Melting Heat Transfer and Radiation Effects on Jeffrey Fluid Flow over a Continuously Moving Surface with a Parallel Free Stream

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Abstract. This article is proposed to address the melting heat transfer of a Jeffrey fluid in Blasius and Sakiadis flow caused due to a moving surface. Thermal radiation and a constant free stream are considered in this mathematical model. The non-linear coupled dimensionless equations from the governing equations are attained by employing appropriate similarity transformations. The resulting dimensionless equations are solved by implementing RKF method. The impact of sundry emerging parameters on different flow fields are interpreted with the help of figures and tables. For augmented values of Deborah number, the velocity profile diminishes in the case of Blasius flow and the reverse behavior in the Sakiadis flow is observed. Moreover, the velocity of non-Newtonian liquid in case of Blasius flow is superior to that of the Sakiadis flow. The present work is demonstrated by matching with the computational results in the literature and found to be outstanding agreement.

Keywords: Jeffrey fluid, Blasius flow, Sakiadis flow, Melting heat transfer, Radiation.

1. Introduction

The non-Newtonian fluids have been widely established by many engineers and scientists because of their appropriate industrial and engineering applications. The examples of those fluid models are applesauce, waste fluids, food products, condensed milk and toothpaste. In particular, these fluids are involved in the melting of plastics, cosmetics and beautifying agents, polymers and nourishment preparing, coal slurries, etc. The rheological behavior of such fluids can be expressed as many constituent relationships due to their versatile nature. Therefore, a diversity of non-Newtonian liquids exhibiting distinct rheological behavior has been examined by many researchers [1-5]. In fact, several models are established to recognize the performance of the non-Newtonian fluids such as viscoelastic fluid, Oldroyd-B, nanofluid, Maxwell fluid, Sisko fluid, Jeffrey fluid. Amongst, Jeffrey fluid model is the most often used model due to its time derivative or rather converted derivative. The Jeffrey fluid model is employed to formulate the flows associated with modern industrial materials such as polymer solutions (melts & solutions), blood model and multi-phase systems (foams, emulsion, slurries, etc). This liquid model describes the notable attributes of relaxation and retardation times. Recently, the influence of heat transfer on the Jeffrey non-Newtonian fluid model properties is described by many researchers [6-12].

The non-Newtonian liquid flows generated by a continuously moving surface are of massive interest in many engineering and industrial applications, such as polymer sheet, fiber glass production, melt-spinning process, spinning of fibers, continuous casting and hot rolling. For instance, the quality and physical characteristics of each and every product depend on rate of heat transfer along the surface. In view of all these practical claims, Blasius [13] examined numerical solutions of a power law fluid flow towards sheet moving with uniform velocity. Later, the same work was extended by Sakiadis [14] considering
continuously moving surface. Further, this work was numerically and theoretically studied by many researches [15–18] under various aspects. Hayat et al. [19] characterized the heat transfer flow in a Jeffrey liquid over a continuously moving surface by implementing the HAM. Jilal et al. [20] have utilized a Powell-Eyring fluid past a continuously moving permeable surface. Anjali Devi et al. [21] explored the impact of Blasius and Sakiadis flow of the MHD mixed convection nano-liquid flow past a plate. Mustafa et al. [22] developed the Sakiadis flow of a non-Newtonian (Maxwell fluid) fluid using convective boundary conditions. Ramesh et al. [23] reported the numerical solution to Blasius and Sakiadis flows of Williamson non-Newtonian liquid under composite velocity and convective surface conditions. Some other latest works have been done on various flow models under different heat and mass transfer effects [24-29].

In the past few decades, the massive utility of heat transfer accompanied by melting phenomenon is considered in many researchers due to its numerous applications, for example, casting and welding process, latent heat storage, crystal growth, material processing, purification of materials, heat transportation melting of permafrost, preparation of semiconductors material and others. Roberts [30] first proposed to scrutinize the shielding effect on melting body of ice in a plane surface to hot stream of air. Epstein et al. [31] inspected persuade of melting heat processes in steady flow owing a flat plate. Some other progressive work contributed for melting heat transfer effects on Newtonian fluids along different channels by [32–33]. Recently, Azizah et al. [34] discussed the micropolar fluid in the direction of a linearly stretching and shrinking sheet with the melting process. They found the dual solutions which are unique for both the shrinking and stretching cases. Mabood et al. [35] investigated the impacts of variable fluid properties on heat transfer in a Casson-liquid over a permeability moving surface. They observed that the heat transfer rate rises as the Casson-fluid parameter increases. Khan et al. [36] deliberated the melting heat transfer influence on generalized Burgers fluid. Investigations related to the melting heat transfer effect on non-Newtonian fluid models was reported in [37- 39].

