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Determination of Periodic Solution for Tapered Beams with Modified Iteration Perturbation Method

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Abstract

In this paper, we implemented the Modified Iteration Perturbation Method (MIPM) for approximating the periodic behavior of a tapered beam. This problem is formulated as a nonlinear ordinary differential equation with linear and nonlinear terms. The solution is quickly convergent and does not need complicated calculations. Comparing the results of the MIPM with the exact solution shows that this method is effective and convenient. Also, it is predicated that MIPM can be potentially used in the analysis of strongly nonlinear oscillation problems accurately.

Keywords: Periodic behavior, Tapered beam, Modified Iteration Perturbation Method (MIPM), Nonlinear ordinary differential equation, Nonlinear oscillation.

1. Introduction

The demand for engineering structures is continuously increasing. Aerospace vehicles, bridges, and automobiles are examples of these structures. Many aspects have to be taken into consideration in the design of these structures to improve their performance and extend their life. One aspect of the design process is the dynamic response of structures. The dynamics of distributed parameter and continuous systems, like beams, were governed by linear and nonlinear partial differential equations in space and time. It was difficult to find the exact or closed-form solutions for nonlinear problems. Consequently, researchers have used two classes of approximate solutions of initial boundary-value problems: numerical techniques [1, 2], and approximate analytical methods [3, 4]. Beside all of the advantages of using numerical methods, closed-form solutions appear more appealing because they reveal physical insights through the physics of the problem. Also, by applying analytical methods, parametric studies become more convenient. Moreover, analytical solutions are generally required for the validation of numerical methods and computer softwares.

Approximate methods for studying non-linear vibrations of beams are important for investigating and designing purposes. During the recent years, some promising approximate analytical solutions have been proposed, such as Frequency Amplitude Formulation [5], Variational Iteration [6,7], Iteration Perturbation Method[8-11], Homotopy-Perturbation [12,13], Parameterized-Perturbation [14], Max-Min [15-17], Newton Harmonic Balance Method [18,19], Differential Transformation Method [20], Energy Balance [21,22], etc.

Some kind of these methods like the DTM is solved based on polynomial trial function and the equation should be solved for specific interval time domain separately in oscillation equations. In addition, some other methods like HPM and VIM are powerful methods and can be used for almost all types of nonlinear equations, which does not need small parameters for solving the equations [23].

Moreover, there are some newer methods introduced for oscillatory equations such as IPM, FAF, MMA, EBM and MIPM. The IPM is very prolific, rapid and does not demand small perturbation and is also sufficiently accurate to both linear and nonlinear problems in engineering [9]. The FAF method is an efficient technique in finding analytical solutions to wide classes of nonlinear oscillator and doesn't need programming, but it is important to choose a correct frequency for solving some complicated problems [5]. The MMA is based on an ancient Chinese mathematics and is valid for weak and strong nonlinear systems for which maximum/minimum thresholds can be found. In max-min method the angular frequency can be readily obtained [15-17]. In EBM, a variational principle for the nonlinear oscillation is established, then a Hamiltonian is constructed, from which the angular frequency can be easily obtained by collocation method [21, 22].

Recently, some researchers have studied the periodic behavior of tapered beams. Goorman [24] derived the differential equation of the tapered beam. Evensen [25] investigated nonlinear vibration of the beam with various boundary conditions by using the perturbation methods. Pillai and Rao [26] applied Galerkin approach and harmonic balance method to investigate vibration of the nonlinear beam. Barari et al.[27] studied the nonlinear vibration of the beams, subjected to the axial loads by using perturbations techniques. Klein [28] used finite element and Rayleigh-Ritz methods for analyzing the vibration of the tapered beams. Sato [29] improved the Ritz method to study a linear tapered beam with restrained ends against rotation and subjected to an axial force.

