Journal of Applied and Computational Mechanics, Vol. 1, No. 3, (2015), 134-144
DOI: 10.22055/jacm.2015.10997

Shahid Chamran University

Journal of
Applied and Computational Mechanics

Fuzzy Modeling and Synchronization of a New Hyperchaotic Complex System with Uncertainties

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Received September 23 2014; revised February 12 2015; accepted for publication February 14 2015.
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Abstract

In this paper, the synchronization of a new hyperchaotic complex system based on T-S fuzzy model is proposed. First, the considered hyperchaotic system is represented by T-S fuzzy model equivalently. Then, by using the parallel distributed compensation (PDC) method and by applying linear system theory and exact linearization (EL) technique, a fuzzy controller is designed to realize the synchronization. Finally, simulation results are carried out to demonstrate the performance of our proposed control scheme, and also the robustness of the designed fuzzy controller to uncertainties.

Keywords: a new hyperchaotic complex system; hyperchaotic synchronization; T-S fuzzy model; parallel distributed compensation (PDC) method; exact linearization (EL).

1. Introduction

Chaos is a very complex nonlinear phenomenon that exhibits some specific features such as crucial dependence on initial conditions, Fourier transform spectra, strange attractors and fractal properties of the motion in phase space. Because of these features, chaos synchronization has a strong application prospect in biology, physics, chemical reactions, ecological systems, secure communication and so on [1,2]. Compared with the chaotic systems, hyperchaotic ones have two or more positive Lyapunov exponents and that clearly improves the security by generating more complex dynamics due to the separation of the tracks in more directions. Therefore, a secure communication system based on hyperchaotic systems in not easy to be broken down and that's why the synchronization of hyperchaotic systems is more meaningful to us. Recently, researchers have found or created many new hyperchaotic systems [3-8].

Controlling synchronization of hyperchaotic system has attracted a great deal of attention from various fields and become a challenging work. In 1990, Carroll and Pecora [9] proposed the concept of chaotic synchronization for the first time. From then on, many kinds of conventional and intelligent control methods have been applied to synchronize two hyperchaotic (or chaotic) systems, such as linear state feedback control [10], active control [11], passive control [12], neural network control [13], adaptive control [14,15], impulsive feedback control [16], delayed feedback control [17], inverse optimal control [18], guaranteed cost control [19], backstepping control [20], fuzzy control [21-24], etc. Among these control methods, some must use high gain in designing parameters, while others need Lipschitz conditions for nonlinear terms to satisfy. Fuzzy logical system can approximate nonlinear functions to arbitrary precision [25], and therefore fuzzy control is suggested as an alternative candidate for its ability of dealing with uncertainties and nonlinearities.

In 1985, a new fuzzy model, the Takagi-Sugeno (T-S) fuzzy model was proposed in [26] and has been widely applied to many fields because of its simple structure with local dynamics. Delavari et al. proposed fuzzy control schemes for controlling and synchronizing uncertain (fractional order) chaotic systems [27-29]. Following the idea of representing a hyperchaotic system via a T-S fuzzy model and developing a fuzzy controller design method for hyperchaotic synchronization, the fuzzy modeling and synchronization of the fourth Rössler system and the MCK circuit was proposed in [30]. A robust fuzzy synchronization control for Lü hyperchaotic system was designed in [31]. The synchronization of hyperchaotic Chen system based on fuzzy state feedback controller was developed in [32]. A fuzzy controller was designed in [33] for the synchronization of Lorenz hyperchaotic system. The T-S fuzzy identical synchronization of Henon hyperchaotic maps was proposed in [34]. A fuzzy modeling method for the generalized asymptotic synchronization between Chen hyperchaotic system and Liu hyperchaotic system was designed in [35]. In [36], a fuzzy robust controller was developed to synchronize Lü hyperchaotic system and Lorenz hyperchaotic system, but only the H_∞ synchronization is derived.

