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Low Velocity Impact Response of Laminated Composite Truncated Sandwich Conical Shells with Various Boundary Conditions Using Complete Model and GDQ Method

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Abstract

In this paper, the dynamic analysis of the composite sandwich truncated conical shells (STCS) with various boundary conditions subjected to the low velocity impact was studied analytically, based on the higher order sandwich panel theory. The impact was assumed to occur normally over the top face-sheet, and the contact force history was predicted using two solution models of the motion which were derived based on Hamilton's principle by considering the displacement continuity conditions between the layers· In order to obtain the contact force and the displacement histories, the differential quadrature method (DQM) was used. In this investigation, the effects of different parameters such as the number of layers of the face sheets, the boundary conditions, the semi vertex angle of the cone, and the impact velocity of the impactor on the impact response of the complete model were studied.

Keywords: Low velocity impact, STCS, DQM, Complete model.

1. Introduction

Today, the sandwich structures consisting of two thin face sheets and a relatively thick lightweight core with various shapes have gained widespread usage because of their higher strength-to-weight and bending stiffness-to-weight ratios [1]. The thin and thick circular conical shell structures rise to optimum conditions for the static and dynamic behavior. They are widely used in mechanical, civil, aerospace, architectural, and marine engineering applications such as hoppers, pressure vessels and tanks, space vehicle and space crafts, sub marines, and reactors nozzles [2].

Since one of the main parameters of designing such structures is the evaluation of their dynamic response to the impact load, the impact of objects on sandwich structures has been an interesting subject for researchers during the last decades. Chai and Zhu [3] reviewed the numerical, mathematical, and experimental methods used for the analysis of the sandwich panels subjected to impact loading. They analyzed the impact responses according to the key parameters and; consequently, identified various classes of impact. The impact responses on the sandwich structures were classified into two main groups: the high velocity and low velocity impacts with the focus on the low-velocity impact. According to the mass ratio, the response under the low-velocity impact was further subdivided into three possible categories, namely, large, small, and medium mass impacts.

A comprehensive review of the analytical models has been provided by Abrate [4] which classifies the previous researches into three categories: the spring-mass models (a combination of the global and local springs) which are used to present the transverse load-formation behavior, the energy-balance models that assume a quasi-static behavior of the structure, and the complete models in which the dynamic behavior of the structure is fully modeled. Shivakumar et al. [5] used a two-degree-of-freedom model that consisted of four springs for bending, shear, membrane, and contact rigidities to predict the impact response of a circular plate. In their model, the contact force and the contact duration for the low-velocity impact on the circular laminates was calculated. Anderson [6] performed an investigation of the single-degree-of-freedom models for a large mass impact on the composite sandwich laminates. The stiffness parameters of the models were derived from the results of the three-dimensional quasi-static contact analyses of a rigid sphere indenting a multi-layered sandwich laminate. Gong and Lam [7] used a spring-mass model having two-degrees-of-freedom to determine the history of the contact force produced during the impact. They also included the structural damping in their model. Malekzadeh et al. [8] presented a new computational method for the face sheets based on the improved higher order sandwich plate theory (IHSAPT) to analyze the transverse low velocity impact on the sandwich panels which was caused by a spherical impactor. In their study, a new three-degree-of-freedom (TDOF) springs-masses-damper (SMD) model is proposed to predict the contact force history for the composite sandwich panels with a transversely flexible core. Khalili et al. [9] presented a new equivalent 3DOF spring-masses (SM) model that accommodated the normal impact at any location and used it to predict the low velocity impact response of the composite sandwich panels with a stiff and flexible core. Their method allowed more than one impactor to act simultaneously on the panel, at different locations, either on the same face sheet or on the opposite sides of the panel. Based on Love's first approximation shell theory, the free vibration analysis of the conical and cylindrical shells with various boundary conditions was performed by Wilkins et al. [10]. In their theory, the transverse shear strain was not ignored. Using the finite deformation theory, Struck [11] studied the buckling analysis of the shallow open conical sandwich shells under uniform external pressure.

Bardell et al. [12] used the h-p finite-element method together with the Love's thin shell equations to investigate the natural frequencies of conical sandwich panels having the full range of classical boundary conditions which includes free, clamped, simply supported and shear diaphragm edges. Malekzadehfard and Livani [13] performed the free vibration analysis of thick truncated conical composite sandwich shells with flexible cores and simply supported boundary conditions based on a new improved and enhanced higher order sandwich shell theory and the first order shear deformation theory for the inner and outer composite face sheets.

In the present paper, the study of the dynamic effects of low velocity impact on the sandwich conical shells with the simply supported and clamp conditions is performed using the higher order sandwich shells theory. In this analysis the elastic region, the displacements, and rotations are assumed to be small. The impact modeling is performed by using the complete model.

Generally, in this paper, the partial differential equations of motion, obtained from Hamilton's concept, are converted into algebraic equations by using the DQ method. Moreover, the core to the face sheet stiffness ratio, the core to the face sheet thickness ratio, semi-vertex angle, large radii of cone-to-length, the boundary conditions, the impactor mass, the impactor velocity, the orientation and stacking sequence, and the symmetric and anti-symmetric angle ply are investigated on the contact force and the maximum deflection of the thick laminated truncated conical sandwich shells.

2. Formulation of conical composite sandwich shells

Consider a STCS which is composed of two orthotropic laminated composite face sheets separated by an orthotropic thick compressible or incompressible core. The thickness of the top face, the core, and the bottom face layers are t^t, t^c, t^b, respectively, and H is the total thickness of STCS. As shown in Fig. 1, r_1 and r_2 indicate the radii of the cone at its small and large ends, respectively, α denotes semi-vertex angle of the cone, and L is the cone length along its generator. The origin of the coordinate system (x, φ, z) is located on one corner of the mid plane of STCS; x is measured along the cone's generator starting at the mid length, φ is the circumferential coordinate, and z is a straight line normal to the mid surface of the shell. The assumptions used in the present analysis are followed by those encountered in the linear elastic small deformation. u, v, and w are the displacement components in the axial, tangential, and radial directions, respectively. Based on the first shear deformation theory, the displacements u, v, and w of the face-sheets with small linear displacements are expressed through the following relations:

$$\begin{aligned} u^i(\alpha,\beta,z,t) &= u_0^i(\alpha,\beta,t) + z^i\theta_\alpha^i(\alpha,\beta,t) \\ v^i(\alpha,\beta,z,t) &= v_0^i(\alpha,\beta,t) + z^i\theta_\beta^i(\alpha,\beta,t)\boldsymbol{(1)} \\ w^i(\alpha,\beta,z,t) &= w^i(\alpha,\beta,t) \end{aligned} \tag{1}$$

where $u^i, v^i and\ w^i$ $(i = t, b)$ denote the displacement components of the face sheets of DCSP. In equation (1) , u_0^i, v_0^i, and $w^i$are the displacements at the mid surface in the α, β, and z directions. θ_α^i and θ_β^i are the rotations of a transverse normal around the α and β curvilinear coordinates, respectively. The kinematic equations for the strains of the face sheets are as follows [14]:

$$\varepsilon_{xx}^i = \varepsilon_{0x}^i + z^i\kappa_x^i \tag{2a}$$

$$\varepsilon_{\varphi\varphi}^i = \left(\varepsilon_{0\varphi}^i + z^i\kappa_\varphi^i\right) \tag{2b}$$

$$\gamma_{x\varphi}^i = \varepsilon_{0x\varphi}^i + z^i\chi_{x\varphi}^i + \left(\varepsilon_{0\varphi x}^i + z^i\chi_{\varphi x}^i\right) \tag{2c}$$

$$\gamma_{xz}^i = \gamma_{oxz}^i \tag{2d}$$

$$\gamma_{\varphi z}^i = \gamma_{o\varphi z}^i \tag{2e}$$

in which:

$$\varepsilon_{0x}^{i} = \frac{\partial u_0^i}{\partial x},\ \kappa_x^i = \frac{\partial \theta_x^i}{\partial x} \tag{3a}$$

$$\varepsilon_{0\varphi}^{i} = \frac{1}{x\sin\alpha}\frac{\partial v_0^i}{\partial \varphi} + \frac{u_0^i}{x} + \frac{w_0^i}{x\tan\varphi},\ \kappa_\varphi^i = \frac{1}{x\sin\alpha}\frac{\partial \theta_\varphi^i}{\partial \varphi} + \frac{\theta_x^i}{x} \tag{3b}$$

$$\varepsilon_{0x\varphi}^{i} = \frac{\partial v_0^i}{\partial x}, \varepsilon_{0\varphi x}^{i} = \frac{1}{x\sin\alpha}\frac{\partial u_0^i}{\partial \varphi} - \frac{v_0^i}{x}, \chi_{x\varphi}^{i} = \frac{\partial \theta_\varphi^i}{\partial x}, \chi_{\varphi x}^{i} = \frac{1}{x\sin\alpha}\frac{\partial \theta_x^i}{\partial \varphi} - \frac{\theta_\varphi^i}{x}. \gamma_{0x\varphi}^{i} = \varepsilon_{0x\varphi}^{i} + \varepsilon_{0\varphi x}^{i} \tag{3c}$$

$$\gamma_{0xz}^{i} = \frac{\partial w_0^i}{\partial x} + \theta_x^i \tag{3d}$$

$$\gamma_{0\varphi z}^{i} = \frac{1}{x\sin\alpha}\frac{\partial w_0^i}{\partial \varphi} - \frac{v_0^i}{x\tan\varphi} + \theta_\varphi^i \tag{3e}$$

where u_0^i , v_0^i, and w^i are the displacements at the mid surface in the α, β, and z directions, respectively; θ_x^i and θ_φ^i are the rotations of a transverse normal around the α and β curvilinear coordinates, respectively. In the above equations, i stands for the face sheets, $i = t$ represents the top face sheets, and $i = b$ represents the bottom face sheet. For the displacement fields of the core, the cubic pattern through the core thickness and the vertical displacement of the core are considered as follows:

$$\begin{aligned} u^c(\alpha,\beta,z,t) &= u_0^c(\alpha,\beta,t) + z u_1^c(\alpha,\beta,t) + z^2 u_2^c(\alpha,\beta,t) + z^3 u_3^c(\alpha,\beta,t) \\ v^c(\alpha,\beta,z,t) &= v_0^c(\alpha,\beta,t) + z v_1^c(\alpha,\beta,t) + z^2 v_2^c(\alpha,\beta,t) + z^3 v_3^c(\alpha,\beta,t) \\ w^c(\alpha,\beta,z,t) &= w_0^c(\alpha,\beta,t) + z w_1^c(\alpha,\beta,t) + z^2 w_2^c(\alpha,\beta,t) + z^3 w_3^c(\alpha,\beta,t) \end{aligned} \tag{4}$$