In this paper, Modified Iteration Perturbation Method is applied to scrutinize the free vibration analysis of the nonlinear tapered beams. This method is obtained by combining the IPM of He and Michens into a new iteration method that valid for small and large oscillation amplitude and can be used for nonlinear problems with a single degree of freedom.

2. Basic idea of Modified Iteration Perturbation Method

The Modified Iteration Perturbation Method (MIPM) is composed of the Mickens and Iteration methods. This method firstly presented by Marinca and Herisanu [30] in 2006. By considering the following equation as the general nonlinear oscillation:

$$\ddot{u}+\omega^2 u = f(u,\dot{u},\ddot{u}),\quad u(0)=A,\quad \dot{u}(0)=0 \tag{1}$$

We rewrite Eq. (1) in the following form:

$$\ddot{u}+\Omega^2 u = u(\Omega^2-\omega^2+\frac{f(u,\dot{u},\ddot{u})}{u}) := ug(u,\dot{u},\ddot{u}), \tag{2}$$

Where Ω is a priori unknown frequency of the periodic solution, u(t) being sought. The proposed iteration scheme is:

$$\begin{aligned}&\ddot{u}_{n+1}+\Omega^2 u_{n+1} = u_{n-1}[g(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})+g_u(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(u_n-u_{n-1})+\ldots\\ &g_{\dot{u}}(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(\dot{u}_n-\dot{u}_{n-1})+g_{\ddot{u}}(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(\ddot{u}_n-\ddot{u}_{n-1})],\quad n=0,1,2,..\end{aligned} \tag{3}$$

Where the inputs of starting functions are [31]:

$$u_{-1}(t)=u_0(t)=A\cos\Omega t. \tag{4}$$

The initial conditions must be satisfied by the solution of Eq. (3) for each n. These initial conditions are as follows:

$$u_n(0)=A,\ \dot{u}_n(0)=0.\ n=1,2,3,... \tag{5}$$

Note that given $u_{n-1}(t)$ and $u_n(t)$, Eq. (3) is second order inhomogeneous differential equation for $u_{n+1}(t)$. The right hand side os Eq. (3) can be expanded into the following Fourier series:

$$\begin{aligned}&u_{n-1}[g(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})+g_u(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(u_n-u_{n-1})+\ldots\\ &g_{\dot{u}}(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(\dot{u}_n-\dot{u}_{n-1})+g_{\ddot{u}}(u_{n-1},\dot{u}_{n-1},\ddot{u}_{n-1})(\ddot{u}_n-\ddot{u}_{n-1})]=a_1(A,\Omega,\omega)\cos\Omega t+\ldots\\ &\sum_{n=2}^{N}a_n(A,\Omega,\omega)\cos n\Omega t+b_1(A,\Omega,\omega)\sin\Omega t+\sum_{n=2}^{N}b_n(A,\Omega,\omega)\sin n\Omega t,\end{aligned} \tag{6}$$

Where the coefficients $a_n(A,\Omega,\omega)$ and $b_n(A,\Omega,\omega)$ are known, and the integer N depends upon the function $g(u,\dot{u},\ddot{u})$ on the right hand side of Eq.(2) in Eq.(6), the requirement of no secular term needs that:

$$a_1(A,\Omega,\omega)=0,\quad b_1(A,\Omega,\omega)=0. \tag{7}$$

The solution of Eq. (3) with the initial conditions, according to Eq. (5), is given by:

$$u_{n+1}(t)=A\cos\Omega t-\sum_{n=2}^{N}\frac{a_n(A,\Omega,\omega)}{(n^2-1)\Omega^2}(\cos n\Omega t-\cos\Omega t)-\sum_{n=2}^{N}\frac{b_n(A,\Omega,\omega)}{(n^2-1)\Omega^2}(\sin n\Omega t-\sin\Omega t). \tag{8}$$

Eq. (7) allows the determination of the frequency Ω as a function of A and ω. This procedure can be performed to any desired iteration step n. As in the following examples will be shown, an excellent approximate analytical representation to the exact solution, valid for small as well as large values of the oscillation amplitude, is obtained.