All the above researchers considered the synchronization of real dynamical systems, i.e., dynamical systems with real variables. Recently, for describing the real world better, many complex dynamical systems are proposed and studied. Since Fowler et al. [37] generalized the real Lorenz model to a complex Lorenz model, which can be used to describe and simulate the physics of a detuned laser and the thermal convection of liquid flows [38,39], complex chaotic and hyperchaotic systems have been intensively studied. In secure communications, doubling the number of variables or using complex variables (which means using higher dimensional chaotic systems) increases the content and security of the transmitted information [40]. Thus, the synchronization in chaotic or hyperchaotic complex-variable systems has been extensively investigated. The Synchronization and control of hyperchaotic complex Lorenz system has been achieved in [41]. The dynamics of a new hyperchaotic complex Lorenz system and its synchronization are introduced in [42].

Therefore, in this paper, the T-S fuzzy model of a new hyperchaotic complex system is presented, which is not considered in any of the previous papers. Then, by using the parallel distributed compensation (PDC) method and by applying linear system theory and exact linearization (EL) technique, a fuzzy controller is designed to realize the synchronization. The problem is also solved in the presence of uncertainties.

The organization of this paper is as follows: section 2 details the representation of a new hyperchaotic complex system by the T-S fuzzy models with six IF-THEN fuzzy rules. Section 3 is devoted to the design of fuzzy controller by using the parallel distributed compensation (PDC) method. Simulation studies are shown in section 4, to demonstrate the effectiveness of our proposed approach, followed by some concluding remarks in section 5.

2. Fuzzy Modeling

2.1 T-S Fuzzy Model

Consider the continuous-time T-S fuzzy rule base described as follows [43]:

$$R^i: \text{IF } p_1(t) \text{ is } M_{i1}, \ldots, \text{and } p_q(t) \text{ is } M_{iq}, \text{THEN } \dot{x}(t) = A_i x(t) + B_i w(t) \quad (1)$$

where $i = 1,2,\ldots,r$ (r is the number of fuzzy rules), $x(t) \in R^n$ and $w(t) \in R^m$ denote the state vector and the control input vector, respectively, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times m}$ are known system matrix and control input coefficient matrix with appropriate dimensions, respectively. $p_1(t)$, $p_2(t)$, … , $p_q(t)$ are premise variables, M_{ij} is a fuzzy set ($j = 1,2,\ldots,q$).

By taking a standard fuzzy inference strategy, i.e., using a singleton fuzzifier, product fuzzy inference and center average defuzzifier, the final continuous-time fuzzy T-S system is inferred as follows:

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} \omega_i(p(t))(A_i x(t) + B_i w(t))}{\sum_{i=1}^{r} \omega_i(p(t))} \quad (2)$$

Where

$$\omega_i(p(t)) = \prod_{j=1}^{q} M_{ij}(p_j(t)) \quad (3)$$

in which $M_{ij}(p_j(t))$ is the degree of membership of $p_j(t)$ in M_{ij} , with

$$\begin{cases} \sum_{i=1}^{r} \omega_i(p(t)) > 0, \\ \omega_i(p(t)) \geq 0, \end{cases} i = 1,2,\ldots,r. \tag{4}$$

By using $\mu_i(p(t)) = \omega_i(p(t)) / \sum_{i=1}^{r} \omega_i(p(t))$ instead of $\omega_i(p(t))$, Equation (2) is rewritten as:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(p(t))(A_i x(t) + B_i w(t)) \tag{5}$$

Note that

$$\begin{cases} \sum_{i=1}^{r} \mu_i(p(t)) > 0, \\ \mu_i(p(t)) \geq 0, \end{cases} i = 1,2,\ldots,r. \tag{6}$$

in which $\mu_i(p(t))$ can be regarded as the firing strength of the i th IF-THEN rules.

2.1 Fuzzy Modeling of a New Hyperchaotic Complex System

After the complex Lorenz model, many other chaotic and hyperchaotic complex-variable systems have been reported, including the complex Chen and complex Lü systems [44], complex detuned laser system [45] and complex modified hyperchaotic Lü system [46]. A novel hyperchaotic complex-variable system [47] which generates 2-, 3- and 4-scroll attractors has recently been presented in the following form:

$$\begin{cases} \dot{x}(t) = y - ax + byz \\ \dot{y}(t) = cy - xz + z \\ \dot{z}(t) = \frac{d}{2}(\bar{x}y + x\bar{y}) - hz \end{cases} \tag{7}$$

