Conjugate and Directional Chaos Control Methods for Reliability Analysis of CNT–Reinforced Nanocomposite Beams under Buckling Forces; A Comparative Study

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Abstract

The efficiency and robustness of reliability methods are two important factors in the first-order reliability method (FORM). The conjugate choice control (CCC) and directional chaos control method (DCC) are developed to improve the robustness and efficiency of the FORM formula using the stability transformation method. In this paper, the CCC and DCC methods are applied for the reliability analysis of a nanocomposite beam as a complex engineering problem, which is reinforced by carbon nanotubes (CNTs) under buckling force. The probabilistic model for nanocomposite beam is developed through the buckling failure mode which is computed by using the Euler-Bernoulli beam model. The robustness and efficiency CCC and DCC are compared using the stable solution and a number of call limit state functions. The results demonstrate that the CCC method is more robust than the DCC in this case, while the DCC method is simpler than the CCC.

Keywords: Reliability analysis; Nanocomposite beam; Conjugate chaos control; Directional chaos control.

1. Introduction

The first order reliability method (FORM) is used widely to approximate the failure probabilities of the complex structural problems due to its simplicity and efficiency compared to the sampling methods, i.e. the Monte Carlo simulation [1, 2]. Based on the FORM, the failure probabilities can be estimated by linearizing the limit state function (LSF) in the standard normal space as follows [3]:

$$P_f = \int\limits_{g(X) \leq 0} ... \int f_X(x_1,...,x_n) dx_1...dx_n \approx \Phi(-\beta) \tag{1}$$

where, $g(X)$ is the LSF, $g(X) \leq 0$ defines the failure region, $f_X(x_1,...,x_n)$ is the joint probability density function for the basic random variables X and Φ is the cumulative distribution function. β is the reliability index, which corresponds to the minimum distance to the origin on the limit state function [4]. Generally, the main effort of FORMs is to search the most probable point (MPP), i.e. U^* which is the point of the minimum distance to the origin on the limit state surface in the standard normal space. Several iterative formulas were developed to enhance the robustness and efficiency of the FORM based on the steepest descent search direction [3, 5-8] and the conjugate search direction [1, 4, 9- 11]. Yang [5] proposed the stability transformation method (STM) of chaos control (CC) for the reliability analysis-based FORM. After that, Keshtegar and Miri [3] proposed a relaxed chaos control method with a dynamic step size named as relaxed HL-RF (RHL-RF). Recently, the FORM formula of STM-based CC was modified using the radial direction in terms of the steepest descent search direction in the directional CC (DCC) method proposed by Meng et al. [8]. The CC approach was improved using the conjugate search direction

through a dynamic line search in the conjugate chaos control (CCC) which was proposed by Keshtegar [9]. Keshtegar [9] showed the CCC method is more robust than the FORM based steepest descent search direction such as CC [5], RHL-RF [3] and more efficient than the finite-step length (FSL) [6] and the conjugate HL-RF (CHL-RF) [11] methods. Consequently, the performance convergences including both robustness and efficiency of CCC and DCC can be investigated in a complex engineering problem.

The nanoparticles can improve the mechanical, electrical, and thermal properties of beams. Moreover, the reliability analysis using a robust and efficient FORM for the nanocomposite beams under axial compressive load can provide a safety level to the successful application of these complex engineering problems. The probabilistic model using the buckling force is important in the reliability analysis of nanocomposite beams. In previous research, a nanocomposite beam reinforced by a single-walled carbon nanotube was analyzed using the airy stress-function method to determine the local buckling [12, 13]. The buckling and bifurcation buckling behavior of composite plates, which are reinforced by carbon nanotubes and armchair double-walled boron nitride nanotubes, were investigated by Ghorbanpour Arani et al. [14], Rafiee et al. [15], Wattanasakulpong and Ungbhakorn [16], Kolahchi et al. [17], and Mosharrafian and Kolahchi [18]. The buckling failure mode is an important issue to evaluate the nonlinear mechanical properties of the nanocomposite. Therefore, the accurate analysis of the buckling force and reliability procedure can provide appropriate reliability results to evaluate the failure probabilities. Moreover, the CCC and DCC can provide stable and efficient results for highly nonlinear problems in comparison with the other reliability methods such as CHL-RF [11], CC [5], FSL [6] and RHL-RF [3].

