



Investigation of the vdW Force-Induced Instability in Nano-scale Actuators Fabricated from Cylindrical Nanowires

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Abstract

The presence of van der Waals (vdW) force can lead to mechanical instability in freestanding nano-scale actuators. Most of the previous researches in this area has exclusively focused on modeling the instability in actuators with one actuating component, while less attention has been paid to actuators that consist of two actuating components. Herein, the effect of the vdW force on the instability of freestanding actuators with two parallel actuating components is investigated. Conventional configurations including cantilever and double-clamped geometries are investigated. A continuum mechanics theory along with Euler-beam model is applied to obtain governing equations of the systems. The nonlinear governing equations of the actuators are solved using two different approaches; the modified Adomian decomposition and the finite difference method. The maximum length of the nanowire and the minimum initial gap which prevents the instability is computed.

Keywords: Freestanding nanoactuators, van der Waals (vdW) force, Instability, Modified Adomian decomposition, Finite difference method.

1. Introduction

Beam-type nano-actuators are widely used in many applications (1&2). The physical behavior of miniature actuators has been investigated for over two decades without considering nano-scale forces (3-5). Among the nano-scale forces, the van der Waals (vdW) attraction is one of the most important forces that highly affect the performance of nano-scale systems (6-16). The vdW force may significantly affect the stable performance when the initial gap between the components of actuator is typically below several ten nanometers (17&18). Indeed, it can induce undesired mechanical instability in freestanding nanostructures during the fabricating or operating stages. Therefore, this force should be considered in the design, manufacture and operation of freestanding nanoscale devices such as nanoactuators.

The effect of the vdW force on the mechanical instability of nano-scale beam-type systems has been simulated by Koochi and his coworkers (2, 11, 12, 18 & 19). Spengen et al. (20) surveyed the instability in electromechanical systems due to the vdW force. The effect of the vdW attraction on the instability voltage of carbon-nanotube-based switches has been calculated by Dequesnes et al. (21). The influence of the size effect on the pull-in instability of beam-type nanostructures under vdW force has been simulated by Tadi Beni et al. (13). An analytical relation to express the effect of the vdW force on the pull-in gap and the voltage of a nano-actuator was obtained by Rotkin (22). Farrokhabadi et al. (14) studied the static response and the pull-in instability of carbon nanotube nanotweezers under the vdW attraction. The effect of the vdW force on the dynamic behavior of a nano-actuator has been considered by Lin and Zhao (23). Abadyan et al. (15) investigated the elastic boundary condition effect on the pull-in instability of

beam-type NEMS under the vdW force. A one degree of freedom lumped parameter model has been proposed by Koochi et al. (18 & 19) to survey the physical behavior of a nanoactuator in the vdW force regime. In another study, Koochi et al. (18) used the homotopy perturbation to survey the pull-in parameters of cantilever beam-type actuators under vdW forces. It should be noted that all the above-mentioned works have exclusively focused on modeling the instability in actuators with two components while less attention has been paid to actuators that consist of three components. Currently, nanoactuators with three components have attracted attentions due to their promising electromechanical performance (24-26). Therefore, in the present study, the authors demonstrate the effect of the vdW force on the instability of freestanding nanoactuators made of a nanowire suspended between two parallel actuating components. Nanowires are of the most common constructive elements in fabricating nanoactuators (14, 17-19 & 27). To investigate the behavior of nano-scale systems, several approaches have been employed. A reliable approach to simulate the instability of a nano-scale system is to apply nano-scale continuum models. Herein, a continuum model is applied to obtain the constitutive governing equation of the actuators. Afterwards, the nonlinear governing equations are solved using two different approaches; the modified Adomian decomposition and the finite difference numerical solution.

2. Theoretical Model

Fig. 1 shows the typical three-electrode nanoactuators made of a movable nanowire suspended between two fixed planes. The nanowire can deflect up/down towards each of the fixed planes.