3. Problem formulation

A schematic design of the cantilever tapered beam is shown in Fig. 1. The dimensionless vibration equation of the tapered beam is represented by Goorman [24] as follows:

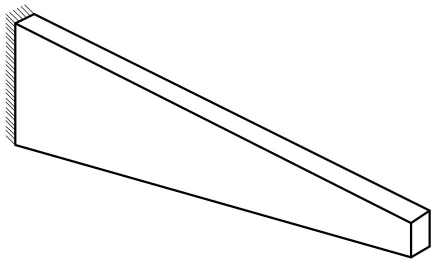

Fig. 1. Schematic illustration of the tapered beam.

$$\ddot{u}+\varepsilon_1(u^2\ddot{u}+u\dot{u}^2)+u+\varepsilon_2 u^3=0 \tag{9}$$

where u is the beam displacement and ε_1 and ε_2 are the arbitrary constants. The initial conditions are considered as follows, $u(0)=A$, $\dot{u}(0)=0$, that A denotes the maximum amplitude of the beam.
We rewrite Eq. (9) as follows:

$$\ddot{u}+\Omega^2 u=u(\Omega^2-1-\varepsilon_1(u\ddot{u}+\dot{u}^2)-\varepsilon_2 u^2) \tag{10}$$

The inputs of the starting function are $u_{-1}(t)=u_0(t)=A\cos\Omega t$ and $g(u,\dot{u},\ddot{u},t)=\Omega^2-1-\varepsilon_1(u\ddot{u}+\dot{u}^2)-\varepsilon_2 u^2$.
So, the first iteration is given by:

$$\ddot{u}_1+\Omega^2 u_1=A(\Omega^2-1+\frac{\varepsilon_1 A^2\Omega^2}{2}-\frac{3}{4}\varepsilon_2 A^2)\cos(\Omega t)+(\frac{\varepsilon_1 A^3\Omega^2}{2}-\frac{\varepsilon_2 A^3}{4})\cos(3\Omega t) \tag{11}$$

In order to ensure that no secular terms appear in the next iteration, the resonance must be avoided. So, the coefficient of $\cos\Omega t$ in Eq. (11) is required to be zero.

$$\Omega_1^2=\frac{4+3\varepsilon_2 A^2}{4+2\varepsilon_1 A^2} \tag{12}$$

So, from Eq. (11), with initial conditions in Eq. (5), we have the following first-order approximate solution:

$$u_1(t)=A\cos(\Omega_1 t)+\frac{A^3(2\varepsilon_1\Omega_1^2-\varepsilon_2)}{32\Omega_1^2}(\cos(\Omega_1 t)-\cos(3\Omega_1 t)) \tag{13}$$

For n=1 into Eq. (3), with the initial functions in Eq. (4) and u_1 given by Eq. (13), the following differential equation is obtained for u_2 :

$$\begin{aligned}\ddot{u}_2+\Omega^2 u_2=A(\Omega^2-1+\frac{\varepsilon_1 A^2\Omega^2}{2}-\frac{3}{4}\varepsilon_2 A^2+\frac{A^4(2\varepsilon_1^2\Omega^4-5\varepsilon_1\varepsilon_2\Omega^2+2\varepsilon_2^2)}{64\Omega^2}-\dots\\ \frac{A^4(2\varepsilon_1^2\Omega^2-\varepsilon_1\varepsilon_2)}{16})\cos(\Omega t)+(\frac{\varepsilon_1 A^3\Omega^2}{2}-\frac{\varepsilon_2 A^3}{4}-\frac{A^5(2\varepsilon_1^2\Omega^4-5\varepsilon_1\varepsilon_2\Omega^2+2\varepsilon_2^2)}{128\Omega^2}-\dots\\ \frac{19A^5(2\varepsilon_1^2\Omega^2-\varepsilon_1\varepsilon_2)}{128})\cos(3\Omega t)-(\frac{A^5(2\varepsilon_1^2\Omega^4-5\varepsilon_1\varepsilon_2\Omega^2+2\varepsilon_2^2)}{128\Omega^2}+\frac{5A^5(2\varepsilon_1^2\Omega^2-\varepsilon_1\varepsilon_2)}{128})\cos(5\Omega t)\end{aligned} \tag{14}$$

