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Reliability analysis of nanocomposite beams-reinforced by CNTs under buckling forces using the conjugate HL-RF

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Abstract

In this paper, the nonlinear conjugate map based on the conjugate Hasofer-Lind and Rackwitz- Fiessler (CHL-RF) method is applied to evaluate the reliability index using first order reliability method of the embedded noanocompsite beam, which is made from polymer reinforced with carbon nanotubs (CNTs). The structure is simulated with Timoshenko beam model. The Mori-Tanaka model is applied for calculating the effective material properties of nanocomposite beam. The surrounding elastic medium is considered by spring and shear constants. Based on energy method and Hamilton's principal, the governing equations are derived. Using an analytical method, the buckling performance function of structure is obtained. The effects of the basic random variables including the length to thickness ratio of beam (L/h), spring constant and shear constant of foundation with respect to the volume fraction of CNTs are investigated on the reliability index of the nanocomposite beam subjected to axial force of 20 GPa. The results indicated that the failure probabilities of the studied the nanocomposite beam are the sensitive to the length to thickness of beam (L/h) and spring constant of foundation variables.

Keywords: Nanocomposite beam; Conjugate HL-RF; first order reliability method; Timoshenko beam model.

1. Introduction

Research and development in composites has advanced at an unimaginable rate in the past decade. Indeed, the nanoscale can lead to new phenomena, providing opportunities for novel multifunctional materials applications. Nanocomposites have become a new class of material that circumvents classic composite material performance by accessing new properties and exploiting unique synergism between materials.

Buckling of beam has been investigated by many researchers. The first who studied theoretically the buckling stability of elasto-plastic beams appeared to be Engesser [1]. He raised the important question how the beam unloads, and suggested that the buckling load of an inelastic beam must be obtained from Euler's formulae. This question was only correctly resolved later on by Shanley [2] who, with the help of the simple theoretical model and various experiments, showed that the buckling of an elastic-plastic beam occurs at the so-called tangent critical load. Mau [3] and Mau and El-Mabsout [4] developed a beam-beam element for the finite element inelastic buckling analysis to determine the beam load-carrying capacity. Pantazopoulou [5] compiled data from the literature of over 300 beam tests and developed requirements for reinforcement stability that recognize the interaction between displacement ductility demand in critical section, tie effectiveness, limiting strain, bar size and tie spacing. Dhakal and Maekawa [6] used fiber finite element analyses to present an average compressive stress-strain relation for reinforcing bars as a function of slenderness ratio and yield strength. Later, Bae et al. [7] conducted an experimental program study on bar buckling and examined the effects of three important bar parameters, the L/D ratio (length over bar diameter), e/D (initial imperfection over bar diameter) ratio and the ratio of ultimate strength to yield strength. Dhakal and Maekawa [8] derived a method to predict the buckling length of longitudinal reinforcing bars using an energy method. The exact analytical solutions for the buckling loads of a reinforced Euler-type beam are presented in, e.g. Krauberger et al. [9], where the effect of the material non-linearity on the buckling load is fully assessed. The development of a finite element model for the geometric and material nonlinear analysis of bonded prestressed continuous beams was presented by Lou et al. [10]. Bajc et al. [11] derived a new semi-analytical procedure for the determination of buckling of the reinforced beam exposed to fire. An experimental investigation on the behaviour of geopolymer composite beams reinforced with conventional steel bars and various types of fibres namely steel, polypropylene and glass in different volume fractions under flexural loading was presented by Vijai et al. [12].