where a, b, c, d and h are positive parameters, and $x = u_1 + iu_2$ and $y = u_3 + iu_4$ are complex variables and $i = \sqrt{-1}$. u_j ($j = 1,2,\ldots,5$) and $z = u_5$ are real variables. The real version of (7) is a 5D hyperchaotic autonomous system as follows:

$$\begin{cases} \dot{u}_1 = u_3 - au_1 + bu_3u_5 \\ \dot{u}_2 = u_4 - au_2 + bu_4u_5 \\ \dot{u}_3 = cu_3 - u_1u_5 + u_5 \\ \dot{u}_4 = cu_4 - u_2u_5 \\ \dot{u}_5 = d(u_1u_3 + u_2u_4) \end{cases} \tag{8}$$

The above system exhibits hyperchaotic behaviour for specific positive values of a, b, c, d and h and different initial conditions. The (u_1, u_3) plane and the (u_1, u_3, u_5) space of the three-scroll hyperchaotic attractor of (8) are represented in Figure 1 and Figure 2. According to the property of boundness of hyperchaotic systems, one can assume that $u_1 \in [-r_1, r_1]$, $u_2 \in [-r_2, r_2]$ and $u_5 \in [-r_5, r_5]$, where the positive constants r_1, r_2 and r_5 are chosen according to the region of interest of the hyperchaotic complex system. It should be noted that since there is no input in the autonomous hyperchaotic systems, the term $B_i w(t)$ in the T-S fuzzy model is always equal to zero in the process of fuzzy modeling.

For the hyperchaotic system (8), we derive the following T-S fuzzy model rule base to represent it:

Rule 1: If u_1 is M_1 and u_2 is M_2 and u_5 is M_{51}, then $\dot{U} = A_1 U$.

Rule 2: If u_1 is M_1 and u_2 is M_2 and u_5 is M_{52}, then $\dot{U} = A_2 U$.

Rule 3: If u_1 is M_{11} and u_2 is M_{21} and u_5 is M_5, then $\dot{U} = A_3 U$.

Rule 4: If u_1 is M_{11} and u_2 is M_{22} and u_5 is M_5, then $\dot{U} = A_4 U$.

Rule 5: If u_1 is M_{12} and u_2 is M_{21} and u_5 is M_5, then $\dot{U} = A_5 U$.

Rule 6: If u_1 is M_{12} and u_2 is M_{22} and u_5 is M_5, then $\dot{U} = A_6 U$.

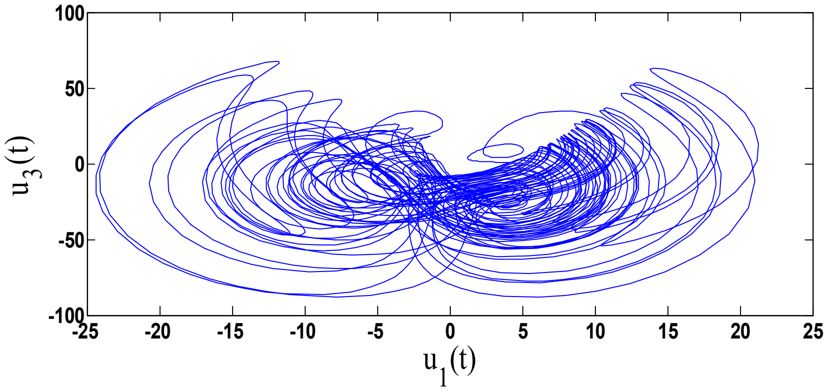

Fig. 1. The (u_1, u_3) plane of the three-scroll hyperchaotic attractor of system (8)

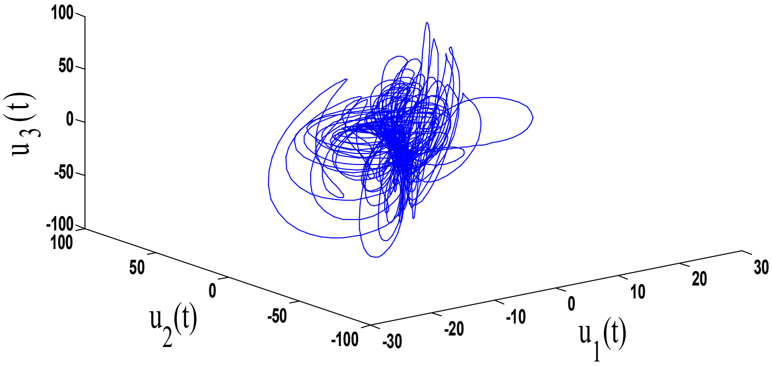

Fig. 2. The (u_1, u_3, u_5) space of the three-scroll hyperchaotic attractor of system (8)

