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Size Effect Impact on the Mechanical Behavior of an Electrically Actuated Polysilicon Nanobeam based NEMS Resonator

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Abstract

In this paper, the dynamic response of resonating nano-beams is investigated using a strain gradient elasticity theory. A nonlinear model is obtained based on the Galerkin decomposition method to find the dynamic response of the investigated beam around its statically deflected position. The mid-plane stretching, axial residual stress and nonlinear interaction due to the electrostatic force on the deflected beam are included in the proposed nonlinear beam model. Comparing the beam natural frequency using strain gradient theory with experimental data shows an excellent agreement among both approaches. The normalized natural frequency is shown to be increasing nonlinearly with the decrease of the applied DC voltage as well as beam thickness. The results also reveal that increasing the tension axial stress increases the natural frequency; however its influence decreases when decreasing the beam thickness. To investigate the effect of AC actuation voltage on the beam resonant frequency, a Lindstedt-Poincare based perturbation method is utilized and validated by comparison with experimental data. The results show that increasing the AC actuation voltage makes the beam stiffer by increasing its resonant frequency.

Keywords: NEMS, Nano-resonator, Strain gradient elasticity theory, Size effect.

1. Introduction

Electrically actuated micro- and nano-beams are among the most used components in several micro- as well as nano-electromechanical systems (MEMS and NEMS) such as switches, sensors, actuators, filters and resonators. A typical electrostatically actuated MEMS\NEMS device includes two parallel conducting electrodes, one is fixed and the other one is movable due to the applied electrostatic force.

The beam mid-plane stretching and the nonlinear coupling between the electrical and the beam restoring forces result in several nonlinear dynamical phenomena such as fractal jumps (hysteresis), pull-in phenomena and chaotic motion [1-5]. To keep MEMS and NEMS devices operating in stable state, analyzing their dynamical behavior is essential during their design and fabrication processes. However, the different sources of nonlinearities appearing essentially in their governing equations open numerous challenging problems in solving their dynamical behavior. In this regards, a number of methods such as shooting method [6], differential quadrature method [7], finite element method [8, 9], perturbation method [10] and homotopy analysis method [11] were developed to get the dynamic responses of some MEMS and NEMS devices.

In all of the above-mentioned investigations, the movable electrode was modeled as an elastic beam based on classical continuum theory. However, the thickness of micro and nano-beams are in the order of microns and submicron, add to that many experimental observations revealed that when the size of some metals and polymers decreases, a size-dependent behavior appears which cannot be explained using conventional continuum theory. For instance, Fleck et al. [11] have reported that the torsion hardening of copper wire increases by a factor of 3 as the wire diameter decreases from 170 to 12 μm. In other experimental investigation [12], it was reported that the bending stiffness of a silicon beam with the dimension of 6 μm × 0.37 μm ×0.255 μm was 2.3 to 4.7 times and 38 times larger than that of a micrometre-sized and a millimetre-sized beams, respectively. In [13] and based on micro-bending test of thin nickel beams, it was reported that the plastic work hardening increased extensively with a decrease of the beam thickness from 50 to 12.5 μm.

To predict the size-dependent behavior of micro- and nano-beams, governing equations of Euler-Bernoulli beams were developed based on non-classical theories such as the modified couple stress theory, the nonlocal and strain gradient elasticity theories and the modified continuum model incorporating surface elasticity [14-17]. Recently, several investigations studied the static and dynamic behavior of micro- and nano-beams based on aforementioned non-classical theories where the significance of the size effect has been compared with the results of classical theories [18-23]. However, investigations of size-dependent behavior of electrostatically actuated devices are uncommon despite their importance to well predict the behavior of electrostatically actuated micro- and nano-beams. Hence, it is necessary to analyze the dynamic behavior of these systems based on the theories which incorporate this effect, which represents the objective of this current research work.

In this paper, first, a nonlinear model is derived to study the dynamic response of nano- and micro- beams around their statically deflected position based on strain gradient theory. The fringing field effect, residual axial stress and mid-plane stretching have been taken into account in the beam model. Variation of natural frequencies and resonant frequencies versus the electric voltage are compared with different experimental results reported in the literature. Based on the proposed method, effects of beam thickness, applied DC voltage, axial residual stress and AC actuation voltage on the beam frequency are then examined.