The reliability analysis can be monitored the safety levels of the complex real engineering problems. The first order reliability method (FORM) is commonly applied to estimate the failure probability based on the reliability index using the most probability failure point (MPP). The FORM is a basic reliability approach for reliability-based design code [13] and reliability –based design optimization [14, 15]. There were developed the robust and efficient algorithms to search the MPP based on the conjugate search direction [13, 16-19]. Keshtegar et al. [13,16-18] introduced the conjugate algorithms-based the Fletcher–Reeves method [20]. The stability transformation method and chaos control approach are improved using the chaotic conjugate search direction [16, 17]. The conjugate search direction is improved based on the limited scalar conjugate factor in the limited conjugate first order reliability method [18]. The sufficient descent condition in the proposed approach in the Refs [16, 18] and Armijo rule in the Ref [13, 17] are applied to achieve the stabilization of iterative FORM formal. Keshtegar [16-18] showed that the conjugate search direction can be improved the robustness and efficiency of FORM formula compared to the reliability algorithm –based steepest descent search direction [21, 22] for highly nonlinear engineering problems. The conjugate Hasofer-Lind and Rackwitz- Fiessler (CHL-RF) method [13] is a simple and robust FORM formula which is developed based on the conjugate discrete map to improve the instability of the HL-RF method. The application of the CHL-RF is applied to approximate the failure probabilities of corroded pipe in Ref [13], successfully.

In this paper, The CHL-RF method is implemented to search MPP of a nanocomposite beam under buckling failure mode due to its simplicity and robustness. The robustness of the CHL-RF is guaranteed using Armijo's rule based on the finite step length. A nanocomposite beam under buckling forces is used to determine the probabilistic model that the implicit buckling performance function of nanocomposite beam is theoretically estimated using Timoshenko beam model and Navier method. The random variables such as the length to thickness of beam, volume fraction of CNT, shear and the spring constant of foundation are applied to reliability analysis of nanocomposite beam, which are simulated with normal and non-normal distributions. The results illustrated that the CHL-RF provided the stable results for iterative formula of FORM. The length to thickness of beam (L/h) and shear constant of foundation are sensitive and insensitive basic variables on reliability index among other basic variables.

2. Conjugate HL-RF

In FORM, structural failure probabilities of the nanocomposite beams are approximated based on the reliability index as follows [13,18]:

$$P_f = \int_{g(X)\le 0} \cdots \int f_X(x_1,\ldots,x_n)dx_1..dx_n \approx \Phi(-\beta) \tag{1}$$

Where, β is the reliability index, which corresponds to minimum distance to the origin on the limit state function in the standard normal space [17]. Generally, the main goal of FORM is to search the most probable point (MPP i.e. $\boldsymbol{U}^*$) search for computing the reliability index as $\beta = \|\boldsymbol{U}^*\|$. $g(\boldsymbol{X})$ is the limit state function or performance function that it can be defined based on the bucking performance for nanocompsite beam in this paper, $f_X(x_1,\ldots,x_n)$ is the joint probability density function for the basic random variables $\boldsymbol{X}$ and Φ is the cumulative distribution function (CDF) of standard normal variables. $g(\boldsymbol{X})<0$ defines the failure region as follows: $P_{cr} - P < 0$ in which P is the external compressive force which is given based on the applied loads and P_{cr} is the theoretical buckling force which is determine based on the capacity nanocompsite beam.

Typically, FORM –based CHL-RF involves the three steps for estimating the probability of failure as follows [13, 17]:

Step 1. Transform random variables in X-space into U-space by the following relation:

$$u = \frac{x - \mu_x^e}{\sigma_x^e} \tag{2}$$

where u is the standard normal variable with the mean and standard deviation equal to zero and one, respectively. μ_x^e and σ_x^e are equivalent mean and standard deviation of the random variable *x*, respectively. It can be obtained for normal random variable as $\mu_x^e = \mu_x$ and $\sigma_x^e = \sigma_x$. The equivalent mean and standard deviation of non-normal random variables can be determined by the following equations [19]:

$$\sigma_x^e = \frac{1}{f_X(x)} \varphi[\Phi^{-1}\{F_X(x)\}] \tag{3}$$

$$\mu_x^e = x - \sigma_x^e \Phi^{-1}[F_X(x)] \tag{4}$$

where $F_X(x)$ is cumulative distribution, $f_X(x)$ is probability distribution, Φ^{-1} is inverse standard normal cumulative distribution and φ is standard normal probability distribution function.