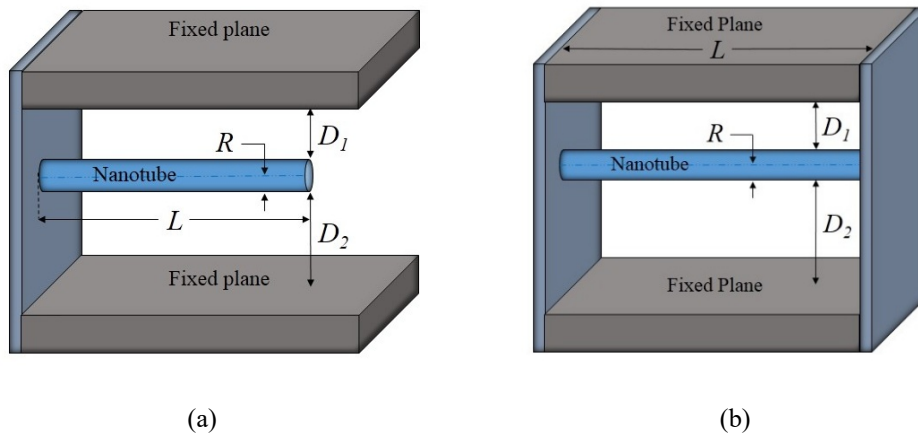


Fig. 1. The schematic representation of typical three-electrode nanoactuators: (a) cantilever, (b) fixed-fixed.

The length and the radius of the nanowire are L and R , respectively. The initial gap between the nanowire and the upper plane is D_1 and the initial separation between the lower plane and the nanowire is D_2 .

2.1. The van der Waals Force

The vdW energy per unit length between a cylinder and a flat plane can be evaluated as (28):

$$E = -\frac{A}{12} \sqrt{\frac{R}{2D^3}}. \quad (1)$$

By differentiating the energy, the vdW force per unit length can be obtained as

$$f = \frac{A}{8} \sqrt{\frac{R}{2D^5}}, \quad (2)$$

where A is the Hamaker constant, R is the nanowire radius and D is the gap distance. Now by considering the deflection of the nanowire and by replacing D_1 with $D_1 - w$ and D_2 with $D_2 + w$, the vdW forces per unit length can be obtained from (2) as

$$f_{vdW1} = \frac{A}{8} \sqrt{\frac{R}{2(D_1 - w)^5}}, \quad (3-a)$$

$$f_{vdW2} = \frac{A}{8} \sqrt{\frac{R}{2(D_2 + w)^5}}, \quad (3-b)$$

where f_{vdW1} and f_{vdW2} denote the vdW attraction between the nanowire/upper plane and the nanowire/lower plane, respectively.

2.2. The Elastic resistance

The elastic resistance of freestanding nanoactuators can be modeled according to the continuum mechanics theory (6, 29 & 30). According to this theory, the strain energy density of a continuum $\tilde{U}$ can be written as

$$\tilde{U} = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} \quad (i, j = 1, 2, 3), \quad (4)$$

where the variables of σ_{ij} and ε_{ij} are the stress tensor and strain tensor, respectively, and defined by the following relations

$$\varepsilon_{ij} = \frac{1}{2} \left((\nabla \vec{u})_{ij} + (\nabla \vec{u})_{ji}^T \right), \quad (5-a)$$

$$\sigma_{ij} = \lambda \varepsilon_{mm} \delta_{ij} + 2\mu \varepsilon_{ij}, \quad (5-b)$$

where λ , μ and $\vec{u}$ are the Lamé constant, shear modulus and displacement vector, respectively. Now, using the Euler-Bernoulli beam model for the nanowires, the displacement vector can be expressed as (6, 29 & 30)

$$\begin{aligned} u_1 &= -z \frac{\partial w(X)}{\partial X}, \\ u_2 &= 0, \\ u_3 &= w(X). \end{aligned} \quad (6)$$

Considering a small deformation and substituting relation (6) in Equation (5-a), it is obtained that:

$$\varepsilon_{11} = -z \frac{\partial^2 w(X)}{\partial X^2}, \quad \varepsilon_{22} = \varepsilon_{33} = \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{13} = 0, \quad (7)$$

By substituting (7) in (5-b) the stress components are obtained as:

$$\sigma_{11} = -E_{eff} z \frac{\partial^2 w(X)}{\partial X^2}, \quad \sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0, \quad (8)$$

where E_{eff} is the effective Young's modulus.

Substituting Equations 7 and 8 into Equation (4), the strain energy can be defined as

$$U_{elas} = \frac{1}{2} \int_0^L \int_A \sigma_{ij} \varepsilon_{ij} dA dX = \frac{1}{2} \int_0^L E_{eff} I \left(\frac{\partial^2 w}{\partial X^2} \right)^2 dX. \quad (9)$$

In the above relation, $E_{eff} I$ and X are the effective flexural rigidity of the nanowire and the distance from the origin, respectively; μ is the shear modulus, and A is the cross-sectional area of the nanowire, respectively.