Avoiding the presence of a secular term requires:

$$
\begin{aligned}
&\Omega_2 = \frac{\sqrt{\alpha \times (\beta + \sqrt{\gamma})}}{\phi} \\
&\alpha = 32 + 16\varepsilon_1 A^2 - 3\varepsilon_1^2 A^4 \\
&\beta = 64 + 48\varepsilon_2 A^2 + \varepsilon_1 \varepsilon_2 A^4 \\
&\gamma = 4096 + 6144\varepsilon_2 A^2 + 128\varepsilon_1\varepsilon_2 A^4 + 1792\varepsilon_2^2 A^4 - 160\varepsilon_1\varepsilon_2^2 A^6 + 49\varepsilon_1^2\varepsilon_2^2 A^8 \\
&\phi = 64 + 32\varepsilon_1 A^2 - 6\varepsilon_1^2 A^4
\end{aligned} \tag{15}
$$

Solving Eq. (14) with the initial conditions in Eq. (5), we obtain the following form for $u_2(t)$:

$$
\begin{aligned}
&u_2(t) = A\cos(\Omega_2 t) + \left(-\frac{34}{768}\varepsilon_1^2 A^5 + \frac{21}{768\Omega_2^2}\varepsilon_1\varepsilon_2 A^4 - \frac{24}{768\Omega_2^2}\varepsilon_2 A^2 - \frac{2}{768\Omega_2^4}\varepsilon_2^2 A^4 + \ldots \right. \\
&\left.\frac{48}{768}\varepsilon_1 A^2\right)\cos(\Omega_2 t) + \frac{A^5(2\varepsilon_1\Omega_2^2 - \varepsilon_2)}{512\Omega_2^4}(10\varepsilon_1 A^2\Omega_2^2 - 16\Omega_2^2 - \varepsilon_2 A^2)\cos(3\Omega_2 t) + \ldots \\
&\frac{A^5(4\varepsilon_1\Omega^2 - \varepsilon_2)}{3840\Omega_2^4}\cos(5\Omega_2 t)
\end{aligned} \tag{16}
$$

4. Results and discussion

In this section the dynamic behavior of a tapered beam such as frequency and displacement is scrutinized. Moreover, the error and phase plane of the beam are compared with the exact solution results. The exact frequency for the tapered beam can be derived as follows [32]:

$$
\omega_{Exact} = 2\pi \Big/ 4\sqrt{2}A\int_0^{\pi/2} \frac{\sqrt{1+\varepsilon_1 A^2\cos^2 t}\,\sin t}{\sqrt{A^2(1-\cos^2 t)(\varepsilon_2 A^2\cos^2 t + \varepsilon_2 A^2 + 2)}}dt \tag{17}
$$

To demonstrate the accuracy of the Modified Iteration Perturbation Method, the procedure explained in the previous section is applied to obtain the natural frequency and the corresponding displacement of tapered beams. A comparison of the obtained results from the MIPM and the exact one is tabulated in Table 1 for different parameters A, ε_1 and ε_2. Also Figs. 2 and 3, show the displacement response to tapered beams, u(t), analytical solution of du/dt, respectively, for $\varepsilon_1 = 0.25$, $\varepsilon_2 = 0.5$ and A=2.