Step 2. Find the most probable point (MPP) based on the iterative formula of CHL-RF.

The MPP $\boldsymbol{U}^* = (u_1^*, u_2^*, ..u_n^*)^T$ can be searched based on the conjugate search direction as follows [13]:

$$\boldsymbol{U}_{k+1} = \beta_{k+1}\alpha_{k+1}^{\lambda} \tag{5}$$

$$\beta_{k+1} = \frac{g(\boldsymbol{U}_k) - \nabla^T g(\boldsymbol{U}_k)\boldsymbol{U}_k}{\nabla^T g(\boldsymbol{U}_k)\alpha_{k+1}^{\lambda}} \tag{6}$$

where $\nabla g(\boldsymbol{U}) = [\partial g/\partial u_1, \partial g/\partial u_2, ..., \partial g/\partial u_n]^T$ is gradient vector of the limit state function at the design point$\boldsymbol{U}$. α_{k+1}^{λ} is direction cosine vector or sensitivity vector of random variables, which can be computed as below:

$$\alpha_{k+1}^{\lambda} = \frac{\boldsymbol{U}_{k+1}^{C\lambda}}{\left\|\boldsymbol{U}_{k+1}^{C\lambda}\right\|} \tag{7}$$

in which, point $\boldsymbol{U}_{k+1}^{C\lambda}$ is along the direction of the negative conjugate gradient vector at design point$\boldsymbol{U}_k$. The point $\boldsymbol{U}_{k+1}^{C\lambda}$ is determined by the following relation

$$\boldsymbol{U}_{k+1}^{C\lambda} = \boldsymbol{U}_k + \lambda \boldsymbol{d}_k \tag{8}$$

where $\lambda > 0$ is the step length and $\boldsymbol{d}_k$ is conjugate search direction, which are computed by the following relations:

$$\boldsymbol{d}_k = -\nabla g(\boldsymbol{U}_k) - \frac{\left\|\nabla g(\boldsymbol{U}_k)\right\|^2}{\left\|\nabla g(\boldsymbol{U}_{k-1})\right\|^2}\boldsymbol{d}_{k-1}$$

$$(9)\ \lambda_k = \begin{cases} \lambda_{k-1} & if \ \left\|\boldsymbol{U}_{k+1} - \boldsymbol{U}_k\right\| > \left\|\boldsymbol{U}_k - \boldsymbol{U}_{k-1}\right\| \\ c\lambda_{k-1} & \text{otherwise} \end{cases} \tag{10}$$

where, $0 < c < 1$ is the adjusted factor which is given as 0.9 in this paper. The step length can be provided a global convergence for CHL-RF algorithm in the complex engineering problems with highly nonlinear performance function. The initial step size is determined based on Armijo-type line search as follows [13, 17]:

$$\lambda_0 = \frac{50}{\left\|\nabla g(\boldsymbol{U}_{\mu})\right\|^2} \tag{11}$$

Based on the Eq. (10), $\boldsymbol{U}_{k+1}^{C\lambda} = \boldsymbol{U}_k$ when the step length is $\lambda = 0$. This means that the $\boldsymbol{U}_{k+1}^{C\lambda}$ point is in a fixed position. According to the above equations, a cycle of the iterative CHL-RF formula is given for reliability analysis as follows:

1- Compute the gradient vector of the performance function at point $\boldsymbol{U}_k$

2- Compute the conjugate search direction $\boldsymbol{d}_k$ using Eq. (9)

3- Determine the point $\boldsymbol{U}_{k+1}^{C\lambda}$ in terms of Eq. (8).

4- Compute the sensitivity vector of random variables based on Eq. (7).

5- Compute the reliability index and new point using Eqs. (6) and (5), respectively.