2.3. The non-dimensional governing equation

In order to derive the governing equation of the nanoactuator, the minimum energy principle is applied. It implies that equilibrium is achieved when the energy reaches a minimum value, i.e:

$$\delta(U_{elas} - W_{vdW}) = 0 \quad (10)$$

In the above-mentioned relation, W_{vdW} is the work done by the vdW attractive force:

$$W_{vdW} = \iint [f_{vdW1} - f_{vdW2}] dX dw \quad (11)$$

Now, by substituting (11) and (9) in (10), the nonlinear governing differential equations of the system can be written as

$$E_{eff} I \frac{d^4 w}{dX^4} = f_{vdW1} - f_{vdW2}, \quad (12)$$

And the boundary conditions are obtained as

$$w(0) = \frac{dw(0)}{dX} = \frac{d^2 w(L)}{dX^2} = \frac{d^3 w(L)}{dX^3} = 0, \quad \text{For cantilever,} \quad (13-a)$$

$$w(0) = \frac{dw(0)}{dX} = w(L) = \frac{dw(L)}{dX} = 0. \quad \text{For fixed-fixed.} \quad (13-b)$$

By substituting relations (3-a) and (3-b) into Eq. (12), using Eq. (10), the dimensionless nonlinear equation for the system is derived as

$$\frac{d^4 \hat{w}}{d\hat{x}^4} = \frac{\gamma}{\sqrt{(1-\hat{w})^5}} - \frac{\gamma}{\sqrt{(\alpha+\hat{w})^5}}, \quad (14-a)$$

$$\hat{w}(0) = \frac{d\hat{w}}{d\hat{x}}(0) = \frac{d^2 \hat{w}}{d\hat{x}^2}(1) = \frac{d^3 \hat{w}}{d\hat{x}^3}(1) = 0, \quad \text{For cantilever,} \quad (14-b)$$

$$\hat{w}(0) = \frac{d\hat{w}}{d\hat{x}}(0) = \hat{w}(1) = \frac{d\hat{w}}{d\hat{x}}(1) = 0. \quad \text{For fixed-fixed.} \quad (14-c)$$

The following dimensionless parameters were used for deriving Eqs. (14)

$$x = \frac{X}{L}, \quad (15-a)$$

$$\alpha = \frac{D_2}{D_1}, \quad (15-b)$$

$$\hat{w} = \frac{w}{D_1}, \quad (15-c)$$

$$\gamma = \frac{AR^{\frac{1}{2}}L^4}{8\sqrt{2}E_{eff}ID_1^{\frac{7}{2}}}, \quad (15-d)$$

3. Solution methods

To solve the governing equation of the systems, we apply two different approaches; analytical and numerical methods.

3.1 Modified Adomian Decomposition (MAD)

In this section, the differential equation is analytically solved using the modified Adomian decomposition (MAD).

The basic idea of the MAD for solving boundary value problems is explained in Refs. (6, 11, 12, 14, 16, 31 and 32).

To analytically solve Eq. (14), we consider the following equation

$$\frac{d^4 \hat{w}(x)}{dx^4} = f(x, \hat{w}), \quad (16-a)$$

$$\hat{w}(0) = C_1 \quad \text{and} \quad \hat{w}'(0) = C_2. \quad (16-b)$$

Using the MAD method, the nonlinear function on the right-hand of the Equation (16-a) comprise a series of Adomian polynomials which are tailored to the nonlinearity $f(x, \hat{w})$. Thereby, the solution of (16-a) can be represented as

$$\hat{w}(x) = \sum_{n=0}^{\infty} \hat{w}_n(x) = C_1 + C_2 x + \frac{1}{2!} C_3 x^2 + \frac{1}{3!} C_4 x^3 + \int_0^x \int_0^x \int_0^x \int_0^x \left(\sum_{n=0}^{\infty} f_n(x) \right) dx dx dx dx, \quad (17)$$

where C_1 , C_2 , C_3 and C_4 are constants of integration. Note that the values of C_1 and C_2 are obtained from Equation (16-b). The $f_n(x)$, which comprises the nonlinear function of $f(x, \hat{w})$, is determined from the following formula (11, 12, 31, 32)

