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An Analytical Technique for Solving Nonlinear Oscillators of the Motion of a Rigid Rod Rocking Bock and Tapered Beams

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Abstract

In this paper, a new analytical approach is presented for solving strongly nonlinear oscillator problems. The iteration perturbation method leads us to a high accurate solution. Two different high nonlinear examples are also presented to show the application and accuracy of the presented method. The results are compared with analytical methods and with the numerical solution using Runge-Kutta method in different figures. It is shown that the iteration perturbation approach doesn't need any small perturbation and is accurate for nonlinear oscillator equations.

Keywords: Periodic solution, Nonlinear oscillators, Motion of a rigid rod rocking back, Tapered beams.

1. Introduction

The solution of differential equations in physics and engineering, especially some oscillation equations, is nonlinear, and in most cases it is difficult to solve such equations, especially analytically. Recently, several scientific papers were devoted to estimate analytical approximate solutions for nonlinear oscillators. Some approximate approaches have proposed to solve strongly nonlinear differential equations such as the homotopy perturbation method [1-3], the energy balance method [4-6], the frequency amplitude formulation [7, 8], the parameter expansion method [3, 9], the variational iteration method [10, 11], the max min approach [12, 13], the hamiltonian approach [14-16], the variational approach [17, 18], and other new methods [19-27].

The main propose of this paper is to obtain a highly accurate analytical solution for free vibrations of strongly nonlinear oscillators. The iteration method solution has been compared with other methods and the numerical solution using Runge-Kutta method of order four. Consequently, the results show its effective and convenient approximate solution.

The paper has been organized as follows: In Section 2, we present the analytical procedure. In Section 3, we apply iteration procedure to solve two important applications. Section 4 provides the comparison between analytical and numerical solutions, and in the last section the most important findings of the paper are presented.

2. Solution Procedure

We consider a generalized nonlinear oscillator in this form

$$\ddot{u} + f\,(u\,,\, \dot{u}\,,\, \ddot{u}) = 0, \tag{1}$$

with initial conditions

$$u(0) = A, \qquad \dot{u}(0) = 0. \tag{2}$$

Based on He's frequency-amplitude formulation approach [28, 29], the trial function to determine the angular frequency ω is given by

$$u = A\cos\omega t. \tag{3}$$

Substituting Eq. (3) into Eq. (1), one can obtain the following residual as

$$R(t) = -A\omega^2\cos\omega t + f(A\cos\omega t, -A\omega\sin\omega t, -A\omega^2\cos\omega t). \tag{4}$$

Introducing a new function, $H(t)$, defined as [30]

$$H(t) = \int_0^T R(t)\cos(\omega t)dt = 0, \qquad T = \frac{2\pi}{\omega}. \tag{5}$$

Solving the above equation, the relationship between the frequency and the amplitude of the oscillator can be obtained.

3. Applications

In order to assess the advantages and also the accuracy of the iteration procedure, the following two examples are considered.

3.1 The Motion of a Rigid Rod Rocking Back

The motion of a rigid rod rocking back and forth on the circular surface without slipping is considered here. The governing equation of motion can be expressed as [31-34].

$$\left(\frac{1}{12}+\frac{1}{16}u^2\right)\frac{d^2u}{dt^2}+\frac{1}{16}u\left(\frac{du}{dt}\right)^2+\frac{g}{4L}u\cos u = 0, \quad u(0)=A, \quad \frac{du}{dt}(0)=0, \tag{6}$$

where $g > 0, L > 0$ are known positive constants.

Using the following trial function to determine the angular frequency ω,

$$u = A\cos\omega t. \tag{7}$$

Substituting Eq. (7) into Eq. (6) results in the following residual;

$$\begin{aligned} R(t) = \tfrac{A}{1536L}\Big[&\left(384g - 144A^2g + 10A^4g - 128L\omega^2 - 48L\omega^2A^2\right)\cos\omega t \\ &-\left(48A^2g - 5A^4g + 48A^2L\omega^2\right)\cos 3\omega t + A^4g\cos 5\omega t.\Big] \end{aligned} \tag{8}$$

Using Eq. (8) into Eq. (5), we can easily obtain

$$H(t) = \int_0^{2\pi/\omega} R(t)\cos\omega t dt = \frac{A\pi}{768L\omega}\left[(192 - 72A^2 + 5A^4)g - 8(8+3A^2)L\omega^2\right] = 0. \tag{9}$$

