Journal of Applied and Computational Mechanics, Vol. 3, No. 1, (2017), 16-24
DOI: 10.22055/jacm.2017.12548

Journal of
Applied and Computational Mechanics

# Free Vibration of a Thick Sandwich Plate Using Higher Order Shear Deformation Theory and DQM for Different Boundary Conditions

**M. Nasihatgozar[a], S.M.R. Khalili [b, *1]**

[a] Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran

[b] Centre of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanical Engineering, K.N. Toosi University of Technology, Tehran, Iran

Received November 24 2016; revised January 2 2017; accepted for publication January 15 2017.
Corresponding author: S. Mohammad Reza Khalili, smrkhalili2005@gmail.com

**Abstract**

In this paper, the effect of different boundary conditions on the free vibration analysis response of a sandwich plate is presented using the higher order shear deformation theory. The face sheets are orthotropic laminated composites that follow the first order shear deformation theory (FSDT) based on the Rissners-Mindlin (RM) kinematics field. The motion equations are derived considering the continuity boundary conditions between the layers based on the energy method and Hamilton's principle. The frequency and mode shapes of the structure are obtained using the differential quadrature method (DQM). The effects of different parameters such as the face sheet-to-core stiffness ratio, the boundary conditions, and the core-to-face sheet thickness ratio on the frequency of the sandwich plate are shown. Moreover, the numerical results indicate that the frequency of the CCCC and CFFF sandwich plates predict the higher and lower frequency, respectively.

**Keywords**: Sandwich plate, Vibration, DQM, Higher order theory, FSDT.

## 1. Introduction

During the past decades, sandwich structures have been of great interest among researchers due to their lightweight and high stiffness. These structures have many applications in different industries such as the aerospace engineering. With respect to the mechanical analysis of the sandwich structures, there are many research studies in the literature. Noor et al. [1] focused on the hierarchy of the computational models for the sandwich plates and shells, the predictor-corrector procedures, and the sensitivity of the sandwich response to variations in the different geometric and material parameters. The free undamped vibration of an isotropic circular cylindrical shell was analyzed by Bhimaraddi [2] using the higher order displacement model which gave rise to a more realistic parabolic variation of the transverse shear strains. A rigorous theory was derived by Leissa and Chang [3], which governs the linearly elastic deformation of the shells which are made of the laminated composite materials, including the shear deformation and the rotary inertia effects. The closed-form formulation of the three-dimensional (3-D) refined higher-order shear deformation theory (RHOST) was presented by Khalili et al. [4] for the free vibration analysis of simply supported-simply supported and clamped-clamped homogenous isotropic circular cylindrical shells. Thai et al. [5] presented a novel numerical procedure based on the framework of the isogeometric analysis for the static, free vibration, and buckling analysis of

*1 Corresponding author: Center of Excellence for Research in Advanced Materials and Structures, Faculty of Mechanic al Engineering, K.N. Toosi University of Technology, Pardis Street, Molassadra Avenue, Vanak Square, Tehran, Iran. Tel./fa x: +98 2188674747.

the laminated composite plates using the first-order shear deformation theory. A non-uniform rational B-spline based isogeometric finite element method was used by Valizadeh et al. [6] to study the static and dynamic characteristics of the functionally graded material (FGM) plates. Kapoor and Kapania [7] presented the development of geometrically nonlinear NURBS isogeometric finite element analysis of the laminated composite plates. Viola et al. [8] provided a general framework for the formulation and the dynamic analysis computations of moderately thick laminated doubly-curved shells and panels. The numerical results of the analysis of the axisymmetric vibrations of the annular sandwich plates with a relatively stiff core of non-uniform thickness were presented by Lal and Rani [9] using a refined theory. The static and vibration analysis of the isotropic and functionally graded sandwich plates was presented by Nguyen et al. by using the higher-order shear deformation theory [10].