In view of all the cited studies above, it is concluded that the thermal radiation and melting heat transfer effects on the flow of a Jeffrey liquid past a continuously moving surface is not scrutinized up to yet. Thus, to fill this gap, the prime object of this article is to explore the impact of melting processes and thermal radiation on flow of Jeffrey liquid over a moving surface with free stream velocity. The nonlinear coupled dimensionless equations from the governing equations are attained by employing appropriate similarity transformations. Finally, the resulting dimensionless equations are solved computationally by implementing the RKF method through bvc45 in MATLAB software. The graphs for velocity component, skin-friction coefficient, temperature field and local Nusselt number are exhibited and deliberated.

2. Mathematical formulations of the problem

A steady flow of a Jeffrey non-Newtonian fluid towards a moving surface under uniform velocity \( u_w \) and the uniform free stream velocity \( u_\infty \) in the similar direction are displayed in Fig. 1. The melting surface of wall subjected to fixed temperature \( T_m \) and the free stream condition \( T_\infty \) with \( T_\infty > T_m \) is considered. Also, the viscous dissipation and thermal radiation effects are taken into account. Under these assumptions, the governing equations can be reported in Cartesian system as listed in [12, 19]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\nu}{1 + \lambda} \left[ \frac{\partial^2 u}{\partial y^2} + \lambda \left( \frac{\partial u}{\partial y} \right)^2 + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial x^2} \right] + \frac{\partial q_r}{\partial y}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p (1 + \lambda)} \left( \frac{\partial u}{\partial y} \right)^2 + \lambda \left( \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial x^2} \right) \right)
\]
The boundary conditions are (see ref. [3, 31, 39-42]):

\[ u = u_w, \quad v = 0, \quad T = T_w \quad \text{at} \quad y = 0 \]  (4)

\[ u \rightarrow u_\infty, \quad \frac{\partial u}{\partial y} = 0, \quad T \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty \]  (5)

here

\[ k \left( \frac{\partial T}{\partial y} \right)_{y=0} = \rho \left\{ \lambda_2 + c_r (T_w - T_\infty) \right\} v (x, 0) \]  (6)

Utilizing Rosseland approximation of thermal radiation, we obtain the resultant expression

\[ \frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_w^3}{3K_s} \frac{\partial T}{\partial y}^2 , \quad \text{where} \quad q_r = \frac{4\sigma^* \partial T^4}{3K_s} \frac{\partial T}{\partial y} \]  (7)

Equation (7) is considered in Eq.(3) to attain the below equation

\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \alpha + \frac{16\sigma^* T_w^3}{3pc_k K_s} \right) \frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p (1 + \lambda)} \left[ \frac{\partial u}{\partial y} \right]^2 + \lambda_1 \left( u \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^2 u}{\partial y^2} \right) \]  (8)

Equations (1)-(3) can be easily transformed into a simpler form by establishing the following similarity transformation:

\[ u = U f'(\eta), \quad v = -\frac{U \nu}{2 \lambda} \left[ f(\eta) - \eta f'(\eta) \right], \quad \theta(\eta) = \frac{T - T_m}{T_w - T_m}, \quad \eta = \sqrt{\frac{U}{2 \nu \lambda}} y \]  (9)

where \( U = u_w + u_\infty \). Further, the stream function \( \psi \) is defined as \( u = \partial \psi / \partial y \) and \( v = -\partial \psi / \partial x \), which is inevitably satisfies Eq. (1). Utilizing aforementioned similarity transformation (i.e Eq.9), Eqs. (2) -(3) becomes

\[ (\eta \beta f' - \beta f f''') + (1 - 2 \beta f') f'' + (1 + \lambda) f f' - \beta f f'' = 0 \]  (10)

\[ (1 + \lambda)(1 + R)\beta f'' + Pr(1 + \lambda) f \theta' + Pr Ec \left[ f'' - \beta f' f'' - \beta f f'' \right] = 0 \]  (11)

where primes denote differentiation with respect to \( \eta \).

