A New Quasi-3D Model for Functionally Graded Plates

Shantaram M. Ghumare\textsuperscript{1}, Atteshamuddin S. Sayyad\textsuperscript{2}

\begin{itemize}
  \item \textsuperscript{1} Department of Civil Engineering, SRES’s Sanjivani College of Engineering, Savitribai Phule Pune University, Kopargaon-423601, Maharashtra, India, smghumare@rediffmail.com
  \item \textsuperscript{2} Department of Civil Engineering, SRES’s Sanjivani College of Engineering, Savitribai Phule Pune University, Kopargaon-423601, Maharashtra, India, attu_sayyad@yahoo.co.in
\end{itemize}

Received August 08 2018; Revised September 29 2018;Accepted for publication October 01 2018.
Corresponding author: Atteshamuddin S. Sayyad, attu_sayyad@yahoo.co.in

© 2019 Published by Shahid Chamran University of Ahvaz and International Research Center for Mathematics & Mechanics of Complex Systems (M&MoCS)

Abstract. This article investigates the static behavior of functionally graded plate under mechanical loads by using a new quasi 3D model. The theory is designated as fifth-order shear and normal deformation theory (FOSNDT). Properties of functionally graded material are graded across the transverse direction by using the rule of mixture i.e. power-law. The effect of thickness stretching is considered to develop the present theory. In this theory, axial and transverse displacement components respectively involve fifth-order and fourth-order shape functions to evaluate shear and normal strains. The theory involves nine unknowns. Zero transverse shear stress conditions are satisfied by employing constitutive relations. Analytical solutions are obtained by implementing the double Fourier series technique. The results predicted by the FOSNDT are compared with existing results. It is pointed out that the present theory is helpful for accurate structural analysis of isotropic and functionally graded plates compared to other plate models.

Keywords: FOSNDT, FG plate, Static behavior, Shear deformation, Thickness stretching.

1. Introduction

Nowadays, functionally graded (FG) materials are being used in many advanced and important engineering structures. The material composition and volume fraction vary according to the simple rule of mixture i.e. power-law through the thickness. Wide applications of FGM in various industries forced researchers to develop accurate analytical and numerical techniques. This can be achieved by selecting a proper structural theory. Modeling of plate structures is based on either classical and refined computational models or three-dimensional elasticity theories. However, exact 3D elasticity theories for the FG plates are not found in the whole variety of literature. Therefore, researchers have hired various approximate plate theories for predicting the structural behavior of FG plates. Approximate theories reduce the 3D problem to a 2D problem. Various investigations on FG plate which are based on approximate theories are well documented in Jha et al. [1], Swaminathan et al. [2], Swaminathan and Sangeeta [3], etc.

Classical plate theory (CPT) predicts zero values for strains in the transverse direction (z-direction). Therefore, it is not suitable for thick FG plates wherein these strains are more pronounced. The first-order shear deformation theory (FSDT) considered these strain components, but shows constant variation of transverse strains in the transverse direction. These drawbacks of CPT and FSDT forced the researchers to develop refined plate theories. Several higher-order shear deformation theories (HSDTs) are developed by different scientists for predicting the structural behavior of FG plates. These theories are systematically documented by Sayyad and Ghugal [4, 5]. Reddy [6] analyzed the FG plates by his well-known polynomial type model. Reddy and Cheng [7] presented 3D asymptotic theory for FG plates. Zenkour [8] studied the behavior of FG plates under uniform load. Zhong and Shang [9] presented 3D analysis of FG plates using Plevako’s solution. Lu et al. [10] obtained natural frequencies of FG thick plates using 3D elasticity theory. Ameur et al. [11] developed a trigonometric theory containing...

Recently, few research papers have been published on the applications of analytical [32-38] and numerical methods [39-50] for the analysis of functionally graded beams, plates, and shells. However, in most of the literature, the effect of transverse normal deformation is neglected to minimize unknown variables in the displacement field.

1.1 Present Contribution

In the current contribution, FOSNDT investigated by Ghumare and Sayyad [51] is extended to examine structural behavior of the FG plates under transverse loadings. The novelty and contribution of the present theory are summarized as follows:

1) For the accurate description of the bending behavior of the thick FGM plates, shear and normal deformations play important roles. Thus, their effects are considered. Many published theories neglect the effect of transverse normal deformations. Hence, in this work, a new quasi-3D model is presented for FG plate including normal deformation along with shear deformation.

2) To account for the effects of cross-sectional warping and thickness stretching, a polynomial type shearing strain function expanded up to fifth-order is chosen. Zero transverse shear stress conditions are satisfied by using constitutive relations.

3) Since the current developed theory is a polynomial type, it is computationally simpler than non-polynomial plate theories which are mathematically more cumbersome.

4) Since 3D Hooke’s law is used to obtained stresses associated with the present theory, it accurately describes the state of stress in 3D continuum.

5) The developed theory shows improvements in results when compared to the other HSDTs found in the literature [52-57].

2. Problem Formulation

The transversely loaded FG plate which is presented in Fig. 1 is considered for the mathematical formulation and numerical study. Properties of material graded in z-direction using the power-law relation stated in Eq. (1), wherein top face is made of metal and bottom face is of purely ceramic.

\[
E(z) = E_m V_m + E_c V_c, \quad V_m = 1 - V_c \quad V_c = (0.5 + z/h)^p
\]

where subscript \(m\) stands for metal and subscript \(c\) refers to ceramic. \(E\) is the elastic modulus, \(V\) is the volume fraction, and \(P\) is the power-law coefficient/index. Fig. 2 plots the elastic modulus in z-direction.
2.1 Kinematics of the present model

The displacement field of the present theory is defined as,

\[
\begin{align*}
  u &= u_0 - z \frac{\partial w_0}{\partial x} + z \left[ 1 - \frac{4 z^2}{3 h^3} \right] \phi_x + z \left[ 1 - \frac{16 z^4}{5 h^4} \right] \psi_x \\
  v &= v_0 - z \frac{\partial w_0}{\partial y} + z \left[ 1 - \frac{4 z^2}{3 h^3} \right] \phi_y + z \left[ 1 - \frac{16 z^4}{5 h^4} \right] \psi_y \\
  w &= w_0 + \left[ 1 - \frac{4 z^2}{h^2} \right] \phi_z + \left[ 1 - \frac{16 z^4}{h^4} \right] \psi_z 
\end{align*}
\]

where \( u_0, v_0, w_0 \) are the displacements of a mid-plane. \( \phi_x, \psi_x, \phi_y, \psi_y, \phi_z, \psi_z \) are the unknown rotations. \( f_1(z) \) and \( f_2(z) \) are assumed to get the parabolic variation of shear strains. The non-zero normal and shear strain \( (\varepsilon_x, \varepsilon_y, \gamma_{xy}, \gamma_{xz}, \gamma_{yz}) \) components are as follows:

\[
\begin{align*}
  \varepsilon_x &= \varepsilon_x^0 + z k^b + \left[ \frac{1}{3} \left( \frac{z}{h^3} \right) \right] \varepsilon_x^1 + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \varepsilon_x^2 \\
  \varepsilon_y &= \varepsilon_y^0 + z k^b + \left[ \frac{1}{3} \left( \frac{z}{h^3} \right) \right] \varepsilon_y^1 + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \varepsilon_y^2 \\
  \varepsilon_z &= \left( -\frac{8z}{h^2} \right) \phi_z + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \psi_z \\
  \gamma_{xy} &= \gamma_{xy}^0 + z k^b + \left[ \frac{1}{3} \left( \frac{z}{h^3} \right) \right] \gamma_{xy}^1 + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \gamma_{xy}^2 \\
  \gamma_{xz} &= \left[ 1 - 4 \left( \frac{z^2}{h^2} \right) \right] \gamma_{xz} + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \gamma_{xz}^2 \\
  \gamma_{yz} &= \left[ 1 - 4 \left( \frac{z^2}{h^2} \right) \right] \gamma_{yz} + \left[ \frac{1}{5} \left( \frac{z}{h^4} \right) \right] \gamma_{yz}^2
\end{align*}
\]

where

\[
\begin{align*}
  \varepsilon_x^0 &= \frac{\partial u_0}{\partial x} \\
  k^b &= - \frac{\partial^2 w_0}{\partial z^2} \\
  \varepsilon_y^0 &= \frac{\partial v_0}{\partial y} \\
  \varepsilon_z^0 &= \frac{\partial w_0}{\partial z} \\
  \varepsilon_x^1 &= \frac{\partial \phi_x}{\partial x} \\
  \varepsilon_y^1 &= \frac{\partial \phi_y}{\partial y} \\
  \varepsilon_z^1 &= \frac{\partial \psi_z}{\partial z} \\
  k^b &= - \frac{\partial^2 w_0}{\partial z^2} \\
  \varepsilon_x^2 &= \frac{\partial \psi_y}{\partial y} \\
  \gamma_{xy}^0 &= \left( \phi_x + \frac{\partial \phi_x}{\partial x} \right) y_x + \left( \psi_y + \frac{\partial \psi_y}{\partial y} \right) x_x + \left( \psi_x + \frac{\partial \psi_x}{\partial x} \right) y_y + \left( \phi_y + \frac{\partial \phi_y}{\partial y} \right) x_y \\
  \gamma_{xz}^0 &= \left( \phi_x + \frac{\partial \phi_x}{\partial x} \right) y_x + \left( \psi_x + \frac{\partial \psi_x}{\partial x} \right) y_y + \left( \phi_z + \frac{\partial \phi_z}{\partial z} \right) x_z + \left( \psi_z + \frac{\partial \psi_z}{\partial z} \right) y_z \\
  \gamma_{yz}^0 &= \left( \phi_x + \frac{\partial \phi_x}{\partial x} \right) y_x + \left( \psi_x + \frac{\partial \psi_x}{\partial x} \right) y_y + \left( \psi_y + \frac{\partial \psi_y}{\partial y} \right) y_y + \left( \phi_y + \frac{\partial \phi_y}{\partial y} \right) x_y
\end{align*}
\]
$\begin{align*}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{xz}
\end{bmatrix} &= 
\begin{bmatrix}
Q_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 \\
Q_{12} & Q_{22} & Q_{23} & 0 & 0 & 0 \\
Q_{13} & Q_{23} & Q_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & Q_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & Q_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & Q_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{yz} \\
\gamma_{xz}
\end{bmatrix} \\
\text{where}
Q_{11} = Q_{22} = Q_{33} &= \frac{E(z)(1-\mu)}{(1+\mu)(1-2\mu)} \\
Q_{12} = Q_{13} = Q_{23} &= \frac{E(z)\mu}{(1+\mu)(1-2\mu)} \\
Q_{44} = Q_{55} = Q_{66} &= \frac{E(z)}{2(1+\mu)}
\end{align*}$

2.3 Governing differential equations

The principle of virtual work stated in Eq. (7) is applied to derive the variationally consistent governing differential equations.

$\int_a^b \int_{a/2}^{b/2} \left( \sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \right) dz \ dy \ dx = \int_a^b \int_{a/2}^{b/2} q(x, y) \delta w \ dy \ dx \tag{7}$

where $\delta$ represents the virtual operator. Substituting non-zero strains from Eqs. (3) and (4) into Eq. (7) one can write:

$\int_a^b \int_{a/2}^{b/2} \left( N_x \frac{\partial \delta u_x}{\partial x} - M_y \frac{\partial^2 \delta w_y}{\partial x^2} + M_z \frac{\partial^2 \delta w_z}{\partial x^2} + M_{y} \frac{\partial^2 \delta w_y}{\partial x \partial y} + M_{z} \frac{\partial^2 \delta w_z}{\partial x \partial y} - N_y \frac{\partial \delta v_y}{\partial y} + N_z \frac{\partial \delta v_z}{\partial y} - 2M_x \frac{\partial^2 \delta w_x}{\partial y^2} + M_{y} \frac{\partial^2 \delta w_y}{\partial y^2} + M_{z} \frac{\partial^2 \delta w_z}{\partial y^2} \right) \ dy \ dx = \int_a^b \int_{a/2}^{b/2} q(x, y) \delta w \ dy \ dx \tag{8}$

$\text{where} \quad N_x = \int_{-a/2}^{a/2} \sigma_x \ dz, \quad M_y = \int_{-a/2}^{a/2} \sigma_y \ z \ dz, \quad M_z = \int_{-a/2}^{a/2} \sigma_z \ z \ dz, \quad M_{y} = \int_{-a/2}^{a/2} \frac{\partial^2 \delta w_y}{\partial x \partial y} \ dz, \quad N_y = \int_{-a/2}^{a/2} \sigma_y \ dz, \quad M_{z} = \int_{-a/2}^{a/2} \frac{\partial^2 \delta w_z}{\partial x \partial y} \ dz,$

$\quad M_y = \int_{-a/2}^{a/2} \sigma_y \ z \ dz, \quad M_z = \int_{-a/2}^{a/2} \frac{\partial^2 \delta w_z}{\partial x \partial y} \ dz, \quad N_y = \int_{-a/2}^{a/2} \tau_{xy} \ dz, \quad M_{y} = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz, \quad M_{z} = \int_{-a/2}^{a/2} \tau_{xz} \ z \ dz,$

$\quad M_y = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz, \quad Q_{x} = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz, \quad Q_{z} = \int_{-a/2}^{a/2} \tau_{xz} \ z \ dz, \quad Q_{y} = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz,$