Also, the kinematic relations of the core for STCS are based on small deformations [14]:

$$\varepsilon_{xx}^{c} = \varepsilon_{0x}^{c} + z^c\kappa_x^c + z^{c2}\varepsilon_{0x}^{*c} + z^{c3}\kappa_x^{*c} \tag{5a}$$

$$\varepsilon_{\varphi\varphi}^{c} = \varepsilon_{0\varphi}^{c} + z^c\kappa_\varphi^c + z^{c2}\varepsilon_{0\varphi}^{*c} + z^{c3}\kappa_\varphi^{*c} \tag{5b}$$

$$\varepsilon_{zz}^{c} = \varepsilon_{0z}^{c} + z^c\kappa_z^c + z^{c2}\varepsilon_{0x}^{*c} \tag{5c}$$

$$\varepsilon_{x\varphi}^{c} = \varepsilon_{0x\varphi}^{c} + z^c\chi_{x\varphi}^c + z^{c2}\varepsilon_{0x\varphi}^{*c} + z^{c3}\chi_{x\varphi}^{*c} \tag{5d}$$

$$\varepsilon_{\varphi x}^{c} = \varepsilon_{0\varphi x}^{c} + z^c\chi_{\varphi x}^c + z^{c2}\varepsilon_{0\varphi x}^{*c} + z^{c3}\chi_{\varphi x}^{*c} \tag{5e}$$

$$\gamma_{xz}^{c} = \left[\varepsilon_{0xz}^{c} + z^c\chi_{xz}^c + z^{c2}\varepsilon_{0xz}^{*c} + z^{c3}\chi_{xz}^{*c}\right] + \left[\varepsilon_{0zx}^{c} + z^c\chi_{zx}^c + z^{c2}\varepsilon_{0zx}^{*c}\right] \tag{5f}$$

$$\gamma_{\varphi z}^{c} = \left[\varepsilon_{0\varphi z}^{c} + z^c\chi_{\varphi z}^c + z^{c2}\varepsilon_{0\varphi z}^{*c} + z^{c3}\chi_{\varphi z}^{*c}\right] + \left[\varepsilon_{0z\varphi}^{c} + z^c\chi_{z\varphi}^c + z^{c2}\varepsilon_{0z\varphi}^{*c}\right] \tag{5g}$$

in which:

$$\begin{aligned} \varepsilon_{0x}^{c} &= u_{0,x}^{c} \\ \kappa_x^c &= u_{1,x}^c \\ \varepsilon_{0x}^{*c} &= u_{2,x}^c \\ \kappa_x^{*c} &= u_{3,x}^c \end{aligned} \quad \begin{aligned} \varepsilon_{0\varphi}^{c} &= \frac{1}{x\sin\alpha}v_{0,\varphi}^c + \frac{u_0^c}{x} + \frac{w_0^c}{x\tan\alpha} \\ \kappa_\varphi^c &= \frac{1}{x\sin\alpha}v_{1,\varphi}^c + \frac{u_1^c}{x} + \frac{w_1^c}{x\tan\alpha} \\ \varepsilon_{0\varphi}^{*c} &= \frac{1}{x\,si}v_{2,\varphi}^c + \frac{u_2^c}{x} + \frac{w_2^c}{x\tan\alpha}\kappa_\varphi^{*c} \\ &= \frac{1}{x\sin\alpha}v_{3,\varphi}^c + \frac{u_3^c}{x} \\ &+ \frac{w_3^c}{x\tan\alpha} \end{aligned} \quad \begin{aligned} \varepsilon_{0z}^{c} &= w_1^c \\ \kappa_z^c &= 2w_2^c \\ \varepsilon_{0z}^{*c} &= 3w_3^c \end{aligned} \tag{6}$$

$$\begin{aligned} \varepsilon_{0x\varphi}^{c} &= v_{0,x}^{c} \\ \chi_{x\varphi}^c &= v_{1,x}^c \\ \varepsilon_{0x\varphi}^{*c} &= v_{2,x}^c \\ \chi_{x\varphi}^{*c} &= v_{3,x}^c \end{aligned} \quad \begin{aligned} \varepsilon_{0\varphi x}^{c} &= \frac{1}{x\sin\alpha}u_{0,\varphi}^c - \frac{v_0^c}{x} + \frac{w_0^c}{x\tan\alpha} \\ \chi_{\varphi x}^c &= \frac{1}{x\sin\alpha}u_{1,\varphi}^c - \frac{v_1^c}{x} + \frac{w_1^c}{x\tan\alpha} \\ \varepsilon_{0\varphi x}^{*c} &= \frac{1}{x\sin\alpha}u_{2,\varphi}^c - \frac{v_2^c}{x} + \frac{w_2^c}{x\tan\alpha} \\ \chi_{\varphi x}^{*c} &= \frac{1}{x\sin\alpha}u_{3,\varphi}^c - \frac{v_3^c}{x} + \frac{w_3^c}{x\tan\alpha} \end{aligned} \quad \begin{aligned} \varepsilon_{0xz}^{c} &= w_{0,x}^c \\ \chi_{xz}^c &= w_{1,x}^c \\ \varepsilon_{0xz}^{*c} &= w_{2,x}^c \\ \chi_{xz}^{*c} &= w_{3,x}^c \end{aligned} \tag{7}$$

$$\begin{array}{lll} \varepsilon_{0zx}^{c}=u_1^c & \varepsilon_{0\varphi z}^{c}=\dfrac{1}{x\sin\alpha}w_{0,\varphi}^{c}-\dfrac{v_0^c}{x\tan\alpha} & \varepsilon_{0\varphi z}^{c}=v_1^c \\ \chi_{zx}^{c}=2u_2^c & \chi_{\varphi z}^{c}=\dfrac{1}{x\sin\alpha}w_{1,\varphi}^{c}-\dfrac{v_1^c}{x\tan\alpha} & \chi_{\varphi z}^{c}=2v_2^c \\ \varepsilon_{0z}^{*c}=3u_3^c & \varepsilon_{0\varphi z}^{*c}=\dfrac{1}{x\sin\alpha}w_{2,\varphi}^{c}-\dfrac{v_2^c}{x\tan\alpha} & \varepsilon_{0\varphi z}^{*c}=3v_3^c \\ & \chi_{\varphi z}^{*c}=\dfrac{1}{x\sin\alpha}w_{3,\varphi}^{c}-\dfrac{v_3^c}{x\tan\alpha} & \end{array} \tag{8}$$

where u_1^c, v_1^c, and w_1^c functions are rotational; the parameters $u_2^c, u_3^c, v_2^c, v_3^c, w_2^c$, and w_3^c are the higher-order terms in the Taylor's series expansion. Reminding that there is no slipping between the face sheets and the core, the following relations are written [15]:

$$\begin{cases} u^c|_{z=\frac{t^c}{2}}=u^t|_{z=\frac{t^c}{2}} \\ u^c|_{z=-\frac{t^c}{2}}=u^b|_{z=-\frac{t^c}{2}} \end{cases},\quad \begin{cases} v^c|_{z=\frac{t^c}{2}}=v^t|_{z=\frac{t^c}{2}} \\ v^c|_{z=-\frac{t^c}{2}}=v^b|_{z=-\frac{t^c}{2}} \end{cases},\quad \begin{cases} w^c|_{z=\frac{t^c}{2}}=w^t|_{z=\frac{t^c}{2}} \\ w^c|_{z=-\frac{t^c}{2}}=w^b|_{z=-\frac{t^c}{2}} \end{cases} \tag{9}$$

3. Governing equations

In order to derive the motion equations of the laminated STCS, the energy method is used. The first variation of the strain energy for STCS during the elastic deformation is:

$$\begin{aligned} &\int_{t_1}^{t_2}\delta\, U^c\,dt+\sum_i^{t,b}\int_{t_1}^{t_2}\delta U^i dt \\ &=\int_0^t\int_z\int_A\left(h_x\sigma_x^c\delta\varepsilon_x^c+h_\varphi\sigma_\varphi^c\delta\varepsilon_\varphi^c+\sigma_z^c\delta\varepsilon_z^c+h_{x\varphi}\tau_{x\varphi}^c\delta\gamma_{x\varphi}^c+\tau_{x\varphi}^c\delta\gamma_{x\varphi}^c\right. \\ &\left.+\tau_{\varphi z}^c\delta\gamma_{\varphi z}^c\right)dA^c\,dz\,dt \\ &+\sum_i^{t,b}\left\{\int_0^t\int_{z^i}\int_A\left(\sigma_x^i\delta\varepsilon_x^i+\sigma_\varphi^i\delta\varepsilon_\varphi^i+\tau_{x\varphi}^i\delta\gamma_{x\varphi}^i+\tau_{xz}^i\delta\gamma_{xz}^i+\tau_{\varphi z}^i\delta\gamma_{\varphi z}^i\right)dA^i\,dz^i\,dt\right\} \end{aligned} \tag{10}$$