6- Up-to-date the new step length based on the Armijo's rule and Eq. (10).

Step 3. Calculate the failure probability as $P_f \approx \Phi(-\beta)$.

It can be funded from the reliability analysis that the iterative approach of the FORM and the performance function are two important factors to evaluate the failure probability of the nanocomposite beams. The iterative formula is laded to the stable results; consequently, the CHL-RF approach can be provided stable solutions for highly nonlinear performance functions [14]. The performance function is an essential key for accurate reliability analysis of the nanocomposite beams, thus it can be defined as follows:

Performance function of the nanocomposite beam

The limit state function is separated the design domain into the failure and safe regions based on the buckling mode of the nanocomposite beams. It can be supposed that the nanocomposite beams may be failed when the external compressive load (P) is more than the buckling forces (P_{cr}) of the beams. Consequently, the limit state function of nanocomposite beams under buckling failure mode can be given as follows:

$$g(L,h,kw,kg,rho) = P_{cr}(L,h,kw,kg,rho) - P < 0 \tag{12}$$

where, $P_{cr}(L,h,kw,kg,rho)$ is the buckling force which is can be obtained based on the Timoshenko beam theory. The displacements of an arbitrary point in the Timoshenko beam are [18]

$$\begin{aligned} u_1(x,z) &= U(x) + z\,\psi(x), \\ u_2(x,z) &= 0, \\ u_3(x,z) &= W(x) \end{aligned} \tag{13}$$

where ψ is the rotation of beam cross-section.

$$\varepsilon_{xx} = \frac{\partial U}{\partial x} + z\frac{\partial \psi}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2, \tag{14}$$

$$\gamma_{xz} = \frac{\partial W}{\partial x} + \psi. \tag{15}$$

The stress-strain relations can be written as

$$\sigma_{xx} = C_{11}\varepsilon_x, \tag{16}$$

$$\sigma_{xz} = C_{55}\left[\frac{\partial W}{\partial x} + \psi\right], \tag{17}$$

where C_{11} and C_{55} are elastic constants which can be calculated by Mori-Tanaka model as follows

$$C_{11} = k + m, \tag{18}$$

$$C_{55} = m, \tag{19}$$

where

$$k = \frac{E_m\{E_m c_m + 2k_r(1+\nu_m)[1+c_r(1-2\nu_m)]\}}{2(1+\nu_m)[E_m(1+c_r-2\nu_m) + 2c_m k_r(1-\nu_m-2\nu_m^2)]}, \tag{20}$$

$$m = \frac{E_m[E_m c_m + 2m_r(1+\nu_m)(3+c_r-4\nu_m)]}{2(1+\nu_m)\{E_m[c_m+4c_r(1-\nu_m)] + 2c_m m_r(3-\nu_m-4\nu_m^2)\}}, \tag{21}$$

where c_m and c_r are the volume fractions of the polymer and the nano-fibers, respectively and kr ،l_r ، n_r ، p_r , m_r are the Hills elastic modulus for the nano-fibers. In addition, E_m and υ_m are the Young's modulus and Poisson's ratio of matrix.

The strain energy of the structure can be expressed as

$$U = \frac{1}{2}\int_0^L \int_A (\sigma_{xx}\,\varepsilon_{xx} + 2\sigma_{xz}\,\varepsilon_{xz})\,dA\,dx \tag{22}$$

Submitting Eqs. (13) to (15) into (22) yields

$$U = \frac{1}{2}\int_0^L \left\{ N_x\frac{\partial U}{\partial x} + M_x\frac{\partial \psi}{\partial x} + \frac{1}{2}N_x\left(\frac{\partial W}{\partial x}\right)^2 + Q_x\frac{\partial W}{\partial x} + Q_x\psi \right\} dx \tag{23}$$