In this paper, the CCC and DCC methods of FORM are applied for the reliability analysis of a nanocomposite beam reinforced by the carbon nanotubes (CNTs). The probabilistic model of the buckling failure mode of the nanocomposite beams is developed using the Euler-Bernoulli beam model, which involves various uncertainties including the length and thickness of the beam, the spring constant and the shear constant of the foundation, the applied voltage, and the volume fraction of CNTs. The efficiency and robustness of the CCC and DCC methods are investigated in a real complex nonlinear engineering problem under the bucking performance function. The reliability index is extracted using the DCC and CCC methods for two random variables of the applied voltage ranging from -200 to 200 volts and the volume fraction of nanoparticles ranging from 0 to 0.5. The results indicate that the CCC method provided more stable results for nanocomposite beams under buckling forces than the DCC method, and the applied voltage has performed better than the volume fraction of nanoparticles to improve the confidence levels of the beam.

2. Reliability methods

2.1. Directional chaos control (DCC) method

The efficiency of the stability transformation method (STM) is improved through the directional sensitivity vector of random variables using the chaos control approach, named as the directional chaos control (DCC). The DCC iterative formula of FORM is proposed based on the STM as follows [8]:

$$U_{k+1} = \beta_k \frac{n_k}{\|n_k\|} \tag{2}$$

where, n_k is radial direction which is along the reliability index β_k. The reliability index can be obtained as follows [1, 10]:

$$\beta_{k+1} = \frac{g(U_k) - \nabla^T g(U_k) U_k}{\|\nabla g(U_k)\|} \tag{3}$$

Where, U is the standard normal variable with the mean and standard deviation equal to zero and one, respectively. $\nabla g(U_k)$ is the gradient vector of the LSF in the standard normal space at the point U_k as $\nabla g(U) = [\partial g / \partial u_1, \partial g / \partial u_2, ..., \partial g / \partial u_n]^T$. The radial directional vector n_k is computed based on the STM of the chaos control by the following discrete map [8]:

$$n_k = U_k + \xi C(f(U_k) - U_k) \tag{4}$$

where C is the involuntary matrix (namely, only one element in each row and each column of this matrix is 1 or -1 and the others are 0. C is usually the orthogonal matrix, i.e. $C = I$). ξ is the chaos control factor which is given as $0 < \xi < 1$. To achieve the stabilization for the highly nonlinear LSFs, a small constant value (i.e. 0.1–0.001 [8, 5]) should be selected by the factor ξ. The $f(U_k)$ is the discrete nonlinear map which is computed as

$$f(U_k) = \frac{\nabla^T g(U_k) U_k - g(U_k)}{\nabla^T g(U_k) \nabla g(U_k)} \nabla g(U_k) \tag{5}$$

The control factor of the radial direction can control the instability of FORM formula and this radial directional vector in Eq. (4) –based chaos control remarkably improves the computational cost of STM for the highly nonlinear performance functions.

2.2 Conjugate chaos control (CCC) method

The chaos control iterative FORM-based STM is improved based on the conjugate search by Keshtegar as follows [9]:

$$U_{k+1}^{CCC} = U_k^{CCC} + \xi[f^C(U_k^{CCC}) - U_k^{CCC}] \tag{6}$$

Where, U^{CCC} is the point which is determined based on the iterative formula of conjugate FORM with the chaos feedback control; λ is the control factor which can be generated as $0 < \xi < 1$; $f^C(U_k^{CCC})$ is the solution of the kth iteration based on the conjugate search direction at the point U_k^{CCC}; $f^C(U_k^{CCC})$ is determined based on a discrete nonlinear map as follows:

$$f^C(U_k^{CCC}) = \frac{\nabla^T g(U_k^{CCC})U_k^{CCC} - g(U_k^{CCC})}{\nabla^T g(U_k^{CCC})\alpha_k^C}\alpha_k^C \tag{7}$$

where, α_k^C can be defined as the normalized conjugate search direction vector or the conjugate unit vector which is computed as follows [1, 11]:

$$\alpha_k^C = \frac{U_{k+1}^{C\lambda}}{\left\|U_{k+1}^{C\lambda}\right\|} \tag{8}$$