The kinetic energy for STCS is given by:

$$E=\frac{1}{2}\sum_i^{t,b,c}\int_{V^i}\rho^i(\dot{u}^{i^2}+\dot{v}^{i^2}+\dot{w}^{i^2})dV^i \tag{11}$$

where$\rho^i(i=t,b,c)$ is the mass per unit volume of the top and the bottom face sheets and the core, respectively; $\dot{u}^i,\dot{v}^i,$and $\dot{w}^i(i=t,b,c)$ are the velocities in the x, φ, and z directions, respectively; "**.**" denotes the first time derivative; $V^i(i=t,b,c)$ is the volume of the top and the bottom face sheets and the core, respectively. The first variation of the kinetic energy can be written as:

$$\int_0^t\delta E dt=\sum_i^{t,b}\int_0^t\delta E^i dt+\int_0^t\delta E^c dt \tag{12a}$$

$$\begin{aligned} &=\sum_i^{t,b}\left\{-\int_0^t\int_A\left[(I_0^i\ddot{u}_0^i+I_1^i\ddot{\theta}_x^i)\delta u_0^i+(I_1^i\ddot{u}_0^i+I_2^i\ddot{\theta}_x^i)\delta\theta_x^i+(I_0^i\ddot{v}_0^i+I_1^i\ddot{\theta}_\varphi^i)\delta v_0^i+(I_1^i\ddot{u}_0^i+I_2^i\ddot{\theta}_\varphi^i)\delta\theta_\varphi^i\right.\right. \\ &\left.\left.+(I_0^i\ddot{w}^i)\delta w^i\right](x\sin\alpha\,dxd\varphi)dt\right\}-\int_0^t\int_A\left[(I_0^c\ddot{u}_0^c+I_1^c\ddot{u}_1^c+I_2^c\ddot{u}_2^c+I_3^c\ddot{u}_3^c)\delta u_0^c\right. \\ &+(I_1^c\ddot{u}_0^c+I_2^c\ddot{u}_1^c+I_3^c\ddot{u}_2^c+I_4^c\ddot{u}_3^c)\delta u_1^c+(I_2^c\ddot{u}_0^c+I_3^c\ddot{u}_1^c+I_4^c\ddot{u}_2^c+I_5^c\ddot{u}_3^c)\delta u_2^c+(I_3^c\ddot{u}_0^c \\ &+I_4^c\ddot{u}_1^c+I_5^c\ddot{u}_2^c+I_6^c\ddot{u}_3^c)\delta u_3^c+(I_0^c\ddot{v}_0^c+I_1^c\ddot{v}_1^c+I_2^c\ddot{v}_2^c+I_3^c\ddot{v}_3^c)\delta v_0^c+(I_1^c\ddot{v}_0^c+I_2^c\ddot{v}_1^c \\ &+I_3^c\ddot{v}_2^c+I_4^c\ddot{v}_3^c)\delta v_1^c+(I_2^c\ddot{v}_0^c+I_3^c\ddot{v}_1^c+I_4^c\ddot{v}_2^c+I_5^c\ddot{v}_3^c)\delta v_2^c+(I_3^c\ddot{v}_0^c+I_4^c\ddot{v}_1^c+I_5^c\ddot{v}_2^c \\ &+I_6^c\ddot{v}_3^c)\delta v_3^c+(I_0^c\ddot{w}_0^c+I_1^c\ddot{w}_1^c+I_2^c\ddot{w}_2^c+I_3^c\ddot{w}_3^c)\delta w_0^c+(I_1^c\ddot{w}_0^c+I_2^c\ddot{w}_1^c+I_3^c\ddot{w}_2^c \\ &+I_4^c\ddot{w}_3^c)\delta w_1^c+(I_2^c\ddot{w}_0^c+I_3^c\ddot{w}_1^c+I_4^c\ddot{w}_2^c+I_5^c\ddot{w}_3^c)\delta w_2^c+(I_3^c\ddot{w}_0^c+I_4^c\ddot{w}_1^c+I_5^c\ddot{w}_2^c \\ &\left.+I_6^c\ddot{w}_3^c)\delta w_3^c\right](x\sin\alpha\,dxd\varphi)dt \end{aligned} \tag{12b}$$

in which:

$$I_n^i = \int_z \rho^i\,(z^n)dz\,, \quad i = t, b, c \; and \; n = 1 \; to \; 6 \tag{13}$$

However, substituting Eqs. (10) and (12) into the following equation [14] yields:

$$\delta \int_{t_1}^{t_2} (L)dt = \delta \int_{t_1}^{t_2} [E - (U + W)]\,dt = 0 \tag{14}$$

By integrating the parts and collecting the coefficients of independent variations in $\delta u_0^t\,, \delta v_0^t\,, \delta w^t\,, \delta \theta_x^t\,, \delta \theta_\varphi^t\,, \delta u_2^c\,, \delta v_2^c\,, \delta w_2^c\,, \delta u_3^c\,, \delta v_3^c\,, \delta w_3^c\,, \delta u_0^b\,, \delta v_0^b\,, \delta w^b\,, \delta \theta_x^b\,, \delta \theta_\varphi^b$, sixteen equations of motion for STCS may be expressed as:

$$\begin{aligned} &-\left(\frac{1}{t^c}\right)\{Q_{zx}^c\} + \left(\frac{1}{x}\right)\{N_x^t\} + \{N_{x,x}^t\} - \left(\frac{1}{x}\right)\{N_\varphi^t\} + \left(\frac{1}{x\sin\alpha}\right)\{N_{\varphi x,\varphi}^t\} \\ &\quad = \left[\left(\frac{1}{4}I_0^c + \frac{1}{t^c}I_1^c + \frac{1}{t^{c2}}I_2^c\right) + I_0^t\right]\ddot{u}_0^t + \left[\left(-\frac{t^t}{8}I_0^c - \frac{t^t}{2t^c}I_1^c - \frac{t^t}{2t^{c2}}I_2^c\right) + I_1^t\right]\ddot{\theta}_x^t \\ &\quad + \left[-\frac{t^{c2}}{8}I_0^c - \frac{t^c}{4}I_1^c + \frac{1}{2}I_2^c + \frac{1}{t^c}I_3^c\right]\ddot{u}_2^c + \left[-\frac{t^{c2}}{8}I_1^c - \frac{t^c}{4}I_2^c + \frac{1}{2}I_3^c + \frac{1}{t^c}I_4^c\right]\ddot{u}_3^c \\ &\quad + \left[\frac{1}{4}I_0^c - \frac{1}{t^{c2}}I_2^c\right]\ddot{u}_0^b + \left[\frac{t^b}{8}I_0^c - \frac{t^b}{2t^{c2}}I_2^c\right]\ddot{\theta}_x^b \end{aligned} \tag{15a}$$

$$\begin{aligned} &\left(\frac{1}{2x\tan\alpha}\right)\{Q_{\varphi z}^c\} + \left(\frac{1}{t^c x\tan\alpha}\right)\{S_{\varphi z}^c\} - \left(\frac{1}{t^c}\right)\{Q_{z\varphi}^c\} + \left(\frac{1}{x\sin\alpha}\right)\{N_{\varphi,\varphi}^t\} + \left(\frac{1}{x}\right)\{N_{x\varphi}^t\} + \{N_{x\varphi,x}^t\} \\ &\quad + \left(\frac{1}{x}\right)\{N_{\varphi x}^t\} + \left(\frac{k_s}{x\tan\alpha}\right)\{Q_{\varphi z}^t\} \\ &\quad = \left[\left(\frac{1}{4}I_0^c + \frac{1}{t^c}I_1^c + \frac{1}{t^{c2}}I_2^c\right) + I_0^t\right]\ddot{v}_0^t + \left[\left(-\frac{t^t}{8}I_0^c - \frac{t^t}{2t^c}I_1^c - \frac{t^t}{2t^{c2}}I_2^c\right) + I_1^t\right]\ddot{\theta}_\varphi^t \\ &\quad + \left[-\frac{t^{c2}}{8}I_0^c - \frac{t^c}{4}I_1^c + \frac{1}{2}I_2^c + \frac{1}{t^c}I_3^c\right]\ddot{v}_2^c + \left[-\frac{t^{c2}}{8}I_1^c - \frac{t^c}{4}I_2^c + \frac{1}{2}I_3^c + \frac{1}{t^c}I_4^c\right]\ddot{v}_3^c \\ &\quad + \left[\frac{1}{4}I_0^c - \frac{1}{t^{c2}}I_2^c\right]\ddot{v}_0^b + \left[\frac{t^b}{8}I_0^c - \frac{t^b}{2t^{c2}}I_2^c\right]\ddot{\theta}_\varphi^b \end{aligned} \tag{15b}$$

$$\begin{aligned} &-\left(\frac{1}{t^c}\right)\{N_z^c\} + \left(\frac{1}{2x}\right)\{Q_{xz}^c\} + \left(\frac{1}{2}\right)\{Q_{xz,x}^c\} + \left(\frac{1}{xt^c}\right)\{S_{xz}^c\} + \left(\frac{1}{t^c}\right)\{S_{xz,x}^c\} + \left(\frac{1}{2x\,sin\,\varphi}\right)\{Q_{\varphi z,\varphi}^c\} \\ &\quad + \left(\frac{1}{x\,sin\,\varphi\,t^c}\right)\{S_{\varphi z,\varphi}^c\} - \left(\frac{1}{x\,sin\,\varphi}\right)\{N_\varphi^t\} + \left(\frac{k_s}{x}\right)\{Q_{xz}^t\} + k_s\{Q_{xz,x}^t\} \\ &\quad + \left(\frac{k_s}{x\,sin\,\varphi}\right)\{Q_{\varphi z,\varphi}^t\} + N_x^m\left({w^t}_{,xx} + 0.25({w^t}_{,xx} + {w^b}_{,xx} - \frac{t_c^2}{2}{w^c}_{2,xx})\right) \\ &\quad - (A^t B^t)q_w^t \\ &\quad = \left[\left(\frac{1}{4}I_0^c + \frac{1}{t^c}I_1^c + \frac{1}{t^{c2}}I_2^c\right) + I_0^t\right]\ddot{w}^t + \left[-\frac{t^{c2}}{8}I_0^c - \frac{t^c}{4}I_1^c + \frac{1}{2}I_2^c + \frac{1}{t^c}I_3^c\right]\ddot{w}_2^c \\ &\quad + \left[-\frac{t^{c2}}{8}I_1^c - \frac{t^c}{4}I_2^c + \frac{1}{2}I_3^c + \frac{1}{t^c}I_4^c\right]\ddot{w}_3^c + \left[\frac{1}{4}I_0^c - \frac{1}{t^{c2}}I_2^c\right]\ddot{w}^b \end{aligned} \tag{15c}$$

