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Free Vibration of Annular Plates by Discrete Singular Convolution and Differential Quadrature Methods

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Abstract

Plates and shells are significant structural components in many engineering and industrial applications. In this study, the free vibration analysis of annular plates is investigated. To this aim, two different numerical methods including the differential quadrature and the discrete singular convolution methods are performed for numerical simulations. Moreover, the Frequency values are obtained via these two methods and finally, the performance of these methods is investigated.

Keywords: Differential quadrature; Discrete singular convolution; Annular plate; Free vibration.

1. Introduction

There are several applications for circular and annular plates in civil, aerospace, petroleum, nuclear, and mechanical engineering. They are used as aircraft fuselages, rockets and turbo jets, reactor walls, ship and submarine parts, holding tanks, etc. This paper deals with the application of the Discrete Singular Convolution (DSC) and the Differential Quadrature (DQ) methods for the free vibration analysis of thin annular plates with clamped and simply supported boundary conditions. The solutions of plate and shell problems have been an interesting issue for engineers and researchers since past 100 years. Therefore, different analytical and numerical methods have been used for this analysis [1-9]. This study focused on DSC and DQ methods.

2. Discrete singular convolution (DSC)

Consider a distribution, T, and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined as follows [10-15] :

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx\,, \tag{1}$$

where $T(t-x)$ is a singular kernel. The mathematical property or requirement of f(x) is determined by the approximate kernel T_α. The Shannon's kernel is regularized as follows [16-28]:

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)}\exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right];\ \sigma>0. \tag{2}$$

where Δ is the grid spacing. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$. This can be expressed as follows [29-46]:

$$\left.\frac{d^n f(x)}{d x^n}\right|_{x=x_i}=f^{(n)}(x)\approx\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(n)}(x_i-x_k)f(x_k);\quad (n=0,1,2,...,). \tag{3}$$

where superscript n denotes the nth-order derivative with respect to x.

3. Differential Quadrature Method (DQM)

Similar to the other numerical analysis techniques, such as the finite element or finite difference methods, the DQM also transforms the given differential equation into a set of analogous algebraic equations in terms of the unknown function values at the reselected sampling points in the domain. The problem areas, in which the applications of differential quadrature method may be found in the available literature, include static and dynamic structural mechanics and the stability analysis of structures [47-55]. Recently, the DQM has been largely developed by Bert et al. [53] who were the first to suggest the method as a tool for the structural analysis. In fact, they had made most of effective supplements to the theory and application of the DQM. Mainly on the vibration analysis of plates, the recent works of Bert et al. [53] have contributed significantly to the development of the DQM [52, 53].

It has been claimed that the DQM has the capability of producing highly accurate solutions with minimal computational effort. All this work has demonstrated that the application of the DQ methods leads to accurate results with less computational effort and there is a potential that the method may become an alternative to the conventional methods such as finite differences and finite elements. Therefore, research on the extension and application of the method becomes an important endeavor. In the differential quadrature method, a partial derivative of a function with respect to a space variable at a discrete point is approximated as a weighted linear sum of the function values at all discrete points in the region of that variable. For simplicity, a one-dimensional function $\psi(x)$ in the [-1, 1] domain and N discrete points are considered. Then, the first derivatives at point i as $x = x_i$ is given as follows: [53]

$$\Psi_x(x_i)=\left.\frac{\partial\Psi}{\partial x}\right|_{x=x_i}=\sum_{j=1}^{N}A_{ij}\,\Psi((x_j);\qquad i=1,2,........,N \tag{4}$$

where x_j are the discrete points in the variable domain, $\psi(x_j)$ are the function values at these points and A_{ij} are the weighting coefficients for the first order derivative attached to these function values. Two methods have been suggested to determine the weighting coefficients. The first one is to let Eq. (1) be exact for the test functions as

$$\Psi_k(x)=x^{k-1},\qquad k=1,2,...,N \tag{5}$$

which leads to a set of linear algebraic equations as

$$(k-1)x_i^{k-2}=\sum_{j=1}^{N}A_{ij}x_j^{k-1};\ \text{for}\quad i=1,2,........,N\quad\text{and}\quad k=1,2,........,N \tag{6}$$

