Generalized 2-Unknown’s HSDT to Study Isotropic and Orthotropic Composite Plates

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Abstract. The present study introduces a generalized 2-unknown’s higher order shear deformation theory (HSDT) for isotropic and orthotropic plates. The well-known Shimpi’s two-unknown's HSDT is reproduced as a special case. Reddy’s shear strain shape function (SSSF) is also adapted to the present generalized theory. The results show that both Shimpi and the adapted Reddy’ HSDT are essentially the same, i.e., both present the same static results. This is due to the fact that both theories use polynomial SSSFs. This study presents a new optimized cotangential SSSF. The generalized governing equation obtained from the principle of virtual displacement is solved via the Navier closed-form solution. Results show that transverse shear stresses can be improved substantially when non-polynomial SSSFs are utilized. Finally, this theory is attractive and has the potential to study other mechanical problems such as bending in nanoplates due to its reduced number of unknown's variables.

Keywords: Layered structures; Plates; Elasticity; Analytical modeling.

1. Introduction

Natural and artificial composite materials are present everywhere. Industrial classical composite structures such as the single skin and sandwich-structured composites are common in many industries. Composite materials are attractive and demanding nowadays due to their increased reliability, fatigue resistance, and more importantly their high performance applications (weight reduction), and thereby such materials are increasing the speed of vehicles with more efficient power plants (lower fuel consumption). Moreover, the launch of the first commercial airplane (see Boing 787 and Airbus A350) along with the representative amount of composite structures usage are speeding up the uses of composite materials in other industries. Composite structures are kinds of multilayered structures that exhibit a different mechanical behavior of metals. Therefore, the need for a clear understanding of the mechanics of the material through experimental and numerical studies is of vital importance.

From the analytical point of view, several plate theories were developed. For example the Classical Plate Theory and First order Shear Deformation Theory (CPT, FSDT) are relevant ones. Some disadvantages of them are as follows: the first one has limitation to estimate transverse shear stresses; the second one is not capable to correctly model transverse shear stresses. Fortunately, the HSDT can overcome such limitations and improve the results from moderate thick to very thick plates. Nowadays, there exist several HSDTs developed on the top of two variational statements: Principle of virtual displacement (PVD) and Reissner mixed variational theorem (RMVT). However, very few theories that consider limited number of
unknown variables exist, i.e. computational effective.

In this sense, a remarkable study on isotropic and layered structures (plates) by Shimpi [1] and Shimpi and Patel [2, 3] is introduced. Based on this interesting work, Mechab et al. [4] studied the static behavior of advanced composites by using 4-unknown plate theory. Then, Abdelaziz et al. [5] studied the static analysis of advanced composite sandwich plates with the same plate theory. Houari et al. [6] and Hamidi [7] analyzed the thermoelastic behavior of advanced composite plates by the 4-unknown plate theory. Recently, Mechab et al. [8] studied the static and dynamic analysis of advanced composite plates with a hyperbolic shear strain shape function (SSSF).

Thailand and Kim [9] developed a 5-unknown trigonometric plate theory (TPT) with thickness stretching effect having a good accuracy regarding its counterpart, the TPT, with 6-unknown. Based on this work and previous experience on non-polynomial HSDTs, Mantari and Guedes Soares [10-12] developed an optimized TPT with an stretching effect (5 and 6-unknown) having improved results compared with the 5 and 6-unknown quasi-3D trigonometric plate theories (TPT) [9][13]. Recently, Zenkour [14] developed an interesting HSDT with 4-unknown and a thickness stretching effect. Based on non-polynomial HSDT, Mantari and Guedes Soares [15-17] developed an optimized 4-unknown’s quasi-3D HSDT having improved results. The elastic foundation of FGPs modeled as a two-parameter Pasternak foundation on the bases of a 4-unknown shear deformation theory was introduced by Thai and Choi [44]. Overall, several new and optimized shear strain shape functions adopted by shear deformation theories did not include the HSDT of two-unknown’s variables. The present paper attempts to fill this gap.

Applications of the HSĐT can be used to solve mechanical problems in nonlinearities of material or/and geometry. For example, readers may found nonlinear thermo-stability of classical and advanced composites (Refs. [45, 46]) an extension of this work to the nonlinear thermal stability of the composite plate with embedded and through-the-width delamination under a uniform temperature rise (Ref. [47]). Moreover, the nonlinear thermal instability of moving shape memory alloy (SMA) sandwich plates subjected to a constant moving speed was investigated by using the HSĐT in Ref. [48]. Further studies on deformation theories and nonlinearities can be found in Refs. [49, 50].