Solving the above equation, an approximate frequency ω as a function of amplitude A is as follows:

$$\omega = \frac{\sqrt{\left(192 - 72A^2 + 5A^4\right)g}}{\sqrt{8L(8+3A^2)}}. \tag{10}$$

Hence, the approximate solution can be readily obtained as

$$u(t) = A\cos\left(\frac{\sqrt{\left(192-72A^2+5A^4\right)g}}{\sqrt{8L(8+3A^2)}}t\right). \tag{11}$$

3.2 Tapered Beams

Tapered beams can model engineering structures which require a variable stiffness along the length, such as moving arms and turbine blades. In the dimensionless form, the governing differential equation corresponding to the fundamental vibration mode of tapered beams is given by [35, 36]. As can be seen in the geometry of the problem in Fig. 1, m_1 is the mass of the block on the horizontal surface, m_2 is the mass of the block which is just slipped in the vertical and is linked to m_1, L is the length of the link, g is the gravitational acceleration, and k is the spring constant [37, 38].

Assuming $u = \frac{x}{L}, |u| \ll 1$, the equation of motion can yield as follows:

$$\left(\frac{d^2u}{dt^2}\right)+\left(\frac{m_2}{m_1}\right)u^2\left(\frac{d^2u}{dt^2}\right)+\left(\frac{m_2}{m_1}\right)u\left(\frac{du}{dt}\right)^2+\left(\frac{k}{m_1}+\frac{m_2g}{Lm_1}\right)\mathrm{u}+\left(\frac{m_2g}{2Lm_1}\right)u^3=0. \tag{12}$$

The initial conditions for Eq. (12) are given by $u(0)=A$ and $\dot{u}(0)=0$. Here u and t are the generalized dimensionless displacement and time variables.

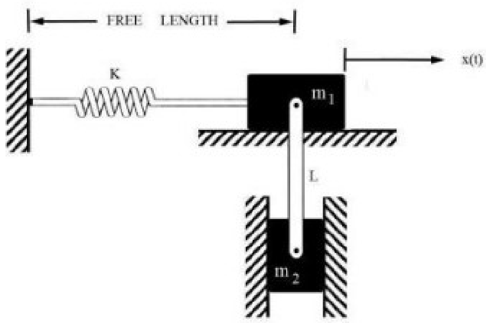


Fig. 1. Geometric of the tapered beams

Using Eqs. (3-5), and (12) leads to the relationship between the amplitude and angular frequency.

$$\omega = \sqrt{\frac{8kL+8gm_2+3A^2gm_2}{8m_1L+4A^2m_2L}}. \tag{13}$$

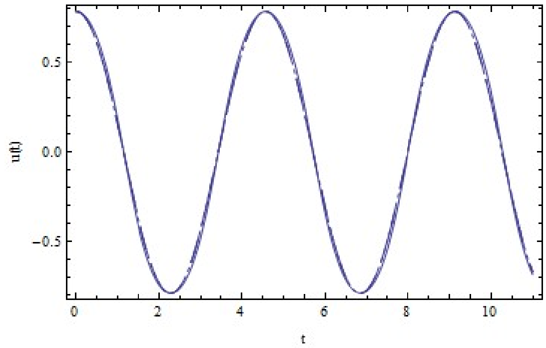
Hence, the approximate solution can be readily obtained as

$$u(t) = A\cos\left(\sqrt{\frac{8kL+8gm_2+3A^2gm_2}{8m_1L+4A^2m_2L}}t\right). \tag{14}$$

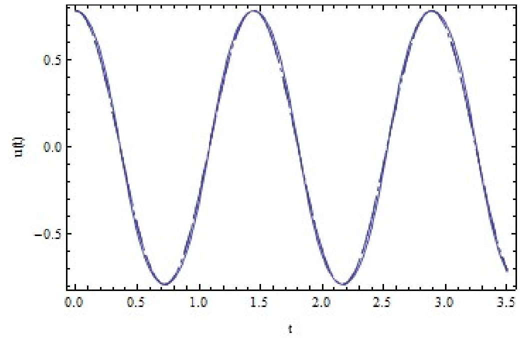
4. Results and Discussion

In this section, an approximate technique is developed based on He's frequency-amplitude formulation and He's energy balance method to solve strongly nonlinear differential equations. The solutions for two nonlinear problems show a good agreement with the numerical solutions using the Runge-Kutta method.