$$\begin{aligned} &\left(\frac{t^t}{2t^c}\right)\{Q_{zx}^c\} + \left(\frac{1}{x}\right)\{M_x^t\} + \{M_{x,x}^t\} - \left(\frac{1}{x}\right)\{M_\varphi^t\} + \left(\frac{1}{x\sin\alpha}\right)\{M_{\varphi x,\varphi}^t\} - k_s\{Q_{zx}^t\} \\ &\quad = \left[\left(-\frac{t^t}{8}I_0^c - \frac{t^t}{2t^c}I_1^c - \frac{t^t}{2t^{c2}}I_2^c\right) + I_1^t\right]\ddot{u}_0^t + \left[\left(\frac{t^{t2}}{16}I_0^c + \frac{t^{t2}}{4t^c}I_1^c + \frac{t^{t2}I_2^c}{4t^{c2}}\right) + I_2^t\right]\ddot{\theta}_x^t \\ &\quad + \left[\frac{t^t t^{c2}}{16}I_0^c + \frac{t^t t^c}{8}I_1^c - \frac{t^t}{4}I_2^c - \frac{t^t}{2t^c}I_3^c\right]\ddot{u}_2^c + \left[\frac{t^t t^{c2}}{16}I_1^c + \frac{t^t t^c}{8}I_2^c - \frac{t^t}{4}I_3^c - \frac{t^t}{2t^c}I_4^c\right]\ddot{u}_3^c \\ &\quad + \left[-\frac{t^t}{8}I_0^c + \frac{t^t}{2t^{c2}}I_2^c\right]\ddot{u}_0^b + \left[-\frac{t^t t^b}{16}I_0^c + \frac{t^t t^b}{4t^{c2}}I_2^c\right]\ddot{\theta}_x^b \end{aligned} \tag{15d}$$

$$
\begin{aligned}
&-\left(\frac{t^t}{4x\,tan\,\varphi}\right)\{Q_{\varphi z}^c\}-\left(\frac{t^t}{2t^c x\,tan\,\varphi}\right)\{S_{\varphi z}^c\}+\left(\frac{t^t}{2t^c}\right)\{Q_{z\varphi}^c\}+\left(\frac{1}{x\,sin\,\varphi}\right)\{M_{\varphi,\varphi}^t\}+\left(\frac{1}{x}\right)\{M_{\varphi x}^t\}\\
&\quad+\left(\frac{1}{x}\right)\{M_{x\varphi}^t\}+\{M_{x\varphi,x}^t\}+\left(\frac{k_s}{x\,tan\,\varphi}\right)\{S_{\varphi z}^t\}-k_s\{Q_{z\varphi}^t\}\\
&\quad=\left[\left(-\frac{t^t}{8}I_0^c-\frac{t^t}{2t^c}I_1^c-\frac{t^t}{2t^{c2}}I_2^c\right)+I_1^t\right]\ddot{v}_0^t+\left[\left(\frac{t^{t2}}{16}I_0^c+\frac{t^{t2}}{4t^c}I_1^c+\frac{t^{t2}I_2^c}{4t^{c2}}\right)+I_2^t\right]\ddot{\theta}_\varphi^t\\
&\quad+\left[\frac{t^tt^{c2}}{16}I_0^c+\frac{t^tt^c}{8}I_1^c-\frac{t^t}{4}I_2^c-\frac{t^t}{2t^c}I_3^c\right]\ddot{v}_2^c+\left[\frac{t^tt^{c2}}{16}I_1^c+\frac{t^tt^c}{8}I_2^c-\frac{t^t}{4}I_3^c-\frac{t^t}{2t^c}I_4^c\right]\ddot{v}_3^c\\
&\quad+\left[-\frac{t^t}{8}I_0^c+\frac{t^t}{2t^{c2}}I_2^c\right]\ddot{v}_0^b+\left[-\frac{t^tt^b}{16}I_0^c+\frac{t^tt^b}{4t^{c2}}I_2^c\right]\ddot{\theta}_\varphi^b
\end{aligned}
\tag{15e}
$$

$$
\begin{aligned}
-2\{S_{zx}^c\}&=\left[-\frac{t^{c2}}{8}I_0^c-\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c+\frac{1}{t^c}I_3^c\right]\ddot{u}_0^t+\left[\frac{t^{c2}t^t}{16}I_0^c+\frac{t^ct^t}{8}I_1^c-\frac{t^t}{4}I_2^c-\frac{t^t}{2t^c}I_3^c\right]\ddot{\theta}_x^t\\
&\quad+\left[\frac{t^{c4}}{16}I_0^c-\frac{t^{c2}}{2}I_2^c+I_4^c\right]\ddot{u}_2^c+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{u}_3^c\\
&\quad+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{u}_0^b+\left[-\frac{t^{c2}t^b}{16}I_0^c+\frac{t^ct^b}{8}I_1^c+\frac{t^b}{4}I_2^c-\frac{t^b}{2t^c}I_3^c\right]\ddot{\theta}_x^b
\end{aligned}
\tag{15f}
$$

$$
\begin{aligned}
&-\left(\frac{t^{c2}}{4x\tan\alpha}\right)\{Q_{\varphi z}^c\}+\left(\frac{1}{x\tan\alpha}\right)\{Q_{\varphi z}^{*c}\}-2\{S_{z\varphi}^c\}\\
&\quad=\left[-\frac{t^{c2}}{8}I_0^c-\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c+\frac{1}{t^c}I_3^c\right]\ddot{v}_0^t+\left[\frac{t^{c2}t^t}{16}I_0^c+\frac{t^ct^t}{8}I_1^c-\frac{t^t}{4}I_2^c-\frac{t^t}{2t^c}I_3^c\right]\ddot{\theta}_\varphi^t\\
&\quad+\left[\frac{t^{c4}}{16}I_0^c-\frac{t^{c2}}{2}I_2^c+I_4^c\right]\ddot{v}_2^c+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{v}_3^c\\
&\quad+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{v}_0^b+\left[-\frac{t^{c2}t^b}{16}I_0^c+\frac{t^ct^b}{8}I_1^c+\frac{t^b}{4}I_2^c-\frac{t^b}{2t^c}I_3^c\right]\ddot{\theta}_\varphi^b
\end{aligned}
\tag{15g}
$$

$$
\begin{aligned}
&-2\{M_z^c\}-\left(\frac{t^{c2}}{4x}\right)\{Q_{xz}^c\}-\left(\frac{t^{c2}}{4}\right)\{Q_{xz,x}^c\}+\left(\frac{1}{x}\right)\{Q_{xz}^{*c}\}+\{Q_{xz,x}^{*c}\}-\left(\frac{t^{c2}}{4x\sin\alpha}\right)\{Q_{\varphi Z,\varphi}^c\}\\
&\quad+\left(\frac{1}{x\sin\alpha}\right)\{Q_{\varphi z,\varphi}^{*c}\}\\
&\quad=\left[-\frac{t^{c2}}{8}I_0^c-\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c+\frac{1}{t^c}I_3^c\right]\ddot{w}^t+\left[\frac{t^{c4}}{16}I_0^c-\frac{t^{c2}}{2}I_2^c+I_4^c\right]\ddot{w}_2^c\\
&\quad+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{w}_3^c+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{w}^b
\end{aligned}
\tag{15h}
$$

$$
\begin{aligned}
&\left(\frac{t^{c2}}{4}\right)\{Q_{zx}^c\}-3\{Q_{zx}^{*c}\}\\
&=\left[-\frac{t^{c2}}{8}I_1^c-\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c+\frac{1}{t^c}I_4^c\right]\ddot{u}_0^t+\left[\frac{t^{c2}t^t}{16}I_1^c+\frac{t^ct^t}{8}I_2^c-\frac{t^t}{4}I_3^c-\frac{t^t}{2t^c}I_4^c\right]\ddot{\theta}_x^t\\
&+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{u}_2^c+\left[\frac{t^{c4}}{16}I_2^c-\frac{t^{c2}}{2}I_4^c+I_6^c\right]\ddot{u}_3^c\\
&+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{u}_0^b+\left[-\frac{t^{c2}t^b}{16}I_1^c+\frac{t^ct^b}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{\theta}_x^b
\end{aligned}
\tag{15i}
$$

$$
\begin{aligned}
&-\left(\frac{t^{c2}}{4x\tan\alpha}\right)\{S_{\varphi z}^c\}+\left(\frac{t^{c2}}{4}\right)\{Q_{z\varphi}^c\}+\left(\frac{1}{x\tan\alpha}\right)\{S_{\varphi z}^{*c}\}-3\{Q_{z\varphi}^{*c}\}\\
&=\left[-\frac{t^{c2}}{8}I_1^c-\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c+\frac{1}{t^c}I_4^c\right]\ddot{v}_0^t+\left[\frac{t^{c2}}{16}I_1^c+\frac{t^c}{8}I_2^c-\frac{t^t}{4}I_3^c-\frac{t^t}{2t^c}I_4^c\right]\ddot{\theta}_\varphi^t
\end{aligned}
$$