2. Mathematical Modeling

Figure 1 shows a micro/nano-beam under parallel-plates electrostatic actuation method. As seen in the figure, this system includes two parallel conductive plates: a movable and a fixed electrode. By applying a DC voltage V_p superimposed to a small AC voltage $V_{ac}(t)$, and based on both the Euler-Bernoulli beam assumption and the strain gradient theory, the governing equation of the micro- and nano-beam can be written as [16, 24]:

$$-K\frac{\partial^6 w}{\partial x^6}+S\frac{\partial^4 w}{\partial x^4}-(N+\alpha_1\int_0^1(\frac{\partial w}{\partial x})^2dx)\frac{\partial^2 w}{\partial x^2}+\rho A\frac{\partial^2 w}{\partial t^2}-\varepsilon b(V_p+V_{ac}(t))^2(\frac{1}{(g-w)^2}+\frac{\beta}{b}\frac{1}{g-w})=0 \quad (1)$$

where

$$K=\mu I(2l_0^2+\frac{4}{5}l_1^2) \qquad S=EI+\mu A(2l_0^2+\frac{8}{15}l_1^2+l_2^2) \quad (2)$$

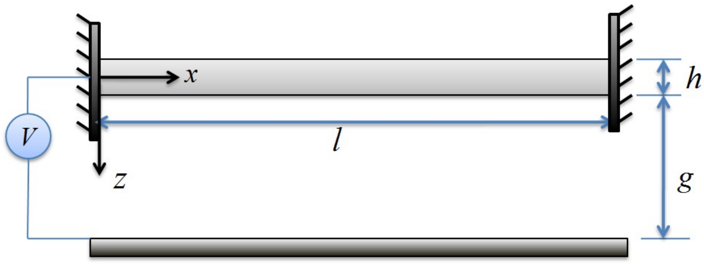

Fig.1. Schematic of an electrostatically actuated micro/nano-beam

In the above equation of motion, the parameters E, A, ρ, I, ε, N, g, μ and w are the beam Young's modulus, its cross section area, its mass density, its momentum of inertia, the air dielectric constant, the axial force, the gap distance between the two electrodes, the shear modules and micro-beam deflection, respectively. Note that the beam deflection is a function of the axial position, x and time, t. The independent length scale parameters l_0, l_1 and l_2 are associated to the dilatation gradient, the deviatoric stretch gradient and the rotation gradient, respectively.

To solve Eq. (1), it is more convenient to use non-dimensional parameters introduced as follows:

$$\hat{x} = \frac{x}{l}; \ \hat{w} = \frac{w}{g}; \ \hat{t} = \frac{t}{T} \tag{3}$$

Substituting Eq. (3) in Eq. (1), Eq. (1) can be rewritten as

$$T(\hat{w}) = \alpha_0 \frac{\partial^6 \hat{w}}{\partial \hat{x}^6} + \frac{\partial^4 \hat{w}}{\partial \hat{x}^4} - (\hat{N} + \alpha_1 \int_0^1 (\frac{\partial \hat{w}}{\partial \hat{x}})^2 d\hat{x}) \frac{\partial^2 \hat{w}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{w}}{\partial \hat{t}^2} - \alpha_2 (V_p + V_{ac})^2 (\frac{1}{(1-\hat{w})^2} + \frac{\beta g}{b} \frac{1}{1-\hat{w}}) = 0 \tag{4}$$

The non-dimensional parameters in Eq. (4) are defined as follows:

$$\alpha_0 = -\frac{K}{Sl^2}, \quad \alpha_1 = \frac{EAg^2}{2S}, \quad \alpha_2 = \frac{\varepsilon b l^4}{2Sg^3}, \quad \hat{N} = \frac{l^2}{S} N, \quad T = \sqrt{\frac{\rho b h l^4}{S}} \tag{5}$$

The classical and non-classical clamped-clamped beam boundary conditions based on the strain gradient theory are as follows:

$$\hat{w}(0,t) = \hat{w}(1,t) = \frac{\partial \hat{w}(0,t)}{\partial \hat{x}} = \frac{\partial \hat{w}(1,t)}{\partial \hat{x}} = \frac{\partial^3 \hat{w}(0,t)}{\partial \hat{x}^3} = \frac{\partial^3 \hat{w}(1,t)}{\partial \hat{x}^3} = 0 \tag{6}$$