This study presents a generalized two-unknown HSĐT in which polynomial and non-polynomial SSSFs can be arbitrarily introduced, and, if desired, the theory can be optimized for further improvement of the bending mechanical problems. Moreover, an unavailable optimized cotangential SSSF within the generalized theory is introduced. Results show that the transverse shear stresses can be improved substantially with the optimization. The present theory is excellent for isotropic plates and some types of orthotropic materials. Additionally, it has the potential to study nanosheets.

2. Analytical modelling

The displacement field, satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point \((x, y, \pm h/2)\) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows:

\[
\begin{align*}
\bar{u} &= z \left[ y'' \frac{\partial w}{\partial x} + q'' \frac{\partial \theta}{\partial x} + f(z) \frac{\partial \theta}{\partial x} \right], \\
\bar{v} &= z \left[ y'' \frac{\partial w}{\partial y} + q'' \frac{\partial \theta}{\partial y} + f(z) \frac{\partial \theta}{\partial y} \right], \\
\bar{w} &= w + \theta,
\end{align*}
\]

where \(w(x, y)\) and \(\theta (x, y)\) are the two unknown displacement functions of the middle surface of the plate, \(f(z)\) allows to model the in-plane displacements using the plate thickness, whilst \(y''\), \(q''\), \(y'\) and \(q'' = y' + q'\) are constant values depending of the selected SSSF obtained following the same strategy presented in Reddy and Liu [18] (see Table 1 for details). By considering the kinematic and constitutive relations in the linear regimen, the principle of virtual works can be formulated as in Mantari and Guedes Soares [17].

### Table 1. SSSFs of classical and new theories.

<table>
<thead>
<tr>
<th>SSSFs</th>
<th>(f(z))</th>
<th>(m)</th>
<th>(y')</th>
<th>(q')</th>
<th>(y'')</th>
<th>(q'')</th>
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<tbody>
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<td>Polynomial Shimpi</td>
<td>(-sz''^2/3h^2)</td>
<td>NA</td>
<td>5/4</td>
<td>-1</td>
<td>-1</td>
<td>-1/4</td>
</tr>
<tr>
<td>Polynomial Reddy</td>
<td>(z^3)</td>
<td></td>
<td></td>
<td>-3/4</td>
<td>-1</td>
<td>-3/4</td>
</tr>
<tr>
<td>Non-polynomial</td>
<td>(\arctan(mz/h)^1)</td>
<td>1/4</td>
<td>-4m/(h(m^2+4))</td>
<td>-1</td>
<td>-1-4m/(h(m^2+4))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\arctan(mz/h)^1)</td>
<td>-11/2</td>
<td>-4m/(h(m^2+4))</td>
<td>-1</td>
<td>-1-4m/(h(m^2+4))</td>
<td></td>
</tr>
</tbody>
</table>

\(^1\)Obtained through optimization procedure mentioned in the manuscript (Opt1, \(m=1/4\); Opt2, \(m=-11/2\)).

2.1. The principle of virtual works

Considering the static version of the principle of virtual work, the following expressions can be obtained:

\[
0 = \int_{h/2}^{h/2} \left[ \int_{\Omega} \left[ \sigma_{xx}^{(k)} \tilde{\delta}e_{xx} + \sigma_{yy}^{(k)} \tilde{\delta}e_{yy} + \sigma_{xy}^{(k)} \tilde{\delta}e_{xy} + \sigma_{xz}^{(k)} \tilde{\delta}e_{xz} + \sigma_{yz}^{(k)} \tilde{\delta}e_{yz} \right] \, dx \, dy \right] \, dz - \int_{\Omega} q \, \tilde{\delta}w \, dx \, dy
\]

(2)
where $\varepsilon^{(k)}$ or $\sigma^{(k)}$ are the stress and the strain vectors of the $k^{th}$ layer and $q$ is the distributed transverse load. Figure 1 show the layer and laminate coordinate system.