$$f_n = \sum_{v=0}^{\infty} C(v, n) h_v(\hat{w}_0), \quad (18)$$

where

$$C(v, n) = \sum_{p_i} \prod_{i=1}^v \frac{1}{k_i!} \hat{w}_{p_i}^{k_i}, \quad \sum_{i=1}^v k_i p_i = n, \quad n > 0, \quad 0 \leq i \leq n, \quad 1 \leq p_i \leq n - v + 1, \quad (19)$$

$$h_v(\hat{w}_0) = \frac{d^v}{d\lambda^v} [f(\hat{w}(\lambda))]_{\lambda=0}. \quad (20)$$

In the above relations, k_i is the number of repetitions in the $\hat{w}_{p_i}$, the values of p_i are selected from the above range by combination without repetition, $h_v(\hat{w}_0)$ is calculated by differentiating the nonlinear term $f(\hat{w})$ v times with respect to $\hat{w}$ and evaluated at $\lambda = 0$. Using relations (18)-(20), the Adomian polynomials are obtained as

$$\begin{aligned} f_0 &= h_0(\hat{w}_0) \\ f_1 &= C(1, 1) h_1(\hat{w}_0) = \hat{w}_1 h_1(\hat{w}_0) \\ f_2 &= C(1, 2) h_1(\hat{w}_0) + C(2, 2) h_2(\hat{w}_0) = \hat{w}_2 h_1(\hat{w}_0) + \frac{1}{2!} \hat{w}_1^2 h_2(\hat{w}_0) \\ f_3 &= C(1, 3) h_1(\hat{w}_0) + C(2, 3) h_2(\hat{w}_0) + C(3, 3) h_3(\hat{w}_0) = \hat{w}_3 h_1(\hat{w}_0) + \hat{w}_1 \hat{w}_2 h_2(\hat{w}_0) + \frac{1}{3!} \hat{w}_1^3 h_3(\hat{w}_0) \\ &\dots \end{aligned} \quad (21)$$

By using relations (18) in conjunction with Eq. (17), the solution of Eqs. (14) is obtained as

$$\begin{aligned} \hat{w} &= \frac{1}{2!} C_3 x^2 + \frac{1}{3!} C_4 x^3 + \frac{1}{4!} \left(\frac{\gamma}{(1 - \hat{w}_0)^{\frac{5}{2}}} - \frac{\gamma}{(\alpha + \hat{w}_0)^{\frac{5}{2}}} \right) x^4 \\ &+ \left[\frac{5\gamma}{2(1 - \hat{w}_0)^{\frac{7}{2}}} + \frac{5\gamma}{2(\alpha + \hat{w}_0)^{\frac{7}{2}}} \right] \times \left[\frac{1}{6!} C_3 x^6 + \frac{1}{7!} C_4 x^7 + \frac{1}{8!} \left(\frac{\gamma}{(1 - \hat{w}_0)^{\frac{5}{2}}} - \frac{\gamma}{(\alpha + \hat{w}_0)^{\frac{5}{2}}} \right) x^8 \right] \\ &+ \dots \end{aligned} \quad (22)$$

Finally, the constants C_3 and C_4 are obtained from the boundary conditions of (14-b) and (14-c) for cantilever and fixed-fixed double-sided nanoactuators, respectively.

3.2. Numerical finite difference method (FDM)

In this section, a numerical algorithm based on the finite difference method (FDM) is used to solve these nonlinear boundary value problems. According to the standard FDM procedure, the domain is divided into n equal

$$\frac{\hat{w}_{i-2} - 4\hat{w}_{i-1} + 6\hat{w}_i - 4\hat{w}_{i+1} + \hat{w}_{i+2}}{\Delta x^4} = F_i, \quad (23)$$

where Δx , is the grid spacing, $\hat{w}_i$ is the deflection of the i^{th} grid and

$$F_i = \frac{\gamma}{\sqrt{(1-\hat{w}_i)^5}} - \frac{\gamma}{\sqrt{(\alpha+\hat{w}_i)^5}}. \quad (24)$$