In general, there are three main approaches to analyze the sandwich structures: a) the 3D elasticity approaches in which the equations of motion expressed without considering any assumption for the displacement field are solved, and the stresses, the strains, and the displacement components are obtained; therefore, this theory is the most exact theory for analyzing the mechanical behavior of the constructions, b) the Equivalent single layer (ESL) theories in which all the unknown displacement field functions do not depend on the considered layer, meaning that all the layers have the same degrees of freedom (DOF), and c) the Layer wise (LW) theories in which the unknown displacement field functions depend on the considered layer.

To the best of the authors' knowledge, no research study on the free vibrations of the sandwich plates using the higher order shear deformation theory considering different boundary conditions is reported adequately in the existing literature. The current research study presents the free vibration of the thick orthotropic laminated plates using a new type of higher order sandwich panel theory. By applying the energy method and Hamilton's principle, the motion equations are derived. DQM is used for obtaining the frequency of the structure for the different boundary conditions.

## 2. Definition of the 3D displacement field in the face sheets and the core

The displacement field for an arbitrary point within the top and bottom face sheets based on the Mindlin–Reissner shell theory can be written as [11]:

$$\begin{aligned} u^i(x,y,z,t) &= u_0^i(x,y,t) + z^i\theta_x^i(x,y,t) \\ v^i(x,y,z,t) &= v_0^i(x,y,t) + z^i\theta_y^i(x,y,t) \\ w^i(x,y,z,t) &= w^i(x,y,t) \end{aligned} \tag{1}$$

Where $u^i, v^i,$ and $w^i (i = t, b)$ denote the displacement components of the face sheets of DCSP. In equation (1) , $u_0^i$, $v_0^i$, and $w^i$ are the displacements at the mid surface in the α, β, and $z$ directions. $\theta_x^i$ and $\theta_y^i$ are the rotations of a transverse normal around α and β curvilinear coordinates, respectively. The displacement components $u^c, v^c$ and $w^c$ of a generic point in the core are related to the mid surface displacement ($u_0^c$ , $v_0^c$ , $w_0^c$ ) by:

$$\begin{aligned} u^c(x,y,z,t) &= u_0^c(x,y,t) + zu_1^c(x,y,t) + z^2u_2^c(x,y,t) + z^3u_3^c(x,y,t) \\ v^c(x,y,z,t) &= v_0^c(x,y,t) + zv_1^c(x,y,t) + z^2v_2^c(x,y,t) + z^3v_3^c(x,y,t) \\ w^c(x,y,z,t) &= w_0^c(x,y,t) + zw_1^c(x,y,t) + z^2w_2^c(x,y,t) + z^3w_3^c(x,y,t) \end{aligned} \tag{2}$$

In equation (2), $u_1^c$, $v_1^c$, and $w_1^c$ functions are rotational; the parameters $u_2^c, u_3^c, v_2^c, v_3^c, w_2^c$, and $w_3^c$ are the higher-order terms in the Taylor's series expansion. The strains are found to be [11]:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x}\ , \qquad \varepsilon_y = \frac{\partial v}{\partial y}\ , \quad \varepsilon_z = \frac{\partial w}{\partial z} \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad ,\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z},\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \end{aligned} \tag{3}$$

The above equations can be easily applied for the flat plate, cylindrical, spherical shells, etc. The kinematic relations for the top and bottom face sheets and the core in terms of the mid surface displacement are obtained by substituting the displacement field relationships into Eq. (3) which yields the Eqs. (4) and (5) for the face sheets and the core, respectively:

$$\begin{aligned} \varepsilon_x^i &= \left(\frac{\partial u_0^i}{\partial x} + z^i\frac{\partial \theta_x^i}{\partial x}\right) \quad ,\varepsilon_y^i = \left(\frac{\partial v_0^i}{\partial y} + z^i\frac{\partial \theta_y^i}{\partial y}\right) \\ \gamma_{xy}^i &= \left(\frac{\partial v_0^i}{\partial x} + z^i\frac{\partial \theta_y^i}{\partial x}\right) + \left(\frac{\partial u_0^i}{\partial y} + z^i\frac{\partial \theta_x^i}{\partial y}\right) \quad , \quad \gamma_{xz}^i = \frac{\partial w_0^i}{\partial x} + \theta_x^i \quad , \quad \gamma_{yz}^i = \frac{\partial w_0^i}{\partial y} + \theta_y^i \end{aligned} \tag{4}$$

In the above equations $i$ stands for the face sheets, $i = t$ represents the top face sheets, and $i = b$ represents the bottom face sheet.