where $U = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5]$. M_{ij} are the membership functions chosen as follows:

$$M_1(u_1) = M_2(u_2) = M_5(u_5) = 1$$

$$M_{11}(u_1) = \frac{1}{2}\left(1 + \frac{u_1}{r_1}\right), M_{12}(u_1) = \frac{1}{2}\left(1 - \frac{u_1}{r_1}\right)$$

$$M_{21}(u_2) = \frac{1}{2}\left(1 + \frac{u_2}{r_2}\right), M_{22}(u_2) = \frac{1}{2}\left(1 - \frac{u_2}{r_2}\right)$$

$$M_{51}(u_5) = \frac{1}{2}\left(1 + \frac{u_5}{r_5}\right), M_{52}(u_5) = \frac{1}{2}\left(1 - \frac{u_5}{r_5}\right)$$

Therefore, the fuzzy modeling matrices A_i are as follows:

$$A_1=\begin{bmatrix}-a & 0 & 1+2br_5 & 0 & 0\\ 0 & -a & 0 & 1+2br_5 & 0\\ -2r_5 & 0 & c & 0 & 1\\ 0 & -2r_5 & 0 & c & 0\\ 0 & 0 & 0 & 0 & -h\end{bmatrix} A_2=\begin{bmatrix}-a & 0 & 1-2br_5 & 0 & 0\\ 0 & -a & 0 & 1-2br_5 & 0\\ 2r_5 & 0 & c & 0 & 1\\ 0 & 2r_5 & 0 & c & 0\\ 0 & 0 & 0 & 0 & -h\end{bmatrix}$$

$$A_4=\begin{bmatrix}-a & 0 & 1 & 0 & 0\\ 0 & -a & 0 & 1 & 0\\ 0 & 0 & c & 0 & 1\\ 0 & 0 & 0 & c & 0\\ 0 & 0 & 2dr_1 & 2dr_2 & -h\end{bmatrix}, A_3=\begin{bmatrix}-a & 0 & 1 & 0 & 0\\ 0 & -a & 0 & 1 & 0\\ 0 & 0 & c & 0 & 1\\ 0 & 0 & 0 & c & 0\\ 0 & 0 & 2dr_1 & -2dr_2 & -h\end{bmatrix}$$

$$A_5=\begin{bmatrix}-a & 0 & 1 & 0 & 0\\ 0 & -a & 0 & 1 & 0\\ 0 & 0 & c & 0 & 1\\ 0 & 0 & 0 & c & 0\\ 0 & 0 & -2dr_1 & 2dr_2 & -h\end{bmatrix} A_6=\begin{bmatrix}-a & 0 & 1 & 0 & 0\\ 0 & -a & 0 & 1 & 0\\ 0 & 0 & c & 0 & 1\\ 0 & 0 & 0 & c & 0\\ 0 & 0 & -2dr_1 & -2dr_2 & -h\end{bmatrix}$$

Because the summation of the membership functions selected is $\sum_{j=1}^{2} M_{1j}=1$, $\sum_{j=1}^{2} M_{2j}=1$ and $\sum_{j=1}^{2} M_{5j}=1$, the output of the system namely the fuzzy model of hyperchaotic system (8) is shown as follows:

$$\begin{aligned}\dot{U}=&\sum_{j=1}^{2} M_1(u_1).M_2(u_2).M_{5j}(u_5).A_j.U\\ &+\sum_{j=1}^{2} M_{11}(u_1).M_{2j}(u_2).M_5(u_5).A_{j+2}.U\\ &+\sum_{j=1}^{2} M_{12}(u_1).M_{2j}(u_2).M_5(u_5).A_{j+4}.U\end{aligned} \tag{9}$$

