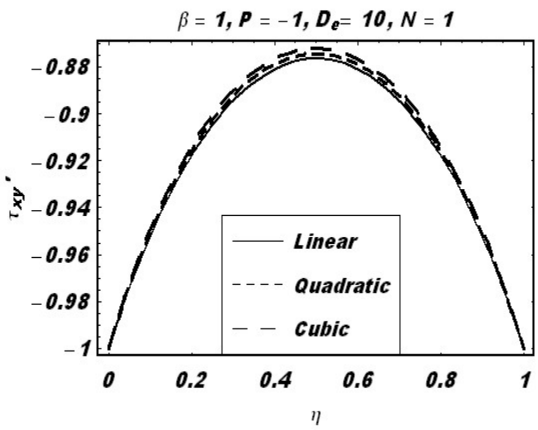

Fig. 7. Effect of large D_e for different models of PTT fluid

The graphs for the function $\tau_{xy}^{/}$ are plotted against η for linear, quadratic and cubic PTT models for different values of the parameters appearing in Eqs. 28-0. From Fig. 2 we depict that there is change in the velocity profile as we move from linear to quadratic and then from quadratic to cubic model. Fig. 3 explains that for a higher value of N the effects of linear, quadratic and cubic model reduced as compared to Fig. 2. Furthermore, we increase β corresponding to small N and by keeping other parameters constant. Increasing the effect of β is prominent on all three models (see Fig. 4). In Fig. 5, we study the behavior of the flow for increased β and N. The effect of increased β and N is consistent with the last three observations of Figs. 2 to 4. The distance between the lines representing linear, quadratic and cubic flow reduces with the increasing value of β and . The effect of β is same as that of N . As we increase β or N , the flow behavior becomes identical for all three models, which is obvious as $\beta = 2\varepsilon N$.

Afterwards, we investigate the effect of Deborah number. In Fig. 6, we can see that for small D_e , we can identify the flow behavior for all three models but as long as we increase the value of all three parameters the behavior of flow becomes identical (see Fig. 7).

By comparing the scales of all graphical presentation, we observe that the velocity profiles are more pronounced in the viscoelastic case than in the Newtonian case. Furthermore, the effect of the magnetic parameter N is to increase the value of the velocity profile and since β and magnetic parameter is directly proportional to each other, so the same would be the case for β .

5. Concluding Remarks

After a careful investigation we conclude with the following very useful observations for engineers:

1. Nonlinearity does not play a vital role in the flow phenomena of all three models.
2. Behavior of flow becomes identical at a certain level (see Fig. 7).
3. Velocity profiles are easily noticeable in the viscoelastic case than in the Newtonian case.
4. We can predict that the behavior of flow even for higher order models (tetra or penta model) would be the same as it is in linear, quadratic or cubic for higher values of parameters.

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Nomenclature

λ	Relaxation time	μ	dynamic viscosity
τ	Extra stress tensor	I	Identity tensor
p	pressure	tr	Trace of a matrix
$\overset{\triangledown}{\tau}$	Oldroyd's upper-convected derivative	ε	Elongation behavior of the model
D_e	Deborah number	N	Magnetic parameter

References

[1] L-M. Maria, M. Hana, N. Sarka, Global existence and uniqueness result for the diffusive peterlin viscoelastic model, Nonlin. Anal. Meth. Appl.120 (2015) 154-170.

[2] Z. Ting, Global strong solutions for equations related to the incompressible viscoelastic fluids with a class of large initial data, Nonlin. Anal. Meth. Appl. 100 (2014) 59-77.

[3] G. Matthias, G. Dario, N. Manuel, L-p-theory for a generalized nonlinear viscoelastic fluid model of differential type in various domains, Nonlin. Anal. Meth. Appl. 75 (2012) 5015-5026.

[4] F. Ettwein, M. Ruzicka, B. Weber, Existence of steady solutions for micropolar electrorheological fluid flows, Nonlin. Anal. Meth. Appl. 125 (2015) 1-29.

[5] F. J. Suarez-Grau, Asymptotic behavior of a non-Newtonian flow in a thin domain with Navier law on a rough boundary, Nonlin. Anal. Meth. Appl. 117 (2015) 99-123.

[6] Y. Ye, Global existence and blow-up of solutions for higher-order viscoelastic wave equation with a nonlinear source term, Nonlin. Anal. Meth. Appl.112 (2015) 129-46.

[7] R. B. Bird, R. C. Armstrong and O. Hassager, Dynamics of polymeric liquids, 1 Fluid Mechanics second edition, John Wiley & Sons, Inc. 1987.

[8] N. Phan-Thien and R. I. Tanner, A new constitutive equation derived from network theory, J. Non-Newtonian Fluid Mech. 2 (1977) 353–365.