Employing Equation (23) to all of the elements and using the boundary conditions (Equations (14-b) & (14-c)), we create a matrix system of algebraic equations

$$[A]\{\hat{w}\} = \{F\}, \quad (25)$$

where $\{\hat{w}\} = [\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n]^T$, $\{F\} = [F_1, F_2, \dots, F_n]^T$ and $[A]$ are the displacement vector, force vector and the stiffness matrix, respectively. According to the boundary conditions, the evaluation of $[A]$ the stiffness matrix yields

$$[A] = \begin{bmatrix} 7 & -4 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 6 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -4 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -4 & 5 & -2 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & -2 & 1 \end{bmatrix}_{30 \times 30} \quad \text{cantilever,} \quad (26-a)$$

$$[A] = \begin{bmatrix} 7 & -4 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 6 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -4 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -4 & 6 & -4 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & -4 & 7 \end{bmatrix}_{30 \times 30} \quad \text{fixed-fixed.} \quad (26-b)$$

The Maple commercial software is used to numerically solve the system of equations (25).

3.3. Instability Analysis

For any given γ , where $\gamma \leq \gamma_{cr}$, one can find solutions for $\hat{w}$. However, when $\gamma > \gamma_{cr}$, no solution exists for $\hat{w}$. This means that the instability occurs and the nanowire collapses onto the fixed substrate (2 & 33). For both numerical and analytical solutions, the values of γ_{cr} and the corresponding nanowire critical deflection, $\hat{w}_{cr}$, can be determined by plotting the dimensionless vdW attraction parameter, γ , vs. the dimensionless deflection at the tip point of the nanowire, $\hat{w}_{tip} = \hat{w}(x=1)$, and the dimensionless deflection at the middle point of the nanowire, $\hat{w}_{mid} = \hat{w}(x=0.5)$, for cantilever and fixed-fixed double-sided nanoactuators, respectively.

4. Results and Discussion

4.1. Deflection of the movable electrode

Figs. 2 and 3 show the variation of the deflection of the cantilever nanowire in the presence of the vdW force obtained by MAD and numerical solutions, respectively. Similarly, the variation of the deflection of the fixed-fixed

structure determined by MAD and numerical solutions are shown in Figs. 4 and 5, respectively. Note that the parameter γ denotes the dimensionless value of the vdW attraction. Any decrease in the gap (D_l) as well as any increase in the diameter (R) or length (L) leads to an increase in the γ value.

As seen above, the deflection of the nanowire increases by any increase in the vdW force, and at the instability point the value of the vdW force (γ) reaches its critical value (γ_{cr}).

It is worth noting that the maximum differences between the numerical finite difference and MAD solution methods in predicting γ_{cr} and $\hat{w}_{cr}$ values are less than 2% and 7%, respectively which demonstrates a good agreement between the numerical and MAD solution methods.

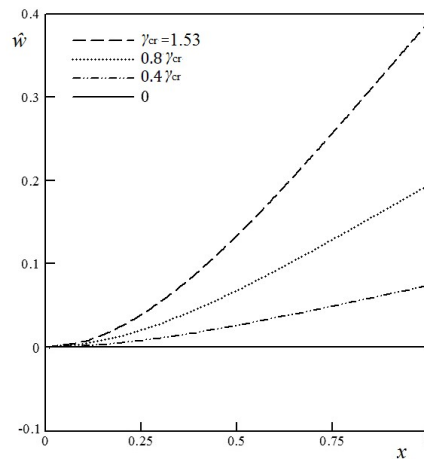


Fig. 2. The deflection of the cantilever nanowire for different values of γ and $\alpha = 2$ using MAD.

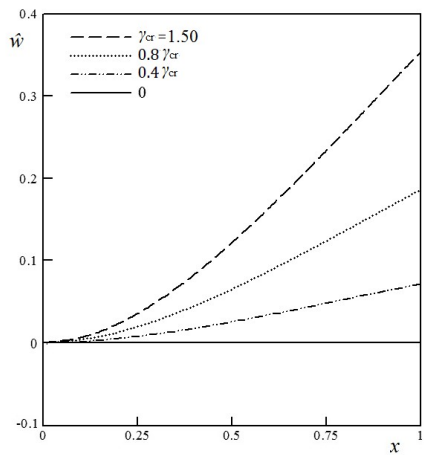


Fig. 3. The deflection of the cantilever nanowire for different values of γ and $\alpha = 2$ using the numerical solution.