$$
\begin{aligned}
&-+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{v}_2^c+\left[\frac{t^{c4}}{16}I_2^c-\frac{t^{c2}}{2}I_4^c+I_6^c\right]\ddot{v}_3^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{v}_0^b\\
&\qquad+\left[-\frac{t^{c2}t^b}{16}I_1^c+\frac{t^ct^b}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{\theta}_\varphi^b
\end{aligned}
\tag{15j}
$$

$$
\begin{aligned}
&\left(\frac{t^{c2}}{4}\right)\{N_z^c\}-3\{N_z^{*c}\}-\left(\frac{t^{c2}}{4x\sin\alpha}\right)\{S_{\varphi z,\varphi}^c\}+\left(\frac{1}{x\sin\alpha}\right)\{S_{\varphi z,\varphi}^{*c}\}-\left(\frac{t^{c2}}{4x}\right)\{S_{xz}^c\}-\left(\frac{t^{c2}}{4}\right)\{S_{xz,x}^c\}\\
&\qquad+\left(\frac{1}{x}\right)\{S_{xz}^{*c}\}+\{S_{xz,x}^{*c}\}\\
&\qquad=\left[-\frac{t^{c2}}{8}I_1^c-\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c+\frac{1}{t^c}I_4^c\right]\ddot{w}^t+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{w}_2^c\\
&\qquad+\left[\frac{t^{c4}}{16}I_2^c-\frac{t^{c2}}{2}I_4^c+I_6^c\right]\ddot{w}_3^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{w}^b
\end{aligned}
\tag{15k}
$$

$$
\begin{aligned}
&\left(\frac{1}{t^c}\right)\{Q_{zx}^c\}+\left(\frac{1}{x}\right)\{N_x^b\}+\{N_{x,x}^b\}-\left(\frac{1}{x}\right)\{N_\varphi^b\}+\left(\frac{1}{x\,sin\,\varphi}\right)\{N_{\varphi x,\varphi}^b\}\\
&\qquad=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{u}_0^t+\left[-\frac{t^t}{8}I_0^c+\frac{t^t}{2t^{c2}}I_2^c\right]\ddot{\theta}_x^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{u}_2^c\\
&\qquad+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{u}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{u}_0^b\\
&\qquad+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{\theta}_x^b
\end{aligned}
\tag{15l}
$$

$$
\begin{aligned}
&\left(\frac{1}{2x\tan\alpha}\right)\{Q_{\varphi z}^c\}-\left(\frac{1}{t^cx\tan\alpha}\right)\{S_{\varphi z}^c\}+\left(\frac{1}{t^c}\right)\{Q_{z\varphi}^c\}+\left(\frac{1}{x\sin\alpha}\right)\{N_{\varphi,\varphi}^b\}+\left(\frac{1}{x}\right)\{N_{x\varphi}^b\}+\{N_{x\varphi,x}^b\}\\
&\qquad+\left(\frac{1}{x}\right)\{N_{\varphi x}^b\}+\left(\frac{k_s}{x\tan\alpha}\right)\{Q_{\varphi z}^b\}\\
&\qquad=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{v}_0^t+\left[-\frac{t^t}{8}I_0^c+\frac{t^t}{2t^{c2}}I_2^c\right]\ddot{\theta}_\varphi^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{v}_2^c\\
&\qquad+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{v}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{v}_0^b\\
&\qquad+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{\theta}_\varphi^b
\end{aligned}
\tag{15m}
$$

$$
\begin{aligned}
&\left(\frac{1}{t^c}\right)\{N_z^c\}+\left(\frac{1}{2x}\right)\{Q_{xz}^c\}+\left(\frac{1}{2}\right)\{Q_{xz,x}^c\}-\left(\frac{1}{xt^c}\right)\{S_{xz}^c\}-\left(\frac{1}{t^c}\right)\{S_{xz,x}^c\}+\left(\frac{1}{2x\sin\alpha}\right)\{Q_{\varphi z,\varphi}^c\}\\
&\qquad-\left(\frac{1}{x\sin\alpha\,t^c}\right)\{S_{\varphi z,\varphi}^c\}-\left(\frac{1}{x\sin\alpha}\right)\{N_\varphi^b\}+\left(\frac{k_s}{x}\right)\{Q_{xz}^b\}+k_s\{Q_{xz,x}^b\}\\
&\qquad+\left(\frac{k_s}{x\tan\alpha}\right)\{Q_{\varphi z,\varphi}^b\}\\
&\qquad=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{w}^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{w}_2^c\\
&\qquad+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{w}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{w}^b
\end{aligned}
\tag{15n}
$$

$$
\begin{aligned}
&\left(\frac{t^b}{2t^c}\right)\{Q_{zx}^c\}+\left(\frac{1}{x}\right)\{M_x^b\}+\{M_{x,x}^b\}-\left(\frac{1}{x}\right)\{M_\varphi^b\}+\left(\frac{1}{x\sin\alpha}\right)\{M_{\varphi x,\varphi}^b\}-k_s\{Q_{zx}^b\}\\
&\qquad=\left[\frac{t^b}{8}I_0^c-\frac{t^b}{2t^{c2}}I_2^c\right]\ddot{u}_0^t+\left[-\frac{t^tt^b}{16}I_0^c+\frac{t^tt^b}{4t^{c2}}I_2^c\right]\ddot{\theta}_x^t\\
&\qquad+\left[-\frac{t^bt^{c2}}{16}I_0^c+\frac{t^bt^c}{8}I_1^c+\frac{t^b}{4}I_2^c-\frac{t^b}{2t^c}I_3^c\right]\ddot{u}_2^c\\
&\qquad+\left[-\frac{t^bt^{c2}}{16}I_1^c+\frac{t^bt^c}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{u}_3^c+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{u}_0^b\\
&\qquad+\left[\left(\frac{t^{b2}}{16}I_0^c-\frac{t^{b2}}{4t^c}I_1^c+\frac{t^{b2}I_2^c}{4t^{c2}}\right)+I_2^b\right]\ddot{\theta}_x^b
\end{aligned}
\tag{15o}
$$

$$\left(\frac{t^b}{4x\tan\alpha}\right)\{Q_{\varphi z}^c\}-\left(\frac{t^b}{2t^c x\tan\alpha}\right)\{S_{\varphi z}^c\}+\left(\frac{t^b}{2t^c}\right)\{Q_{z\varphi}^c\}+\left(\frac{1}{x\sin\alpha}\right)\{M_{\varphi,\varphi}^b\}+\left(\frac{1}{x}\right)\{M_{\varphi x}^b\}+\left(\frac{1}{x}\right)\{M_{x\varphi}^b\}$$
$$+\{M_{x\varphi,x}^b\}+\left(\frac{k_s}{x\tan\alpha}\right)\{S_{\varphi z}^b\}-k_s\{Q_{z\varphi}^b\}$$
$$=\left[\frac{t^b}{8}I_0^c-\frac{t^b}{2t^{c2}}I_2^c\right]\ddot{v}_0^t+\left[-\frac{t^t t^b}{16}I_0^c+\frac{t^t t^b}{4t^{c2}}I_2^c\right]\ddot{\theta}_\varphi^t$$
$$+\left[-\frac{t^b t^{c2}}{16}I_0^c+\frac{t^b t^c}{8}I_1^c+\frac{t^b}{4}I_2^c-\frac{t^b}{2t^c}I_3^c\right]\ddot{v}_2^c \tag{15p}$$
$$+\left[-\frac{t^b t^{c2}}{16}I_1^c+\frac{t^b t^c}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{v}_3^c+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{v}_0^b$$
$$+\left[\left(\frac{t^{b2}}{16}I_0^c-\frac{t^{b2}}{4t^c}I_1^c+\frac{t^{b2}I_2^c}{4t^{c2}}\right)+I_2^b\right]\ddot{\theta}_\varphi^b$$

In the above relations, the components of the resultant forces and moments per unit of the length which act along the lines of the constant x or φ in the face sheets and the core of STCS can be defined as [16]:

$$\begin{bmatrix} N_x^c & N_x^{*c} & M_x^c & M_x^{*c} \\ N_\varphi^c & N_\varphi^{*c} & M_\varphi^c & M_\varphi^{*c} \\ N_z^c & N_z^{c*} & M_z^c & 0 \\ N_{x\varphi}^c & N_{x\varphi}^{*c} & M_{x\varphi}^c & M_{x\varphi}^{*c} \\ N_{\varphi x}^c & N_{\varphi x}^{*c} & M_{\varphi x}^c & M_{\varphi x}^{*c} \end{bmatrix}=\int_z \begin{Bmatrix}\sigma_x\\ \sigma_\varphi\\ \sigma_z\\ \tau_{x\varphi}\\ \tau_{\varphi x}\end{Bmatrix}^c (1,z^2,z,z^3)dz \tag{16}$$

$$\begin{bmatrix} Q_{\varphi z}^c & Q_{\varphi z}^{*c} & S_{\varphi z}^c & S_{\varphi z}^{*c} \\ Q_{z\varphi}^c & Q_{z\varphi}^{*c} & S_{z\varphi}^c & 0 \\ Q_{xz}^c & Q_{xz}^{*c} & S_{xz}^c & S_{xz}^{*c} \\ Q_{zx}^c & Q_{zx}^{*c} & S_{zx}^c & 0 \end{bmatrix}=\int_z \begin{Bmatrix}\tau_{\varphi z}\\ \tau_{z\varphi}\\ \tau_{xz}\\ \tau_{zx}\end{Bmatrix}^c (1,z^2,z,z^3)dz \tag{17}$$

$$\begin{bmatrix} N_x^i & M_x^i \\ N_\varphi^i & M_\varphi^i \\ N_{x\varphi}^i & M_{x\varphi}^i \\ N_{\varphi x}^i & M_{\varphi x}^i \end{bmatrix}=\int_{z^i} \begin{Bmatrix}\sigma_x\\ \sigma_\varphi\\ \tau_{x\varphi}\\ \tau_{\varphi x}\end{Bmatrix}^i (1,z^i)dz^i \tag{18}$$

$$\begin{bmatrix} Q_{\varphi z}^i & S_{\varphi z}^i \\ Q_{z\varphi}^i & 0 \\ Q_{xz}^i & S_{xz}^i \\ Q_{zx}^i & 0 \end{bmatrix}=\int_{z^i} \begin{Bmatrix}\tau_{\varphi z}\\ \tau_{z\varphi}\\ \tau_{xz}\\ \tau_{zx}\end{Bmatrix}^i (1,z^i)dz^i \tag{19}$$