The error signal is obtained by the following equation:

$$e=U-U'$$

where U denotes the drive system state vector and $U'=[u_1' \quad u_2' \quad u_3' \quad u_4' \quad u_5']$ is the response system state vector. So, by considering M_{ij}' as membership functions and A_j' as the fuzzy modeling matrices of the response system:

$$\begin{aligned}\dot{U}'=&\sum_{j=1}^{2} M_1'(u_1').M_2'(u_2').M_{5j}'(u_5').A_j'.U'+B_j w(t)\\ &+\sum_{j=1}^{2} M_{11}'(u_1').M_{2j}'(u_2').M_5'(u_5').A_{j+2}'.U'+B_{j+2}w(t)\\ &+\sum_{j=1}^{2} M_{12}'(u_1').M_{2j}'(u_2').M_5'(u_5').A_{j+4}'.U'+B_{j+4}w(t)\end{aligned} \tag{10}$$

Here we divide the controller $u(t)$ into two subcontrollers by using the PDC method, and as it follows, a fuzzy controller can be designed to realize the synchronization:

Subcontroller $w_d(t)$:

Rule 1: If u_1 is M_1 and u_2 is M_2 and u_5 is M_{51} , then $w_d(t)=K_1U$.

Rule 2: If u_1 is M_1 and u_2 is M_2 and u_5 is M_{52} , then $w_d(t)=K_2U$.

Rule 3: If u_1 is M_{11} and u_2 is M_{21} and u_5 is M_5 , then $w_d(t)=K_3U$.

Rule 4: If u_1 is M_{11} and u_2 is M_{22} and u_5 is M_5 , then $w_d(t)=K_4U$.

Rule 5: If u_1 is M_{12} and u_2 is M_{21} and u_5 is M_5 , then $w_d(t)=K_5U$.

Rule 6: If u_1 is M_{12} and u_2 is M_{22} and u_5 is M_5 , then $w_d(t)=K_6U$.

Subcontroller $w_r(t)$:

Rule 1: If u_1' is M_1' and u_2' is M_2' and u_5' is M_{51}' , then $w_r(t)=-\hat{K}_1U'$.

Rule 2: If u_1' is M_1' and u_2' is M_2' and u_5' is M_{52}' , then $w_r(t)=-\hat{K}_2U'$.

Rule 3: If u_1' is M_{11}' and u_2' is M_{21}' and u_5' is M_5' , then $w_r(t)=-\hat{K}_3U'$.

Rule 4: If u_1' is M_{11}' and u_2' is M_{22}' and u_5' is M_5' , then $w_r(t)=-\hat{K}_4U'$.

Rule 5: If u_1' is M_{12}' and u_2' is M_{21}' and u_5' is M_5' , then $w_r(t)=-\hat{K}_5U'$.

Rule 6: If u_1' is M_{12}' and u_2' is M_{22}' and u_5' is M_5' , then $w_r(t)=-\hat{K}_6U'$.

The total fuzzy controller is constructed by the parallel connection method:

$$\begin{aligned} w(t)=\;& w_d(t)+w_r(t)\\ &+\sum_{j=1}^{2} M_1(u_1).M_2(u_2).M_{5j}(u_5).K_j.U\\ &+\sum_{j=1}^{2} M_{11}(u_1).M_{2j}(u_2).M_5(u_5).K_{j+2}.U\\ &+\sum_{j=1}^{2} M_{12}(u_1).M_{2j}(u_2).M_5(u_5).K_{j+4}.U\\ &-\sum_{j=1}^{2} M_1'(u_1').M_2'(u_2').M_{5j}'(u_5').\hat{K}_j'.U'\\ &-\sum_{j=1}^{2} M_{11}'(u_1').M_{2j}'(u_2').M_5'(u_5').\hat{K}_{j+2}'.U'\\ &-\sum_{j=1}^{2} M_{12}'(u_1').M_{2j}'(u_2').M_5'(u_5').\hat{K}_{j+4}'.U' \end{aligned} \tag{11}$$