which represents N sets of N linear algebraic equations. This equation system has a unique solution because its matrix is a Vandermonde matrix. Regarding the weighting coefficients, this equation may be solved analytically by using the Hamming's method [51-54] or the numerical method which applies certain special algorithms for Vandermonde equations, such as the methods proposed by Bjorck and Pereyra [47-54]. Similar to the first order, the second order derivative can be written as

$$\Psi_{xx}(x_i) = \left.\frac{\partial^2 \Psi}{\partial x^2}\right|_{x = x_i} = \sum_{j=1}^{N} B_{ij}\, \Psi(x_j); \qquad i = 1, 2, \ldots, N \tag{7}$$

where the B_{ij} is the weighting coefficients for the second order derivative. Equation (4) also can be written as

$$\Psi_{xx}(x_i) = \left.\frac{\partial^2 \Psi}{\partial x^2}\right|_{x = x_i} = \sum_{j=1}^{N} A_{ij} \sum_{k=1}^{N} A_{jk}\, \Psi(x_k); \qquad i = 1, 2, \ldots, N \tag{8}$$

4. Free vibration analysis of annular plate

The governing equation for a thin and isotropic circular/annular plate (Fig.1) under the axisymmetric motion is given as

$$\frac{\partial^4 u}{\partial r^4} + \frac{2}{r}\left(\frac{\partial^3 u}{\partial r^3}\right) - \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial r^2}\right) + \frac{1}{r^3}\left(\frac{\partial u}{\partial r}\right) + \frac{\rho h}{D}\left(\frac{\partial^2 u}{\partial t^2}\right) = 0 \tag{9}$$

For the free vibration, the transverse displacement u is assumed as

$$u(r,t) = U(R)\, e^{i\omega t} \tag{10}$$

Substituting the above-mentioned equation into equation (4) and its normalizing yields

$$\frac{\partial^4 U}{\partial R^4} + \frac{2}{R}\left(\frac{\partial^3 U}{\partial R^3}\right) - \frac{1}{R^2}\left(\frac{\partial^2 U}{\partial R^2}\right) + \frac{1}{R^3}\left(\frac{\partial U}{\partial R}\right) - \Omega^2 U = 0 \tag{11}$$

where $R = r/a$, a is the outside radius of the plate, h is the thickness of the plate, D is the flexural rigidity, and Ω is the dimensionless frequency and given $\Omega^2 = \rho h\, \omega^2 a^4 / D$. Equation (11) can be written by applying the DSC as

$$\sum_{k=-M}^{M} \delta^{(4)}_{\Delta,\sigma}(k\Delta x) U_{k,j} + \frac{2}{R_i} \sum_{k=-M}^{M} \delta^{(3)}_{\Delta,\sigma}(k\Delta x) U_{k,j} - \frac{1}{R^2_i} \sum_{k=-M}^{M} \delta^{(2)}_{\Delta,\sigma}(k\Delta x) U_{k,j} + \frac{1}{R^3_i} \sum_{k=-M}^{M} \delta^{(1)}_{\Delta,\sigma}(k\Delta x) U_{k,j} = \Omega^2 U_{k,j} \tag{12}$$

The boundary and regularity conditions can be written in a similar way. Consequently, the remaining eigenvalue problem is solved to obtain the natural frequencies.

Equation (11) can be easily written in DQ form as

$$\sum_{k=-M}^{M} D_{ij} U_{k,j} + \frac{2}{R_i} \sum_{k=-M}^{M} \delta^{(3)}_{\Delta,\sigma} C_{ij} U_{k,j} - \frac{1}{R^2_i} \sum_{k=-M}^{M} B_{ij} U_{k,j} + \frac{1}{R^3_i} \sum_{k=-M}^{M} A_{ij} U_{k,j} = \Omega^2 U_{k,j} \tag{13}$$

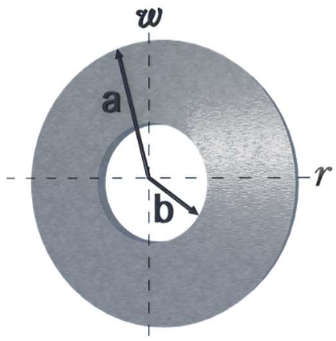

Fig.1. A typical annular plate

5. Results and discussions

The following combination of boundary conditions have been considered while both inner and outer edges are clamped (C-C). Table 1 summarizes the numerical results of first three frequencies by using DSC for the clamped circular plates. By using the generalized differential quadrature (GDQ) method, the first three frequencies obtained by Du et al. [54] are also presented in Table 1. It is apparent that the results were analyzed very well through analytical [7] and GDQ solutions [54]. As can be seen in Table 1, the obtained natural frequencies found by DSC and DQ are very accurate.