The beam deflection under an electrostatic actuation is composed of a static component caused by the DC voltage and a dynamic component induced by the AC harmonic voltage, denoted by $w_s(\hat{x})$ and $w_d(\hat{x},\hat{t})$ respectively. Thus, the total deflection of beam can be written as follows:

$$w(\hat{x},\hat{t}) = w_s(\hat{x}) + w_d(\hat{x},\hat{t}) \tag{7}$$

By ignoring the dynamic terms in Eq. (4), the governing static equation of micro- and nano-beam can be obtained. Substituting Eq. (7) into Eq. (4), dropping the static equilibrium deflection terms and neglecting the nonlinear higher order terms (V_{ac}^2, w_d^2 and $V_{ac} w_d$), the dynamic response of beam around its statically deflected profile can be obtained as [6]:

$$\begin{aligned} T(\hat{w}) = {} & \frac{\partial^2 \hat{w}_d}{\partial \hat{t}^2} + \alpha_0 \frac{\partial^6 \hat{w}_d}{\partial \hat{x}^6} + \frac{\partial^4 \hat{w}_d}{\partial \hat{x}^4} - N_1 \frac{\partial^2 \hat{w}_d}{\partial \hat{x}^2} - N_2 (\frac{d^2 \hat{w}_s}{d\hat{x}^2} + \frac{\partial^2 \hat{w}_d}{\partial \hat{x}^2}) - N_3 (\frac{d^2 \hat{w}_s}{d\hat{x}^2} + \frac{\partial^2 \hat{w}_d}{\partial \hat{x}^2}) \ldots \\ & -2\alpha_2 V_p^2 (\frac{\hat{w}_d}{(1-\hat{w}_s)^3} + \frac{\beta g}{2b} \frac{\hat{w}_d}{(1-\hat{w}_s)^2}) - 2\alpha_2 V_p V_{ac}(t) (\frac{1}{(1-\hat{w}_s)^2} + \frac{\beta g}{b} \frac{1}{(1-\hat{w}_s)}) = 0 \end{aligned} \tag{8}$$

where

$$N_1 = (\hat{N} + \alpha_1 \int_0^1 (\frac{d\hat{w}_s}{d\hat{x}})^2 d\hat{x}); \qquad N_2 = 2\alpha_1 \int_0^1 (\frac{d\hat{w}_s}{d\hat{x}})(\frac{\partial \hat{w}_d}{\partial \hat{x}}) d\hat{x}; \qquad N_3 = \alpha_1 \int_0^1 (\frac{\partial \hat{w}_d}{\partial \hat{x}})^2 d\hat{x} \tag{9}$$

Based on the separation of variable principle, the solution of Eq. (4) can be written in the following form:

$$w_d(\hat{x},\hat{t}) = \varphi(\hat{x}) u(\hat{t}) \tag{10}$$

where, $\varphi(\hat{x})$ is the first mode shape of the considered beam. Based on the strain gradient theory, $\varphi(\hat{x})$ is expressed as [25]:

$$\varphi(\hat{x}) = C_1 \cosh(\omega_1 \hat{x}) + C_2 \sinh(\omega_1 \hat{x}) + C_3 \cos(\omega_2 \hat{x}) + C_4 \sin(\omega_2 \hat{x}) + C_5 \frac{e^{-\omega_3 \hat{x}}}{\omega_3^2} + C_6 \frac{e^{\omega_3 \hat{x} - \omega_3}}{\omega_3^2} \tag{11}$$

where, C_1 to C_6 and ω_1 to ω_3 are constants which depends on the value of the parameter α_0.