\[ A = \frac{u_w}{U}, \quad \beta = \frac{\lambda_1 U}{2x} \quad \text{See ref. [7]}, \quad \Pr = \frac{\nu}{\alpha}, \quad Ec = \frac{U^2}{c_p (T_w - T_m)} \quad \text{and} \quad R = \frac{16\sigma^* T_w^3}{3K_s k} \]  (12)

Here \( Ec > 0 \) indicates that the heated wall and when \( Ec = 0 \), the viscous dissipation term is absence in energy Eq. (3). The boundary conditions (4) and (5) become

\[ f'(\eta) = A, \quad Pr f + H \theta'(\eta) = 0, \quad \theta(\eta) = 1 \quad \text{at} \quad \eta = 0 \]  (13a)

\[ f'(\eta) = 1 - A, \quad f''(\eta) = 0 \quad \text{at} \quad \eta \rightarrow \infty \]  (13b)

The boundary condition \( A = 0 \) represents the Blasius flow (i.e. laminar boundary layer flow induced by a stationary surface) and \( A = 1 \) indicates the Sakiadis flow (i.e. moving surface in absence of free steam velocity). When \( A < 1 \), the free stream velocity is positive \( x \)-direction whereas the plate moves to the negative \( x \)-direction (See ref. [43]). Here \( H = [c_r (T_w - T_\infty)] / [\lambda_2 + (T_w - T_\infty)] \). The skin friction coefficient and the local Nusselt number are required engineering physical quantities of the problem and these are defined as

\[ C_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_{w_\infty}}{k (T_w - T_\infty)} \]  (14)

where

\[ \tau_w = \frac{\mu}{1 + \lambda} \left[ \frac{\partial u}{\partial y} + \lambda_1 \left( u \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^2 u}{\partial y^2} \right) \right]_{y=0} \]  (15)
Using (9), we obtain the following

$$\text{Re}^{\frac{1}{2}} C_f = \frac{1}{1+\lambda} \left[ f''(0) - \beta f'(0) f''(0) - \beta f(0) f'''(0) \right], \quad \text{Nu}_x \text{Re}^{\frac{1}{2}} = -\theta(0)$$

(17)

where $\text{Re}_x = [u_a(x), x]/\nu$ is the local Reynolds number.

3. Results and Discussion

The Figures 2-12 are depicted for the heterogeneous physical quantities on the velocity, local skin-friction coefficient, temperature and the Nusselt number in presence of both Blasius and Sakiadis flow. The comparability between local Nusselt number for different values of Prandtl number with those of Refs. [41, 42] are arrayed in Table 1 and the results are in good alliance with existing. Also, the local Nusselt number magnitude rises with an increase of $Pr$.

<table>
<thead>
<tr>
<th>$Pr$</th>
<th>Present Results</th>
<th>Hayat et al. [41] ($K=0$)</th>
<th>Bianachi &amp; Viskanta [42]</th>
</tr>
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<tr>
<td>0.01</td>
<td>-0.0519</td>
<td>-0.0519</td>
<td>-0.0519</td>
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<td>-0.728</td>
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</table>

The effects of Deborah number $\beta$, ratio of relaxation and retardation times $\lambda$ on the velocity profile for both Blasius and Sakiadis flows are presented respectively in Figs. 2 and 3. The velocity of the fluid declines with rising values of $\beta$ in case of Blasius flow, and opposite behavior is seen in the rising values of $\lambda$. Both $\beta$ and $\lambda$ show the opposite nature in case Sakiadis flow. Physically, in case of Blasius flow ($A=0$), there is no moment in the surface and hence the moment of the fluid is less compare to that of Sakiadis flow ($A=1$). Also, Deborah number $\beta$ is proportional to the rate of sheet. As $\beta$ rises, the stretching rate increases which leads to fluid flow increases.