Table 1. Comparison between MIPM frequency with exact and other analytical frequencies.

constant parameters			MIPM Eq. (15)	MMA & HPM [31]	IAFF & VAM [30]	Exact solution
A	ε_1	ε_2				
0.1	0.1	0.5	1.001622	1.001498	1.001622	1.001621
	0.5	1	1.002490	1.002492	1.002490	1.002489
	5	10	1.024112	1.024295	1.024099	1.024105
0.5	0.1	0.5	1.039349	1.036523	1.039349	1.039241
	0.5	1	1.057188	1.058300	1.057188	1.057272
	5	10	1.331477	1.371988	1.330124	1.345552
1	0.1	0.5	1.144347	1.135923	1.144344	1.143338
	0.5	1	1.183251	1.195228	1.183215	1.185919
	5	10	1.604034	1.695582	1.558387	1.640213

The effect of parameter "A" has been studied in the phase plane figures, that is shown in Fig. 4 for $\varepsilon_1 = \varepsilon_2 = 0.75$.

Fig. 5 shows the MIPM and the Max-Min Approach errors against Amplitude at t=2s. Moreover, as it can be seen in this figure, error percentage obtained from MIPM is less than the MMA for the same Amplitude. The effects of different parameters $\varepsilon_1, \varepsilon_2$ and A are studied in Figs. 6 and 7 simultaneously. Finally, Fig. 8 shows the displacement behavior of the system versus time and the initial amplitude.

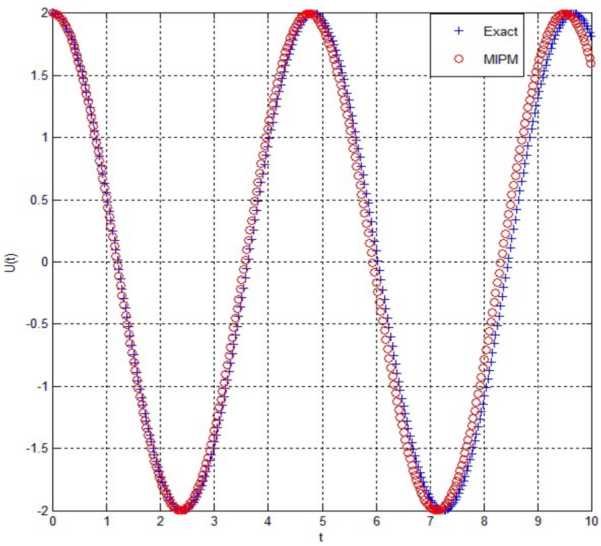

Fig. 2. Comparison of the analytical solution of u(t) based on time with the exact solution for $\varepsilon_1 = 0.25, \varepsilon_2 = 0.5, A = 2$

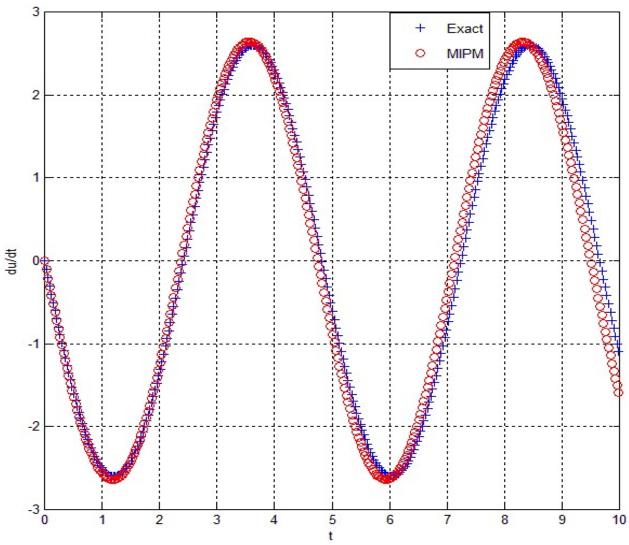

Fig. 3. Comparison of the analytical solution of du/dt based on time with the exact solution for $\varepsilon_1 = 0.25, \varepsilon_2 = 0.5, A = 2$