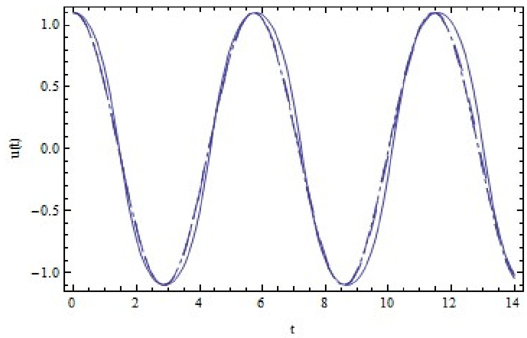
Fig. 2 shows the comparison between Analytical solutions and the Runge-Kutta method. As we see, the results are compared with the amplitude frequency formulation [32], the energy balance method [33, 34] and an accurate numerical solution, using fourth order Runge-Kutta method to show the accuracy of the method. It has been indicated that the present method has an excellent agreement with the numerical solution. It is a simple method and easy to apply to any kind of nonlinear vibration problems.



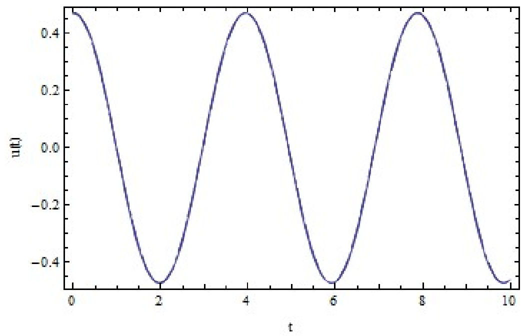
$A = 0.25\pi, g = 1, L = 1$



$A = 0.25\pi, g = 10, L = 1$



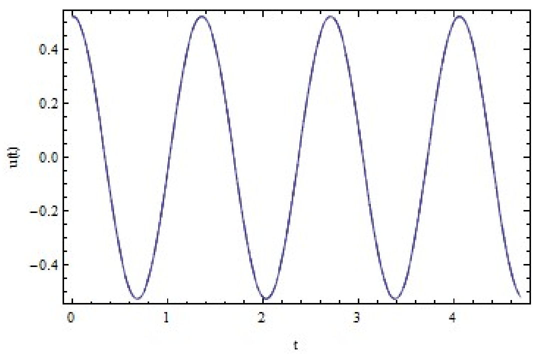
$A = 0.35\pi, g = 1, L = 1$



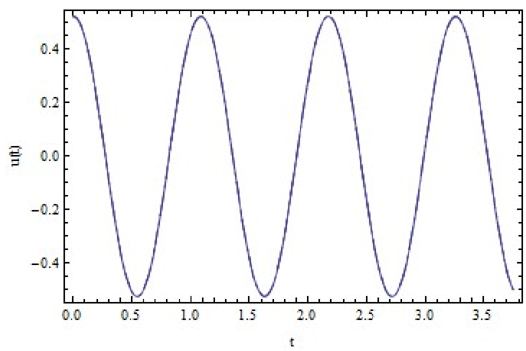
$A = 0.15\pi, g = 1, L = 1$

Fig. 2. The comparison between the analytical solution (….), the energy balance method (- - -), the amplitude frequency formulation (— —) and the numerical solution (——).

Fig. 3 represents the comparison of the analytical and numerical solutions for different parameters in the two cases to show the accuracy of the method. It has been shown that the results of the analytical approximate solution is similar to those obtained from the results of the max-min approach [37], the amplitude frequency formulation [37, 38], and the energy balance method [38], and therefore it indicates a high validity in comparison with the numerical solution using fourth order Runge-Kutta method.



$A = \frac{\pi}{6}, g = 9.81, k = 100, m_1 = 5, m_2 = 1, L = 1$



$A = \frac{\pi}{6}, g = 9.81, k = 300, m_1 = 10, m_2 = 2, L = 0.5$

Fig. 3. The comparison between the analytical and numerical solutions (——).

5. Conclusion

Based on He’s frequency-amplitude formulation and He’s energy balance method, a new analytical technique has been presented to determine approximate solutions of some strongly nonlinear differential equations. Compared to the previously published methods, the determination of solutions is straightforward and simple. In comparison with fourth-order Runge-Kutta method, which is a powerful numerical solution, the results show that the present method is very convenient for solving nonlinear equations and also can be used for strong nonlinear oscillators.

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