$\quad Q_{x} = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz, \quad Q_{z} = \int_{-a/2}^{a/2} \tau_{xz} \ z \ dz, \quad Q_{y} = \int_{-a/2}^{a/2} \tau_{xy} \ z \ dz,$

$\text{where} \quad (N, M, Q) \text{ are the resultant in-plane forces, moments, and shear forces, respectively. The superscript } b \text{ is associated with the terms analogous to classical theory, whereas } S_1 \text{ and } S_2 \text{ are the superscripts associated with the transverse shear deformation effect. Additionally, Superscripts 1 and 2 are associated with the shearing strain functions } f_s(z) \text{ and } f_s(z). \text{ The governing differential equations are obtained by integration of Eq. (8) and setting the coefficients of unknown equal to zero.}$
\[
\delta u_{a} : \frac{\partial N_{ax}}{\partial x} + \frac{\partial N_{ay}}{\partial y} = A_{1} \frac{\partial^{2} u_{a}}{\partial x^{2}} - B_{1} \frac{\partial^{2} w_{a}}{\partial x^{2}} + C_{11} \frac{\partial^{4} \psi_{a}}{\partial x^{4}} + D_{12} \frac{\partial^{4} \phi_{a}}{\partial x^{2} \partial y^{2}} + A_{12} \frac{\partial^{2} \phi_{a}}{\partial x^{2} \partial y^{2}} + D_{12} \frac{\partial^{2} \psi_{a}}{\partial x \partial y^{2}} + D_{12} \frac{\partial^{2} \psi_{a}}{\partial x^{2} \partial y} + D_{12} \frac{\partial^{2} \psi_{a}}{\partial x \partial y^{2}} + D_{12} \frac{\partial^{2} \psi_{a}}{\partial x^{2} \partial y} = 0
\]

\[
\delta v_{b} : \frac{\partial N_{bx}}{\partial x} + \frac{\partial N_{by}}{\partial y} = A_{2} \frac{\partial^{2} u_{b}}{\partial x^{2}} - B_{2} \frac{\partial^{2} w_{b}}{\partial x^{2}} + C_{12} \frac{\partial^{4} \psi_{b}}{\partial x^{4}} + D_{21} \frac{\partial^{4} \phi_{b}}{\partial x^{2} \partial y^{2}} + A_{22} \frac{\partial^{2} \phi_{b}}{\partial x^{2} \partial y^{2}} + D_{21} \frac{\partial^{2} \psi_{b}}{\partial x \partial y^{2}} + D_{21} \frac{\partial^{2} \psi_{b}}{\partial x^{2} \partial y} + D_{21} \frac{\partial^{2} \psi_{b}}{\partial x \partial y^{2}} + D_{21} \frac{\partial^{2} \psi_{b}}{\partial x^{2} \partial y} = 0
\]

\[
\delta w_{g} : \frac{\partial M_{gx}}{\partial x} + \frac{\partial M_{gy}}{\partial y} = Q = B_{1} \frac{\partial^{2} u_{g}}{\partial x^{2}} - C_{11} \frac{\partial^{4} \psi_{g}}{\partial x^{4}} + C_{12} \frac{\partial^{4} \phi_{g}}{\partial x^{2} \partial y^{2}} + B_{12} \frac{\partial^{2} \phi_{g}}{\partial x^{2} \partial y^{2}} + B_{12} \frac{\partial^{2} \psi_{g}}{\partial x \partial y^{2}} + B_{12} \frac{\partial^{2} \psi_{g}}{\partial x^{2} \partial y} + B_{12} \frac{\partial^{2} \psi_{g}}{\partial x \partial y^{2}} = 0
\]

\[
\delta \phi_{c} : \frac{\partial M_{cx}}{\partial x} + \frac{\partial M_{cy}}{\partial y} = -Q^2_{c} = C_{11} \frac{\partial^{4} \psi_{c}}{\partial x^{4}} + C_{12} \frac{\partial^{4} \phi_{c}}{\partial x^{2} \partial y^{2}} + C_{12} \frac{\partial^{4} \phi_{c}}{\partial x^{2} \partial y^{2}} + C_{12} \frac{\partial^{4} \phi_{c}}{\partial x^{2} \partial y^{2}} + C_{12} \frac{\partial^{4} \phi_{c}}{\partial x^{2} \partial y^{2}} = 0
\]

\[
\delta \psi_{d} : \frac{\partial M_{dx}}{\partial x} + \frac{\partial M_{dy}}{\partial y} = -Q^2_{d} = D_{12} \frac{\partial^{2} u_{d}}{\partial x^{2}} - D_{12} \frac{\partial^{2} w_{d}}{\partial x^{2}} + C_{12} \frac{\partial^{4} \phi_{d}}{\partial x^{2} \partial y^{2}} + D_{12} \frac{\partial^{4} \phi_{d}}{\partial x^{2} \partial y^{2}} + D_{12} \frac{\partial^{4} \phi_{d}}{\partial x^{2} \partial y^{2}} + D_{12} \frac{\partial^{4} \phi_{d}}{\partial x^{2} \partial y^{2}} = 0
\]
\[ \begin{align*}
\delta\phi : \quad & \frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} - Q = C_{SSS14} \frac{\partial^2 \phi}{\partial x^2} + C_{SSS14} \frac{\partial^2 \phi}{\partial y^2} + C_{SSS244} \frac{\partial^2 \psi}{\partial x^2} + C_{SSS244} \frac{\partial^2 \psi}{\partial y^2} + C_{SSS155} \frac{\partial \phi}{\partial y} + C_{SSS155} \frac{\partial \phi}{\partial y^2} + C_{SSS235} \frac{\partial \psi}{\partial y} + C_{SSS235} \frac{\partial \psi}{\partial y^2} - I_{133} \frac{\partial u}{\partial x} + I_{313} \frac{\partial u}{\partial y} - I_{133} \frac{\partial v}{\partial x} + I_{133} \frac{\partial u}{\partial y} + I_{313} \frac{\partial v}{\partial x} + I_{313} \frac{\partial u}{\partial y} - I_{133} \frac{\partial w}{\partial y} + I_{313} \frac{\partial w}{\partial y} - I_{133} \frac{\partial \phi}{\partial y} + I_{133} \frac{\partial \phi}{\partial y} + I_{313} \frac{\partial \phi}{\partial y} + I_{313} \frac{\partial \phi}{\partial y} + I_{313} \frac{\partial \phi}{\partial y} + I_{313} \frac{\partial \phi}{\partial y} = 0 \\
\delta\psi : \quad & \frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial y^2} - Q = C_{SSS14} \frac{\partial^2 \phi}{\partial x^2} + C_{SSS14} \frac{\partial^2 \phi}{\partial y^2} + C_{SSS244} \frac{\partial^2 \psi}{\partial x^2} + C_{SSS244} \frac{\partial^2 \psi}{\partial y^2} + C_{SSS155} \frac{\partial \phi}{\partial y} + C_{SSS155} \frac{\partial \phi}{\partial y^2} + C_{SSS235} \frac{\partial \psi}{\partial y} + C_{SSS235} \frac{\partial \psi}{\partial y^2} + D_{SSS221} \frac{\partial u}{\partial x} + D_{SSS221} \frac{\partial u}{\partial y} + D_{SSS221} \frac{\partial v}{\partial x} + D_{SSS221} \frac{\partial v}{\partial y} + D_{SSS221} \frac{\partial w}{\partial y} + D_{SSS221} \frac{\partial w}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} + D_{SSS221} \frac{\partial \phi}{\partial y} = 0
\end{align*}\]