Substituting Eqs. (2), (3), (5)-(8) into Eqs. (1) and (4) and combining them with Eqs. (16) to (19) yields:

$$\{\bar{F}\}=[D]\{\Xi\} \tag{20}$$

in which:

$$[D]=\begin{bmatrix}[D_f^i]_{a\times a} & 0\\ 0 & k_o[D_s^i]_{b\times b}\end{bmatrix} \quad , \ [D_f^i]=\begin{bmatrix}[A]_{c\times c}^i & [B]_{c\times d}^i\\ [E]_{d\times c}^i & [D]_{d\times d}^i\end{bmatrix} \tag{21}$$

The dimensions of the matrix D for the core are: $a=19$, $b=14$, $c=10$, and $d=9$, and for the face sheets are: $a=14$, $b=6$, $c=4$, and $d=4$. Furthermore, the elements of $[D]$ for the core and the face sheets are given in appendix A. In addition, parameter k_o is called the shear correction factor of FSDT which is equal to $\frac{5}{6}$[17]. Components of $\bar{F}$ and Ξ for the face sheets and the core are defined as:

$$\{\Xi_c\}=\{\varepsilon_{0x}^c,\varepsilon_{0\varphi}^c,\varepsilon_{0\varphi x}^c,\varepsilon_{0x\varphi}^c,\varepsilon_{0x}^{*c},\varepsilon_{0\varphi}^{*c},\varepsilon_{0\varphi x}^{*c},\varepsilon_{0x\varphi}^{*c},\varepsilon_{0z}^c,\varepsilon_{0z}^{*c},\kappa_x^c,\kappa_\varphi^c,\chi_{\varphi x}^c,\chi_{x\varphi}^c,\kappa_x^{*c},\kappa_\varphi^{*c},\chi_{\varphi x}^{*c},\chi_{x\varphi}^{*c},\kappa_z^c,$$
$$\varepsilon_{0zx}^c,\varepsilon_{0xz}^c,\varepsilon_{0z\varphi}^c,\varepsilon_{0\varphi z}^c,\varepsilon_{0zx}^{*c},\varepsilon_{0z\varphi}^{*c},\varepsilon_{0xz}^{*c},\varepsilon_{0\varphi z}^{*c},\chi_{zx}^c,\chi_{xz}^c,\chi_{z\varphi}^c,,\chi_{\varphi z}^c,\chi_{xz}^{*c},\chi_{\varphi z}^{*c}\}^T \tag{22}$$

$$\{\bar{F}_c\}=\{N_x^c,N_\varphi^c,N_{\varphi x}^c,N_{x\varphi}^c,N_x^{*c},N_\varphi^{*c},N_{\varphi x}^{*c},N_{x\varphi}^{*c},N_z^c,N_z^{*c},M_x^c,M_\varphi^c,M_{\varphi x}^c,M_{x\varphi}^c,M_x^{*c},M_\varphi^{*c},M_{\varphi x}^{*c},M_{x\varphi}^{*c},M_z^c$$
$$,Q_{zx}^c,Q_{xz}^c,Q_{z\varphi}^c,Q_{\varphi z}^c,Q_{zx}^{*c},Q_{z\varphi}^{*c},Q_{xz}^{*c},Q_{\varphi z}^{*c},S_{zx}^c,S_{xz}^c,S_{z\varphi}^c,S_{\varphi z}^c,S_{xz}^{*c},S_{\varphi z}^{*c}\}^T \tag{23}$$

$$\{\Xi_i\} = \{\varepsilon_{0x}^i, \varepsilon_{0\varphi}^i, \varepsilon_{0\varphi x}^i, \varepsilon_{0x\varphi}^i, \kappa_x^i, \kappa_\varphi^i, \chi_{\varphi x}^i, \chi_{x\varphi}^i, \varepsilon_{0zx}^i, \varepsilon_{0xz}^i, \varepsilon_{0z\varphi}^i, \varepsilon_{0\varphi z}^i, \chi_{xz}^i, \chi_{z\varphi}^i\}^T thati = t, b \tag{24}$$

$$\{\bar{F}_i\} = \{ N_x^i, N_\varphi^i, N_{\varphi x}^i, N_{x\varphi}^i, M_x^i, M_\varphi^i, M_{\varphi x}^i, M_{x\varphi}^i, Q_{zx}^i, Q_{xz}^i, Q_{z\varphi}^i, Q_{\varphi z}^i, S_{xz}^i, S_{z\varphi}^i\}^T thati = t, b \tag{25}$$

The superscript T denotes the transpose. The considered boundary conditions (BCs) in this study are

- **Simply-Simply BC**

$$v^i = w^i = v_2^c = v_3^c = w_2^c = w_3^c = N_x^j = M_x^j = N_x^{*c} = M_x^{*c} = 0 \quad , \quad \text{on both ends}$$

$$j = t, b, c \quad \text{and} \quad i = t, b \tag{26}$$

- **Clamped-Clamped BC**

$$u^i = v^i = w^i = u_2^c = u_3^c = v_2^c = v_3^c = w_2^c = w_3^c = \frac{\partial w^i}{\partial x} = \frac{\partial w_2^c}{\partial x} = \frac{\partial w_3^c}{\partial x} = 0 \quad , \quad \text{on both ends}$$

$$j = t, b, c \quad \text{and} \quad i = t, b \tag{27}$$

- **Clamped-Simply BC**

$$u^i = v^i = w^i = u_2^c = u_3^c = v_2^c = v_3^c = w_2^c = w_3^c = \frac{\partial w^i}{\partial x} = \frac{\partial w_2^c}{\partial x} = \frac{\partial w_3^c}{\partial x} = 0 \quad , \quad \text{at x=0}$$

$$v^i = w^i = v_2^c = v_3^c = w_2^c = w_3^c = N_x^j = M_x^j = N_x^{*c} = M_x^{*c} = 0 \quad , \quad \text{at x=L} \tag{28}$$

$$j = t, b, c \quad \text{and} \quad i = t, b$$

where $j = t, b, c$ and $i = t, b$.

4. Estimation of contact force history

To predict the impact force, different methods such as the finite element code, the spring-mass model, and the Hertz's perfect model could be used. The complete model for estimation of the contact force history is presented herein. In the present study, the effects of the strain rate and the wave propagation on the impact response and the parts of the impactor body, the thermal effects, the acoustic emission, and the local damage are assumed to be neglected.

4.1. Hertz's complete model

In this model, the improved nonlinear Hertzian contact law is used for obtaining the contact force history. Hertzian contact law is presented for the static loading on an isotropic linear elastic half-space [18]. It is also used for the case of the impact on the composite structures as follows [19]:

$$F(t) = k_c \alpha^{1.5} = k_c (w_i \quad (t) - w_s(t))^{1.5} \tag{29}$$

where α *is* the indentation, W_i is the displacement of the impactor, w_s is the displacement of the shell, and k_c is the Hertzian contact stiffness that is presented for the contact between the rigid spherical impactor and STCS as follows [20]:

$$K_c = \left(\frac{4}{3}\right) E R_i^{1/2}, \qquad \frac{1}{E} = \frac{1 - \nu_i^2}{E_i} + \frac{1 - \nu_s^2}{E_s}$$

$$\frac{1}{R*} = \frac{1}{Ri} + \frac{1}{2}\left(\frac{1}{R1} + \frac{1}{R2}\right) \tag{30}$$

where *i* and *s* are related to the impactor and target, respectively. The equations of motion of STCS may be expressed in the compact matrix form as in Eq. (31). In the complete model, the equations of motion and the impactor are obtained as a set of nonlinear coupled ordinary differential equations in which w_i and w_s are obtained, then, the contact force history and the dynamic response are obtained from Eq. (29) as:

$$[M]\{\ddot{\chi}\}[K]\{\chi\} = \{Q\} \qquad , X(t = 0) = 0 \tag{31}$$

$$mi\, \ddot{W\iota} + F(t) = 0 \;, \quad Wi(t = 0) = 0 \;, \quad \dot{W}i(t = 0) = V0 \tag{32}$$

$$\{Q\} = \{0,0, Qmn, 0,0,0,0,0,0,0,0,0,0,0,0\}$$

where $[M]$ and $[K]$ are the square mass matrix and the stiffness of the shell , respectively. Q is the impact force vector, and W_i and m_i are the displacement and mass of the impactor. For a concentrated load, the time coefficient is as follows:

$$Qmn(t) = \frac{4F(t)}{ab} \sin \alpha_m x1c \sin \beta n\, x2c \tag{33}$$

5. DQM

DQM approximates the partial derivative of a function F, with respect to two spatial variables (x and ϕ) at a given discrete point (x_i, ϕ_i) as a weighted linear sum of the function values at all discrete points chosen in the solution domain ($0<x<L$, $0<\phi<2\pi$) with $N_x \times N_\phi$ grid points along x and ϕ axes, respectively. Then, the n[th]-order partial derivative of $F(x,\phi)$ with respect to x, the m[th]-order partial derivative of $F(x,\phi)$ with respect to ϕ, and the (n + m)[th]-order partial derivative of $F(x,\phi)$ with respect to both x and ϕ are expressed discretely at the point (x_i, ϕ_i) as [21-23]:

$$\frac{d^n F(x_i,\phi_j)}{dx^n} = \sum_{k=1}^{N_x} A_{ik}^{(n)} F(x_k,\phi_j) \qquad n = 1,\ldots,N_x - 1, \tag{34}$$