The governing equations of structure can be derived from the Hamilton's principle as

$$\frac{\partial N_x}{\partial x} = 0, \tag{24}$$

$$\frac{\partial Q_x}{\partial x} - \frac{\partial}{\partial x}\left(N_x^M\frac{\partial W}{\partial x}\right) + q = 0, \tag{25}$$

$$\frac{\partial M_x}{\partial x} - Q_x = 0, \tag{26}$$

Based on dimensionless parameters as follows

$$\begin{aligned} &\xi = \frac{x}{L},\ (\overline{W},\overline{U}) = \frac{(W,U)}{h},\ \eta = \frac{h}{L},\ \psi = \overline{\psi},\ \overline{K}_w = \frac{K_w hL}{C_{11}A}, \\ &\overline{G}_p = \frac{G_p}{C_{11}A},\ \overline{N}_x^M = \frac{N_x^M}{C_{11}A},\ \overline{I} = \frac{I}{AL^2},\ C_5 = \frac{C_{55}}{C_{11}}. \end{aligned} \tag{27}$$

the governing equations can be rewritten as

$$\frac{\partial^2 \overline{U}}{\partial \xi^2} + \eta \frac{\partial^2 \overline{W}}{\partial \xi^2}\frac{\partial \overline{W}}{\partial \xi} = 0, \tag{28}$$

$$C_5\left[\eta \frac{\partial^2 \overline{W}}{\partial \xi^2} + \frac{\partial \overline{\psi}}{\partial \xi}\right] - \overline{N}_x^M \eta \frac{\partial^2 \overline{W}}{\partial \xi^2} - \overline{K}_w \overline{W} + \overline{G}_p \frac{\partial^2 \overline{W}}{\partial \xi^2} = 0, \tag{29}$$

$$\bar{I}\frac{\partial^2 \overline{\psi}}{\partial \xi^2} + K_s C_5\left[\eta \frac{\partial \overline{W}}{\partial \xi} + \overline{\psi}\right] = 0. \tag{30}$$

Based on DQM, the functions f and their kth derivatives with respect to x can be approximated as [21, 22]

$$\frac{d^n f(x_i)}{dx^n} = \sum_{j=1}^{N} C_{ij}^{(n)} f(x_j) \qquad n = 1,\ldots,N-1, \tag{31}$$

where N is the total number of nodes distributed along the x-axis and C_{ij} is the weighting coefficients, the recursive formula for which can be found in [21-24]. Using DQM, the governing equations can be expressed in matrix form as

$$\left(\left[\underbrace{K_L + K_{NL}}_{K}\right] + P\left[K_g\right]\right)\begin{Bmatrix}\{d_b\}\\ \{d_d\}\end{Bmatrix} = 0, \tag{32}$$

where K_L is the linear stiffness matrix; K_{NL} is the nonlinear stiffness matrix and K_g is geometric stiffness matrix. Also, d_b and d_d represent boundary and domain points. Finally, based on an iterative method and eigenvalue problem, the buckling load of structure may be obtained.

3. Results and discussion

A nanocomposite beam with limit state function in Eq. (12) under the external compressive load of 20 GPa i.e. P=20 GPa and buckling capacity which is computed using Eq. (32) is applied to reliability analysis based on the CHL-RF method. The reliability index of nanocomposite beam is obtained based on the five basic random with normal and non-normal distributions, whose their statistical properties are tabulated in Table 1.

Table 1: Statistical properties of the basic random variables for nanocoposite beam

Variable	Unite	Description	Mean	Coefficient of variation	Distribution
L	m	Length of beam	0.4,0.8,2,4	0.1	Normal
h	m	Thickness of beam	0.4	0.1	Normal
kw	GPa	Stiffness constant of foundation	100, 200, 500,1000	0.12	Gumbel
kg	N	shear constant of foundation	0, 10, 20, 50, 100	0.12	Gumbel
rho	--	volume fraction of CNT nanoparticles	0-0.5	0.15	Lognormal