The importance of melting parameter $H$ on the horizontal velocity and temperature profiles is plotted, respectively, in Figs. 4 and 5. As melting parameter $H$ increases, $f'(|\eta|)$ rises for the case of Blasius flow, while the reverse trend is seen in the case of Sakiadis flow. In addition, much heat transfer to cold melting surface due to high convection is marked by greater melting parameter values.

From Fig.5 it is noticed that $\theta(\eta)$ decelerates for higher values of $H$ and the thickness of the thermal boundary layer increase for an increase in $H$. Virtually, the transfer of more heat on the melting surface from heated fluid is proven greater $H$ values. Therefore, temperature distribution decreases. Also, it is noticed that the decay in the thermal boundary layers for positive values of Sakiadis flow in the case of non-Newtonian and Newtonian ($\lambda = 0$ and $\beta = 0$) fluids.

The deviations of $\lambda$ on $\theta(\eta)$ for both cases of Blasius and Sakiadis flow are demonstrated in Fig. 6. $\theta(\eta)$ decelerates with the rising values $\lambda$ for the Blasius and Sakiadis flows. Fig.7 draws to examine the behavior of Prandtl number on the temperature profile. In rising values of Prandtl number, there is a diminishing pattern in the temperature distribution for both the Blasius and Sakiadis flows. Fundamentally, the fluids thermal diffusivity and Prandtl number are inversely related. Due to
this, the temperature and thermal boundary layer thickness decreases.

![Graph](image1)

**Fig. 4.** Effect of $H$ on Velocity, when $\beta = 0.1; \lambda = 0.1; Ec = 1; Pr = 1; R = 0.1$

![Graph](image2)

**Fig. 5.** Effect of $H$ on Temperature, when $\beta = 0.1; \lambda = 0.1; A = 1; Ec = 0.3; Pr = 1; R = 0.1$

The influence of the thermal radiation $R$ on temperature distribution is shown in Fig. 8. The temperature distribution declines for larger values of thermal radiation $R$. In reality, increase in $R$ enhances thickness of thermal boundary layer and flux of energy transport to the fluid temperature. The impact of Eckert number $Ec$ on $\theta(\eta)$ is exposed in Fig. 9. It is indicated that temperature profile $\theta(\eta)$ is an increasing function of $Ec$. The boundary layer viscosity rises when $Ec$ increases. Literally, the essential energy to enhance the temperature is attributed by the fluid particles which more active at higher $Ec$.

![Graph](image3)

**Fig. 6.** Effect of $\lambda$ on Temperature, when $\beta = 0.1; A = 0.1; Ec = 2; H = 1.0; Pr = 1; R = 0.1$

![Graph](image4)

**Fig. 7.** Effect of $Pr$ on Temperature, when $\beta = 0.1; \lambda = 0.1; Ec = 1; H = 1.0; R = 0.1$

Figs.10 and 11, respectively illustrate the impact of $\beta$ and $H$ against $A$ on skin friction coefficient. It is noticed that the exclarating values of $\beta$ is resposible for hike of skin friction coefficient in the range $0 \leq A \leq 0.5$ and later it shows opposie
effect as \( A \) incrases. On the other hand \( H \) exhibits opposite effect for \( f''(0) \). The drag force employed on the solid surface by the liquid is expressed by \( f''(0) > 0 \) and \( f''(0) < 0 \) implies the drag force of the fluid utilized by the solid surface. From both the figures it is observed that \( f''(0) \) has reverse nature at \( A=0.5 \), this is due to the presence of equal stretching and free stream velocities.

The drag force employed on the solid surface by the liquid is expressed by

\[
(0)\quad f''(0) > 0 \quad \text{and} \quad (0)\quad f''(0) < 0
\]

implies the drag force of the fluid utilized by the solid surface. From both the figures it is observed that \( (0)\quad f''(0) \) has reverse nature at \( A=0.5 \), this is due to the presence of equal stretching and free stream velocities.