The point $U_{k+1}^{C\lambda}$, which is along the conjugate direction at the design point U_k^{CCC}, is computed as

$$U_{k+1}^{C\lambda} = U_k^C + \lambda_k d_k \tag{9}$$

where, d_k, which is the conjugate search direction vector, is defined as follows [10, 11]:

$$d_k = -\nabla g(U_k^{CCC}) - \frac{\left\|\nabla g(U_k^{CCC})\right\|^2}{\left\|\nabla g(U_{k-1}^{CCC})\right\|^2}d_{k-1} \tag{10}$$

If λ is well-defined, the convergence of the conjugate FORM can be obtained accurately in the reliability analysis. The finite- step size λ_k in Eq. (9) is determined as follows:

$$\lambda_k = \begin{cases} \lambda_{k-1} & if \quad \left\|U_{k+1}^{CCC} - U_k^{CCC}\right\| < \left\|U_k^{CCC} - U_{k-1}^{CCC}\right\| \\ 0.9\lambda_{k-1} & o.w \end{cases} \tag{11}$$

The initial finite-step size in Eq. (11) is defined based on the Armijo-line search rule as follows:

$$\lambda_0 = \frac{25}{\left\|\nabla g(U^{CCC}{}_{|U=\mu})\right\|} \tag{12}$$

The above-mentioned step size implies the sufficient descent condition as $\left\|U_{k+1}^{CCC} - U_k^{CCC}\right\| < \left\|U_k^{CCC} - U_{k-1}^{CCC}\right\|$, therefore, it can be concluded that $\left\|U_{k+1}^{CCC} - U_k^{CCC}\right\| \approx 0$ when $k \to \infty$. This means that $U_{k+1}^{CCC} \approx U_k^{CCC}$ and a fixed point is obtained based on the conjugate iterative formula.

3. Probabilistic model of the nanocomposite beam

The reliability analysis of the nanocomposite beam can be done based on a probabilistic buckling model which includes various uncertainties of the beams such as geometric optics of a beam, nanoparticles, and mechanical properties of the foundation. Therefore, a limit state function which separates the design domain into the failure and safe regions based on the buckling model of the nanocomposite beams can be defined as follows:

$$g(L, h, kw, kg, V0, rho) = P_{cr}(L, h, kw, kg, V0, rho) - P \tag{13}$$

It is supposed that the nanocomposite beams may be failed when the obtained buckling force (P_{cr}) is less than the external compressive load (P) of the beams (P is given as 50GPa in this study). $P_{cr}(L, h, kw, kg, V0, rho)$ is the buckling force which depends on the length of the beam (L), the thickness of the beam (h), the spring constant of the foundation (kw), the shear constant of the foundation (kw), the applied voltage ($V0$), and the volume fraction of CNTs in the beam (rho). A polymeric-reinforced CNTs beam under externally applied voltage, which is shown in Fig. 1, is considered for comparing the CCC and DCC methods in the reliability analysis based on the limit state function in Eq. (13).
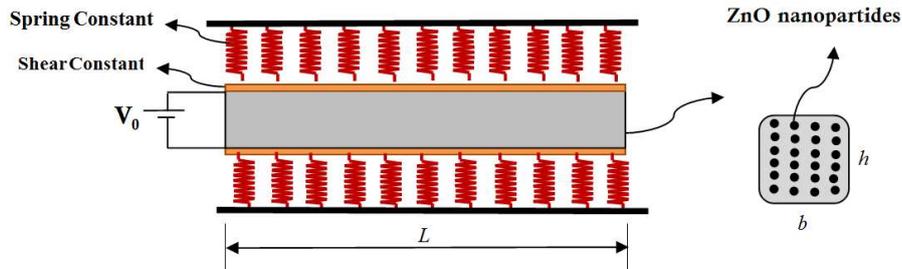
Fig. 1. The schematic view of the nanocomposite beam

The structure is modeled based on the Euler-Bernoulli beam theory where the displacement field can be given as follows [13]:

$$u_1(x,z) = U(x) - z\frac{\partial W(x)}{\partial x},$$
$$u_2(x,z) = 0,$$
$$u_3(x,z) = W(x),$$

(14)

where $U(x)$ and $W(x)$ are displacement components in the mid-plane. The von Karman type nonlinear strain–displacement relations are given by

$$\varepsilon_x = \left(\frac{\partial U}{\partial x}\right) + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 - z\left(\frac{\partial^2 W}{\partial x^2}\right).$$

(15)