2.2. Plate governing equations

Using the generalized kinematic and constitutive relations, applying integration of parts and collecting the coefficients of $w$ and $\theta$, the equations of motion are obtained as follows:

$$
\delta w: y'' \left( \frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + 2 \frac{\partial^2 M_3}{\partial x \partial y} \right) = q,
$$

$$
\delta \theta: q'' \left( \frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + 2 \frac{\partial^2 M_3}{\partial x \partial y} + \frac{\partial^2 P_1}{\partial x^2} + \frac{\partial^2 P_2}{\partial y^2} + 2 \frac{\partial^2 P_3}{\partial x \partial y} - q'' \left( \frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial x} \right) - \frac{\partial K_1}{\partial x} \right) = q'' q
$$

where $N_i, M_i, P_i,$ and $K_i$ are the resultants of the following integrations:

$$
(M_i, P_i) = \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma_i (1, z, f(z)) dz, \quad (i=1,2,6)
$$

$$
N_i = \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma_i dz, \quad (i=4,5)
$$

$$
K_i = \int_{-\frac{a}{2}}^{\frac{a}{2}} \sigma_i f'(z) dz, \quad (i=4,5)
$$

Details of the formulation presented above along with the methodology used to solve the simple supported advanced composites, and the optimization procedure used in this study can be found in Refs. [15-17, 52-54]. Readers are invited to consult these papers for further details.

3. Solution procedure

Exact solutions of the partial differential equations (3a-b) on the arbitrary domain and for general boundary conditions are easy to be found. General boundary conditions require solution strategies involving deeper mathematical formulations [19-20]. Results based on HSDT using such strategies are available in the literature for laminated shells [21-26]. However, in the case of simply supported plates, the above-mentioned constitutive equations can be solved in a simpler manner. The drawback of the present adopted solution is that the lamination scheme must be cross-ply $[0^\circ/90^\circ...]$ type and symmetric.

Solution functions of the partial differential Equations (3a-b) of a cross-ply plate for simply supported boundary conditions are assumed as follows:

$$
w(x,y) = \sum_{m=1}^{m=n} \sum_{n=1}^{n=m} W_{mn} \sin(\alpha x) \sin(\beta y), \quad 0 \leq x \leq a; \quad 0 \leq y \leq b
$$

$$
\theta(x,y) = \sum_{m=1}^{m=n} \sum_{n=1}^{n=m} \theta_{mn} \sin(\alpha x) \sin(\beta y), \quad 0 \leq x \leq a; \quad 0 \leq y \leq b
$$

where

$$
\alpha = \frac{m \pi}{a}, \quad \beta = \frac{n \pi}{b}
$$

while $m=n=1$ and $m=n=101$ are the terms of harmonic series for sinusoidal and uniform distributed load, respectively. Substituting Eqs. (5a-b) into Eqs. (4a-b), the following equations are obtained:
Elements of $K_j$ in Eq. (7) can be derived as follows (Refs. [15-17]):

$$\{d_j\}^T = \{W_{mn}\ \theta_{mn}\} ,$$

$$\{F_j\}^T = \{Q_{mn} - q*Q_{mn}\} ,$$  \hspace{1cm} (8a-b)

where $Q_{mn}$ is the coefficient in the double Fourier expansion of the transverse load,

$$q(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin(\alpha x) \sin(\beta y)$$  \hspace{1cm} (9)

4. Numerical Results and Discussion

The bending analyses of isotropic and orthotropic plates are presented in this section. Shimpi’s HSDT and a HSDT containing strong similarities with the one developed by Reddy and Liu [18] are also introduced in the context of this generalized formulation. In addition, a non-polynomial SSSF is utilized and optimized following the recommendation in Ref. [15] and implemented in the present generalized formulation (see Fig. 2). Three different case problems are presented. Figure 2 shows that it is possible to achieve excellent solutions for transverse shear stresses without sacrificing much accuracy in deflection and normal stresses by selecting the proper argument of the shear strain shape function. This is something that should be further studied. Results show that the generalized theory with cotangential SSSF performs well for isotropic and orthotropic materials, but for some orthotropic materials they are just acceptable. The reason behind that fact can be difficult to understand in one glance, but it may be related to the reduced number of unknown’s variables and the strong dependency on the shear strain shape function in order to model six strains and six stresses. This could be really difficult to handle just for one function while a couple of them could make a remarkable difference. Consequently, the case dependent problem may be generalized. Overall, the transverse shear stress results can be substantially improved after running optimization procedures.