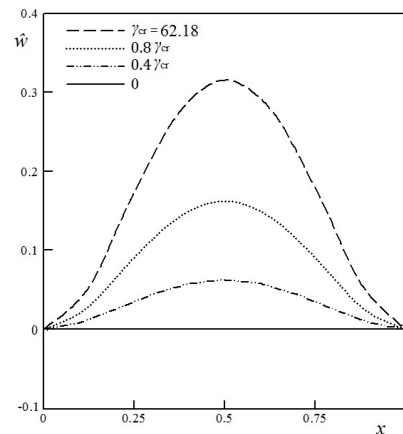


Fig. 4. The deflection of the fixed-fixed nanowire for different values of γ and $\alpha = 2$ using MAD.

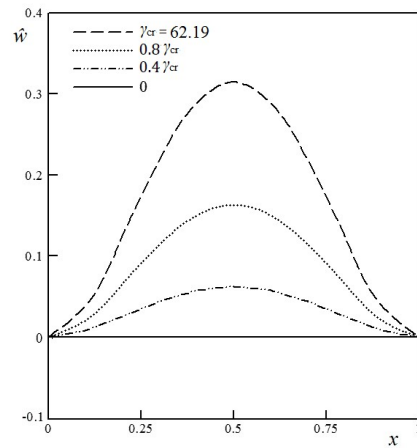


Fig. 5. The deflection of the fixed-fixed nanowire for different values of γ and $\alpha = 2$ using the numerical solution.

4.2. Instability analysis

Fig. 6 and Fig. 7 show the relation between the parameter γ and the tip displacement of the cantilever nanowire; $\hat{w}_{tip} = \hat{w}(x=1)$. Figs. 6 and 7 present the analytical and the numerical results, respectively. As shown below, when the parameter γ reaches its critical value γ_{cr} , the tip deflection of the nanowire reaches its maximum value $\hat{w}_{cr}$. No equilibrium exists for higher values of the vdW force ($\gamma > \gamma_{cr}$). From physical point of view, the vdW attraction overcomes the elastic resistance of the nanowire; hence, the nanowire collapses onto and sticks to the upper plane.

Furthermore, the relation between the dimensionless vdW force γ and the deflection at the middle point of the nanowire $\hat{w}_{mid} = \hat{w}(x=0.5)$ are shown in Fig. 8 and Fig. 9 using MAD and numerical solutions, respectively. These figures reveal that when γ exceeds its critical value γ_{cr} , the nanowire collapses onto and sticks to the upper plane.

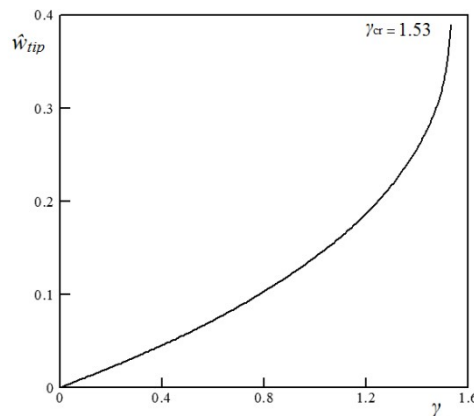


Fig. 6. The variation of the tip displacement of the cantilever nanowire versus γ for $\alpha = 2$ using MAD.

4.3. Impact of geometry of the structures

In order to examine the impact of geometry on the behavior of the system, variation of the critical value of the vdW parameter, γ_{cr} , as a function of the geometrical parameter, α , is presented in Figs. 10-13. Note that α represents the ratio between D_1 and D_2 . Figs. 10 and 11 depict the results obtained by MAD and numerical methods, respectively.

As shown below, any decrease in α increases the critical value of the vdW force. This means that reduction of the difference between D_1 and D_2 can stabilize the structure, because the attractive effect of the upper surface neutralizes the attractive effect of the lower surface.

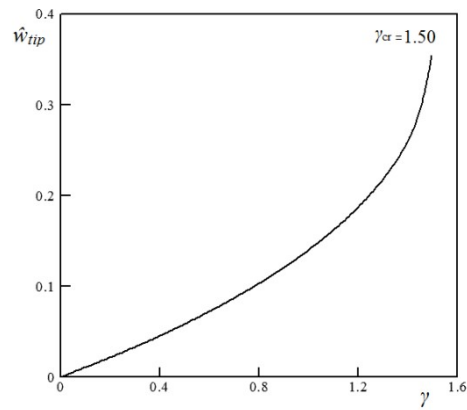


Fig. 7. The variation of the tip displacement of the cantilever nanowire versus γ for $\alpha = 2$ using the numerical solution.