The stress-strain relations can be written as

$$\sigma_{xx} = C_{11}\varepsilon_x$$

(16)

where C_{11} is the elastic constant which can be calculated by the Mori-Tanaka model. The strain energy of the structure can be expressed as

$$U = \frac{1}{2}\int_0^L \int_A (\sigma_{xx}\varepsilon_{xx})\; dAdx.$$

(17)

Substituting Eqs. (14) and (15) into Eq. (17) gives

$$U = \frac{1}{2}\int_0^L \left\{ N_x \frac{\partial U}{\partial x} + \frac{1}{2} N_x \left(\frac{\partial W}{\partial x}\right)^2 - M_x \frac{\partial^2 W}{\partial x^2} \right\} dx,$$

(18)

where N_x is the resultant force and M_x is the bending moment. The external work due to the foundation can be written as [14]

$$\Omega = \int_0^L \left(\underbrace{-K_w W + G_p \nabla^2 W}_{q} \right) W dx.$$

(19)

The governing equations of the structure can be derived from the Hamilton's principle as

$$\delta U : \frac{\partial N_x}{\partial x} = 0,$$

(20)

$$\delta W : \frac{\partial^2 M_x}{\partial x^2} - \frac{\partial}{\partial x}\left(N_x^M \frac{\partial W}{\partial x} \right) + q = 0,$$

(21)

where N_x^M is the axial load applied to the concrete column. Introducing the following dimensionless quantities

$$\xi = \frac{x}{L}, \quad (\overline{W}, \overline{U}) = \frac{(W,U)}{h}, \quad \eta = \frac{h}{L}, \psi = \overline{\psi}, \quad \overline{K}_w = \frac{K_w h L}{C_{11}A},$$

$$\overline{G}_p = \frac{G_p}{C_{11}A}, \overline{N}_x^M = \frac{N_x^M}{C_{11}A}, \overline{I} = \frac{I}{AL^2}, \quad C_5 = \frac{C_{55}}{C_{11}}.$$

(22)

and substituting Eqs. (21)-(22) into the governing equations yields

$$\frac{\partial^2 \overline{U}}{\partial \xi^2} + \eta \frac{\partial^2 \overline{W}}{\partial \xi^2}\frac{\partial \overline{W}}{\partial \xi} = 0,$$

(23)

$$-\overline{I}\frac{\partial^4 \overline{W}}{\partial \xi^4} - \overline{N}_x^M \eta \frac{\partial^2 \overline{W}}{\partial \xi^2} - \overline{K}_w \overline{W} + \overline{G}_p \eta \left(\frac{\partial^2 \overline{W}}{\partial \xi^2}\right) = 0.$$

(24)

Finally, by using DQM we have

$$\left(\left[\underbrace{K_L + K_{NL}}_{K} \right] + P[K_g] \right) \begin{Bmatrix} \{d_b\} \\ \{d_d\} \end{Bmatrix} = 0, \tag{25}$$

where K_L is the linear stiffness matrix, K_{NL} is the nonlinear stiffness matrix and K_g is the geometric stiffness matrix. Moreover, d_b and d_d represent the boundary and domain points. Finally, based on an iterative method and eigenvalue problem, the buckling load of the structure may be obtained. However, the critical buckling load (Pcr) is theoretically obtained based on the Eq. (25). It is supposed that a nanocomposite beam is under an external comparative force, i.e. P=50 GPa and the capacity of beam is obtained based on the buckling force in Eq. (25). Six basic random variables which are extracted from Table 1 are given for this nanocomposite beam in Fig. 1.

Table 1: the basic random variables of the nanocomposite beam

Random variable	h (m)	L (m)	kw (N/m²)	kg (N)	rho	$V0$ (volte)
Mean	0.4	0.6	3.5×10^{12}	5	0–0.5	-200 – 200
Coefficient of variation	0.1	0.1	0.12	0.12	0.15	0.2
Distribution	Lognormal	Normal	Gumbel	Gumbel	Lognormal	Normal

4. Reliability results of the nanocomposite beam

A nanocomposite beam under the buckling failure mode is selected to illustrate the convergence performances including both the robustness and the efficiency of the FORM-based CCC and DCC methods. The stable convergence and the number of call functions to evaluate the buckling forces based on the Eq. (25) are used to compare the convergence performances of the nanocomposite beam in Fig. 1. The MATLAB programs are coded for the reliability analysis of this complex example while the MATLAB codes can evaluate the sensitivity vectors and simulate the normal and non-normal random variables. The gradient vector of the limit state function in Eq. (25) is determined based on the finite difference method.

