

# A Method for Determination of the Fundamental Period of Layered Soil Profiles

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**Abstract.** In this study, a method is proposed to determine the fundamental period of layered soil profiles. A model considering the layered soil as shear type structure is used. At first, the soil profile is divided into substructures. Then, the stiffness matrices of the substructures considered as the equivalent shear structures are assembled according to the Finite Element Method. Thereinafter, the stiffness matrices of the substructures are transformed into the Modified Finite Element Transfer Matrices, which take part in the literature. Finally, the system matrix is assembled using matrices of the substructures. The proposed method provides reduction in the size of the matrix. Therefore, analysis time is remarkably reduced. At the end of the study, the accuracy of the method is presented by the examples. Consequently, the proposed method offers a practical method for determination of the fundamental period of the soil.

**Keywords:** Finite element; Soil profile; Fundamental period; Transformation; Transfer matrix.

## 1. Introduction

For the analysis of the earthquake induced structures, it is important to know the dynamic properties of soil. Therefore, determination of the fundamental period of layered soil profiles is important. Determination of the fundamental period of soil profiles has been the subject of numerous studies [1- 10]. In this study, a practical method is proposed for the determination of the fundamental period of layered soil profiles. Modified Finite Element Transfer Matrix Method, which takes part in the literature [11, 12, 13] is used for the proposed method in the study. The assumptions made for the study are that the soil behaves linear elastic and it is modeled as a shear type structure.

## 2. The Method

One of the simplified methods for modeling the layered soil is idealization it as a shear type structure. Accordingly, the soil profile can be represented as a shear beam of distributed mass (Fig. 1). In this study, the Modified Finite Element- Transfer Matrix Method [11-12-13] proposed for the determination of the fundamental period of the soil profile. According to the model of shear beam of distributed mass, for the layer  $i$  equation of motion can be written as below

$$G_i \frac{\partial^2 y_i}{\partial x_i^2} - \rho_i \frac{\partial^2 y_i}{\partial t^2} = 0 \quad (1)$$



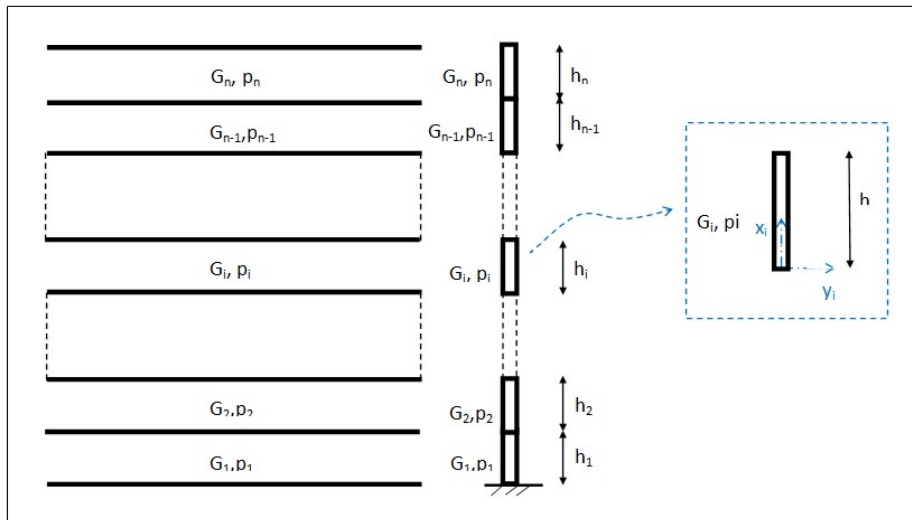


Fig. 1. Soil profile and the equivalent model.

In Equation (1) and in Fig. 1,  $G_i$  denotes the shear modulus,  $\rho_i$  is the density of layer  $i$ ,  $y_i$  is the lateral displacement function and  $x_i$  is the vertical axis of layer  $i$ . Depending on the shear wave velocity ( $v_s$ ), eq. (1) can be written as below

$$\frac{\partial^2 y_i}{\partial x_i^2} - \frac{1}{V_{si}^2} \frac{\partial^2 y_i}{\partial t^2} = 0 \tag{2}$$

In this equation  $V_{si}$  which is the shear wave velocity of layer  $i$ , is obtained by eq. (3).

