

Journal of

Applied and Computational Mechanics



Global Finite Time Synchronization of Two Nonlinear Chaotic Gyros Using High Order Sliding Mode Control

Mohammad Reza Behjameh¹, Hadi Delavari², Ahmadreza Vali³

¹MSc, Faculty of Electrical and Electronic Engineering, Department of Control Engineering, Malek Ashtar University of Technology Shahid Babayi Highway, Lavizan, Tehran, 15875-1774, Iran, r.Behjameh@gmail.com

²Assistant Professor, Department of Electrical and Electronic Engineering, Hamedan University of Technology Shahid Fahmideh Street, Hamedan, 65155-579, Iran, hdelavari@gmail.com

³Assistant Professor, Faculty of Electrical and Electronic Engineering, Department of Control Engineering, Malek Ashtar University of Technology Shahid Babayi Highway, Lavizan, Tehran, 15875-1774, Iran, vali@mut.ac.ir

Received April 27 2014; revised June 01 2014; accepted for publication June 01 2014. Corresponding author: r.Behjameh@gmail.com , hdelavari@gmail.com

Abstract

In this paper, under the existence of system uncertainties, external disturbances, and input nonlinearity, global finite time synchronization between two identical attractors, which belong to a class of second-order chaotic nonlinear gyros, are achieved by considering a method of continuous smooth second-order sliding mode control (HOAMSC). It is proved that the proposed controller is robust to mismatch parametric uncertainties. Also it is shown that the method have excellent performance and more accuracy in comparison with conventional sliding mode control. Based on Lyapunov stability theory, the proposed controller and some generic sufficient conditions for global finite time synchronization are designed such that the errors dynamic of two chaotic behavior satisfy stability in the Lyapunov sense. The numerical results demonstrate the efficiency of the proposed scheme to synchronize the chaotic gyro systems using a single control input.

Keywords: smooth second-order sliding mode control (SSOSMC), chaos synchronization, Lyapunov stability, uncertainty, finite time converges, chaotic gyros stability.

1. Introduction

Since Pecora and Carroll first demonstrated the synchronization of two identical chaotic systems under different initial conditions [1], the feasibility of synchronizing chaotic systems of various types has attracted significant interest. In essence, the chaos synchronization problem entails controlling the dynamic behavior of a "slave" system by means of a control input, such that its oscillation, following a period of transition, mimics that of the "master" system. Nowadays, chaos and its synchronization have found application in many fields of engineering and science such as in secure communications, chemical reactions, power converters, biological systems, and information processing, mechanics, etc. [2-3]. Many methods have been presented for the control and synchronization of chaotic systems [4–9]. Gyros are a particularly interesting form of nonlinear system, and have attracted intensive study in recent decades due to their utility in the navigational, aeronautical and space engineering domains. Recent research has identified various forms of gyro systems with linear or nonlinear damping characteristics and has shown that these systems exhibit a diverse range of dynamic behavior, including both sub- harmonic and chaotic motions [10–13]. Zheng [14] extended the findings of Chen [15] and applied active control theory to synchronize two identical chaotic gyros with nonlinear damping. Various approaches for achieving chaos synchronization using fuzzy systems have been proposed [16]. Furthermore, adaptive fuzzy controllers have been used to control and synchronize chaotic systems [17-18].

In [19] higher order sliding mode control techniques, in specific "prescribed convergence law", "quasi continuous" and "super twisting" control algorithms are used to robustly stabilize the glucose concentration level of a diabetic patient in presence of the parameter variations and meal disturbance. In [20] finite time second order sliding mode is proposed for removing chattering in systems with relative degree two. In [21] a second order sliding mode controller is designed for uncertain linear systems with parametric uncertainty and [22] studies the applicability of "sub-optimal", "twisting", "super-twisting" and "with a prescribed law of variation" algorithms of second order sliding mode control strategies to a wind energy conversion system, which present finite time convergence, robustness, chattering and mechanical stress reduction, and are of quite simple online operation and implementation.