where

\[\begin{align*}
A_q &= Q_y \int_{a/2}^{b/2} z \, dz, \quad B_q = Q_y \int_{a/2}^{b/2} z^2 \, dz, \quad A_{uy} = Q_y \int_{a/2}^{b/2} z \, dz, \\
C_q &= Q_y \int_{a/2}^{b/2} z \left( -\frac{4}{3} \right) \left( z^2 / h^2 \right) \, dz, \quad C_{uy} = Q_y \int_{a/2}^{b/2} z \left[ z - \left( -\frac{4}{3} \right) \left( z^2 / h^2 \right) \right] \, dz, \\
C_{SSS1q} &= Q_y \int_{a/2}^{b/2} \left[ z - \left( -\frac{4}{3} \right) \left( z^2 / h^2 \right) \right] \left[ z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right] \, dz, \\
C_{SSS1uy} &= Q_y \int_{a/2}^{b/2} \left[ z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right] \left[ z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right] \, dz, \\
D_{yy} &= Q_y \int_{a/2}^{b/2} \left[ 1 - 4 \left( z^2 / h^2 \right) \right] \left( z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right) \, dz, \\
D_{uy} &= Q_y \int_{a/2}^{b/2} \left[ z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right] \left[ -8 z / h^2 \right] \, dz, \\
D_{uyu} &= Q_y \int_{a/2}^{b/2} \left[ z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right] \left( z - \left( -\frac{16}{5} \right) \left( z^5 / h^4 \right) \right) \, dz, \\
I_{SSS1} &= Q_y \int_{a/2}^{b/2} \left[ -8 \left( z / h^2 \right) \right] \, dz, \quad I_{SSS1uy} = Q_y \int_{a/2}^{b/2} \left[ -8 \left( z / h^2 \right) \right] \left( -8 \left( z / h^2 \right) \right) \, dz, \\
I_{SSS2} &= Q_y \int_{a/2}^{b/2} \left[ -8 \left( z / h^2 \right) \right] \, dz, \quad I_{SSS2uy} = Q_y \int_{a/2}^{b/2} \left[ -8 \left( z / h^2 \right) \right] \left( -8 \left( z / h^2 \right) \right) \, dz, \\
J_{SSS2} &= Q_y \int_{a/2}^{b/2} \left[ -64 z^3 / h^4 \right] \, dz, \\
J_{SSS2uy} &= Q_y \int_{a/2}^{b/2} \left[ -64 z^3 / h^4 \right] \left( -64 z^3 / h^4 \right) \, dz
\end{align*}\]

Associated boundary conditions are as:

at \( x = 0 \) and \( x = a \)

\[\begin{align*}
N_x &= 0 \quad \text{or} \quad u_x = 0 \\
N_{xy} &= 0 \quad \text{or} \quad v_y = 0 \\
M_x^b &= 0 \quad \text{or} \quad w_y = 0 \\
M_{ux} &= 0 \quad \text{or} \quad \frac{\partial w_y}{\partial x} = 0 \\
M_{ux}^b &= 0 \quad \text{or} \quad \phi_x = 0
\end{align*}\]
A New Quasi-3D Model for Functionally Graded Plates


\[ M_{x}^S = 0 \quad \text{or} \quad \psi_x = 0 \]  
(25)

\[ M_{y}^S = 0 \quad \text{or} \quad \phi_y = 0 \]  
(26)

\[ M_{y}^S = 0 \quad \text{or} \quad \psi_y = 0 \]  
(27)

\[ Q_{xy} = 0 \quad \text{or} \quad \phi_y = 0 \]  
(28)

\[ Q_{zz} = 0 \quad \text{or} \quad \psi_y = 0 \]  
(29)

at \( y = 0 \) and \( y = b \)

\[ N_{x} = 0 \quad \text{or} \quad u_x = 0 \]  
(30)

\[ N_{y} = 0 \quad \text{or} \quad v_y = 0 \]  
(31)

\[ M_{x}^b = 0 \quad \text{or} \quad w_0 = 0 \]  
(32)

\[ M_{y}^b = 0 \quad \text{or} \quad \partial w_0 / \partial y = 0 \]  
(33)

\[ M_{z} = 0 \quad \text{or} \quad \phi_z = 0 \]  
(34)

\[ M_{z} = 0 \quad \text{or} \quad \psi_z = 0 \]  
(35)

\[ M_{x}^t = 0 \quad \text{or} \quad \phi_x = 0 \]  
(36)

\[ M_{y}^t = 0 \quad \text{or} \quad \psi_x = 0 \]  
(37)

\[ Q_{x}^t = 0 \quad \text{or} \quad \phi_z = 0 \]  
(38)

\[ Q_{zz} = 0 \quad \text{or} \quad \psi_z = 0 \]  
(39)

2.4 Closed-form solutions

A Navier’s solution procedure is implemented to obtain static solutions for the FG plates. The displacement variables are assumed to be in the following trigonometric form.

\[ (u, v, w, \phi_x, \phi_y, \psi_z) = \sum_{m=1,3,5, n=1,3,5} q_{mn} \sin \alpha x \sin \beta y \]  
(40)

where \( \alpha = m \pi / a, \quad \beta = n \pi / b \) and \( u_{mn}, v_{mn}, w_{mn}, \phi_{xmn}, \phi_{ymn}, \psi_{zmn}, \phi_{xmn}, \phi_{ymn}, \psi_{zmn} \) are the unknown coefficients. The transverse load is also considered to be in trigonometric form.

\[ q(x, y) = \sum_{m=1,3,5, n=1,3,5} q_{mn} \sin \alpha x \sin \beta y \]  
(41)