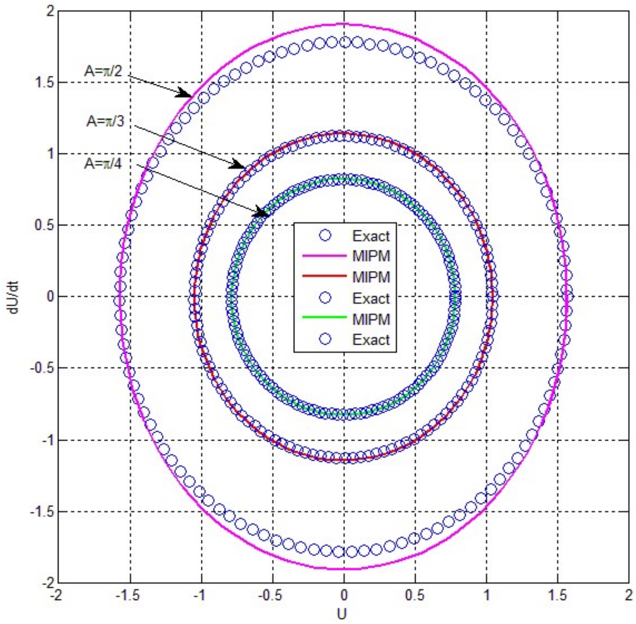

Fig. 4. Comparison of the analytical solution of du/dt based on u(t) with the exact solution for $\varepsilon_1 = \varepsilon_2 = 0.75$.

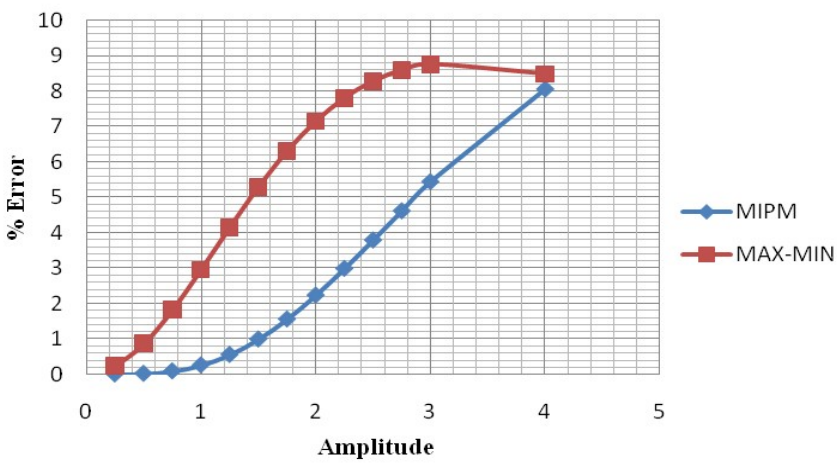

Fig. 5. Comparison of the MIPM and Max-Min Approach results at t=2s.