$$V_{si} = \sqrt{\frac{G_i}{\rho_i}} \tag{3}$$

In the geotechnical earthquake engineering, as an important feature in determining the dynamic behavior of the grounds the shear wave velocity and the change of the shear wave velocity by depth, can be measured by the field seismic test methods applied in the Cross-Hele and Down- Hele methods in general [14]. Equation (4) is written according to the Separation of Variables Method to solve eq. (2). Here,  $u_i$  is the mode shape function and  $r(t)$  is the time function

$$y(x_i, t) = u_i(x_i)r(t) \tag{4}$$

Thus, eq. (5) is obtained using eq. (4).

$$\frac{d^2 u_i}{dx_i^2} + \frac{\omega^2}{v_{si}^2} u_i = 0 \tag{5}$$

Mode shape function is obtained as in eq. (6) by solving eq. (5).

$$u_i(x_i) = c_1 \cos(a_i x_i) + c_2 \sin(a_i x_i) \tag{6}$$

Here,  $a_i$  is obtained by eq. (7) where  $\omega$  denotes the angular frequency.

$$a_i = \frac{\omega}{v_{si}} \tag{7}$$

For the initial part of the point;

$$x_i = 0, u_i(0) = c_1 \tag{8}$$

As for the end part of the point;

$$x_i = h_i \quad u_i(h_i) = c_1 \cos(a_i h_i) + c_2 \sin(a_i h_i) \tag{9}$$

Where  $h_i$  is the thickness of layer  $i$ , eq. (8) and eq. (9) can be expressed as in the matrix Equation (10)

$$\begin{bmatrix} u_i(0) \\ u_i(h_i) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \cos(a_i h_i) & \sin(a_i h_i) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{10}$$

For the shear stress ( $\tau_i$ ) of layer  $i$ .

$$\tau_i = G_i \frac{du_i}{dx_i} \tag{11}$$

is written as:

$$\tau_i(x_i) = G [-c_1 a_i \sin(a_i x_i) + c_2 a_i \cos(a_i x_i)] \tag{12}$$

Shear stresses are as below for the initial part where  $x=0$  and the end part where  $x = h_i$ .

$$\tau_i(0) = G_i c_2 a_i \tag{13}$$

$$\tau_i(h_i) = -G_i c_1 a_i \sin(a_i h_i) + G_i c_2 a_i \cos(a_i h_i) \tag{14}$$

Equation (13) and eq. (14) are expressed as a matrix equation in eq. (15).

$$\begin{bmatrix} \tau_i(0) \\ \tau_i(h_i) \end{bmatrix} = \begin{bmatrix} 0 & G_i a_i \\ -G_i a_i \sin(a_i h_i) & G_i a_i \cos(a_i h_i) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \tag{15}$$

Equation (16) is obtained by writing the vector which consists of integration constants in eq. (10), into eq. (15).

$$\begin{bmatrix} \tau_i(0) \\ \tau_i(h_i) \end{bmatrix} = \begin{bmatrix} G_i a_i \cot(a_i h_i) & -G_i a_i \operatorname{cosec}(a_i h_i) \\ -G_i a_i \operatorname{cosec}(a_i h_i) & G_i a_i \cot(a_i h_i) \end{bmatrix} \begin{bmatrix} u(0) \\ u(h_i) \end{bmatrix} \tag{16}$$

Equation below can be written for the  $i^{th}$  part.

$$\begin{bmatrix} G_i a_i \cot(a_i h_i) & -G_i a_i \operatorname{cosec}(a_i h_i) \\ -G_i a_i \operatorname{cosec}(a_i h_i) & G_i a_i \cot(a_i h_i) \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} = \begin{bmatrix} \tau_{i-1} \\ \tau_i \end{bmatrix} \tag{17}$$

$\tau_{i-1}$  and  $\tau_i$  are written as below using the transformation which is taken from literature [12]

$$\tau_i = T_i u_i \tag{18}$$

$$\tau_{i-1} = -T_{i-1} u_{i-1} \tag{19}$$

Equation (20) can be written according to the eq. (17).

$$\begin{bmatrix} G_i a_i \cot(a_i h_i) & -G_i a_i \operatorname{cosec}(a_i h_i) \\ -G_i a_i \operatorname{cosec}(a_i h_i) & G_i a_i \cot(a_i h_i) \end{bmatrix} \begin{bmatrix} u_{i-1} \\ u_i \end{bmatrix} = \begin{bmatrix} -T_{i-1} u_{i-1} \\ T_i u_i \end{bmatrix} \tag{20}$$

From the first row of eq. (20)

$$G_i a_i \cot(a_i h_i) u_{i-1} - G_i a_i \operatorname{cosec}(a_i h_i) u_i = -T_{i-1} u_{i-1} \tag{21}$$