The Fourier coefficients \( q_{mn} \) for different loading conditions are as follows,

\[ q_{mn} = q_0 \quad (m = 1, n = 1) \quad \text{(Sinusoidal distributed load)} \]  
(42)

\[ q_{mn} = \frac{16q_0}{\pi^2 m n^2} \quad (m = 1, 3, 5 \ldots n = 1, 3, 5 \ldots) \quad \text{(Uniformly distributed load)} \]  
(43)

where \( q_0 \) is the maximum intensity. Substituting Eqs. (40)-(43) into the Eqs. (11)-(18) leads to the following equations:

\[ [K] [\Delta] = [Q] \]  
(44)

where the elements of Matrix \( K \) are described in the Appendix and also we have:

\[ [Q] = \{0,0,q_{mn},0,0,0,0,0\} \]

\[ [\Delta] = \{u_{mn},v_{mn},w_{mn},\phi_{xmn},\phi_{ymn},\psi_{zmn},\phi_{xmn},\phi_{ymn},\psi_{zmn}\}^T \]  
(45)

3. Illustrative examples and validation

The bending analysis of the FG plates is presented herein to prove validity of the theory. The properties of the metal and ceramic are; Metal: \( E_m = 70 \text{ GPa} \) and \( \mu = 0.3 \), Ceramic: \( E_c = 380 \text{ GPa} \) and \( \mu = 0.3 \). The non-dimensional quantities are presented in the following form.
Isotropic Plate:

\[
\bar{w} \left( \frac{a}{2}, \frac{b}{2}, 0 \right) = \frac{100Eh^3}{q_0a^4} w, 
\quad \bar{\sigma}_{x} \left( \frac{a}{2}, \frac{b}{2}, \frac{h}{2} \right) = \frac{h^2}{q_0a^2} \sigma_{x} \quad \bar{\tau} \left( \frac{a}{2}, 0, \frac{z}{h} \right) = \frac{h}{q_0a} \tau_{xy}
\]  

(46)

FG Plate:

\[
\bar{w} \left( \frac{a}{2}, \frac{b}{2}, 0 \right) = \frac{100Eh^3}{q_0a^4} w, 
\quad (\bar{\sigma}_{x}, \bar{\sigma}_{y}) \left( \frac{a}{2}, \frac{b}{2}, \frac{h}{2} \right) = \frac{h}{q_0a} (\sigma_{x}, \sigma_{y}), 
\quad \bar{\tau} \left( 0, 0, \frac{z}{h} \right) = \frac{h}{q_0a} \tau_{xy}, 
\quad \bar{\tau} \left( \frac{a}{2}, 0, \frac{z}{h} \right) = \frac{h}{q_0a} \tau_{xz}
\]  

(47)

3.1 Validation and discussion of results

Due to the unavailability of the exact 3D solutions for FG plates, the present FOSNDT is applied to isotropic plates (E = 210 GPa, μ = 0.3) to prove its accuracy and validity. Non-dimensional quantities are shown in Table 1. Exact solution of Pagano [52], CPT of Kirchhoff [53], PSDT of Reddy [55], and sinusoidal shear and normal deformation plate theory (SSNPT) of Sayyad and Ghugal [56] are used for the comparison purpose. The CPT neglects the shear effect, the FSSTDT considers the first-order shear effect, the PSDT considers the third-order shear effect. SSNPT considers the sinusoidal type shear effect. In the comparison of these theories, the present theory considers the fifth-order shear effect along with thickness stretching effect. Table 1 shows that the present FOSNDT yields accurate predictions of displacements and stresses for all aspect ratios compared to other well-known plate theories. This is mainly due to the inclusion of fifth-order variation of displacements and thickness stretching effect. In many cases, percentage error predicted by the FOSNDT is lower than other existing plate theories. For a/h = 4, the percentage error in transverse deflection predicted by the present theory is 0.076% whereas PSDT, SSNPT, FSDT, and CPT show error of 3.380, -0.262, -1.010, 23.47%, respectively. A similar type of error difference can be observed for in-plane normal and transverse shear stresses. CPT predicted by the present theory is 0.076% whereas PSDT, SSNPT, FSDT, and CPT show error of 3.380, -0.262, -1.010, 23.47%. For a/h = 10, the percentage error in transverse deflection is 0.214, the percentage error in transverse shear, normal shear, and in-plane shear is 0.238, 0.249, and 0.000, respectively.

<table>
<thead>
<tr>
<th>a/h</th>
<th>Model</th>
<th>% error</th>
<th>% error</th>
<th>% error</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>Present (FOSNDT)</td>
<td>0.076</td>
<td>0.2060</td>
<td>0.2356</td>
<td>0.211</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>0.2090</td>
<td>-2.450</td>
<td>0.2260</td>
<td>-4.277</td>
</tr>
<tr>
<td></td>
<td>SSNPT [56]</td>
<td>-0.262</td>
<td>-11.12</td>
<td>0.2355</td>
<td>-0.254</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>0.2267</td>
<td>-3.430</td>
<td>0.2390</td>
<td>1.220</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>23.47</td>
<td>-3.430</td>
<td>0.2380</td>
<td>0.804</td>
</tr>
<tr>
<td></td>
<td>Exact [52]</td>
<td>0.000</td>
<td>0.2040</td>
<td>0.2361</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>Present (FOSNDT)</td>
<td>0.242</td>
<td>0.2000</td>
<td>0.2383</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>0.1990</td>
<td>0.100</td>
<td>0.2290</td>
<td>3.900</td>
</tr>
<tr>
<td></td>
<td>SSNPT [56]</td>
<td>0.2125</td>
<td>6.890</td>
<td>0.2380</td>
<td>-0.126</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>0.1970</td>
<td>-3.430</td>
<td>0.2390</td>
<td>0.293</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>0.2340</td>
<td>-3.430</td>
<td>0.2380</td>
<td>0.804</td>
</tr>
<tr>
<td></td>
<td>Exact [52]</td>
<td>0.000</td>
<td>0.2040</td>
<td>0.2361</td>
<td>0.000</td>
</tr>
<tr>
<td>20</td>
<td>Present (FOSNDT)</td>
<td>0.123</td>
<td>0.355</td>
<td>0.2387</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>-0.320</td>
<td>6.360</td>
<td>0.2384</td>
<td>-0.125</td>
</tr>
<tr>
<td></td>
<td>SSNPT [56]</td>
<td>0.000</td>
<td>0.000</td>
<td>0.2387</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>0.1981</td>
<td>0.253</td>
<td>0.2387</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>-0.324</td>
<td>6.270</td>
<td>0.2385</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>Exact [52]</td>
<td>0.000</td>
<td>0.1976</td>
<td>0.2386</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>Present (FOSNDT)</td>
<td>0.092</td>
<td>0.096</td>
<td>0.2388</td>
<td>0.041</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>0.000</td>
<td>0.1980</td>
<td>0.2390</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>SSNPT [56]</td>
<td>-0.324</td>
<td>0.202</td>
<td>0.2390</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>0.1980</td>
<td>0.202</td>
<td>0.2390</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>-0.035</td>
<td>0.202</td>
<td>0.2390</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>Exact [52]</td>
<td>0.000</td>
<td>0.1976</td>
<td>0.2387</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 2 presents values of non-dimensional quantities of FGM plate for p = {0, 1, 5, 10} and a/h = 10. The top surface, i.e., metal surface, is subjected to mechanical load. The present results are compared with those presented by PSDT of Reddy [55], trigonometric, hyperbolic, and exponential shear deformation theories (TSST, HSDT, and ESST) developed by Sayyad and Ghugal [56], Mindlin [54] and Kirchhoff [53]. It is observed from Table 2 that PSDT, TSST, HSDT, and ESST overestimate the results for FG plate under sinusoidal load. This is in fact due to the neglect of transverse normal deformations, i.e., thickness stretching effect. It is important to note that the nondimensional displacements and stresses are increasing with growth in the power-law index. This is due to increases in the power-law index which reduces the stiffness of the plate. Distributions of stresses in z-direction are plotted in Fig. 3. From these figures, it is observed that the variations of in-plane normal and shear
stresses \( (\sigma_x, \tau_{xy}) \) are linear for \( p = 0 \) and nonlinear for \( p = \{1, 5, 10\} \). It is also observed from Fig. 3 that the maximum compressive in-plane normal stress is increased with an increase in the power-law index. Moreover, it is observed that the transverse shear stresses are maximum at mid-plane \((z/h = 0)\) when \( p = 0 \) and maximum at \( z = +0.14h \) and \(+0.22h\) when \( p = 1 \) and 5, respectively. This is due to the fact that mid-plane shifted toward ceramic face due to an increase in power-law index.