The parameters of the CHL-RF for reliability analysis are given as c=0.9 and the stopping criterion of $\left\|\boldsymbol{U}_k - \boldsymbol{U}_{k-1}\right\| < 10^{-5}$. Based on the Mean and COV from Table 1, the reliability indexes for different Means of the basic random variables are determined with respect to the various volume fractions of CNT nanoparticles, which are changed in the range from 0 to 0.5 for nanocomposite beams. The reliability index is computed based on the applied compressive load of 20 GPa for this nanocomposite beam. The MPP and reliability index are obtained using the CHL-RF method based on the statistical properties of random variables in the Table 1 with mean values of L=0.8m, h=0.5m, kw=500GPa, kg=50N and rho=0.25 as $\boldsymbol{X}$*=(L*=0.6241391m, h*=0.3990155m, kw*=452.592126 GPa, kg*= 48.475307N, rho*=0.2467846) and β= 2.340344 after 31 iterations and 5203 numbers of call probabilistic function, respectively. Therefore, the CHL-RF can be provided stable results for this nanocomposite beam.

The reliability index of the probabilistic model in the Eq. (12) are estimated using the CHL-RF for nanocomposite beam with the mean values of the basic random variables as h=0.4 m, kw=500GPa, and kg=50N that the results of the reliability indexes for different length to thickness of beam are plotted in Fig. 1. The extracted results from Fig. 1 showed that the reliability index is increased by increasing the length to thickness (L/h) of beam. Increment rate of the reliability index with respect to the L/h is a nonlinear form so that the differences between the reliability indexes of L/h=1 and L/h= 2 are obtained more than the differences between reliability indexes of L/h=2 and L/h= 5. The reliability index corresponding to the rho is not significantly changed compared to the variable L/h. It can be concluded from the results of Fig. 1 that very weak safety levels for nanocomposite beam are obtained with respect to the L/h less than 1 (reliability index less than -3). A poor safety levels can be obtained for nanocomposite beams when variable L/h is selected as 2. Good safety levels are obtained with respect to the L/h=5 and the excellent safety levels can be provided when the L/h is selected more than 10 for nanocomposite beam.

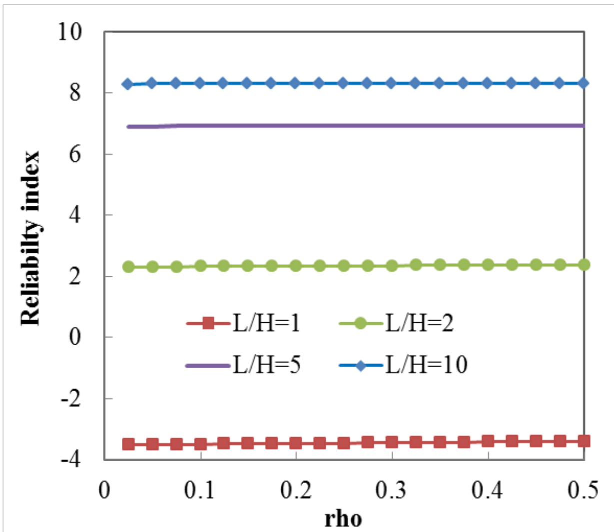

Fig. 1. The reliability indexes of beam for different length to thickness

The effects of the different spring constants of foundation (kw) are evaluated on the reliability indexes on the nanocomposite beam for the mean values of random variables as h=0.4 m, L=1.2m, and kg=50N. The results of the reliability index with respect to various volume fractions of CNT nanoparticles (rho) for different kws are shown in Fig. 2. As seen, the variable kw is significantly affected on the safety levels on the nanocomposite beam compared to the rho. For L/h=3, the kw should be selected more than the 200 GPa to obtain the reliability indexes larger than 2. The reliability indexes are increased by increasing the spring constants of foundation as well as the length to thickness of nanocomposite beams. A good safety laves (reliability index more than 4) are obtained for the studied nanocomposite beam when the kw is given more than 500 GPa. The increment rates of the reliability index corresponding to the spring constants of foundation are obtained a nonlinear rate. It can be seen, the reliability index is improved from 4.8 to 6.4 (relative increment is 25%) when the kw is increased from 500 to 1000 GPa while the reliability indexes are increased from 1.9 to 4.8 (relative increment is about 60%) when kw is changed from 200 to 500 GPa. It can be concluded that an excellent safety levels can be provided for nanocomposite beam in the following conditions: L/h=3; h=0.4m, kg=50N, rho=0-0.5 and kw= 1000Gpa.