Equation (21) is obtained.  $u_{i-1}$  in eq. (21) is

$$u_{i-1} = \frac{G_i a_i \operatorname{cosec}(a_i h_i)}{[T_{i-1} + G_i a_i \cot(a_i h_i)]} u_i \tag{22}$$

Equation (23) is obtained from the second row of the matrix in eq. (20).

$$-G_i a_i \operatorname{cosec}(a_i h_i) u_{i-1} + G_i a_i \cot(a_i h_i) u_i = T_i u_i \tag{23}$$

If  $u_{i-1}$  in eq. (23) is replaced by  $u_{i-1}$  in eq. (22), eq. (24) is obtained.

$$\frac{-G_i^2 a_i^2 \operatorname{cosec}^2(a_i h_i)}{[T_{i-1} + G_i a_i \cot(a_i h_i)]} u_i + G_i a_i \cot(a_i h_i) u_i = T_i u_i \tag{24}$$

If necessary arrangements are made

$$T_i = \frac{-G_i^2 a_i^2 + T_{i-1} G_i a_i \cot(a_i h_i)}{[T_{i-1} + G_i a_i \cot(a_i h_i)]} \tag{25}$$

Equation (26) is obtained. As for the initial part,  $u_{i-1} = 0$  in eq. (20)

$$G_1 a_1 \cot(a_1 h_1) u_1 = T_1 u_1 \tag{26}$$

Equation (27) is obtained. Thereby

$$T_1 = G_1 a_1 \cot(a_1 h_1) \tag{27}$$

is obtained. For the top point shear stress is zero. Therefore, the values of  $\omega$  satisfying eq. (29) are the angular frequency values.

$$T_n = 0 \tag{28}$$

The procedure of the method is expressed with the flow chart given in Fig. 2.

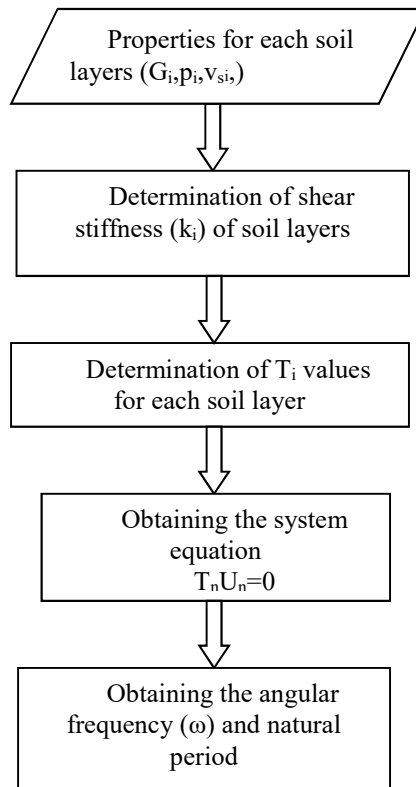


Fig. 2. Flow chart of the method

As presented in Fig. 3, in Classical Finite Element Method, the dimension of system matrix is equal to the number of layers, while in Modified Finite Element Transfer Matrix Method the dimension of system matrix is scalar and is independent of the number of layers.

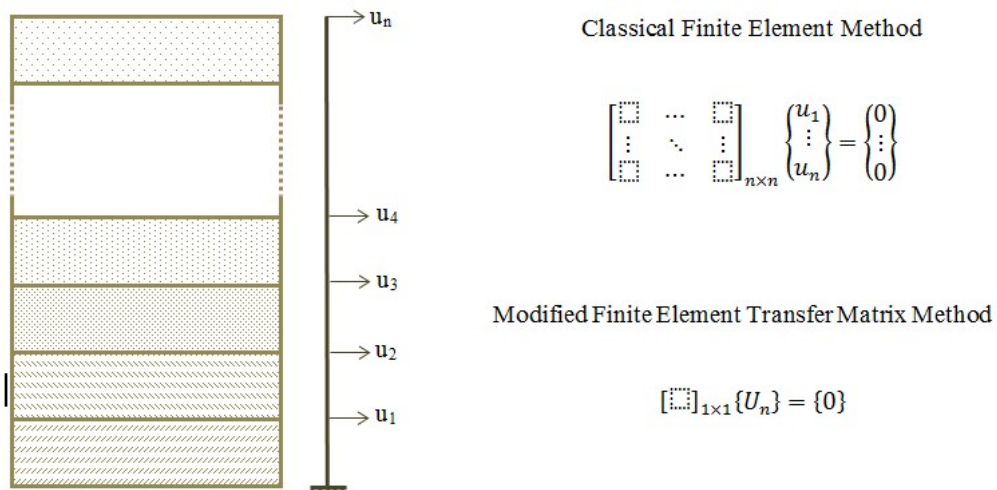


Fig. 3. Comparison of system matrices of the soil layers by the methods

### 3. Examples

In this section, two examples taken from the literature are solved to demonstrate the suitability of the method and the results are compared.