The obtained results of the annular plate are presented in Table 2 for the different radius ratios of *b/a* and S-S type edges. Table 2 presents the non-dimensional fundamental frequencies for the annular plates. In this table, *a* is the inner radius and *b* is the outer radius of annular plates. It is observed that a good agreement exists between DQ and DSC results.

Table 1. First three frequencies of the C-C circular plates

Natural frequencies	DQ N=17 This study	GDQ (Ref.54)	Exact (Ref.7)	DSC (N=17) This study
Ω_1	10.21	10.20	10.22	10.23
Ω_2	39.77	44.68	39.77	40.01
Ω_3	89.10	80.24	89.10	89.11

Table 2. Non-dimensional fundamental frequency of the S-S annular plate

b/a	DQ (N=17)	DSC (N=17)	DSC (N=19)	DQ (N=19)
0.1	14.54	14.56	14.56	14.56
0.2	16.83	16.85	16.85	16.85
0.3	21.10	21.11	21.11	21.11
0.4	28.15	28.17	28.17	28.17
0.5	40.09	40.11	40.11	40.11

6. Conclusion

In the present study, the free vibration analysis of thin circular plates and annular plates has been briefly discussed. The approach has been validated through the comparison with convergence studies and existing results in the literature. It is found that the convergence of DSC and DQ methods are very good and the results agree well with those obtained by other researchers. For higher modes (after the 20th modes), which has not been presented here, the method of DSC gives good results than DQ.

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References

[1] Qatu, M., Vibration of Laminated Shells and Plates, Academic Press, U.K., 2004.
[2] Soedel, W., Vibrations of shells and plates, Third Edition, CRC Press, 2004.
[3] Leissa, A.W., Vibration of shells, Acoustical Society of America, 1993.
[4] Reddy, J.N., Mechanics of laminated composite plates and shells: theory and analysis. (2nd ed.) New York: CRC Press, 2003.
[5] Tornabene, F., Fantuzzi, N., "Mechanics of laminated composite doubly-curved shell structures, the generalized differential quadrature method and the strong formulation finite element method", Società Editrice Esculapio, 2014.
[6] Leissa AW., Vibration analysis of plates. NASA, SP-160, 1969.
[7] Blevins R.D., "Formulas for natural frequency and mode shapes." Malabur-Florida: Robert E. Krieger, 1984.
[8] Vogel, S.M. and Skinner, D.W., "Natural frequencies of transversely vibrating uniform annular plates", J. Appl. Meh., Vol. 32, pp. 926-931, 1965.
[9] Civalek, Ö., "Finite Element analyses of plates and shells". Elazığ: Fırat University, (in Turkish) 1998.