By setting $V_{ac} = |V_{ac}| \cos(\Omega t)$ and using Galerkin's decomposition, the governing differential equation of $u(\hat{t})$, Eq. (10), can be expressed as:

$$\int_0^1 \varphi(\hat{x})T(\hat{w})dx = 0 \quad \Rightarrow \quad M\ddot{u} + K_1 u + K_2 u^2 + K_3 u^3 = F_V \cos(\Omega t) \tag{12}$$

Equation (12) represents the first mode of vibration of the considered beam under electrostatic force. The parameters used in Eq. (12) are given as follows:

$$\begin{aligned} M &= \int_0^1 \varphi(\hat{x})^2 d\hat{x}, \\ K_1 &= \int_0^1 (\alpha_0 \frac{\mathrm{d}^6\varphi}{\mathrm{d}\hat{x}^6} + \frac{\mathrm{d}^4\varphi}{\mathrm{d}\hat{x}^4} - N_1 \frac{\mathrm{d}^2\varphi}{\mathrm{d}\hat{x}^2} - N_2 \frac{\mathrm{d}^2\hat{w}_s}{\mathrm{d}\hat{x}^2} - 2\alpha_2 V_p^2 (\frac{\varphi}{(1-\hat{w}_s)^3} + \frac{\beta g}{2b}\frac{\varphi}{(1-\hat{w}_s)^2}))\varphi \mathrm{d}\hat{x} \\ K_2 &= -\int_0^1 (N_2' \frac{\mathrm{d}^2\varphi}{\mathrm{d}\hat{x}^2} + N_3' \frac{\mathrm{d}^2\hat{w}_s}{\mathrm{d}\hat{x}^2})\varphi \mathrm{d}\hat{x} \\ K_3 &= -\int_0^1 (N_3' \frac{\mathrm{d}^2\varphi}{\mathrm{d}\hat{x}^2})\varphi \mathrm{d}\hat{x} \\ F_V &= 2\alpha_2 V_p |V_{ac}| \int_0^1 (\frac{1}{(1-\hat{w}_s)^2} + \frac{\beta g}{b}\frac{1}{(1-\hat{w}_s)})\varphi \mathrm{d}\hat{x} \end{aligned} \tag{13}$$

where

$$N_2' = 2\alpha_1 \int_0^1 (\frac{\mathrm{d}\hat{w}_s}{\mathrm{d}\hat{x}})(\frac{\mathrm{d}\varphi}{\mathrm{d}\hat{x}}) d\hat{x}; \qquad N_3' = \alpha_1 \int_0^1 (\frac{\mathrm{d}\varphi}{\mathrm{d}\hat{x}})^2 d\hat{x} \tag{14}$$

For small AC voltage amplitude, the hardening spring effect can be ignored and hence according to Eq. (12) and Eq. (5), the natural frequency of the assumed beam can be written as

$$f = \frac{1}{2\pi T}\sqrt{\frac{K_1}{M}} \tag{15}$$

3. Results

3.1 Model validation

To validate the proposed model as well as the assumed method, the natural frequency of some beams will be compared to previously reported experimental data. We consider a clamped-clamped polysilicon micro-beam having the following geometrical and material properties: width $b = 100\mu\mathrm{m}$, thickness $h = 1.5\mu\mathrm{m}$ initial gap $g = 1.18\mu\mathrm{m}$, residual axial load $N = 0.0009\,\mathrm{N}$, Young's modulus $E = 158\,\mathrm{GPa}$, Poisson's ratio $\nu = 0.23$ and length scale parameters $l_0 = l_1 = l_2 = Cl = 90\mathrm{nm}$, respectively, which are identified based on the strain gradient theory using different sets of reported experimental data for static pull in voltage [26].

Table 1 compares the resonant frequencies with the experimental data for four microbeams having different lengths. As seen from the table, natural frequencies of statically deflected micro-beams based on the Strain Gradient Theory (SGT) are in excellent agreement with the experimental results.

Table 1. Comparison of the resonant frequency of micro-beams with experiment data.

Case	Beam length (μm)	DC bias (V)	Experiment ([27])	Present wok (SGT)
1	$L_1 = 210$	6	322.045	323.590
2	$L_2 = 310$	3	163.215	162.813
3	$L_3 = 410$	3	102.169	102.245
4	$L_4 = 510$	2	73.791	73.809

The normalized resonant frequencies versus the applied DC voltage of the above cited micro-beams in Table 1 are plotted in Fig.2. The results of present study were plotted along with the experimental data in the same figure. The figure shows that the predicted resonant frequencies based on the strain gradient theory are in an excellent agreement with the reported experimental data.