#### 3.1 Example 1

The profile of the soil in Fig. 4 is taken from the literature [10] and the characteristic values used are given in Table 1. The method proposed in the study is used to solve example.

$v_s=175.26$ m/sec,	$\rho=2.5$ t/m <sup>3</sup>	2.5 m
$v_s=133.50$ m/sec,	$\rho=1.93$ t/m <sup>3</sup>	5.52 m
$v_s=178.00$ m/sec,	$\rho=1.93$ t/m <sup>3</sup>	4.69m
$v_s=178.00$ m/sec,	$\rho=1.87$ t/m <sup>3</sup>	5.61 m
$v_s=207.26$ m/sec,	$\rho=1.86$ t/m <sup>3</sup>	3.05m
$v_s=164.59$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	7.47m
$v_s=317.30$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	4.72m
$v_s=267.31$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	11.06m
$v_s=267.31$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	19.11m
$v_s=267.31$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	11.28m
$v_s=385.88$ m/sec,	$\rho=1.88$ t/m <sup>3</sup>	8.23m
$v_s=385.88$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	4.79m

Fig. 4. Soil profile

Table 1. Details of soil profile (Example 1)

Layer Thickness (m)	Shear wave velocity (m/sec)	Soil density (t/m <sup>3</sup> )
2.5	175.26	1.93
5.52	133.50	1.93
4.69	178.00	1.93
5.61	178.00	1.77
3.05	207.26	1.77
7.47	164.59	1.77
4.72	317.30	1.83
11.06	267.31	1.83
19.11	267.31	2.06
11.28	267.31	1.85
8.23	385.88	1.85
4.79	385.88	2.01

The fundamental period of the soil is obtained by the proposed method in the study and compared with the literature in Table 2.

Table 2. Fundamental period of soil deposit (Example 1)

Presented Method (s)	Lumped Parameters Method [10]	% [(Presn.Meth.) – (Lump. Par. Meth.)] (Lump. Par. Meth.)	From observed data [10]	% [(Presn.Meth.) – (Obs. Data)] (Obs. Data)
1.26	1.28	1.56	1.25	0.8

It is seen from Table 2, the Presented Method gives accurate result as compared with the results of the literature. The difference between the results of Presented Method and the observed data given in the literature [10] is %0.8. Also the difference between the results of Presented Method and the Lumped Parameters Method in literature is %1.56.

In the Lumped Parameters Method, the dimension of system matrix is equal to the number of layers, while in Modified Finite Element Transfer Matrix Method the dimension of system matrix is scalar and is independent of the number of layers.

#### 3.2 Example 2

Layer profile characteristics of soil in Fig.5 are shown in Table 3. This example is investigated by the proposed method to obtain the fundamental period and the result is given in Table 4.

**Table 3.** Details of soil profile (Example 2)

Layer Thickness (m)	Shear wave velocity (m/sec)	Soil density (t/m <sup>3</sup> )
8	252.37	1.92
5	221.28	1.92
7	316.69	1.92
8	251.46	1.92
5	289.86	1.92
65	387.10	1.92
24	324.61	1.92
16	367.28	1.92
9	326.44	1.92
7	497.74	1.92
21	372.77	1.92
25	846.43	1.92

This example is investigated by the proposed method to obtain the fundamental period and the result is given in Table 4.

$v_s=828$ m/sec,	$\rho=120$ t/m <sup>3</sup>	8m
$v_s=726$ m/sec,	$\rho=120$ t/m <sup>3</sup>	5m
$v_s=1039$ m/sec,	$\rho=120$ t/m <sup>3</sup>	7m
$v_s=825$ m/sec,	$\rho=120$ t/m <sup>3</sup>	8m
$v_s=931$ m/sec,	$\rho=120$ t/m <sup>3</sup>	5m
$v_s=164.59$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	65m
$v_s=267.31$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	24m
$v_s=267.31$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	16m
$v_s=317.30$ m/sec,	$\rho=1.83$ t/m <sup>3</sup>	9m
$v_s=267.31$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	7m
$v_s=385.88$ m/sec,	$\rho=1.88$ t/m <sup>3</sup>	8.23m
$v_s=385.88$ m/sec,	$\rho=1.89$ t/m <sup>3</sup>	4.79m

**Fig. 5.** Soil profile of Example 2.

The proposed method gives same result with the literature [6] and it is seen from Table 4, the difference between the results is equal to zero.