[10] Wei, G.W., 2001, "Vibration analysis by discrete singular convolution, Journal of Sound and Vibration", Vol. 244, pp. 535-553, 2001.
[11] Wei, G.W., "Discrete singular convolution for beam analysis, Engineering Structures", Vol. 23, pp.1045-1053, 2001.
[12] Wei, G.W., Zhou Y.C., Xiang, Y., "Discrete singular convolution and its application to the analysis of plates with internal supports. Part 1: Theory and algorithm", Int J Numer Methods Eng., Vol. 55, pp.913-946, 2002.
[13] Wei, G.W., Zhou Y.C., Xiang, Y., "The determination of natural frequencies of rectangular plates with mixed boundary conditions by discrete singular convolution", Int J Mechanical Sciences, Vol. 43, pp.1731-1746, 2001.
[14] Wei, G.W., "A new algorithm for solving some mechanical problems", Comput. Methods Appl. Mech. Eng., Vol. 190, pp. 2017-2030, 2001.
[15] Lim, C.W., Li Z.R., and Wei, G.W., "DSC-Ritz method for high-mode frequency analysis of thick shallow shells", International Journal for Numerical Methods in Engineering, Vol. 62, pp.205-232, 2005.
[16] Civalek, Ö., "An efficient method for free vibration analysis of rotating truncated conical shells", Int. J. Pressure Vessels and Piping, Vol. 83, pp. 1-12, 2006.
[17] Civalek, Ö., Gürses, M. "Free vibration analysis of rotating cylindrical shells using discrete singular convolution technique". Int J Press Vessel Pip, Vol. 86, No. 10, pp. 677-683, 2009.
[18] Civalek, Ö., "Vibration analysis of laminated composite conical shells by the method of discrete singular convolution based on the shear deformation theory", Compos Part B Eng, Vol. 45, No. 1, pp. 1001-1009, 2013.
[19] Civalek, Ö., "Free vibration analysis of composite conical shells using the discrete singular convolution algorithm", Steel Compos Struct, Vol. 6, No. 4, pp.353-366, 2006.
[20] Civalek, Ö., "The determination of frequencies of laminated conical shells via the discrete singular convolution method", J Mech Mater Struct, Vol. 1, No. 1, pp. 163-182, 2006.
[21] Civalek, Ö., "A four-node discrete singular convolution for geometric transformation and its application to numerical solution of vibration problem of arbitrary straight-sided quadrilateral plates", Appl Math Model, Vol. 33, pp. 300–314, 2009.
[22] Civalek, Ö., "Three-dimensional vibration, buckling and bending analyses of thick rectangular plates based on discrete singular convolution method", Int J Mech Sci, Vol. 49, No.6, pp.752-765, 2007.
[23] Demir, Ç., Mercan, K., Civalek, Ö., "Determination of critical buckling loads of isotropic, FGM and laminated truncated conical panel", Compos Part B: Eng Vol. 94, pp. 1-10, 2016.
[24] Civalek, Ö., "Fundamental frequency of isotropic and orthotropic rectangular plates with linearly varying thickness by discrete singular convolution method", Appl Math Model, Vol. 33, No. 10, pp. 3825-3835, 2009.
[25] Civalek, Ö., "Free vibration analysis of symmetrically laminated composite plates with first-order shear deformation theory (FSDT) by discrete singular convolution method", Finite Elem Anal Des Vol. 44, pp. 725-731, 2008.
[26] Civalek, Ö., "Vibration analysis of conical panels using the method of discrete singular convolution", Commun Numer Methods Eng, Vol. 24, pp. 169-181, 2008.
[27] Civalek, Ö., Korkmaz, A, Demir, Ç., "Discrete singular convolution approach for buckling analysis of rectangular Kirchhoff plates subjected to compressive loads on two opposite edges". Adv Eng Softw, Vol. 41, pp. 557-560, 2010.
[28] Civalek, Ö., "Analysis of thick rectangular plates with symmetric cross-ply laminates based on first-order shear deformation theory", J Compos Mater, Vol. 42, pp. 2853–2867, 2008.
[29] Seçkin, A., Sarıgül, A.S., "Free vibration analysis of symmetrically laminated thin composite plates by using discrete singular convolution (DSC) approach: algorithm and verification", J Sound Vib, Vol. 315, pp. 197-211, 2008.
[30] Seçkin, A., "Modal and response bound predictions of uncertain rectangular composite plates based on an extreme value model", J Sound Vib, Vol. 332, pp.1306-1323, 2013.
[31] Xin, L., Hu, Z., "Free vibration of layered magneto-electro-elastic beams by SSDSC approach", Compos Struct, Vol. 125, pp. 96-103, 2015.
[32] Xin. L., Hu, Z., "Free vibration of simply supported and multilayered magnetoelectro-elastic plates", Comp Struct, Vol. 121, pp. 344-350, 2015.
[33] Wang, X., Xu, S., "Free vibration analysis of beams and rectangular plates with free edges by the discrete singular convolution", J Sound Vib, Vol. 329, pp. 1780-1792, 2010.
[34] Wang, X., Wang, Y., Xu, S., "DSC analysis of a simply supported anisotropic rectangular plate", Compos Struct, Vol. 94, pp. 2576-2584, 2012.
[35] Duan, G., Wang, X., Jin, C., "Free vibration analysis of circular thin plates with stepped thickness by the DSC element method", Thin Walled Struct, Vol. 85, pp. 25-33, 2014.
[36] Baltacıoğlu, A.K., Civalek, Ö., Akgöz, B., Demir, F., "Large deflection analysis of laminated composite plates resting on nonlinear elastic foundations by the method of discrete singular convolution", Int J Pres Vessel Pip, Vol. 88, pp. 290-300, 2011.
[37] Civalek, Ö., Akgöz, B., "Vibration analysis of micro-scaled sector shaped graphene surrounded by an elastic matrix", Comp. Mater. Sci., Vol. 77, pp. 295-303, 2013.