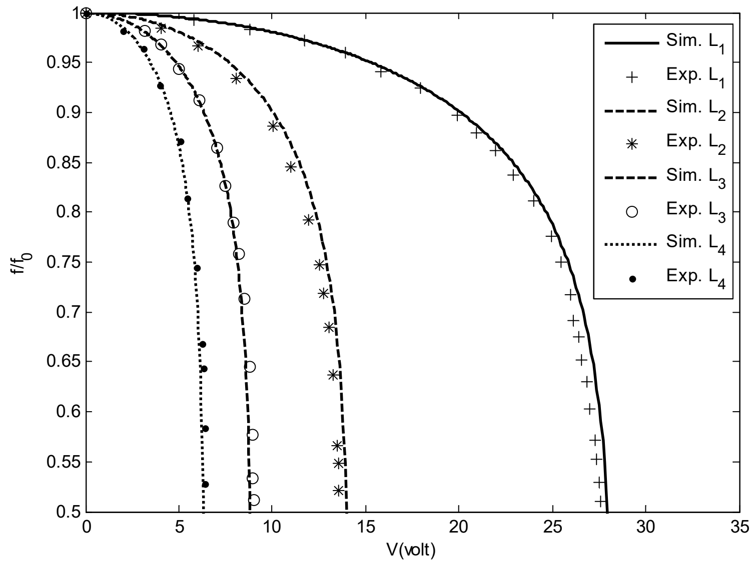

Fig.2. Comparison of analytical and experimental results of normalized frequency of micro-beams of Table 1 as a function of the applied DC voltage.

3.2 Effect of size and DC voltage

To demonstrate the size effect as well as the applied DC voltage on the natural frequency of the assumed micro- and nano-resonators, the thickness of the considered beam is reduced while keeping unchanged the beam dimensional ratio ($L/h, b/h, g/h$). Furthermore, for a beam with thickness, h, the applied polarization DC voltage is varied such as $0 < V_p < (1.4h/Cl)$ allowing the voltage to thickness ratio (V_p/h) unchanged for all assumed beam thicknesses. Fig.3. show the effects of beam thickness and DC voltage on the normalized natural frequencies of the examined nano- and micro-resonators. For an applied voltage, V_P, f_0 is the frequency of resonator based on classical model while ignoring the size effect.

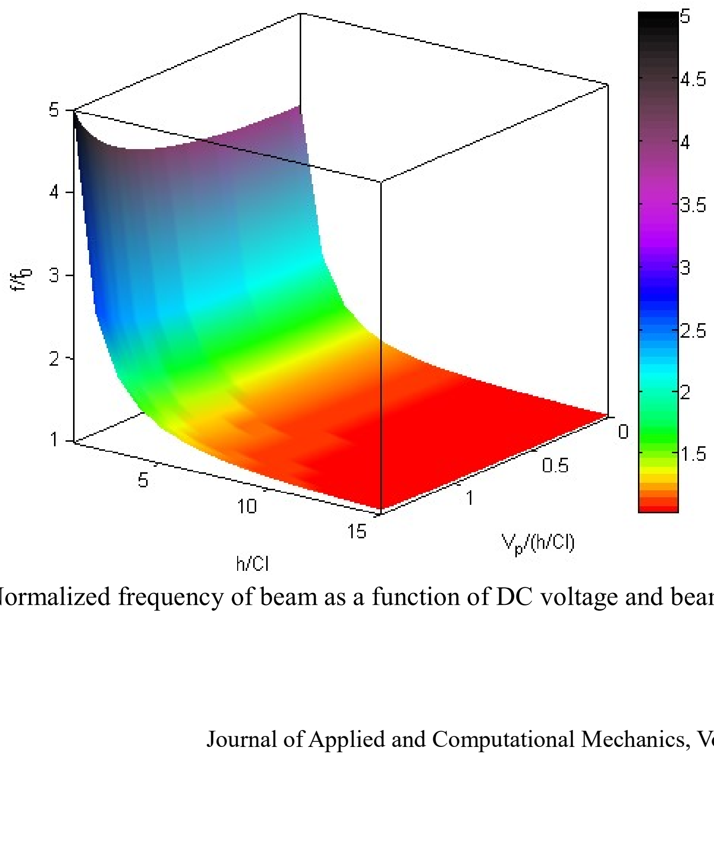

Fig.3. Normalized frequency of beam as a function of DC voltage and beam thickness.