**Fig. 3.** Distribution of stresses in \( z \)-direction for sinusoidal load

**Table 2.** Non-dimensional deflection and stresses for the functionally graded square plate subjected to sinusoidal load \((a=b \text{ and } a/h=10)\)

<table>
<thead>
<tr>
<th>( P )</th>
<th>Model</th>
<th>( w(0) )</th>
<th>( \sigma_x(h/2) )</th>
<th>( \sigma_y(h/2) )</th>
<th>( \tau_{xy}(h/2) )</th>
<th>( \tau_{xz}(0) )</th>
<th>( \tau_{yz}(0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Present (FOSNDT)</td>
<td>2.9425</td>
<td>1.9964</td>
<td>1.9964</td>
<td>1.0634</td>
<td>0.2384</td>
<td>0.2384</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>2.9606</td>
<td>1.9943</td>
<td>1.9943</td>
<td>1.0739</td>
<td>0.2386</td>
<td>0.2386</td>
</tr>
<tr>
<td></td>
<td>TSDT [29]</td>
<td>2.9603</td>
<td>1.9955</td>
<td>1.9955</td>
<td>1.0745</td>
<td>0.2462</td>
<td>0.2462</td>
</tr>
<tr>
<td></td>
<td>HSDT [29]</td>
<td>2.9595</td>
<td>1.9937</td>
<td>1.9937</td>
<td>1.0735</td>
<td>0.2371</td>
<td>0.2371</td>
</tr>
<tr>
<td></td>
<td>ESDT [29]</td>
<td>2.9575</td>
<td>1.9967</td>
<td>1.9967</td>
<td>1.0752</td>
<td>0.2437</td>
<td>0.2437</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>2.9343</td>
<td>1.9758</td>
<td>1.9758</td>
<td>1.0639</td>
<td>0.1592</td>
<td>0.1592</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>2.8026</td>
<td>1.9758</td>
<td>1.9758</td>
<td>1.0639</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>Present (FOSNDT)</td>
<td>5.6956</td>
<td>3.0605</td>
<td>3.0605</td>
<td>1.6715</td>
<td>0.2604</td>
<td>0.2604</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>5.8895</td>
<td>3.0850</td>
<td>3.0850</td>
<td>1.6612</td>
<td>0.2623</td>
<td>0.2623</td>
</tr>
<tr>
<td></td>
<td>TSDT [29]</td>
<td>5.8891</td>
<td>3.0870</td>
<td>3.0870</td>
<td>1.6622</td>
<td>0.2677</td>
<td>0.2677</td>
</tr>
<tr>
<td></td>
<td>HSDT [29]</td>
<td>5.8895</td>
<td>3.0848</td>
<td>3.0848</td>
<td>1.6611</td>
<td>0.2619</td>
<td>0.2619</td>
</tr>
<tr>
<td></td>
<td>ESDT [29]</td>
<td>5.8878</td>
<td>3.0889</td>
<td>3.0889</td>
<td>1.6632</td>
<td>0.2717</td>
<td>0.2717</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>5.8452</td>
<td>3.0536</td>
<td>3.0536</td>
<td>1.6443</td>
<td>0.2688</td>
<td>0.2688</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>5.6228</td>
<td>3.0536</td>
<td>3.0536</td>
<td>1.6443</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>Present (FOSNDT)</td>
<td>8.7493</td>
<td>4.1880</td>
<td>4.1880</td>
<td>2.3219</td>
<td>0.2511</td>
<td>0.2511</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>9.1135</td>
<td>4.2447</td>
<td>4.2447</td>
<td>2.2856</td>
<td>0.2659</td>
<td>0.2659</td>
</tr>
<tr>
<td></td>
<td>TSDT [29]</td>
<td>9.1183</td>
<td>4.2488</td>
<td>4.2488</td>
<td>2.2878</td>
<td>0.2574</td>
<td>0.2574</td>
</tr>
<tr>
<td></td>
<td>HSDT [29]</td>
<td>9.1130</td>
<td>4.2443</td>
<td>4.2443</td>
<td>2.2854</td>
<td>0.2668</td>
<td>0.2668</td>
</tr>
<tr>
<td></td>
<td>ESDT [29]</td>
<td>9.1210</td>
<td>4.2527</td>
<td>4.2527</td>
<td>2.2899</td>
<td>0.2514</td>
<td>0.2514</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>8.9321</td>
<td>4.1848</td>
<td>4.1848</td>
<td>2.2534</td>
<td>0.4971</td>
<td>0.4971</td>
</tr>
</tbody>
</table>
Table 2. Continued

<table>
<thead>
<tr>
<th>P</th>
<th>Model</th>
<th>(w) (0)</th>
<th>(w) (/ 2)</th>
<th>(w) (h / 2)</th>
<th>(w) (h / 2)</th>
<th>(\tau_{xy}) (0)</th>
<th>(\tau_{xy}) (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Present (FOSNDT)</td>
<td>9.8204</td>
<td>5.0845</td>
<td>5.0845</td>
<td>2.7841</td>
<td>0.2218</td>
<td>0.2218</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>10.087</td>
<td>5.0849</td>
<td>5.0849</td>
<td>2.7380</td>
<td>0.2115</td>
<td>0.2115</td>
</tr>
<tr>
<td></td>
<td>TSCT [29]</td>
<td>10.089</td>
<td>5.0890</td>
<td>5.0890</td>
<td>2.7402</td>
<td>0.2198</td>
<td>0.2198</td>
</tr>
<tr>
<td></td>
<td>HSCT [29]</td>
<td>10.086</td>
<td>5.0845</td>
<td>5.0845</td>
<td>2.7378</td>
<td>0.2107</td>
<td>0.2107</td>
</tr>
<tr>
<td></td>
<td>ESCT [29]</td>
<td>10.088</td>
<td>5.0928</td>
<td>5.0928</td>
<td>2.7423</td>
<td>0.2282</td>
<td>0.2282</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>9.8644</td>
<td>5.0173</td>
<td>5.0173</td>
<td>2.7016</td>
<td>0.6160</td>
<td>0.6160</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>9.3546</td>
<td>5.0173</td>
<td>5.0173</td>
<td>2.7016</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Table 3 shows comparison of non-dimensional displacements and stresses of FGM plate subjected to uniformly distributed load (UDL). Similar trends in results and distributions of stresses (see Fig. 4) are observed when the plate is loaded UDL.