Fig. 10. Effect of \( \beta \) on skin-friction coefficient against \( A \), when \( \lambda =0.1; \ Ec=1; H=1; Pr=1; R=0.1 \)

Fig. 11. Effect of \( H \) on skin-friction coefficient against \( A \), when \( \beta =0.1; \ \lambda =0.1; \ Ec=1; \ Pr=1.0; \ R=0.1 \)

Fig. 12 highlights the influence of \( Pr \) on Nusselt number against radiation parameter \( R \). It is obvious that the Nusselt number gradually rises by an increase in \( Pr \). Further, it is perceived that the heat transfer rate declines with the boost of radiation parameter \( R \) for both Sakiadis & Blasius flows. Also, it is marked that the Sakiadis flow is aloft Blasius flow and are coextending with each other.

Fig. 12. Effect of \( Pr \) on Nusselt Number against \( R \), when \( \beta =0.1; \ \lambda =0.1; \ Ec=1; \ H=1.0 \)

4. Conclusion

The melting heat transfer has several real-time applications like manufacturing processing and spinning of fibers, etc. Owing to this, the stretched flow of a rheological model, i.e. Jeffrey fluid over a continuously moving surface with melting heat transfer and thermal radiation under the impact of Blasius and Sakiadis flow, is examined. The non-linear coupled dimensionless equations from the governing equations are attained by employing appropriate similarity transformations. Finally, the resulting dimensionless equations are solved computationally by implementing Runge-Kutta-Fourth order method through bvc45 in MATLAB software. The main results of the given problem are depicted as follows:

- Melting parameter \( (H) \) has a boosting nature of temperature distributions in case of a non-Newtonian fluid.
- The velocity and the boundary layer thickness are decreasing functions of the Deborah number \( (\beta) \).
- The impact of melting parameter \( (H) \), Prandtl number \( (Pr) \), ratio of relaxation and retardation times \( (\lambda) \) on thermal boundary layer thickness & fluid temperature are the same and the reverse trend is observed in various values of thermal radiation, for both Blasius and Sakiadis flows.
- The velocity of Jeffrey fluid is higher in Blasius flow compared to that of Sakiadis flow.

Conflict of Interest

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Nomenclature

- \( A \) Moving parameter
- \( Ec \) Eckert number
- \( c_f \) Skin-friction coefficient
- \( c_p \) specific heat at constant pressure \([J/kg/K]\)
- \( c_s \) the heat capacity of the solid surface
- \( F \) dimensional stream function
- \( f' \) dimensionless velocity
- \( H \) Melting parameter
- \( k \) thermal conductivity of fluid \([W/m/k]\)
- \( K_r \) Rosseland mean absorption coefficient
- \( R \) radiation parameter
- \( Nu_x \) Nusselt number
- \( Pr \) Prandtl number
- \( q_r \) radiative heat flux \([W/m]\)
- \( q_w \) surface heat flux
- \( Re_x \) local Reynolds number
- \( T \) fluid temperature \((K)\)
- \( T_m \) melting temperature
- \( T_s \) the solid temperature
- \( T_\infty \) temperature far away from wall \((K)\)
- \( u, v \) velocity components in \( x-,y\)-directions, respectively \([m/s]\)
- \( u_w \) shrinking velocity \([m/s]\)
- \( x \) distance along wall \([m]\)
- \( y \) distance normal to wall \([m]\)

Greek symbols

- \( \alpha \) the thermal diffusivity of the fluid
- \( \beta \) Deborah number
- \( \eta \) similarity variable
- \( \lambda \) ratio of relaxation and retardation times
- \( \lambda_1 \) relaxation time \([s]\)
- \( \lambda_2 \) the latent heat of the fluid
- \( \mu \) dynamic viscosity \([Pa/s]\)
- \( \nu \) kinematic viscosity \([m^2/s]\)
- \( \rho \) fluid density \([kg/m^3]\)
- \( \sigma \) electric conductivity \([Sm\)]
- \( \sigma^* \) Stefan-Boltzman constant \([Wm^2K^{-4}]\)
- \( \theta \) non-dimensional temperature
- \( \psi \) steam function

Subscripts

- \( s \) the solid temperature
- \( T_\infty \) temperature far away from wall \((K)\)
- \( w \) sheet surface
- \( \infty \) Infinity

Superscript

- \( w \) shrinking velocity \([m/s]\)
- \( ' \) differentiation with respect to \( \eta \)

References

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