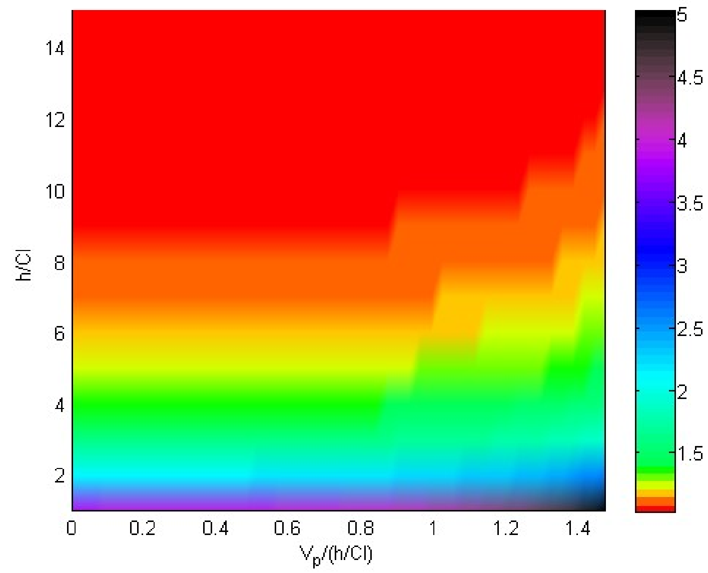

Fig.4. Contour plot of normalized frequency of beam as a function of DC voltage and beam thickness.

As shown from Fig.3 and Fig.4, decreasing the beam thickness results in higher natural frequencies in comparison with the classical continuum theory. Moreover, we can clearly see from the same figure that increasing the applied DC voltage decreases the normalized frequency. The effect of size on micro-beam frequency is the same for different applied DC voltages.

3.3 Effect of axial stress

Effect of axial stress on the natural frequencies of nano- and micro- resonators is investigated in Fig.5, where f_0 is the beam frequency corresponding to axial stress $\sigma_0 = 6\text{MPa}$. The figure indicates that sensitivity of micro- and nano-resonators to axial stress decreases when decreasing the beam thickness.

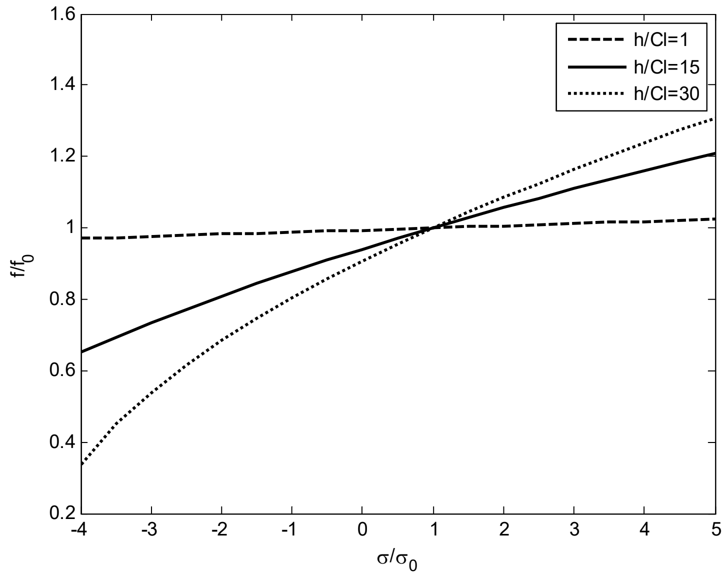

Fig.5. Normalized frequency of beam as a function the axial stress.

3.4 Effect of hardening spring

When increasing the AC voltage amplitude, the spring stiffening effect appears due to nonlinear terms in Eq. (12), thus the quadratic and the cubic nonlinear terms can't be dropped to find the beam resonant frequency. The resonant frequency of nonlinear beam model based on Lindstedt-Poincare method [28, 29] can be expressed as:.