Figure 5 illustrates the variation of transverse normal stress (\(\sigma_z\)) across the thickness of the plate. This variation is rarely available in the whole variety of literature due to the neglect of transverse normal effect. Variations of transverse deflection with respect to aspect ratio are plotted in Fig. 6. Examination of Fig. 6 displays that the non-dimensional transverse deflection is increased with the increase in the power-law index. Moreover, values of transverse deflection are almost constant for higher values of \(a/h\) ratios i.e. for thin plates.

Table 3. Non-dimensional deflection and stresses for the functionally graded square plate subjected to uniformly distributed load (UDL) \((a=b\) and \(a/h=10\)).

<table>
<thead>
<tr>
<th>(P)</th>
<th>Model</th>
<th>(w) (0)</th>
<th>(w) (/ 2)</th>
<th>(w) (h / 2)</th>
<th>(w) (h / 2)</th>
<th>(\tau_{xy}) (0)</th>
<th>(\tau_{xy}) (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Present (FOSNDT)</td>
<td>4.6397</td>
<td>2.8961</td>
<td>2.8961</td>
<td>1.9541</td>
<td>0.4868</td>
<td>0.4868</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>4.6659</td>
<td>2.8928</td>
<td>2.8928</td>
<td>2.0331</td>
<td>0.4925</td>
<td>0.4925</td>
</tr>
<tr>
<td></td>
<td>TSCT [29]</td>
<td>4.6655</td>
<td>2.8940</td>
<td>2.8940</td>
<td>1.9964</td>
<td>0.5077</td>
<td>0.5077</td>
</tr>
<tr>
<td></td>
<td>HSCT [29]</td>
<td>4.6643</td>
<td>2.8921</td>
<td>2.8921</td>
<td>1.9264</td>
<td>0.4890</td>
<td>0.4890</td>
</tr>
<tr>
<td></td>
<td>ESCT [29]</td>
<td>4.6615</td>
<td>2.8943</td>
<td>2.8943</td>
<td>2.0176</td>
<td>0.5023</td>
<td>0.5023</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>4.6277</td>
<td>2.8735</td>
<td>2.8735</td>
<td>1.9473</td>
<td>0.3300</td>
<td>0.3300</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>4.4361</td>
<td>2.8735</td>
<td>2.8735</td>
<td>1.9473</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>Present (FOSNDT)</td>
<td>8.9852</td>
<td>4.4190</td>
<td>4.4190</td>
<td>3.0724</td>
<td>0.5335</td>
<td>0.5335</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>9.2880</td>
<td>4.4738</td>
<td>4.4738</td>
<td>3.1724</td>
<td>0.5414</td>
<td>0.5414</td>
</tr>
<tr>
<td></td>
<td>TSCT [29]</td>
<td>9.2874</td>
<td>4.4758</td>
<td>4.4758</td>
<td>3.0927</td>
<td>0.5501</td>
<td>0.5501</td>
</tr>
<tr>
<td></td>
<td>HSCT [29]</td>
<td>9.2880</td>
<td>4.4736</td>
<td>4.4736</td>
<td>2.9771</td>
<td>0.5407</td>
<td>0.5407</td>
</tr>
<tr>
<td></td>
<td>ESCT [29]</td>
<td>9.2856</td>
<td>4.4777</td>
<td>4.4777</td>
<td>3.1654</td>
<td>0.5598</td>
<td>0.5598</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>9.2234</td>
<td>4.4411</td>
<td>4.4411</td>
<td>3.0097</td>
<td>0.5574</td>
<td>0.5574</td>
</tr>
<tr>
<td></td>
<td>CPT [53]</td>
<td>8.9000</td>
<td>4.4411</td>
<td>4.4411</td>
<td>3.0097</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>Present (FOSNDT)</td>
<td>13.777</td>
<td>6.0400</td>
<td>6.0400</td>
<td>4.3034</td>
<td>0.5166</td>
<td>0.5166</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>14.349</td>
<td>6.1484</td>
<td>6.1484</td>
<td>4.3737</td>
<td>0.5480</td>
<td>0.5480</td>
</tr>
<tr>
<td></td>
<td>TSCT [29]</td>
<td>14.356</td>
<td>6.1526</td>
<td>6.1526</td>
<td>4.2816</td>
<td>0.5298</td>
<td>0.5298</td>
</tr>
<tr>
<td></td>
<td>HSCT [29]</td>
<td>14.348</td>
<td>6.1480</td>
<td>6.1480</td>
<td>4.0138</td>
<td>0.5499</td>
<td>0.5499</td>
</tr>
<tr>
<td></td>
<td>ESCT [29]</td>
<td>14.360</td>
<td>6.1565</td>
<td>6.1565</td>
<td>4.3615</td>
<td>0.5167</td>
<td>0.5167</td>
</tr>
<tr>
<td>10</td>
<td>Present (FOSNDT)</td>
<td>15.457</td>
<td>7.3217</td>
<td>7.3217</td>
<td>5.1493</td>
<td>0.4522</td>
<td>0.4522</td>
</tr>
<tr>
<td></td>
<td>PSDT [55]</td>
<td>15.872</td>
<td>7.3672</td>
<td>7.3672</td>
<td>5.2118</td>
<td>0.4357</td>
<td>0.4357</td>
</tr>
<tr>
<td></td>
<td>TSCT [29]</td>
<td>15.875</td>
<td>7.3713</td>
<td>7.3713</td>
<td>5.1209</td>
<td>0.4524</td>
<td>0.4524</td>
</tr>
<tr>
<td></td>
<td>HSCT [29]</td>
<td>15.872</td>
<td>7.3668</td>
<td>7.3668</td>
<td>4.7875</td>
<td>0.4343</td>
<td>0.4343</td>
</tr>
<tr>
<td></td>
<td>ESCT [29]</td>
<td>15.874</td>
<td>7.3751</td>
<td>7.3751</td>
<td>5.1953</td>
<td>0.4690</td>
<td>0.4690</td>
</tr>
<tr>
<td></td>
<td>FSDT [54]</td>
<td>15.548</td>
<td>7.2970</td>
<td>7.2970</td>
<td>4.9450</td>
<td>1.2773</td>
<td>1.2773</td>
</tr>
</tbody>
</table>
4. Conclusions

A new quasi-3D fifth-order displacement based model was developed and presented for the bending analysis of FGM plates. The theory considered the effects of transverse normal strain/stress on the bending of a plate. Governing equations were obtained using the principle of virtual work. Closed-form solutions were presented based on Navier’s technique. Numerical results were presented when the plate is subjected to sinusoidal and distributed loads. From the numerical study and discussion of the results following conclusions were drawn.
1) The present theory yields accurate predictions of displacements and stresses compared to other well-known higher order plate theories. This is mainly due to the inclusion of fifth-order variation of displacements and thickness stretching effect.