Regarding the reliability analysis, the mean values for the applied voltage and the volume fraction of CNTs are given as $V0$=100 volt and rho=0.3, respectively. Based on the statistical properties of random variables in Table 1, the CCC and DCC methods are applied to approximate the reliability index. The results of the reliability analysis including the reliability index (β), the numbers of evaluating the gradient vector of the limit state function (Iter), the numbers of calling the limit state function (Call) for the CCC and DCC methods with respect to different control factors as λ=0.05, 0.1, 0.5, and 1, are tabulated in Table 2. The reliability index histories of CCC and DCC methods for λ=0.1 and 0.5 are shown in Fig. 2. The results from the Table 2 and Fig. 2 show that the convergence performances including both efficiency and robustness of CCC and DCC methods are strongly depended on the chaos control factor. The CCC method yields unstable results as chaotic solutions when the chaos control factor is given less than 0.1 for this nanocomposite beam. However, the CCC method provides stable results because the initial finite-step size in Eq. (12) is adjusted based on the Eq. (11) for each iteration when the chaotic results are obtained from the iterative formula of CCC method. By selecting a large number for the chaos control factor in CCC method, the number of evaluations of the gradient vector (Iterations) for CCC method is slightly decreased to achieve the stabilization. The CCC method is converged more efficiently when the λ is given as equal to 1. The DCC is more efficient than the CCC method regarding the λ=0.5. According to λ=1, the CCC is converged with the call functions about three times less than the DCC method. In addition, the CCC formula is more robust than the DCC method and the nonlinear dynamic map can provide stable solutions to search MPP for chaos control factor between 0.05 and 1 based on the CCC method. However, the stability of the DCC depends on the appropriate selection of the control factor. The obtained MPP using the DCC and CCC based on λ=1 is X^* (L^* = 0.395245m, h^* = 0.3959143m, kw = 3112.0935GPa, kg =19.5625N, $\rho = 0.29562479$, $V0$ =100.7514volte).

Table 2: The converged results of CCC and DCC for different chaos control factors

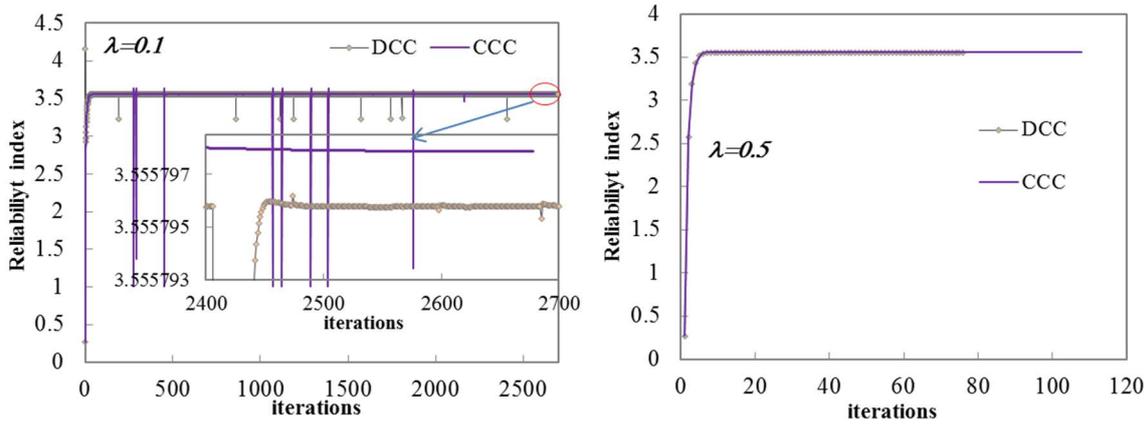| Method	λ=0.05		λ=0.1		λ=0. 5		λ=1	
	β	Iter\Call	β	Iter\Call	B	Iter\Call	β	Iter\Call
CCC	3.555806	4559\523664	3.555796	2677\307211	3.555796	107\11983	3.555796	8\857
DCC	Chaos	Not converged	Chaos	Not converged	3.555798	75\17153	3.555796	13\3085

Fig. 2. Reliability index histories of CCC and DCC methods for the chaos control factor 0.1 and 0.5