For the core:

$$\varepsilon_x^c = \left[u_{0,x}^c + z^c u_{1,x}^c + z^{c2} u_{2,x}^c + z^{c3} u_{3,x}^c\right] \tag{5}$$

$$\varepsilon_y^c = \left[v_{0,x}^c + z^c v_{1,x}^c + z^{c2} v_{2,x}^c + z^{c3} v_{3,x}^c\right]$$
$$\varepsilon_z^c = w_1^c + 2\, z^c w_2^c + 3\, z^{c2} w_3^c$$
$$\varepsilon_{xy}^c = \left[v_{0,x}^c + z^c v_{1,x}^c + z^{c2} v_{2,x}^c + z^{c3} v_{3,x}^c\right]$$
$$\varepsilon_{yx}^c = \left[u_{0,y}^c + z^c u_{1,y}^c + z^{c2} u_{2,y}^c + z^{c3} u_{3,y}^c\right]$$
$$\gamma_{xz}^c = \left[w_{0,x}^c + z^c w_{1,x}^c + z^{c2} w_{2,x}^c + z^{c3} w_{3,x}^c\right] + \left[u_1^c + 2z^c u_2^c + 3z^{c2} u_3^c\right]$$

$$\gamma_{yz}^c = \left[w_{0,y}^c + z^c w_{1,y}^c + z^{c2} w_{2,y}^c + z^{c3} w_{3,y}^c\right] + \left[v_1^c + 2z^c v_2^c + 3z^{c2} v_3^c\right]$$

Reminding that there is no slipping between the face sheets and the core, the following relations can be written [12]:

$$\begin{cases} u^c|_{z=\frac{t^c}{2}} = u^t|_{z=\frac{t^c}{2}} \\ u^c|_{z=-\frac{t^c}{2}} = u^b|_{z=-\frac{t^c}{2}} \end{cases}, \begin{cases} v^c|_{z=\frac{t^c}{2}} = v^t|_{z=\frac{t^c}{2}} \\ v^c|_{z=-\frac{t^c}{2}} = v^b|_{z=-\frac{t^c}{2}} \end{cases}, \begin{cases} w^c|_{z=\frac{t^c}{2}} = w^t|_{z=\frac{t^c}{2}} \\ w^c|_{z=-\frac{t^c}{2}} = w^b|_{z=-\frac{t^c}{2}} \end{cases} \tag{6}$$

By substituting Eq. (1) and the displacement fields, $u_0^c$ , $u_1^c$ , $v_0^c$ , $v_1^c$ , $w_0^c$ , $w_1^c$ are obtained as follows:

$$u_0^c = \frac{u_0^t + u_0^b}{2} + \frac{1}{4}(t^b\theta_x^b - t^t\theta_x^t) - (\frac{t^c}{2})^2 u_2^c$$
$$u_1^c = \frac{u_0^t - u_0^b}{t^c} - \frac{1}{2t^c}(t^b\theta_x^b + t^t\theta_x^t) - \left(\frac{t^c}{2}\right)^2 u_3^c$$
$$v_0^c = \frac{v_0^t + v_0^b}{2} + \frac{1}{4}\left(t^b\theta_y^b - t^t\theta_y^t\right) - (\frac{t^c}{2})^2 v_2^c \tag{7}$$
$$v_1^c = \frac{v_0^t - v_0^b}{t^c} - \frac{1}{2t^c}\left(t^b\theta_y^b + t^t\theta_y^t\right) - (\frac{t^c}{2})^2 v_3^c$$
$$w_0^c = \frac{w^t + w^b}{2} - \left(\frac{t^c}{2}\right)^2 w_2^c\,, \quad w_1^c = \frac{w^t - w^b}{t^c} - \left(\frac{t^c}{2}\right)^2 w_3^c$$