Accordingly, the current study develops a smooth second order sliding mode control to synchronize two identical chaotic gyros systems with different initial conditions, system uncertainties and external disturbances. Based on the proposed control, scheme convergence is proved using a Lyapunov function in a way that the error state trajectories converge in finite time to zero.

The rest of the paper is organized as follow: Section 2 explores the dynamic of nonlinear gyros system. In section 3, the definition of chaos synchronization of the two coupled gyro systems is obtained and the current synchronization problem is formulated. Section 4 for chaos synchronization of the gyro systems in general masterslave configuration a smooth sliding mode control is introduced and designed. Section 5 discusses stability condition. Section 6 presents the results of simulations designed to evaluate the effectiveness of the proposed scheme. Finally, section 7 draws some brief conclusions.

2. Description of Nonlinear Gyro Modeling

The equation governing the motion of a symmetric gyro mounted on a vibrating base is shown in Fig.1 in terms of the notation angle θ , i.e. the angle which the spin axis of the gyro has with the vertical axis, is studied by Chen [15]. The sketch of a single axis, rate-integrating floated gyro is shown in Fig.2. This system is used in aircraft and ships to afford to wait while the gyros are warmed up and could supply the necessary power, which continue to be the case in strategic defense systems. Also, the dynamic of a symmetrical gyro with linear-plus-cubic damping of the angle θ can be expressed as:

$$\theta + \alpha^2 ((1 - \cos\theta)^2 / \sin^3\theta) - \beta \sin(\theta) + c_1 \theta + c_2 \theta^3 = f \sin wt \sin \theta$$
⁽¹⁾

Where $f \sin wt$ represents a parametric excitation, $c_1 \dot{\theta}$ and $c_2 \dot{\theta}^3$ are linear and nonlinear damping terms, respectively, and $\alpha^2 ((1-\cos\theta)^2 / \sin^3\theta) - \beta \sin(\theta)$ is a nonlinear resilience force.



Fig. 1. A schematic diagram of a symmetric gyroscope.

By giving the states $x_1 = \theta$, $x_2 = \theta$ this system can be transformed into the following nominal. Consider that $g(\theta) = -\alpha^2 ((1 - \cos \theta)^2 / \sin^3 \theta) - \beta \sin(\theta)$ then:

$$x_2 = g(x_1) - c_1 x_2 - c_2 x_2^2 + (\beta + f \sin wt) \sin(x_1)$$

This gyro system exhibits complex dynamics and has been studied by Chen [15] for values of $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$ and w = 2. The values of f is in the range 32 < f < 36. Figs.3 and 4 illustrate the irregular

Journal of Applied and Computational Mechanics, Vol. 1, No. 1, (2015), 26-34

motion exhibited by this system for f = 35.5 and initial conditions of $(x_1, x_2) = (1, -1)$.



The next section examines the problem of synchronizing two identical gyros with different initial conditions and parameter uncertainties and introduces the HOSMC to cope with the system uncertainties and external disturbances appearing in the slave system.



Fig. 4. Time history of chaotic gyro. X_2 versus time t.

3. Formulation and Model of Error Dynamical System

In this part, the synchronization model between the master system and the slave system will be studied. For this purpose, we choose the system (3) as the drive system and system (4) as the response system. This implies that when the drive-response system is synchronized, system (4) will trace the dynamics of system (3). Our aim is to design

Journal of Applied and Computational Mechanics, Vol. 1, No. 1, (2015), 26-34

controllers u(t) that will make the response system achieve synchronism with the drive system. Consider two coupled, chaotic gyro systems of the form:

$$x_1 = x_2 \tag{3}$$

$$\dot{x}_2 = g(x_1) - c_1 x_2 - c_2 x_2^3 + (\beta + f \sin wt) \sin(x_1)$$

And consider the coupled system of Eq3, chaotic gyro systems of the form [4]

$$y_{1} = y_{2}$$

$$y_{2} = g(y_{1}) - c_{1} y_{2} - c_{2} y_{2}^{3} + (\beta + f \sin wt) \sin(y_{1})$$

$$+ \Delta f(y_{1}, y_{2}) + d(t) + u(t),$$
(4)

where $u \in R$ is the control input, $\Delta f(y_1, y_2)$ is an uncertainty term representing the unmodeled dynamics or structural variation of the system and d(t) is the time-varying disturbance. In general, the uncertainty and the disturbance are assumed to be bounded as $|\Delta f| \leq \alpha$, $|d(t)| \leq b$, where α and b are positive constants.