4.1. Case Problem 1: Isotropic plate

A moderate thick to thick isotropic plate is considered in this section (E=1, $v=0.3$). It is worth mentioning that the present generalized theory contains strong similarities with the CPT; therefore further comparisons with the CPT and the FSDT (Reissner) are performed. Since the theory is capable of reproducing Shimpi’s HSDT [2] results by using the SSSF and related constant parameters shown in Table 1, further comparisons are performed between the following parameters: the present optimized cotangential HSDT (see Fig. 2); the HSDT developed by Shimpi and Patel [2]; the adapted Reddy HSDT with two-unknown variables introduced in this study (see Table 1, where the non-polynomial function with two optimization parameters #1 and #2 is also introduced); the CPT, the FSDT, and the exact elasticity solution [27]. Table 2 indicates that the present HSDT with the selected parameter corresponding to the optimization #2 (Opt2) shows the best accuracy and a good agreement
with the other theories.

### Table 2. Normalized central deflection of simply-supported isotropic rectangular plate subjected to uniformly distributed transverse load.

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%Average error. 0.2, 6.0, 0.8, 0.8, 0.8, 0.3

### 4.2. Case Problem 2: Orthotropic plate

A simply supported orthotropic rectangular plate subjected to the uniform distributed load is considered. Several thickness ratios a/h, and aspect ratios b/a are considered. The orthotropic properties are given in Eq. (10). For this case problem, the 3D elasticity solution was introduced by Srinivas and Rao [28].

\[
Q = \begin{bmatrix}
0.999781 & 0.231192 & 0 & 0 & 0 \\
0.231192 & 0.524886 & 0 & 0 & 0 \\
0 & 0 & 0.262931 & 0 & 0 \\
0 & 0 & 0 & 0.266810 & 0 \\
0 & 0 & 0 & 0 & 0.159914
\end{bmatrix}
\]  

(10)

The present results are compared with the following theories: Srinivas and Rao [28]; the HSDDT developed by Shimpi and Patel [2]; the original theory by Reddy [29]; the adapted Reddy HSDDT with two-unknown introduced in this study (see Table 1); the CPT, and the FSDT. The normalized quantities used in Tables 3-5 are defined in Eqs. (11a-c).

\[
\bar{w} = w \left( \frac{a}{2}, \frac{a}{2}, 0 \right) \frac{Q_{11}}{hq}, \quad \bar{\sigma}_{xx} = \sigma_{xx} \left( \frac{a}{2}, \frac{a}{2}, \frac{h}{2} \right) \frac{1}{q}, \quad \bar{\tau}_{xz} = \tau_{xz} \left( 0, \frac{b}{2}, 0 \right) \frac{1}{q}.
\]  

(11a-c)

No substantial difference between the theories can be found in Tables 3 and 4. The SSSFs and their parameter of optimization, m (see Table 1), yield basically the same results. However, in the case of a higher modular ratio (top or bottom/middle ratio of mechanical properties) as in Table 5, the transverse shear stresses are substantially different and the second parameter of optimization, \(m = -11/2\), appears to be the best choice (see Fig. 2). Consequently, it can be concluded that transverse shear stresses results can be improved when optimized non-polynomial SSSFs are implemented in the present generalized formulation.

### Table 3. Normalized central deflection of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

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%Average error. 0.2, 0.0, 8.0, 0.8, 0.8, 0.8, 2.9
Table 4. Normalized in-plane normal stresses of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

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%Average error. 0.1 1.4 0.9 1.4 1.4 1.4 1.4 1.7

Table 5. Normalized transverse shear stresses of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

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Average error (%) 1.4 2.5 100.0 21.7 21.5 11.3 11.3

4.3. Case Problem 3: Cross-ply laminated composites

The response of square cross-ply laminated composites subjected to the bi-sinusoidal load is discussed in this section. Mechanical properties are given in the corresponding tables (Tables 6 & 7) and normalization formulas are provided in Eqs. (12a-c). The results of the present theory are compared with the refined HSDT by Carrera et al. [30] (Table 6) as well as the other analytical and numerical results in Refs. [31-33] (Table 7). It can be noticed that highly accurate results are obtained for a cross-ply 0°/90°/0° (see Table 6). A cross-ply laminated composite 0°/90°/90°/0° is also studied and the results show less accuracy than the FSDT for thick plates while they are still comparable for thin plates (Table 7). In general, the reason could be due to the fact that results for thicker laminate schemes are more difficult to capture for any kinds of the shear deformation theory.

\[
\begin{align*}
\varepsilon_{zz} &= \frac{a}{2} \frac{a}{h} \frac{1}{a^2} 
\sigma_{xx} &= \frac{a}{2} \frac{b}{2} \frac{h}{2} \frac{h}{a^2}, \\
\sigma_{yy} &= \frac{a}{2} \frac{b}{2} \frac{h}{2} \frac{h}{a^2}, \\
\tau_{xx} &= \tau_{zz}(0, \frac{b}{2}, \frac{h}{2}) \frac{a}{qa}, \quad B = \tau_{zz}(0, \frac{b}{2}, \frac{h}{2}) \frac{a}{qa}.
\end{align*}
\]

(12)

Table 6. Normalized central deflection of simply-supported cross-ply plate (0°/90°/0°) subjected to sinusoidal transverse load.