## 3. Constitutive Equations

Since the face sheets and the core are assumed to have linear elastic behavior, the stress-strain relation according to the Hook's law is:

$$\{\sigma\} = [Q]\{\varepsilon\} \tag{8a}$$

If the principle axes (1, 2, 3) (local axes) coincide with the geometric axes $(x\,, y\,, z)$ (global axes) (i.e. Fig. 1), the constitutive equation for a fiber-reinforced composite lamina can be written as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{12} \\ \tau_{13} \\ \tau_{23} \end{Bmatrix}^i = \begin{bmatrix} C_{11} & C_{21} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}^i \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix}^i \tag{8b}$$

where the [Q] matrix refers to the elastic stiffness in the principle material axes (1,2,3), and $C_{ij}$'s elements are defined as follows [11]:

$$C_{11} = \frac{E_{11}(1 - \nu_{23}\nu_{32})}{\nu^*}\,, C_{12} = \frac{E_{11}(\nu_{21} - \nu_{31}\nu_{23})}{\nu^*}\,, C_{13} = \frac{E_{11}(\nu_{31} + \nu_{21}\nu_{32})}{\nu^*}$$
$$C_{22} = \frac{E_{22}(1 - \nu_{13}\nu_{31})}{\nu^*}\,, C_{23} = \frac{E_{22}(\nu_{32} + \nu_{12}\nu_{31})}{\nu^*}, C_{33} = \frac{E_{33}(1 - \nu_{12}\nu_{21})}{\nu^*} \tag{9}$$
$$C_{44} = G_{12}, C_{55} = G_{13}, C_{66} = G_{23}$$
$$\nu^* = 1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{13}\nu_{31} - 2\nu_{32}\nu_{13}\nu_{21}$$

The stress resultants can be written as:

$$\begin{bmatrix} N_x^c & N_x^{*c} & M_x^c & M_x^{*c} \\ N_y^c & N_y^{*c} & M_y^c & M_y^{*c} \\ N_z^c & N_z^{c*} & M_z^c & 0 \\ N_{xy}^c & N_{xy}^{*c} & M_{xy}^c & M_{xy}^{*c} \\ N_{yx}^c & N_{yx}^{*c} & M_{yx}^c & M_{yx}^{*c} \end{bmatrix} = \int_z \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yx} \end{Bmatrix}^c (1, z^2, z, z^3)dz, \quad \begin{bmatrix} Q_{yz}^c & Q_{yz}^{*c} & S_{yz}^c & S_{yz}^{*c} \\ Q_{zy}^c & Q_{zy}^{*c} & S_{zy}^c & 0 \\ Q_{xz}^c & Q_{xz}^{*c} & S_{xz}^c & S_{xz}^{*c} \\ Q_{zx}^c & Q_{zx}^{*c} & S_{zx}^c & 0 \end{bmatrix} =$$
$$\int_z \begin{Bmatrix} \tau_{yz} \\ \tau_{zy} \\ \tau_{xz} \\ \tau_{zx} \end{Bmatrix}^c (1, z^2, z, z^3)dz \tag{10}$$

$$\begin{bmatrix} N_x^i & M_x^i \\ N_y^i & M_y^i \\ N_{xy}^i & M_{xy}^i \\ N_{yx}^i & M_{yx}^i \end{bmatrix} = \int_{z^i} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yx} \end{Bmatrix}^i (1, z^i) dz^i \quad , \quad \begin{bmatrix} Q_{yz}^i & S_{yz}^i \\ Q_{zy}^i & 0 \\ Q_{xz}^i & S_{xz}^i \\ Q_{zx}^i & 0 \end{bmatrix} = \int_{z^i} \begin{Bmatrix} \tau_{yz} \\ \tau_{zy} \\ \tau_{xz} \\ \tau_{zx} \end{Bmatrix}^i (1, z^i) dz^i \tag{11}$$