In the discussions above, the systems described in Eq. 3 and 4 correspond to the master system and the slave system, respectively, and the objective of the current control problem is to design an appropriate control signal u(t) such that for any initial conditions of the two systems, the behavior of the slave converges to that of the master, i.e.

$$\left\|\lim_{t \to \infty} (\mathbf{y}(t) - \mathbf{x}(t))\right\| = \left\|\lim_{t \to \infty} e(t)\right\| \to 0 \tag{5}$$

where $\|.\|$ is the Euclidean norm of a vector.

If the error states of the coupled systems are defined as $e_1 = y_1 - x_1$ and $e_2 = y_2 - x_2$, respectively, then the dynamic equations of these errors can be determined directly by subtracting Eq. (3) from Eq. (4), then by rearranging the equation, the following error dynamical system is obtained.

$$e_{1} = e_{2}$$

$$e_{2} = -c_{1}e_{2} + g(e_{1} + x_{1}) - g(x_{1}) - c_{2}(e_{2} + x_{2})^{3}$$

$$+ c_{2}x_{2}^{3} + (\beta + f \sin wt)[\sin(e_{1} + x_{1}) - \sin x_{1}]$$

$$+ \Delta f(e_{1} + x_{1}, e_{2} + x_{2}) + d(t) + u(t)$$
(6)

4. Introduction and Design of Smooth (High Order) Second Order Sliding Mode Control (SSOSMC)

A Consider SISO dynamics:

$$s = g(t) + u, \qquad s \in R \tag{7}$$

which will be further interpreted as the sliding variable dynamics calculated along the system trajectory. The condition s = 0 defines the system motion on the sliding surface, $u \in R$ is a control input that needs to be smooth, and g(t) is a sufficiently smooth uncertain function.

The problem that is addressed in this section is to design smooth control u that drives $s, s \rightarrow 0$ in finite time. The drift term g(t) is to be cancelled by means of a special observer to be developed further. The prescribed compensated s-dynamics in (7) is chosen as:

$$x_{1} = -\alpha_{1} |x_{1}|^{(p-1)/p} sgn(x_{1}) + x_{2}$$

$$x_{2} = -\alpha_{2} |x_{1}|^{(p-2)/p} sgn(x_{1})$$
(8)

Definition. We call a system finite-time stable if it is asymptotically stable with a finite settling time for any solution and initial conditions [24].

Lemma 1. Let $p \ge 2$, $\alpha_1, \alpha_2 > 0$. Then the system (8) is finite time stable with the settling time being a continuous function of the initial conditions, vanishing at the origin.

Proof. Consider the following Lyapunov function candidate:

30 Mohammad Reza Behjameh, Hadi Delavari, Ahmadreza Vali Vol. 1, No. 1, 2015

$$V = \frac{x_2^2}{2} + \int_0^{x_1} |s|^{(p-2)/(p)} sgn(s) ds, \qquad p = m+1$$

$$= \frac{x_2^2}{2} + \frac{p}{2p-2} \alpha_2 x_1^{(2p-2)/p}$$
(9)

Its derivative is:

$$\dot{V} = \frac{\delta V}{\delta x} \dot{x}$$

$$= [\alpha_{2} |x_{1}|^{(p-2)/p} sgnx_{1}, x_{2}] \cdot \begin{bmatrix} x_{2} - \alpha_{1} |x_{1}|^{(p-1)/p} sgnx_{1} \\ -\alpha_{1} |x_{1}|^{(p-2)/p} sgnx_{1} \end{bmatrix}$$

$$= \alpha_{2} x_{2} x_{1}^{(p-2)/p} sgn(x_{1}) - \alpha_{2} x_{2} x_{1}^{(p-2)/p} sgn(x_{1}) - \alpha_{1} \alpha_{2} |x_{1}|^{(p-2)/p+(p-1)/p}$$

$$= -\alpha_{1} \alpha_{2} |x_{1}|^{(2p-3)/p}$$
(10)

Apply the LaSalle theorem. The set $x : \{V(x) = 0\}$ consists of the axis $x_1 = 0$, and the only invariant set inside $x_1 = 0$ is the origin $x_1 = x_2 = 0$. Thus, the finite time convergence of $x_1 = 0$ and $x_2 = 0$ to zero is assured. It is easy to see that system then the finite time stability implies here the finite-time stability and the continuity (and the homogeneity) of the settling time function. Lemma 1 is proven.