$$\omega = \omega_0 (1 + \frac{9\omega_0 \omega_3 - 10\omega_2^2}{24\omega_0^4} A_{ac}) \tag{16}$$

where

$$\omega_0 = \sqrt{\frac{K_1}{M}}; \qquad \omega_2 = \sqrt{\frac{K_2}{M}}; \qquad \omega_3 = \sqrt{\frac{K_3}{M}} \tag{17}$$

and A_{ac} is the maximum deflection of beam due to applied AC voltage and can be obtained as

$$A_{ac} = \frac{F_V Q}{\omega_0^2} \tag{18}$$

where, Q is the quality factor. By substituting Eq. (18) into Eq. (16), the normalized resonant frequency of the investigated micro and nano-beams can be written as

$$\frac{\omega}{\omega_0} = (1 + \frac{9\omega_0 \omega_3 - 10\omega_2^2}{24\omega_0^6} F_V Q) \tag{19}$$

The value of Q are determined in [27] for two different micro-beam lengths $L = 210\mu\text{m}$ and $L = 310\mu\text{m}$ as $Q = 592$ and $Q = 151$, respectively. These values are found by fitting the frequency response predicted curve to experimental results. Because of the sealing pressure variations of the encapsulated micro-beams, this method leads to wrong measurement of the quality factor. Younis et al., [10], determined the value of quality factor by fitting the predicted resonant frequency curve versus the applied AC voltage as $Q = 816.6$ and $Q = 197$ for $L = 210\mu\text{m}$ and $L = 310\mu\text{m}$ beams length. They used perturbation method to solve the nonlinear governing equation. Here, we identify the value of Q based on the strain gradient beam model. Using Eq. (19) for normalized resonant frequency, it is found that by setting $Q = 780$ and $Q = 187$, the calculated results are in a good agreement with experimental ones, as depicted in Fig.6.

It is worth mentioning that neglecting the hardening effect of the spring for high values of harmonic actuation leads to wrong estimation of resonant frequency as shown in Fig.6.

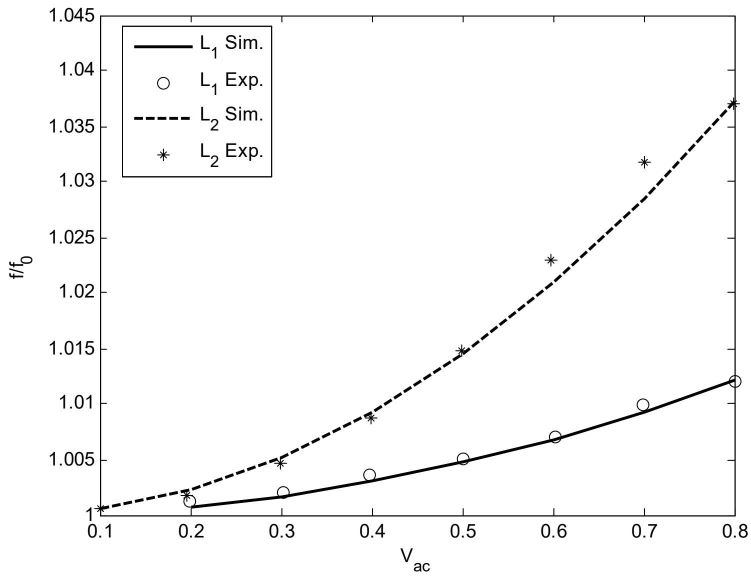

Fig.6. Comparison of analytical and experimental results for normalized frequency of beam as a function of AC actuation voltage.

4. Conclusion

In this study, using the strain gradient theory along with the Galerkin decomposition method, a model to capture the dynamic behavior of nano- and micro beams around its statically deflected shape was presented. This model assumes both quadratic and cubic nonlinearities as consequences of both the nonlinear electrostatic force and the geometric mid-plane stretching effect respectively. Comparison of the calculated micro-beam natural frequencies with experimental data showed excellent agreement and hence confirming the adopted strain gradient theory based model. The results showed that increasing the applied DC voltage leads to lower natural frequencies, hence a softening behavior. Moreover, reducing the beam thickness leads to higher natural frequencies in comparison with classical continuum theory, hence leading to a hardening behavior. However; frequency- thickness curvatures are almost same for wide range of applied DC voltages.

The results presented in this paper also revealed that the sensitivity of the beam frequency to axial stress decreases when decreasing the beam thickness. However, for the beams with thickness in the order of length scale, variation of axial stress has a little effect on the beam dynamics.

Finally, the effect of AC actuation voltage on the micro-beam frequency was investigated using perturbation method and results showed spring hardening effects that were compared then to experimental observations.

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