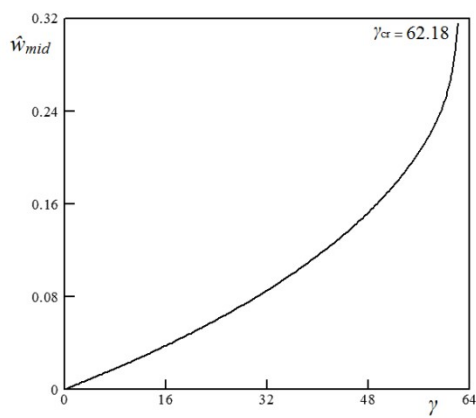


Fig. 8. The variation of the maximum deflection of the fixed-fixed nanowire versus γ for $\alpha = 2$ using MAD.

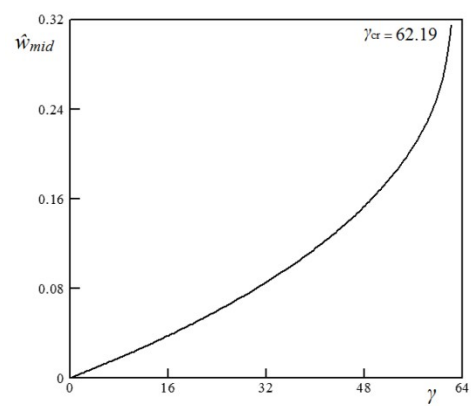


Fig. 9. The variation of the maximum deflection of the fixed-fixed nanowire versus γ for $\alpha = 2$ using the numerical solution.

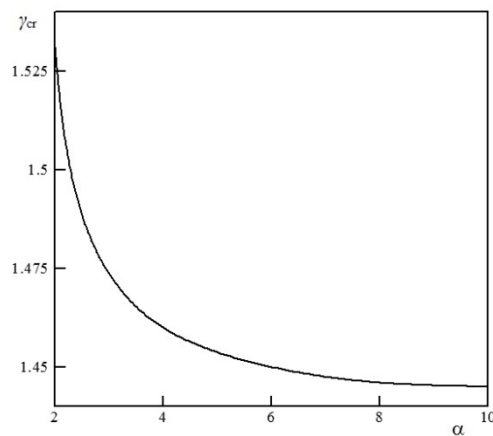


Fig. 10. The variation of the parameter γ_{cr} versus the parameter α for the cantilever nanoactuator using MAD.

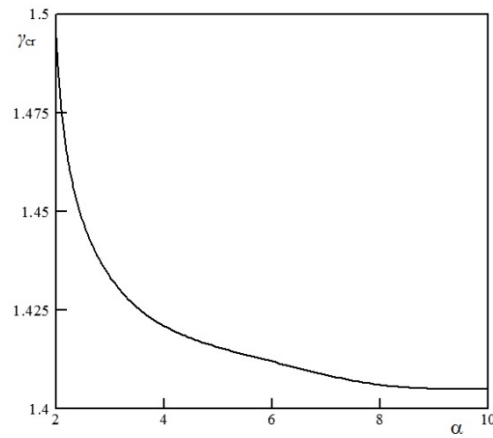


Fig. 11. The variation of the parameter γ_{cr} versus the parameter α for the cantilever nanoactuator using the numerical method.

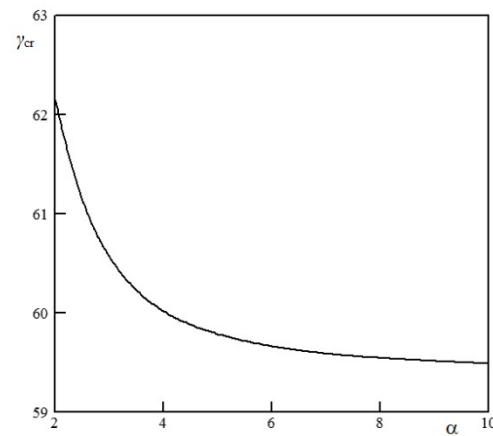


Fig. 12. The variation of the parameter γ_{cr} versus the parameter α for the fixed-fixed nanoactuator using MAD.