The sliding variable dynamics (7) is sensitive to the unknown bounded term g(t). Let the variables s(t) and u(t) be available in real time, g(t) be (m-1)-times differentiable, so that $g^{(m-1)}(t)$ has a known Lipchitz constant L > 0.

The control function u(t) is Lebesgue-measurable. Eq. (7) is understood in the Filippov sense [23], which means in particular that s(t) is an absolutely continuous function defined for $\forall t \ge 0$.

Let the sliding variable dynamics be of the form (7), with g(t) being (m - 1)-smooth with a known Lipchitz constant L > 0 of $g^{(m-1)}(t)$. Control u is easily provided by:

$$u = u_{eq} - \alpha_1 |s|^{m/(m+1)} sgn(s) + w,$$

$$w = -\alpha_2 |s|^{(m-1)/(m+1)} sgn(s).$$
(11)

In the SMC field, the sliding surface is generally taken to be:

$$s = e_2 + \lambda e_1, \tag{12}$$

where λ represents a real number. From [24], the existence of the sliding mode requires the following conditions to be satisfied:

$$s = e_2 + \lambda e_1 = 0 \tag{13}$$

and

$$s = e_2 + \lambda e_1 = 0 \tag{14}$$

(1.4)

Accordingly, the equivalent control law is given by:

$$u_{eq} = -\lambda e_2 + c_1 e_2 - g(e_1 + x_1) + g(x_1) + c_2(e_2 + x_2)^3 - c_2 x_2^3 - (\beta + f \sin wt) [\sin(e_1 + x_1) - \sin x_1] - \Delta f(e_1 + x_1, e_2 + x_2) - d(t) + u(t)$$
(15)

In the sliding mode, the error dynamics become

$$e_1 = e_2 = -\lambda e_1, \tag{15a}$$

$$e_2 = -c_1 e_2.$$
 (15b)

Using SSOSM control (9), with p = 3 and m = 2 the control law is obtained as follow:

Journal of Applied and Computational Mechanics, Vol. 1, No. 1, (2015), 26-34

Global Finite Time Synchronization of Two Nonlinear Chaotic Gyros Using High Order Sliding Mode Control 31

$$u = -\lambda e_2 + c_1 e_2 - g(e_1 + x_1) + g(x_1) + c_2(e_2 + x_2)^3 - c_2 x_2^3 - \alpha_1 |s|^{2/3} sgn(s) + w - \int \alpha_2 |s|^{1/2} sgn(s) ds$$
(16)

The process of synchronization is shown in Fig.5 simply.



Fig. 5. The block diagram of synchronization process

5. Stability Analysis

The ability of the control signal given in Eq. (16) to drive the error of the system in Eq. (6) to the sliding mode s(t) = 0, i.e. to guarantee the attainment of the reaching condition V(t) < 0 can be expressed in the form of the following theorem.

Theorem 1. Consider the master– slave system given in Eqs. (3) and (4). The two systems are synchronized by the control signal u(t) defined in Eq. (16) Thus, the error state trajectory converges to the sliding surface s(t) = 0.