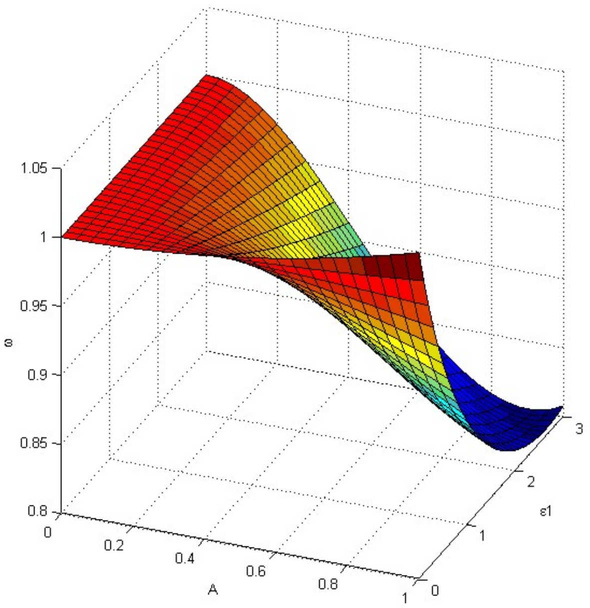

Fig. 6. Sensitivity analysis of the frequency for $\varepsilon_2 = 0.1$

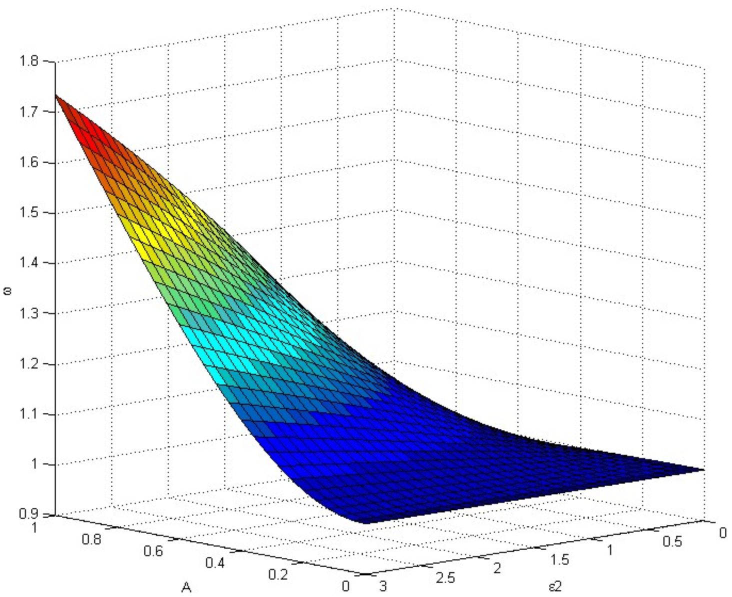

Fig. 7. Sensitivity analysis of the frequency for $\varepsilon_1 = 0.1$

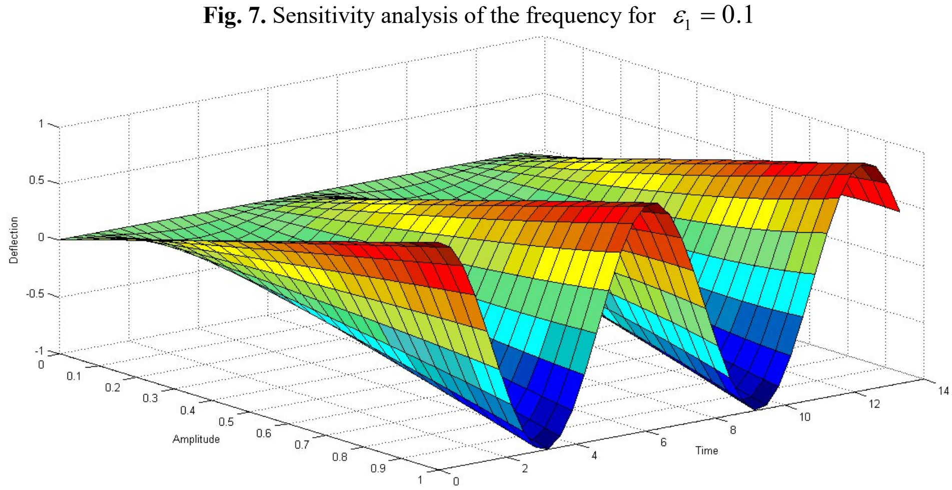

Fig. 8. Influence of initial amplitude on time histories of MIPM response.

5. Conclusions

In this paper, Modified Iteration Perturbation Method has been applied successfully to solve the governing equation of nonlinear oscillation of tapered beams. MIPM is an efficient method and does not require small parameters that are needed by perturbation method. Also, the results of the proposed analytical solution are compared with the exact solution. This comparison confirms the accuracy of MIPM. Moreover, the results reveal that MIPM can be considered as a useful tool for analyzing the periodic behavior of the structures analytically.

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