The reliability indices of the nanocomposite beam with respect to various applied voltages (V0) and volume fractions of CNTs (rho) are computed using the CCC method with the chaos control factor of 1. Figs. 3 and 4 illustrate the reliability index with respect to the various applied voltages and the volume fractions of nanoparticles, respectively. The obtained results in Figs. 3 and 4 show that the reliability indices are decreased by increasing the applied voltage while the volume fractions of nanoparticles can improve the reliability index of nanocomposite beams. The reliability index is reduced from 3.7 to 3.5 when the applied voltages are changed from -200 to 250 volts in rho=0.3. However, the reliability index can be improved from 3.5 to 3.6 based on the volume fractions of nanoparticles 0.05-0.5 in the applied voltage of 100volts. It can be concluded from the results of Figs. 3 and 4 that decreasing the applied voltage and increasing the volume fractions of nanoparticles can improve the confidence level of nanocomposite beams. The applied voltage achieved better reliability indices than the volume fractions of nanoparticles.
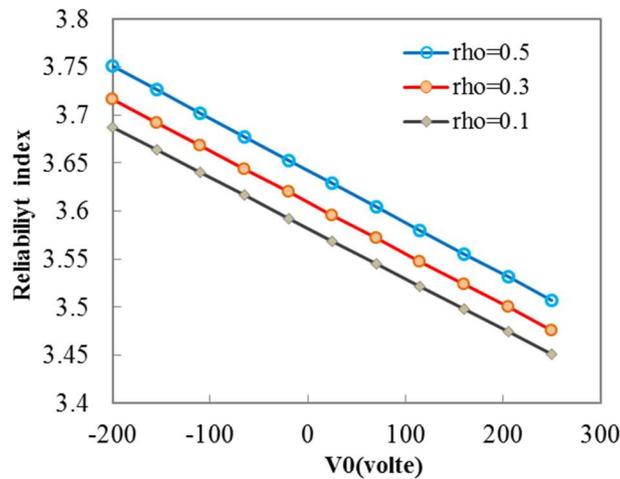

Fig. 3. Reliability index corresponding to various applied voltages based on the CCC for different rho
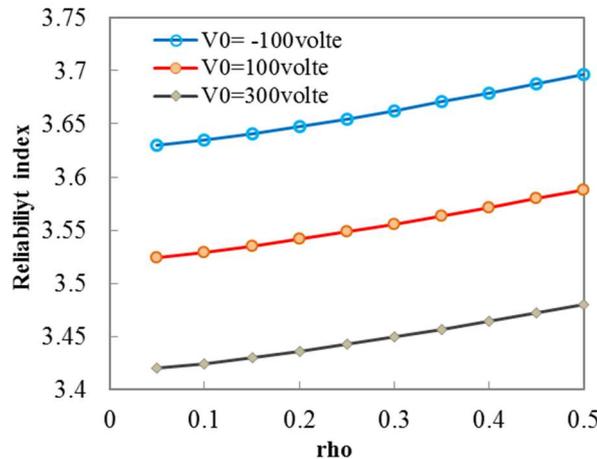

Fig. 4. Reliability index corresponding to various volume fractions of nanoparticles based on the CCC for different V0

5. Conclusion

The conjugate chaos control (CCC) and the directional chaos control (DCC) using stability transformation method were recently developed to improve the efficiency and robustness of the iterative formula of the first-order reliability method (FORM). In the present study, due to the capability and efficiency of these reliability methods, better results is achieved in comparison with the other reliability methods such as HL-RF, relaxed HL-RF, and the stability transformation method (STM). The CCC and DCC methods are utilized for the reliability analysis-based FORM of a nanocomposite beam which is reinforced by the CNTs. The CCC and DCC methods are compared b ased on the nanocomposite beam to illustrate their robustness and efficiency for reliability analysis of the co mplex engineering problems in future. For this purpose, the stable results and a number of evaluations of t he performance function of the buckling failure mode which is determined based on an implicit limit state function using Euler-Bernoulli beam model are applied. A probabilistic model for the buckling force is deve loped theoretically and then the reliability indices of the beam are computed based on different values of a pplied voltages and volume fractions of CNTs. The results of the reliability analysis for nanocomposite beam show that the CCC is a robust and efficient FORM-based method in comparison with the DCC method as long as a large chaos control factor is selected. The CCC is converged to the stable reliability index while the DCC is yielded unstable results as chaotic solutions for the chaos control factor of less than 0.1. Decreasing the applied voltage and increasing the CNTs in the nanocomposite beam can improve the reliability index for this complex reliability problem.

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