[9] L. Quinzani, R. Armstrong, R. Brown, Use of coupled birefringence and LDV studies of flow through a planar contraction to test constitutive equations for concentrated polymer solutions. J. Rheolm. 39 (1955) 1201–1228.

[10] A. Baloch, P. Townsend, M. Webster, On vortex development in viscoelastic expansion and contraction flows. J Non Newton Fluid Mech. 65 (1996) 133–149.

[11] J. Tichy, B. Bou-Said B, (2008) The Phan-Thien and Tannermodel applied to thin film spherical coordinates: applications for lubrication of hip joint replacement. J Biomech Eng, 130 (2008) 021012.

[12] A. M. Siddiqui, Q. A. Azim, A. Ashraf et al, Exact Solution for Peristaltic Flow of PTT Fluid in an Inclined Planar Channel and Axisymmetric Tube, Int.J. Nonlin. Sci. Num. Sim. 10 (2009) 75-91

[13] L. Ferras, J. Nobrega, F. Pinho, Analytical solutions for channel flows of Phan-Thien-Tanner and Giesekus fluids under slip. J. Non Newton Fluid Mech. 171 (2012) 97–105

[14] P. J. Oliveira and F. T. Pinho, Analytical solution for fully-developed channel and pipe flow of Phan-Thien, Tanner fluids, J. Fluid Mech. 387 (1999) 271–280.

[15] F. T. Pinho and P. J. Oliveira, Analysis of forced convection in pipes and channels with simplified Phan-Thien Tanner Fluid, Int. J. Heat Mass Transfer. 43 (2000) 2273–2287.

[16] Hou Lei, V. Nassehi, Evaluation of stress effecting flow in rubber mixing, Nonlin. Anal. Meth. Appl. 47 (2001) 1809-1820.

[17] Hou Lei, Member, IAENG, D.Z. Lin, B.Wang, H.L. Li, L. Qiu,Computational Modelling on the Contact Interface with Boundary-layer Approach , Pro. Worl. Cong. Eng., I (2011) July 6 – 8, London, U.K.

[18] Hou Lei, H. L. Li, H. Wang , L. Qiu, Stochastic Analysis in the Visco-Elastic Impact Condition, Conference on Chemical Engineering and Advanced Materials (CEAM) VIRTUAL FORUM Naples 2009

[19] Hou Lei, J. Zhao and L. Qiu, The non-Newtonian fluid in the collision, Appl. Mech. Mat. 538 (2014) 72-75.

[20] Z. Shaoling, Hou Lei, Decoupled algorithm for solving Phan-Thien-Tanner viscoelastic fluid by finite element method, Comp. Math. App. 69 (2015) 423-437.

[21] Hou Lei, Li, Han-ling, Zhang Jia-jian; et al Boundary-layer eigen solutions for multi-field coupled equations in the contact interface, App. Math. Mech., 31 (2010) 719-732.

[22] N. Faraz, Study of the effects of the Reynolds number on circular porous slider via variational iteration algorithm-II, Comp. Math. App. 61 (2011) 1991-1994.

[23] N. Faraz, Y. Khan, D. S. Shankar, Decomposition-transform method for Fractional Differential Equations, Int. J. Nonl. Sci. Num. Sim. 11 (2010) 305-310.

[24] Y. Khan, N. Faraz, S. Kumar, et al, A Coupling Method of Homotopy Perturbation and Laplace Transformation for Fractional Models, Uni. Pol. Buch. Sci. Bull.-Ser. A-App. Math. Phy. 74 (2012) 57-68.

[25] N. Faraz, Hou Lei, Y. Khan, Homotopy Perturbation Method for Thin Film Flow of a Maxwell Fluid over a Shrinking/Stretching Sheet with Variable Fluid Properties International Conference On Mechanics And Control Engineering, MCE (2015) 52-57.

[26] N. Faraz, Y. Khan, Study of the Rate Type Fluid with Temperature Dependent Viscosity, Zeitschrift Fur Naturforschung Section A-A Journal of Physical Sciences. 67 (2011) 460-468.

[27] Y. Khan, Q. Wu, N. Faraz; et al, Heat Transfer Analysis on the Magnetohydrodynamic Flow of a Non-Newtonian Fluid in the Presence of Thermal Radiation: An Analytic Solution, Zeitschrift Fur Naturforschung Section A-A Journal Of Physical Sciences. 67 (2012) 147-152.

[28] Y. Khan, N. Faraz, Y. Ahmet; et al. A Series Solution of the Long Porous Slider, Tribology Transactions. 54 (2011)187-191.

[29] F. Talay Akyildiz, K. Vajravelu, Magnetohydrodynamic flow of a viscoelastic fluid, Physics Letters A. 372 (2008) 3380-3384.