The governing equations of motion for the free vibration analysis of DCSP and the boundary conditions are obtained using Hamilton's principle [1]:

$$\delta \int_{t_1}^{t_2} (L)\, dt = \delta \int_{t_1}^{t_2} [E - (U + W)]\, dt = 0 \tag{12}$$

where δ is the first variation operator, **E** is the kinetic energy, **U** and **W** denote the total strain energy due to the deformation and the potential of the external loads, respectively, and **t** is the time coordinate. The variation of the kinetic energy for DCSP is given by:

$$\int_0^t \delta E dt = -\sum_i^{t,b,c} \left\{ \int_0^t \int_{V^i} [\rho^i (\ddot{u}^i \delta u^i + \ddot{v}^i \delta v^i + \ddot{w}^i \delta w^i) dV^i]\, dt \right\} \tag{13}$$

where

$$I_n^i = \int_z \rho^i z^n dz\,, \quad i = t, b, c \;\; and \;\; n = 1 \; to \; 6 \tag{14}$$

The first variation of the strain energy for DCSP during the elastic deformation is:

$$\begin{aligned} &\int_{t_1}^{t_2} \delta\, U^c\, dt + \sum_i^{t,b} \int_{t_1}^{t_2} \delta U^i dt \\ &= \int_0^t \int_z \int_A \left(\sigma_z^c \delta\varepsilon_z^c + \tau_{xy}^c \delta\gamma_{xy}^c + \tau_{yz}^c \delta\gamma_{yz}^c\right) dA^c\, dz\, dt \\ &+ \sum_i^{t,b} \left\{ \int_0^t \int_{z^i} \int_A \left(\sigma_x^i \delta\varepsilon_x^i + \sigma_y^i \delta\varepsilon_y^i + \tau_{xy}^i \delta\gamma_{xy}^i + \tau_{xz}^i \delta\gamma_{xz}^i + \tau_{yz}^i \delta\gamma_{yz}^i\right) dA^i\, dz^i\, dt \right\} \end{aligned} \tag{15}$$

By applying Hamilton's principle, the motion equations become:

$$\begin{aligned} &-\left(\frac{1}{t^c}\right)\{Q_{zx}^c\} + \{N_{x,x}^t\} + \{N_{yx,y}^t\} \\ &= \left[\left(\frac{1}{4} I_0^c + \frac{1}{t^c} I_1^c + \frac{1}{t^{c2}} I_2^c\right) + I_0^t\right] \ddot{u}_0^t + \left[\left(-\frac{t^t}{8} I_0^c - \frac{t^t}{2t^c} I_1^c - \frac{t^t}{2t^{c2}} I_2^c\right) + I_1^t\right] \ddot{\theta}_x^t \\ &+ \left[-\frac{t^{c2}}{8} I_0^c - \frac{t^c}{4} I_1^c + \frac{1}{2} I_2^c + \frac{1}{t^c} I_3^c\right] \ddot{u}_2^c + \left[-\frac{t^{c2}}{8} I_1^c - \frac{t^c}{4} I_2^c + \frac{1}{2} I_3^c + \frac{1}{t^c} I_4^c\right] \ddot{u}_3^c \\ &+ \left[\frac{1}{4} I_0^c - \frac{1}{t^{c2}} I_2^c\right] \ddot{u}_0^b + \left[\frac{t^b}{8} I_0^c - \frac{t^b}{2t^{c2}} I_2^c\right] \ddot{\theta}_x^b \end{aligned} \tag{16a}$$