Substituting (11) into (10), and using the above equation, the closed-loop synchronization error system is derived as follows:

$$\begin{aligned} \dot{e}(t)=\;&\sum_{j=1}^{2} M_1(u_1).M_2(u_2).M_{5j}(u_5).(A_j-BK_j).U\\ &+\sum_{j=1}^{2} M_{11}(u_1).M_{2j}(u_2).M_5(u_5).(A_{j+2}-BK_{j+2}).U \end{aligned}$$

$$+\sum_{j=1}^{2} M_{12}(u_1).M_{2j}(u_2).M_5(u_5).(A_{j+4} - BK_{j+4}).U$$

$$-\sum_{j=1}^{2} M_1'(u_1').M_2'(u_2').M_{5j}'(u_5').(A_j' - B\hat{K}_j').U'$$

$$-\sum_{j=1}^{2} M_{11}'(u_1').M_{2j}'(u_2').M_5'(u_5').(A_{j+2}' - B\hat{K}_{j+2}').U'$$

$$-\sum_{j=1}^{2} M_{12}'(u_1').M_{2j}'(u_2').M_5'(u_5').(A_{j+4}' - B\hat{K}_{j+4}').U' \quad (12)$$

Assume that B is nonsingular. So, by choosing appropriately K_i and $\hat{K}_i$ ($i = 1,2,\ldots,6$), a Hurwitz stable matrix L is derived such that:

$$L = diag(l_1, l_2, l_3, l_4) \quad (13)$$

For simplicity, the control input coefficient matrix B is chosen as $B = I$, where I denotes the identity matrix. The feedback gains K_i and $\hat{K}_i$ are derived as follows:

$$A_i - K_i = A'_i - \hat{K}_i = L \quad (14)$$

where $i = 1,2$ and $j = 1,2,3$. Thus the Equation (12) is linearized as:

$$\dot{e}(t) = Le(t) \quad (15)$$

Assuming that L is a Hurwitz stable matrix, Equation (15) is asymptotically stable. Therefore, K_i and $\hat{K}_i$ are denoted as follows:

$$K_i = B^{-1}(A_i - L), \hat{K}_i = B^{-1}(A'_i - L) \quad (16)$$

3. Simulation Results

In this section, numerical simulations are presented to validate the effectiveness and performance of the proposed fuzzy controller. The parameters of the drive system are chosen as $a = 3.5$, $b = 0.06$, $c = 3$, $d = 2$ and $h = 9$. The second parameter is different in the response system and is equal to $b' = 0.599$.

Let the initial values of the drive system be $u_1(0) = 5$, $u_2(0) = 2$, $u_3(0) = 0$, $u_4(0) = 1$ and $u_5(0) = -4$, while those of the response system are $u_1'(0) = 5$, $u_2'(0) = 4$, $u_3'(0) = 3$, $u_4'(0) = 2$ and $u_5'(0) = 1$.

By choosing a Hurwitz stable matrix L such as:

$$L = diag(-5,-4,-3,-2,-1)$$

and according to (16), the feedback gains K_i and $\hat{K}_i$ ($i = 1,2,3,4,5,6$) are calculated. The whole hyperchaotic synchronization system is simulated by Matlab R2012a using the Fuzzy Logic Toolbox. Let the uncertainties be pulse signals with different amplitudes. The synchronization error convergent curves are presented in Figs. 3-7, in which the synchronization between two identical hyperchaotic complex systems in the presence of uncertainties is achieved and the synchronization error ($e_i = u_i - u_i'$) for each state variable converges to zero.

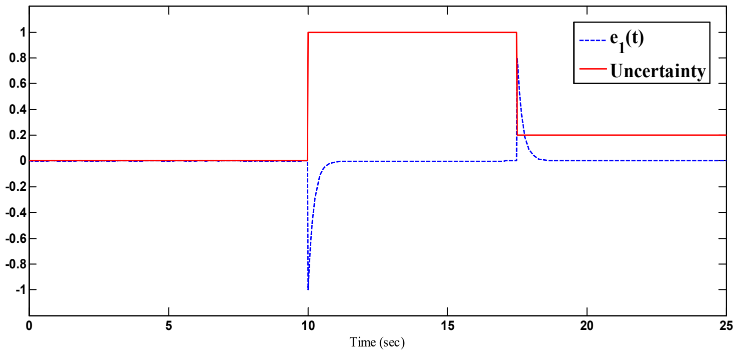

Fig. 3. The hyperchaotic synchronization error (e_1) of the system (8).