$$\frac{d^m F(x_i,\phi_j)}{d\phi^m} = \sum_{l=1}^{N_\phi} B_{jl}^{(m)} F(x_i,\phi_l) \qquad m = 1,\ldots,N_\phi - 1, \tag{35}$$

$$\frac{d^{n+m} F(x_i,\phi_j)}{dx^n d\phi^m} = \sum_{k=1}^{N_x} \sum_{l=1}^{N_\phi} A_{ik}^{(n)} B_{jl}^{(m)} F(x_k,\phi_l), \tag{36}$$

where $A_{ik}^{(n)}$ and $B_{jl}^{(m)}$ are the weighting coefficients which may be defined as:

$$A_{ij}^{(1)} = \begin{cases} \dfrac{M(x_i)}{(x_i - x_j)M(x_j)} & for \quad i \neq j, \quad i,j = 1,2,\ldots,N_x \\ -\sum\limits_{\substack{j=1 \\ i \neq j}}^{N_x} A_{ij}^{(1)} & for \quad i = j, \quad i,j = 1,2,\ldots,N_x \end{cases} \tag{37}$$

$$B_{ij}^{(1)} = \begin{cases} \dfrac{P(\phi_i)}{(\phi_i - \phi_j)P(\phi_j)} & for \quad i \neq j, \quad i,j = 1,2,\ldots,N_\phi, \\ -\sum\limits_{\substack{j=1 \\ i \neq j}}^{N_\phi} B_{ij}^{(1)} & for \quad i = j, \quad i,j = 1,2,\ldots,N_\phi \end{cases} \tag{38}$$

where M and P are the Lagrangian operators which are defined as:

$$M(x_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N_x} (x_i - x_j) \tag{39}$$

$$P(\phi_i) = \prod_{\substack{j=1 \\ j \neq i}}^{N_\phi} (\phi_i - \phi_j) \tag{40}$$

The weighting coefficients for the second, third, and fourth derivatives are determined via matrix multiplication

$$A_{ij}^{(n)} = n\left(A_{ii}^{(n-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(n-1)}}{(x_i - x_j)} \right) \tag{41}$$

$$B_{ij}^{(m)} = m\left(B_{ii}^{(m-1)} B_{ij}^{(1)} - \frac{B_{ij}^{(m-1)}}{(\phi_i - \phi_j)} \right) \tag{42}$$

Using the following rule, the distribution of the grid points in the domain is calculated as:

$$x_i = \frac{L}{2}\left[1-\cos\left(\frac{i-1}{N_x-1}\right)\pi\right] \qquad i=1,...,N_x \tag{43}$$

$$\phi_i = \frac{2\pi}{2}\left[1-\cos\left(\frac{i-1}{N_\phi-1}\right)\pi\right] \qquad i=1,...,N_\phi \tag{44}$$

By using Eqs. (33)-(46), the motion equations for the low velocity impact of STCS can be expressed in the matrix form as:

$$[M]\{\ddot{\chi}\} + [K]\{\chi\} = \{Q\} \tag{45}$$

where $[M]$ and $[K]$ are the mass and stiffness matrixes, respectively; Q is the dynamic load vector, and $\chi = \{u_0^t, v_0^t, w^t, \theta_x^t, \theta_\varphi^t, u_2^c, v_2^c, w_2^c, u_3^c, v_3^c, w_3^c, u_0^b, v_0^b, w^b, \theta_x^b, \theta_\varphi^b\}$ is the displacement vector. The second and first time derivatives are defined using the Teoplitz matrices as follows:

$$\begin{cases} a_{11} = 0, \\ a_{i,1} = (-1)^{i-1}\cot\left(\frac{\pi(i-1)}{n}\right), \\ a_{1,j} = (-1)^{n-j+1}\cot\left(\frac{\pi(n-j+1)}{n}\right), \\ \mathrm{a}_{i+1,j+1} = a_{ij}, \end{cases} \quad i,j = 2,3,4,...,n,\ \tilde{D}_t^1 = 2\pi\left[a_{ij}\right], \tag{46}$$

$$\begin{cases} b_{11} = -\frac{n^2}{12} - \frac{1}{6}, \\ b_{i,1} = \frac{(-1)^{i-1}}{2\sin^2\left(\frac{\pi(i-1)}{n}\right)}, \\ b_{1,j} = \frac{(-1)^{n-j+1}}{2\sin^2\left(\frac{\pi(n-j+1)}{n}\right)}, \\ b_{i+1,j+1} = b_{ij}, \end{cases} \quad i,j = 2,3,4,...,n,\ \tilde{D}_t^2 = (2\pi)^2\left[b_{ij}\right]. \tag{47}$$

Eq. (45) can be written as:

$$\left([D_t^2 \otimes M] + [D_t^1 \otimes C_e] + [I_t \otimes K]\right)\chi = [Q], \tag{48}$$

where $\otimes$ notes the Kronecker product, and I_t is the unit matrix. Finally, solving the above equation yields the deflection and the contact force of the structure which are discussed in the next section.

6. Numerical results and discussion

In this section, the effects of different parameters on the low velocity impact response of the sandwich truncated laminated shell is presented. For this purpose, a truncated laminated shell with the following orthotropic material properties of the face sheets and the core is considered (Table 1).

Table 1. Material property of the face sheets and the core

Material properties	Face sheets	core
(0/90/core/0/90)	$\mathrm{E}_1 = 131\mathrm{GPa}, \mathrm{E}_2 = \mathrm{E}_3 = 10.34\mathrm{GPa}$ $\mathrm{G}_{12} = \mathrm{G}_{13} = 6.895\mathrm{GPa},\ G_{13} = 6.205\mathrm{GPa}$ $\nu_{12} = \nu_{13} = 0.22,\ \nu_{23} = 0.49, \rho = 1627\mathrm{kg/m^3}$	$\mathrm{E}_1 = \mathrm{E}_2 = \mathrm{E}_3 = 0.00689$ GPa $\mathrm{G}_{12} = \mathrm{G}_{13} = \mathrm{G}_{23} = 3.45$ GPa $\nu = 0, \rho = 94.195$ kg / m^3

Fig. 2 shows the effect of the semi vertex angle of the cone on the deflection histories. As can be seen, the central deflection of the laminated structure increases by increasing the semi vertex angle of the cone. This behavior is due to the increase of the stiffness of the structure by increasing the semi vertex angle of the cone.

Fig. 3 presents the effects of different boundary conditions on the deflection histories of the structure. Three cases of the boundary conditions, namely, simply-simply (SS), clamped-clamped (CC), and simply-clamed (SC) are considered. It can be found that considering the CC boundary condition, the central deflection of the top face sheets increases. It is due to the fact that the structure with the CC boundary condition has more rigidity than the other assumed boundary conditions.

The effect of the orientation angle of the face sheets on the deflection histories of the structure is shown in Fig. 4. Four cases of two-layers laminated conical shell are assumed as $\left(0^0,0^0\right)$, the cross ply type of $\left(0^0,90^0\right)$, and the angle-ply types of $\left(30^0,-30^0\right)$ and $\left(45^0,-45^0\right)$. It is evident that the deflection of the angle-ply is higher than the cross-ply type. As a result, the angle-ply type laminated structure leads to higher stiffness, and thus, higher deflection. Furthermore, the angle-ply lamina slightly shortens the contact duration.

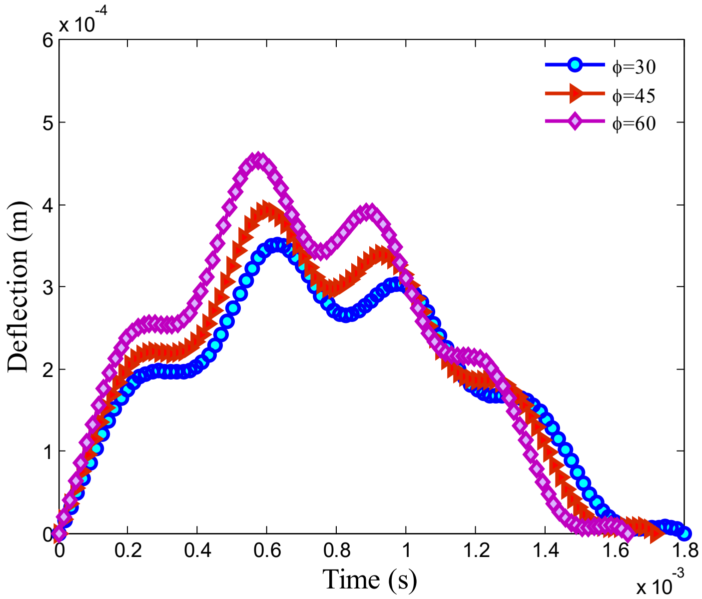

Fig. 2. The effect of the semi vertex angle of the cone on the deflection histories