2) It is deduced that the non-dimensional displacements and stresses are increasing with an increase in the power-law index. This is due to the increases in power-law index which reduces stiffness of the plate and increases its flexibility.

3) It is concluded that the variations of in-plane normal and shear stresses are linear for $p = 0$ and nonlinear for higher values of the power-law index due to gradation of material properties.

4) It is also concluded that mid-plane of the plate is shifted towards ceramic face due to an increase in power-law index. Overall, it is concluded that the present theory is accurate and strongly recommended for the bending analysis of FGM plate.

Conflict of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The author(s) received no financial support for the research, authorship and publication of this article.

References


Appendix

\[ K_{11} = -(A_{11}\alpha^2 + A_{00}\beta^2), \quad K_{12} = -(A_{12} + A_{00})\alpha\beta, \quad K_{13} = B_{11}\alpha^3 + B_{12}\alpha^2\beta + 2B_{00}\alpha\beta^2, \]
\[ K_{14} = -(C_{11}\alpha^2 + C_{00}\beta^2), \quad K_{15} = -(D_{11}\alpha^2 + D_{00}\beta^2), \quad K_{16} = -(C_{12} + C_{00})\alpha\beta, \]
\[ K_{17} = -(D_{12} + D_{00})\alpha\beta, \quad K_{18} = I_{11}\alpha, \quad K_{19} = J_{13}\alpha, \quad K_{22} = -(A_{00}\alpha^2 + A_{12}\beta^2), \]
\[ K_{23} = B_{12}\alpha^2\beta + 2B_{00}\alpha\beta^2 + B_{22}\beta^3, \quad K_{24} = -(C_{12} + C_{00})\alpha\beta, \quad K_{25} = -(D_{12} + D_{00})\alpha\beta, \]
\[ K_{26} = -(C_{22}\beta^2 + C_{00}\alpha^2), \quad K_{27} = -(D_{22}\beta^2 + D_{00}\alpha^2), \quad K_{28} = I_{23}\beta, \quad K_{29} = J_{23}\beta, \]
\[ K_{33} = -(A_{51}\alpha^2 + 2A_{32}\alpha^2\beta^2 + A_{12}\beta^4 + 4A_{16}\alpha^2\beta^2), \quad K_{34} = C_{51}\alpha^3 + C_{52}\alpha^2\beta + 2C_{56}\alpha\beta^2, \]
\[ K_{35} = D_{51}\alpha^3 + D_{52}\alpha^2\beta + 2D_{56}\alpha\beta^2, \quad K_{36} = C_{51}\alpha^2\beta + C_{52}\beta^3 + 2C_{56}\alpha^2\beta, \]
\[ K_{37} = D_{51}\alpha^3 + D_{52}\beta^3 + 2D_{56}\alpha\beta^2, \quad K_{38} = -(I_{51}\alpha^2 + I_{52}\beta^2), \quad K_{39} = -(J_{51}\alpha^2 + J_{52}\beta^2), \]
\[ K_{44} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \quad K_{45} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \]
\[ K_{46} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \quad K_{47} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \quad K_{48} = (I_{51}\alpha^2 - C_{55}\alpha), \]
\[ K_{49} = (J_{51}\alpha^2 - C_{55}\beta), \quad K_{50} = -(C_{51}\alpha^2 + C_{55}\beta^2 + D_{56}\alpha\beta), \quad K_{51} = -(C_{51}\alpha^2 + C_{55}\beta^2 + D_{56}\alpha\beta), \]
\[ K_{57} = -(D_{52}+D_{56}\alpha\beta), \quad K_{58} = (I_{51}\alpha^2 - C_{55}\alpha), \quad K_{59} = (J_{51}\alpha^2 - C_{55}\beta), \]
\[ K_{66} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \quad K_{67} = -(C_{51}\alpha^2 + C_{55}\beta^2 + C_{56}\alpha\beta), \]
\[ K_{68} = (I_{51}\alpha^2 - C_{55}\alpha), \quad K_{69} = (J_{51}\alpha^2 - C_{55}\beta), \quad K_{70} = -(D_{52}+D_{56}\alpha\beta), \quad K_{71} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{72} = -(D_{52}+D_{56}\alpha\beta), \quad K_{73} = -(D_{52}+D_{56}\alpha\beta), \quad K_{74} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{75} = -(D_{52}+D_{56}\alpha\beta), \quad K_{76} = -(D_{52}+D_{56}\alpha\beta), \quad K_{77} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{78} = -(D_{52}+D_{56}\alpha\beta), \quad K_{79} = -(D_{52}+D_{56}\alpha\beta), \quad K_{80} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{81} = -(D_{52}+D_{56}\alpha\beta), \quad K_{82} = -(D_{52}+D_{56}\alpha\beta), \quad K_{83} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{84} = -(D_{52}+D_{56}\alpha\beta), \quad K_{85} = -(D_{52}+D_{56}\alpha\beta), \quad K_{86} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{87} = -(D_{52}+D_{56}\alpha\beta), \quad K_{88} = -(D_{52}+D_{56}\alpha\beta), \quad K_{89} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{90} = -(D_{52}+D_{56}\alpha\beta), \quad K_{91} = -(D_{52}+D_{56}\alpha\beta), \quad K_{92} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{93} = -(D_{52}+D_{56}\alpha\beta), \quad K_{94} = -(D_{52}+D_{56}\alpha\beta), \quad K_{95} = -(D_{52}+D_{56}\alpha\beta), \]
\[ K_{96} = -(D_{52}+D_{56}\alpha\beta), \quad K_{97} = -(D_{52}+D_{56}\alpha\beta), \quad K_{98} = -(D_{52}+D_{56}\alpha\beta), \]

© 2019 by the authors. Licensee SCU, Ahvaz, Iran. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution-NonCommercial 4.0 International (CC BY-NC 4.0 license) (http://creativecommons.org/licenses/by-nc/4.0/).