**Table 4.** Fundamental period of soil deposit (Example 2)

Presented Method (s)	Hadjian [6]	Difference (%)
0.60	0.60	0

#### 4. Conclusion

In this paper, a method for determination of the fundamental period of soil profiles was proposed. Layered soil profile was modeled by using the shear type structure behavior in the study. For each layer of soil, transfer matrix was obtained by applying the transformation used in the literature into the finite element matrix. By using Modified Finite Element Transfer Matrix Method, matrix dimension was reduced to a scalar. Hence, a recurrence relation was obtained for the solution. As the advantage of the Presented Method, the dimension of system matrix was scalar and was independent of the number of layers. At the end of the study the method was tested by two examples taken from the literature. Comparison of the results of the literature and the method, presented the convergence of the proposed method. Consequently, the fundamental periods of the soil layers can be easily obtained by the method proposed in this study.

#### Nomenclature

$G_i$	Shear modulus of layer $i$	$\rho_i$	Density of layer $i$
$u_i$	Mode shape function	$v_{si}$	Shear wave velocity of layer $i$
$\omega$	Angular frequency	$h_i$	Thickness of layer $i$
$\tau_i$	shear stress of layer $i$		

#### References

[1] Dobry, R.I., Oweis, I., Urzua, A. Simplified procedures for estimating the fundamental period of a soil profile, *Bulletin of the Journal of Applied and Computational Mechanics*, Vol. 3, No. 4, (2017), 267-273

*Seismological Society of America*, 66, 1976, pp. 1293-1321.

[2] Gazetas, G. Vibrational characteristics of soil deposits with variable wave velocity, *International Journal for Numerical and Analytical Methods in Geomechanics*, 6, 1982, pp. 1-20.

[3] Medina, F. Modeling of layered soil-structure interaction by infinite elements, *Earthquake Engineering, Tenth World Conference*, Balkema, Rotterdam, 1992.

[4] Singh, Y., Nagpal, A.K. Estimating fundamental period of soil profiles, *Geotechnical Engineering*, 24(2), 1977, pp. 167-174.

[5] Sarma, S.K. Analytical solution to the seismic response of visco-elastic soil layers, *Géotechnique*, 44(2), 1993, pp. 265-275.

[6] Hadjian, A.H. Fundamental period and mode shape of layered soil profiles, *Soil Dynamics and Earthquake Engineering*, 22, 2002, pp. 885-891.

[7] Sawada, S. A simplified equation to approximate natural period of layered ground on the elastic bedrock for seismic design of structures, *13th World Conference on Earthquake Engineering Vancouver*, B.C., Canada, 2004.

[8] Deb, K., Dey, A., Chandra, S. Modeling of layered soil system, *1st Indian Young Geotechnical Engineers Conference*, Hyderabad, India, 2-3rd March, 50-55, 2007.

[9] Ruiz, S., Saragoni, S.R. Free vibration of soils during large earthquakes, *Soil Dynamics and Earthquake Engineering*, 29, 2009, pp. 1-16.

[10] Vijayendra, K.V., Prasad, S.K., Nayak, S. Computation of Fundamental Period of Soil Deposit: A Comparative Study, Oxford, UK, second edition, *Indian Geotechnical Conference*, December 16-18, 2010.

[11] Choi, M.S. Free Vibration Analysis of Plate Structures Using Finite Element-Transfer Stiffness Coefficient Method, *Journal of Mechanical Science and Technology*, 17(6), 2003, pp. 805-815.

[12] Rong, B., Rui, X.T., Wang, G.P. Modified Finite Element Transfer Matrix Method for Eigenvalue Problem of Flexible Structures, *Journal of Applied Mechanics*, 78(2), 2011, 021016.

[13] Ozturk, D., Bozdogan, K., Nuhoglu, A. Modified finite element-transfer matrix method for the static analysis of structures, *Structural Engineering and Mechanics*, 43(6), 2012, pp. 761-769.

[14] Iyisan, R., Hatipoglu, M., Ozudogru, T.Y. Determination of shear wave velocity by suspension ps logging method, *16th National Congress of Soil Mechanics and Geotechnical Engineering*, Erzurum, Turkey, 2016.