Proof. Let the Lyapunov function of the system be defined as $V = 0.5s^2$. The first derivative of this system with respect to time can be expressed as:

$$V = ss$$

$$= s[e_{2} + \lambda e_{1}]$$

$$= s[-c_{1}e_{2} + g(e_{1} + x_{1}) - g(x_{1}) - c_{2}(e_{2} + x_{2})^{3} + c_{2}x_{2}^{3} + (\beta + f \sin wt)[\sin(e_{1} + x_{1}) - \sin x_{1}] + \Delta f(e_{1} + x_{1}, e_{2} + x_{2}) + d(t) + u_{eq}(t) + ku(s) + \lambda e_{2}]$$

$$= s[\Delta f(e_{1} + x_{1}, e_{2} + x_{2}) + d(t) + ku(s)]$$

$$\leq |s|(\beta + \alpha) - k|s| = -(k - (\beta + \alpha))|s|.$$
(17)

Since $k > (\beta + \alpha)$, the reaching condition V(t) < 0 is always satisfied. Thus, the proof is achieved.

6. Simulation

In this section, the synchronization behavior between the two chaotic nonlinear gyros will be studied. For this purpose, we choose the system (3) as the drive system and the system (4) as the response system. This implies that when the drive-response system is synchronized, the system (4) will trace the dynamics of the drive system. Results of the work and discussions are presented here.

The parameters of the nonlinear gyros systems are specified as follows: $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, w = 2 and f = 35. The initial conditions are defined as: $x_1(0) = 1$, $x_2(0) = -1$, $y_1(0) = 1.6$, $y_2(0) = 0.8$. An assumption is made that the uncertainty term, i.e., $\Delta f(e_1 + x_1, e_2 + x_2) = -0.1 \sin(y_1)$ and the disturbance term, i.e. $d(t) = 0.3 \cos(\pi t)$, are bounded by $\Delta f(e_1 + x_1, e_2 + x_2) \le \alpha = 0.1$ and |d(t)| = 0.3 respectively. The simulation results are shown in Figs.6–Figs.11. Figs.6 shows control input signal. Figs.7 and Figs.8 confirm that the master and slave systems achieve a synchronized state following control activation.



Fig. 6. Control input chaotic gyro synchronization system..



Fig. 7. Time responses of controlled chaotic gyro synchronization. (master and slave system output are x_1, y_1)



Fig. 8. Time responses of controlled chaotic gyro synchronization. (master and slave system output are x_2, y_2)

The results demonstrate that the system error states are regulated to zero in finite time following activation of the control signal at t=2s. In addition, it can be seen that the control input is chatter free even though the overall system is subject to uncertainty and disturbance.

In Figure 9, sliding variable trajectory is shown and it is illustrated that reaching phase in proposed controller is less than conventional SMC. As it can be seen in Fig.10 and 11, accuracy and performance of proposed control algorithm (smooth second order sliding mode control) for synchronization of chaotic system is much better than conventional control method (sliding mode algorithm.)



Fig. 11. Time response of error states

7. Conclusion

In this paper, a novel smooth second-order sliding mode (SSOSM) control is studied and its finite-time convergence proved for a system driven by sufficiently smooth uncertainty and disturbances. SSOSM control was applied to make synchronization between two identical chaotic nonlinear systems via smooth second order sliding mode control. It is demonstrated via simulations that the chaotic systems, after activating smooth second order sliding mode controller signal in t=2s, is synchronized as fast as possible. The performance and time of synchronization chaotic systems via smooth second order sliding mode control decrease in comparison with the sliding mode controller. It is also shown synchronization of the chaotic systems is controlled in the presence of uncertainty as well as without parameter uncertainty.

References

Pecora, L.M., Carroll, T.L., 1990. "Synchronization in Chaotic Systems", Physical Review Letters 64, 821–824.
 Nayfeh A.H., Applied Nonlinear Dynamics, Wiley, New York, 1995.

34 Mohammad Reza Behjameh, Hadi Delavari, Ahmadreza Vali Vol. 1, No. 1, 2015

[3] Chen, G., Dong, X., From Chaos to Order: Methodologies, Perspectives and Applications, World Scientific, Singapore, 1998.

[4] Delavari, H., Ghaderi, R., Ranjbar A., Momani, S., 2010. "Fractional order control of a coupled tank," Nonlinear Dynamics, 61, 383–397.

[5] Delavari, H., Ghaderi, R., Ranjbar, A., Momani, S., 2010. "Synchronization of chaotic nonlinear gyro using fractional order controller," Berlin, Springer, 479–485.