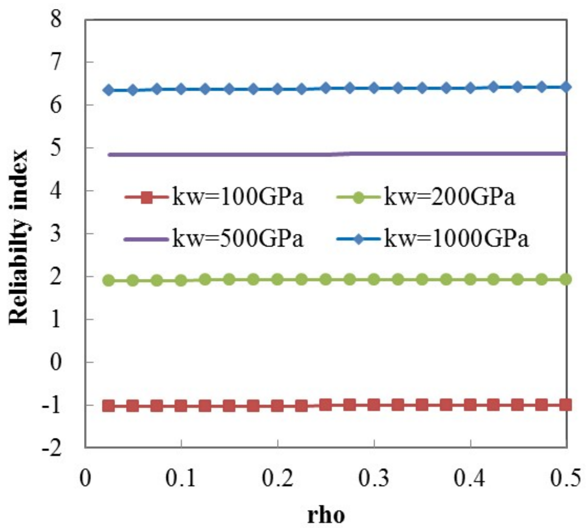

Fig. 2. The reliability indexes of beam for different spring constant of foundation

Figure 3 illustrates the reliability index of nanocomposite beam corresponding to various volume fractions of CNT nanoparticles for different shear constants of foundation (kg). The reliability indexes in Fig. 3 are computed using the CHL-RF method with the mean values of basic random variables as h=0.4 m, L=1.2m, and kw=500GPa. It can be obvious for the reliability indexes in Fig. 3, the reliability indexes are increased by increasing the various volume fractions of CNT nanoparticles (rho) as well as shear constants of foundations. Therefore, the effects of the kg on the reliability index are obtained as well as the rho. Based on the L/H=3, kw=500Gpa, a good confidence levels (reliability index more than 4.5) are obtained for all kg and the reliability indexes are improved from 4.6 to 5.1 when the kg is increased from 0 to 100N. By comparing the results from in Figs. 1-3, the input basic variables length to thickness of nanocomposite beam (L/h) and spring constants of foundation (kw) can be improved the performances of beam under axial compressive loads and are significantly enhanced the capacity of the nanocomposite beams in compression with the volume fractions of CNT nanoparticles (rho) and shear constants of foundation (kg). The reliability indexes are slightly increased by increasing the L/h and kw, while the variables rho and kg are not affected, more reasonably.

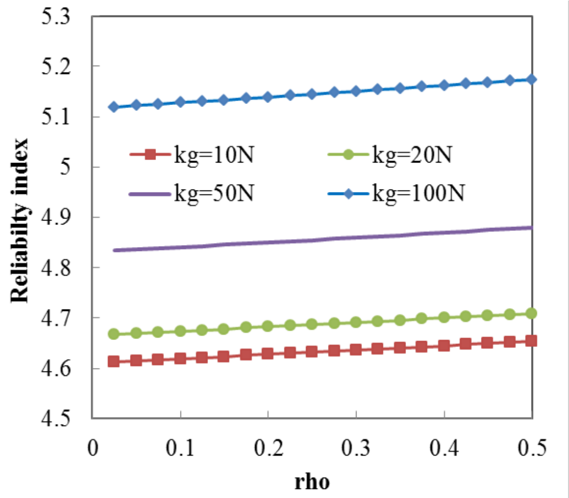

Fig. 3. The reliability indexes of beam for different shear constant of foundation