<table>
<thead>
<tr>
<th>Theory</th>
<th>Strain</th>
<th>DOFs</th>
<th>10</th>
<th>50</th>
<th>100</th>
<th>500</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present non-polynomial Opt 1</td>
<td>εzz=0</td>
<td>2</td>
<td>0.913</td>
<td>0.778</td>
<td>0.774</td>
<td>0.772</td>
<td>0.772</td>
</tr>
<tr>
<td>Present non-polynomial Opt 2</td>
<td>εzz=0</td>
<td>2</td>
<td>0.913</td>
<td>0.778</td>
<td>0.774</td>
<td>0.772</td>
<td>0.772</td>
</tr>
<tr>
<td>Present Polynomial Reddy</td>
<td>εzz=0</td>
<td>2</td>
<td>0.913</td>
<td>0.778</td>
<td>0.774</td>
<td>0.772</td>
<td>0.772</td>
</tr>
<tr>
<td>Analytical LW-4 [30]</td>
<td>εzz=0</td>
<td>2</td>
<td>0.913</td>
<td>0.778</td>
<td>0.774</td>
<td>0.772</td>
<td>0.772</td>
</tr>
<tr>
<td>MITC4 LW-4 [30]</td>
<td>εzz=0</td>
<td>2</td>
<td>0.913</td>
<td>0.778</td>
<td>0.774</td>
<td>0.772</td>
<td>0.772</td>
</tr>
</tbody>
</table>

\[\mathbf{E}_1 = 132.38 \text{ GPa}, \; \mathbf{E}_2 = 10.756 \text{ GPa}, \; \mathbf{G}_{zz} = 3.606 \text{ GPa}, \; \mathbf{G}_{12} = \mathbf{G}_{13} = 5.6537 \text{ GPa}, \; v_{12} = v_{13} = 0.24, \; v_{23} = 0.49\]
4.4. Capability and limitation of this theory

The capability to calculate the bending mechanical response of isotropic plates is demonstrated in this study. However, in case of orthotropic plates and in the case of some laminations schemes, the present theory is satisfactory. But for other lamination schemes, the results are just comparable with existing theories in the thin regimen. The authors believe that this is due to the so-called case dependent problem. However, it should be kept in mind that the present theory contains only two unknown variables much less than that of the FSDT or other HSDTs (Tables 6 & 7). It is important to remark that this theory is valid for symmetric laminated composites since the mathematical model for the displacement field presented in this study cannot capture the membrane effect existing in this type of laminate scheme. Therefore, the response of schemes such as cross-ply 0/90 cannot accurately predict the in-plane deformations and stresses.

Overall, this theory has reduced unknown variables and is highly attractive for the modeling of nanosheets or nanoplates, therefore, it deserves further research and consideration. Readers may consult the relevant and recent research work on this topic is provided in Ref. [34]. Moreover, additional information regarding shear deformation theories with reduced number of unknown variables can be found in Refs. [35-44].

5. Conclusions

An unavailable generalized two-unknown higher order shear deformation theory (HSDT) for isotropic and orthotropic plates was presented and discussed. Shimpi’s HSDT along with a similar theory to Reddy HSDT with two unknown variables were reproduced as special cases. Results showed that both Shimpi and the adapted Reddy’s HSDT are essentially the same. This is due to the fact that both theories use polynomial SSSFs. Moreover, an unavailable optimized cotangential SSSF within the generalized theory was introduced. The generalized governing equation obtained by the principle of virtual displacement was solved via the Navier closed form solution. Results showed that transverse shear stresses can be substantially improved when non-polynomial SSSFs are utilized. Overall, this theory appears to be attractive to nanosheets and nanoplates studies as recently reported in the literature, therefore, the present study deserves attention.

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Conflict of Interest

The authors declare no conflict of interest.

References


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