[6] Delavari, H., Mohammadi Senejohnny, D., Baleanu, D., 2012. "Sliding observer for synchronization of fractional order chaotic systems with mismatched parameter," Central European Journal of Physics ,10(5), 1095-1101.

[7] Faieghi, M.R., Delavari, H., 2012. "Chaos in fractional-order Genesio–Tesi system and its synchronization," Communications in Nonlinear Science and Numerical Simulation, 17, 731-741.

[8] Delavari, H., Lanusse, P., Sabatier, J., 2013. "Fractional Order Controller Design for a Flexible Link Manipulator Robot," Asian Journal of Control, 15(3), 783–795.

[9] Delavari, H., Ghaderi, R., Ranjbar, A., Momani, S., 2010. "A Study on the Stability of Fractional Order Systems," FDA2010, University of Extremadura, Badajoz, Spain, October 18-20.

[10] Salarieh, H., Alastyisc, A., 2008. "Chaos synchronization of nonlinear gyros in presence of stochastic excitation via sliding mode control," Journal of Sound and Vibration, 313, 760–771.

[11] Che, Y.Q., Wang, J., Chan, W., Tsang, K.M., Wei, X.L., Deng, B., 2009. "Chaos Synchronization of Gyro Systems via Variable Universe Adaptive Fuzzy Sliding Mode Control," Proceedings of the 7th Asian Control Conference, 27-29.

[12] Chen, T.U., Zhan, W.I., Lin, C.M., Yeung, D.S., 2010. "Chaos Synchronization of Two Uncertain Chaotic Nonlinear Gyros Using Rotary Sliding Mode Control," Proceedings of the Ninth International Conference on Machine Learning and Cybernetics, July 11-14.

[13] Yang, C.C., Ouech, C.J., 2013. "Adaptive terminal sliding mode control subject to input nonlinearity for synchronization of chaotic gyros," Commun Nonlinear Sci Numer Simulat., 18, 682–691.

[14] Lei, Y., Xu, W., Zheng, H., 2005. "Synchronization of two chaotic nonlinear gyros using active control," Physics Letters A, 343, 153–158.

[15] Chen, H.K. 2002. "Chaos and chaos synchronization of a symmetric gyro with linear-plus-cubic damping," Journal of Sound and Vibration, 255, 719–740.

[16] Tanaka, K., Ikeda, T., Wang, H.O., 1998. "A unified approach to controlling chaos via LMI-based fuzzy control system design," IEEE Transactions on Circuits and Systems I, 45, 1021–1040.

[17] Feng, G., Chen, G., 2005. "Adaptive control of discrete-time chaotic systems: a fuzzy control approach," Chaos Solitons & Fractals, 23, 459–467.

[18] Xue, Y.J., Yang, S.Y., 2003. "Synchronization of generalized Henon map by using adaptive fuzzy controller," Chaos Solitons & Fractals, 17, 717–722.

[19] Kaveh, P., Shtessel, Y.B., 2008. "Blood glucose regulation using higher-order sliding mode control," International Journal of Robust and Nonlinear Control, 18(4-5), 557–569.

[20] Levant, A. 2007. "Principles of 2-slidingmode design," Automatica, 43, 576-586.

[21] Mondal, S., Mahanta, Ch. 2011. "Nonlinear sliding surface based second order sliding mode controller for uncertain linear systems," Commun Nonlinear Sci Numer Simulat, 16, 3760–3769.

[22] Evangelista, C. Puleston, P., Valenciaga, F., 2010. "Wind turbine efficiency optimization. Comparative study of controllers based on second order sliding modes," International Journal of Hydrogen Energy, 35, 5934–5939.

[23] Filippov, A. "Differential equations with discontinuous right-hand side," Dordrecht, Netherlands: Kluwer Academic Publishers, 1988.

[24] Slotine, J. J. E., Li, W., Applied Nonlinear Control, Prentice-Hall, Upper Saddle River, NJ, pp. 276-309. 1991.

[25] Lawrence, A., Modern Internal Technology, navigation, guidance and control, Springer, TL, 588.L38, 1988.