As seen for Fig. 1 and 2, length to thickness of nanocomposite beam (L/h) and spring constants of foundation (kw) are the affective input variables on the reliability index. The reliability indexes of the nanocomposite beam with respect to various L/h (i.e. 1-10) for different kw as 200, 300, 400, 500, 700, 1000Gpa are plotted in Fig. 4 with input variables kw=50N, h=0.4 m, and rho=0.25. It can be concluded that L/h can be improved the safety levels of nanocomposite beam more than the kw. The reliability indexes are obtained more than 4 by increasing the L/h more than 2 for kw in the range between 200 and 1000 GPa. The reliability indexes are significantly increased for domain of the variables L/h less than 3 and kw less than 500GPa. It can be extracted the results from Fig. 4 that an excellent confidence level (reliability index more than 6.5) can be obtained for nanocomposite beams under axial force 20Gpa when the geometrical and mechanical conditions of beams are given as L/h>7 in kw=200Gpa, L/h> 6 in kw=300Gpa, L/h> 5 in kw=400 and 500Gpa, L/h> 4 in kw=700 Gpa and L/h> 3 in kw=1000Gpa. Consequently, the L/h more than 5 and kw more than 400 GPa can be suggested for an excellent robust designs of nanocomposite beams under axial force 20Gpa.

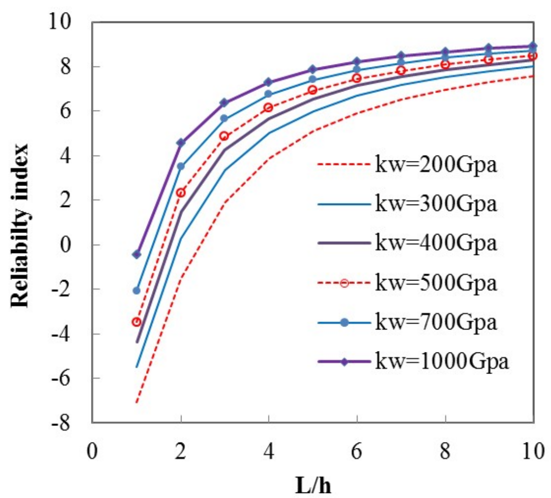

Fig. 4. Reliability indexes of beam for different spring constants of foundation corresponding to various length to thickness ratios

4. Conclusions

The conjugate HL-RF (CHL-RF) with Armijo line search is applied to evaluate the reliability index of the nanocomposite beams- reinforced by the CNT nanoparticles. The conjugate dynamic discrete map is used to improve the robustness of the first order reliability formula in the reliability analysis of nanocomposite beam under buckling failure mode. A probabilistic model is developed based on the buckling forces on the nanocomposite beam using Timoshenko beam theory and Navier method. The input random variables such as length of beam, thickness of beam, volume fractions of CNT nanoparticles, spring constant and shear constant of the foundation are simulated using the normal and non-normal random variables for the reliability analysis of the nanocomposite beam. The results of the reliability analysis for nanocomposite beam illustrated that the CHL-RF can be provided stable solutions and appropriate results for reliability analysis of a complex real example. The shear constant of foundation in the range from 0-100N and the volume fractions of CNT nanoparticles in the range from 0-0.5 can be improved the reliability indexes from 4.6 to 5.2. The spring constant of foundation and the length of the beams are two affective variables to enhance the confidence levels of the nanocomposite beams compared to another random variables i.e. kg and rho, more effectively. The nanaocomposite beam reinforced by CNT nanoparticles under axial force of 20GPa can be provided an excellent confidence level when the beam is designed with the robust conditions as length to thickness of beam more than 5 (i.e. L/h≥5), spring constant of foundation more than 400GPa (i.e. kw≥400GPa), thickness of beam less than 0.4m (i.e. h≤0.4m) and shear constant of foundation more than 10 N (i.e. kw≥10N) with volume fractions of CNT nanoparticles in the range from 0.1-0.5.

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