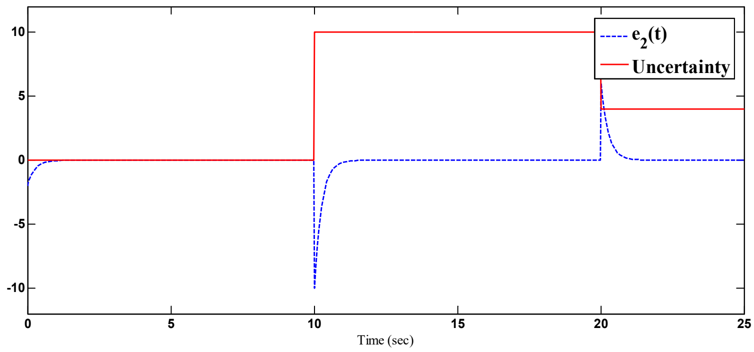

Fig. 4. The hyperchaotic synchronization error (e_2) of the system (8).

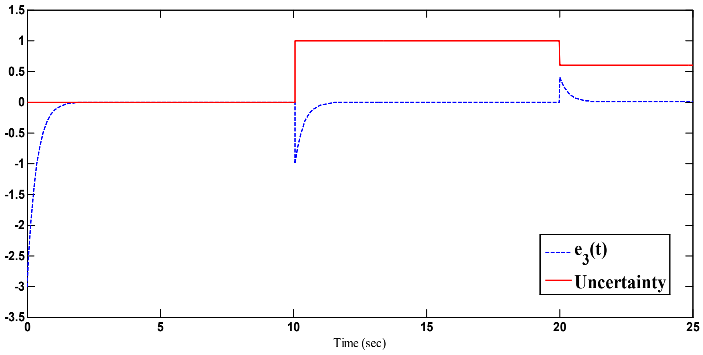

Fig. 5. The hyperchaotic synchronization error (e_3) of the system (8).

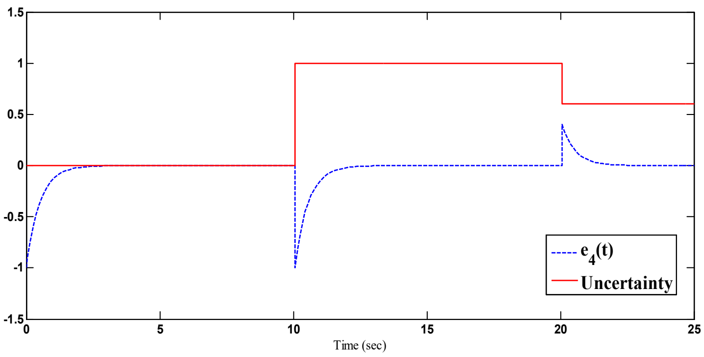

Fig. 6. The hyperchaotic synchronization error (e_4) of the system (8).

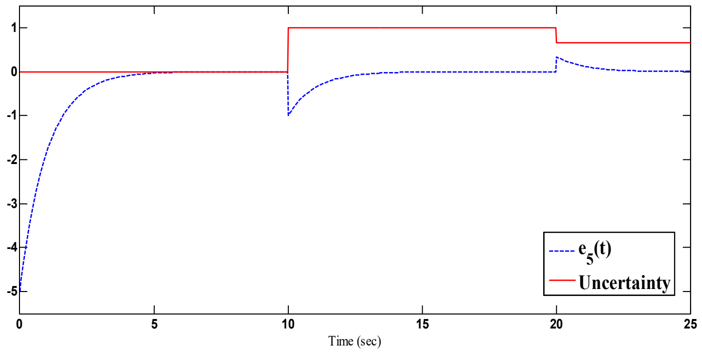

Fig. 7. The hyperchaotic synchronization error (e_5) of the system (8).

4. Conclusion

In this paper, a new hyperchaotic complex system is represented by T-S fuzzy models. The structure of this modeling method is simple and applicable to many hyperchaotic systems. The parallel distributed compensation method is introduced to design the fuzzy controller by which the synchronization is achieved. Simulation results and comparison studies are shown to demonstrate the effectiveness and performance of this method and the robustness of the designed controller against uncertainties considered as pulse signals with different amplitudes.

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