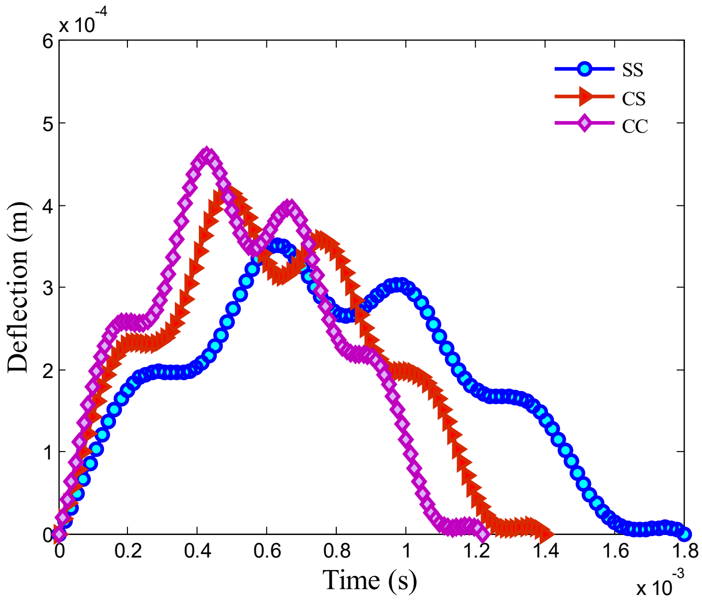

Fig. 3. The effect of the boundary conditions on the deflection histories

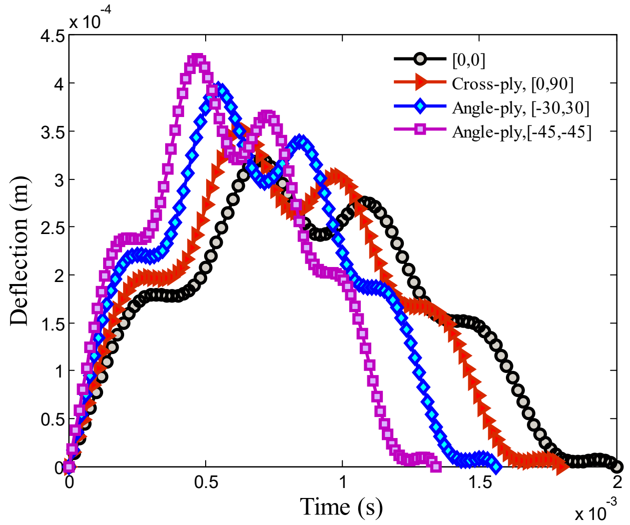

Fig. 4. The effect of the orientation angle of the face sheets on the deflection histories

The impact velocity and the mass effects on the histories of the deflection are demonstrated in Figs. 5 and 6, respectively. As can be seen, the central deflection of the structure increases by increasing the impact velocity and the mass of the impactor. The reason is that the higher impact velocity and mass of the impactor which accompanies higher impact energy requires a larger deflection and an accompanying contact force to dissipate it.

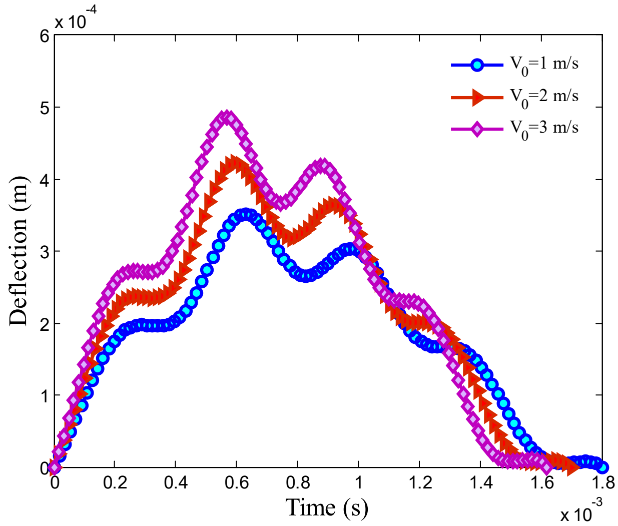

Fig. 5. The effect of the impact velocity on the deflection histories

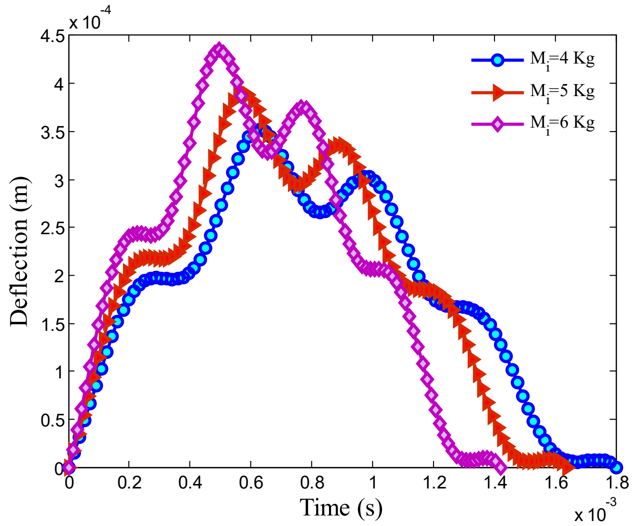

Fig. 6. The effect of the mass of impactor on the deflection histories

7. Conclusion

Based on the higher order sandwich panel theory, the low velocity impact analysis of STCS with various boundary conditions was investigated. The impact was assumed to occur normally over the top face-sheet. The motion equations were derived based on the Hamilton's principle considering the displacement continuity conditions between the layers. In order to obtain the displacement histories, DQM was applied. The effect of different parameters such as the orientation angle of the face sheets, the boundary conditions, the semi vertex angle of the cone, the impact velocity, and the mass of the impactor on the impact response were studied. The numerical result indicated that the central deflection of the laminated structure increased by increasing the semi vertex angle of the cone. Considering the CC boundary condition, the central deflection of the top face sheets increased. In addition, the deflection of the angle-ply was higher than the cross-ply type. Furthermore, the central deflection of the structure increased by increasing the impact velocity and the mass of the impactor.

References

[1] Frostig, Y. and Thomsen, O.T., High-order free vibration of sandwich panels with a flexible core, Int. J. Solids Struct., Vol. 41(5), pp. 1697-1724, 2004.

[2] Sofiyev, A.H., Non-linear buckling behavior of FGM truncated conical shells subjected to axial load, Int. J. Non-Linear Mech., Vol. 46(5), pp. 711-719, 2011.

[3] Chai, G.B. and Zhu, S., A review of low-velocity impact on sandwich structures, Proceed. Inst. Mec. Eng., Part L: J. Mat. Des. Applicat., Vol. 225(4), pp. 207-230, 2011.

[4] Abrate, S., Impact on composite structures, Cambridge university press, 2005.

[5] Shivakumar, K.N., Elber, W. and Illg, W., Prediction of low-velocity impact damage in thin circular laminates, AIAA J., Vol. 23(3), pp. 442-449, 1985.

[6] Anderson, T.A., An investigation of SDOF models for large mass impact on sandwich composites, Compos. Part B: Eng., Vol. 36(2), pp. 135-142, 2005.

[7] Gong, S.W. and Lam, K.Y., Effects of structural damping and stiffness on impact response of layered structure, AIAA J., Vol. 38(9), pp. 1730-1735, 2000.

[8] Malekzadeh, K., Khalili, M.R. and Mittal, R.K., Response of composite sandwich panels with transversely flexible core to low-velocity transverse impact: A new dynamic model, Int. J. Impact Eng., Vol. 34(3), pp. 522-543, 2007.

[9] Khalili, M.R., Malekzadeh, K. and Mittal, R.K., Effect of physical and geometrical parameters on transverse low-velocity impact response of sandwich panels with a transversely flexible core, Compos. Struct., Vol. 77(4), pp. 430-443, 2007.

[10] Wilkins, D.J., Bert, C.W. and Egle, D.M., Free vibrations of orthotropic sandwich conical shells with various boundary conditions, J. Sound Vib., Vol. 13(2), pp. 211-228, 1970.

[11] Struk, R., Non-linear stability problem of an open conical sandwich shell under external pressure and compression, Int. J. Non-linear Mech., Vol. 19(3), pp. 217-233, 1984.

[12] Bardell, N.S., Langley, R.S., Dunsdon, J.M. and Aglietti, G.S., An h–p finite element vibration analysis of open conical sandwich panels and conical sandwich frusta, J. Sound Vib., Vol. 226(2), pp. 345-377, 1999.

[13] Malekzadeh Fard, K.M.and Livani, M., New enhanced higher order free vibration analysis of thick truncated conical sandwich shells with flexible cores, Struct. Eng. Mech., Vol. 55(4), pp. 719-742, 2015.

[14] Reddy J., Mechanics of laminated composite plates and shells, theory and application, CRC Press, Boca Raton FL, 2003.

[15] Kheirikhah, M.M., Khalili, S.M.R. and Malekzadeh Fard, K., Biaxial buckling analysis of soft-core composite sandwich plates using improved high-order theory, Europ. J. Mech. A/Solids, Vol. 31, pp. 54e66, 2012.

[16] Garg, A.K., Khare, R.K. and Kant, T., Higher-order closed-form solutions for free vibration of laminated composite and sandwich shells, J. Sandw. Struct. Mat., Vol. 8(3), pp. 205-235, 2006.

[17] Reissner, E., On a variational theorem for finite elastic deformations, J. Math. Phys, Vol. 32(2-3), pp. 129-135, 1953.

[18] Carvalho, A. and Soares, C.G., Dynamic Response of Rectangular Plates of Composite Materials Subjected to Impact Loads, Compos. Struct., Vol. 34, pp. 55–63, 1996.

[19] Zheng, D. and Binienda, W.K., Analysis of Impact Response of Composite Laminates under Pre-stress, ASCE, Vol. 4, No. 197, pp. 211-219, 2008.

[20] Malekzadeh Fard, K. and Gholami, M., Analysis of Impact Dynamic Response of Doubly Curved Composite Laminated Shell under Initial Stresses, Aerosp. Mech. J., Vol. 10, No. 3, pp. 73–88, 2013.

[21] Kolahchi, R., Safari, M. and Esmailpour, M., Dynamic stability analysis of temperature-dependent functionally graded CNT-reinforced visco-plates resting on orthotropic elastomeric medium, Compos. Struct, Vol. 150, pp. 255–265, 2016.

[22] Ghorbanpour Arani, A., Kolahchi, R. and Zarei, M.Sh., Visco-surface-nonlocal piezoelasticity effects on nonlinear dynamic stability of graphene sheets integrated with ZnO sensors and actuators using refined zigzag theory, Compos. Struct., Vol. 132, pp. 506–526, 2015.

[23] Kolahchi, R. and Moniribidgoli, A.M., Size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes, Appl. Math. Mech. -Engl. Ed., Vol. 37(2), pp. 265–274, 2016.