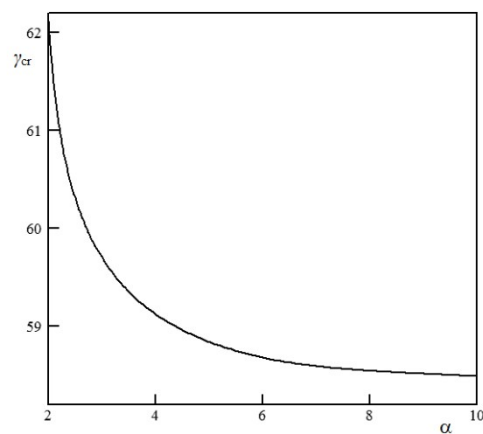


Fig. 13. The variation of the parameter γ_{cr} versus the parameter α for the fixed-fixed nanoactuator using the numerical method.

4.4. Detachment length and minimum gap

As mentioned earlier, the movable elements of nanoactuators (nanowire) may stick to the fixed upper plane due to the strong attractive vdW force which leads to undesired instability in freestanding nanoactuators during the fabricating and manufacturing stages. Therefore, this force should be taken into account in the design, manufacture

In this stage, we determine the detachment length and the minimum gap of nanoactuators which are very important parameters in the design of nanoactuators. The detachment length, L_{max} , is the maximum length of the nanowire which does not stick to the fixed plane (4&5). On the other hand, if the length of the nanowire is known, there is a minimum gap, D_{1min} , which prevents stiction between the nanowire and the plane due to the vdW force.

In order to determine the detachment length and the minimum gap, one can use the Eq. (15-d). By substituting the value of γ_{cr} into the definition of γ (Eq. (15-d)) and considering $A = 50 \times 10^{-20}$ J, the values of L_{max} and D_{1min} , can be easily computed. Note that in general, the value of the detachment length and the minimum gap is a function of geometrical parameters and elastic constants of the nanowire. As a case study, Table 1 shows the comparison between L_{max} and D_{1min} values obtained for a typical nanoactuators (with $\alpha=2$).

Table 1. Critical value of the vdW force and corresponding formulas for computing the detachment length (L_{max}) and the minimum gap (D_{1min}) of freestanding nanoactuators with $\alpha=2$.

	Fixed-Free			Fixed-Fixed		
	γ_{cr}	$L_{max}(m)$	$D_{1min}(m)$	γ_{cr}	$L_{max}(m)$	$D_{1min}(m)$
MAD	1.53	$7.67 \times 10^4 \left(\frac{E_{eff} ID^{3.5}}{\sqrt{R}} \right)^{0.25}$	$2.614 \times 10^{-6} \left(\frac{L^4 \sqrt{R}}{E_{eff} I} \right)^{\frac{2}{7}}$	62.18	$1.94 \times 10^5 \left(\frac{E_{eff} ID^{3.5}}{\sqrt{R}} \right)^{0.25}$	$9.071 \times 10^{-7} \left(\frac{L^4 \sqrt{R}}{E_{eff} I} \right)^{\frac{2}{7}}$
Numerical	1.50	$7.63 \times 10^4 \left(\frac{E_{eff} ID^{3.5}}{\sqrt{R}} \right)^{0.25}$	$2.629 \times 10^{-6} \left(\frac{L^4 \sqrt{R}}{E_{eff} I} \right)^{\frac{2}{7}}$	62.19	$1.94 \times 10^5 \left(\frac{E_{eff} ID^{3.5}}{\sqrt{R}} \right)^{0.25}$	$9.070 \times 10^{-7} \left(\frac{L^4 \sqrt{R}}{E_{eff} I} \right)^{\frac{2}{7}}$

5. Conclusions

Herein, continuum mechanics was applied for modeling the effect of the vdW attraction on the instability of the nanowire-made nanoactuator with three electrodes. It is found that:

- 1- When the vdw force exceeds its critical value, it overcomes the elastic resistance of the nanowire; hence, the nanowire becomes unstable and sticks to the upper plane.
- 2- The detachment length and the minimum gap parameters of the nanoactuator, which are critical parameters in the design and the fabrication of the nanoactuator, were determined.
- 3- A good agreement between the numerical and MAD solutions was observed.

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