[38] Gürses, M., Civalek, Ö., Korkmaz, A., Ersoy, H., "Free vibration analysis of symmetric laminated skew plates by discrete singular convolution technique based on first-order shear deformation theory", Int J Numer Methods Eng, Vol.79, pp. 290-313, 2009.

[39] Baltacıoglu, A.K., Akgöz, B., Civalek, Ö., "Nonlinear static response of laminated composite plates by discrete singular convolution method", Compos Struct, Vol. 93, pp. 153-161, 2010.

[40] Gürses, M., Akgöz, B., Civalek, Ö., "Mathematical modeling of vibration problem of nano-sized annular sector plates using the nonlocal continuum theory via eight-node discrete singular convolution transformation", Appl Math Comput, Vol. 219, pp. 3226–3240, 2012.

[41] Mercan, K., Civalek, Ö., "DSC method for buckling analysis of boron nitride nanotube (BNNT) surrounded by an elastic matrix", Comp Struct, Vol. 143, pp. 300-309, 2016.

[42] Akgöz, B., Civalek, O., "A new trigonometric beam model for buckling of strain gradient microbeams", Int J Mech Sci Vol. 81, pp. 88-94, 2014.

[43] Civalek, Ö., Akgöz, B., "Static analysis of single walled carbon nanotubes (SWCNT) based on Eringen's nonlocal elasticity theory", Int J Eng Appl Sci, Vol. 1, pp. 47-56, 2009.

[44] Demir, Ç., Civalek, Ö., "Torsional and longitudinal frequency and wave response of microtubules based on the nonlocal continuum and nonlocal discrete models", Appl Mathl Model, Vol. 37, pp. 9355-9367, 2013.

[45] Akgöz, B., Civalek, Ö., "Shear deformation beam models for functionally graded microbeams with new shear correction factors", Comp Struct, Vol. 112, pp. 214-225, 2014.

[46] Xin, L., Hu, Z., "Free vibration analysis of laminated cylindrical panels using discrete singular convolution", Comp Struct, Vol. 149, pp. 362-368, 2016.

[47] Tornabene, F., Fantuzzi, N., Viola, E., Ferreira, A.J.M., "Radial basis function method applied to doubly-curved laminated composite shells and panels with a General Higher-order Equivalent Single Layer formulation", Compos Part B: Eng, Vol. 55, pp. 642–659, 2013.

[48] Tornabene, F., Viola, E., "Inman DJ. 2-D differential quadrature solution for vibration analysis of functionally graded conical, cylindrical shell and annular plate structures", J Sound Vib, Vol. 328, pp. 259–290, 2009.

[49] Civalek, Ö., "Application of Differential Quadrature (DQ) and Harmonic Differential Quadrature (HDQ) For Buckling Analysis of Thin Isotropic Plates and Elastic Columns", Eng Struct, Vol. 26, No. 2, pp. 171-186,2004.

[50] Civalek, Ö., "Geometrically non-linear static and dynamic analysis of plates and shells resting on elastic foundation by the method of polynomial differential quadrature (PDQ)", PhD. Thesis, Fırat University, (in Turkish), Elazığ, 2004.

[51] Liew, K.M., Han, J-B, Xiao, Z.M., and Du, H., "Differentiel Quadrature Method for Mindlin plates on Winkler foundations", Int. J. Mech. Sci., Vol. 38, No. 4, pp. 405-421, 1996.

[52] Striz, A.G., Wang, X., Bert, C.W., "Harmonic differential quadrature method and applications to analysis of structural components", Acta Mechanica, Vol. 111, pp. 85-94, 1995.

[53] Bert, C.W., Wang, Z., Striz, A.G., "Static and free vibrational analysis of beams and plates by differential quadrature method", Acta Mechanica, Vol. 102, pp. 11-24, 1994.

[54] Du, H., Lim, M.K., Lin, R.M., "Application of generalized differential quadrature method to vibration analysis", J Sound Vib,Vol. 181, No. 2, 279-93, 1995.

[55] Mercan, K, Demir, Ç., Civalek, Ö., "Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique." Curv Layer Struct, Vol. 3, No.1, pp. 82-90, 2016.