$$\begin{aligned} &-\left(\frac{1}{t^c}\right)\{Q_{zy}^c\} + \{N_{y,y}^t\} + \{N_{xy,x}^t\} \\ &= \left[\left(\frac{1}{4} I_0^c + \frac{1}{t^c} I_1^c + \frac{1}{t^{c2}} I_2^c\right) + I_0^t\right] \ddot{v}_0^t + \left[\left(-\frac{t^t}{8} I_0^c - \frac{t^t}{2t^c} I_1^c - \frac{t^t}{2t^{c2}} I_2^c\right) + I_1^t\right] \ddot{\theta}_y^t \\ &+ \left[-\frac{t^{c2}}{8} I_0^c - \frac{t^c}{4} I_1^c + \frac{1}{2} I_2^c + \frac{1}{t^c} I_3^c\right] \ddot{v}_2^c + \left[-\frac{t^{c2}}{8} I_1^c - \frac{t^c}{4} I_2^c + \frac{1}{2} I_3^c + \frac{1}{t^c} I_4^c\right] \ddot{v}_3^c \\ &+ \left[\frac{1}{4} I_0^c - \frac{1}{t^{c2}} I_2^c\right] \ddot{v}_0^b + \left[\frac{t^b}{8} I_0^c - \frac{t^b}{2t^{c2}} I_2^c\right] \ddot{\theta}_y^b \end{aligned} \tag{16b}$$

$$\begin{aligned} &-\left(\frac{1}{t^c}\right)\{N_z^c\} + \left(\frac{1}{2}\right)\{Q_{xz,x}^c\} + \left(\frac{1}{t^c}\right)\{S_{xz,x}^c\} + \left(\frac{1}{2}\right)\{Q_{yz,y}^c\} + \left(\frac{1}{t^c}\right)\{S_{yz,y}^c\} + (k_s)\{Q_{xz,x}^t\} + (k_s)\{Q_{yz,y}^t\} \\ &= \left[\left(\frac{1}{4} I_0^c + \frac{1}{t^c} I_1^c + \frac{1}{t^{c2}} I_2^c\right) + I_0^t\right] \ddot{w}^t + \left[-\frac{t^{c2}}{8} I_0^c - \frac{t^c}{4} I_1^c + \frac{1}{2} I_2^c + \frac{1}{t^c} I_3^c\right] \ddot{w}_2^c \\ &+ \left[-\frac{t^{c2}}{8} I_1^c - \frac{t^c}{4} I_2^c + \frac{1}{2} I_3^c + \frac{1}{t^c} I_4^c\right] \ddot{w}_3^c + \left[\frac{1}{4} I_0^c - \frac{1}{t^{c2}} I_2^c\right] \ddot{w}^b \end{aligned} \tag{16c}$$

$$\left(\frac{t^t}{2t^c}\right)\{Q^c_{zx}\}+\{M^t_{x,x}\}+\{M^t_{yx,y}\}-k_s\{Q^t_{zx}\}$$
$$=\left[\left(-\frac{t^t}{8}I^c_0-\frac{t^t}{2t^c}I^c_1-\frac{t^t}{2t^{c^2}}I^c_2\right)+I^t_1\right]\ddot{u}^t_0+\left[\left(\frac{t^{t^2}}{16}I^c_0+\frac{t^{t^2}}{4t^c}I^c_1+\frac{t^{t^2}I^c_2}{4t^{c^2}}\right)+I^t_2\right]\ddot{\theta}^t_x$$
$$+\left[\frac{t^tt^{c^2}}{16}I^c_0+\frac{t^tt^c}{8}I^c_1-\frac{t^t}{4}I^c_2-\frac{t^t}{2t^c}I^c_3\right]\ddot{u}^c_2+\left[\frac{t^tt^{c^2}}{16}I^c_1+\frac{t^tt^c}{8}I^c_2-\frac{t^t}{4}I^c_3-\frac{t^t}{2t^c}I^c_4\right]\ddot{u}^c_3 \quad (16)\text{d}$$
$$+\left[-\frac{t^t}{8}I^c_0+\frac{t^t}{2t^{c^2}}I^c_2\right]\ddot{u}^b_0+\left[-\frac{t^tt^b}{16}I^c_0+\frac{t^tt^b}{4t^{c^2}}I^c_2\right]\ddot{\theta}^b_x$$

$$\left(\frac{t^t}{2t^c}\right)\{Q^c_{zy}\}+\{M^t_{y,y}\}+\{M^t_{xy,x}\}-k_s\{Q^t_{zy}\}$$
$$=\left[\left(-\frac{t^t}{8}I^c_0-\frac{t^t}{2t^c}I^c_1-\frac{t^t}{2t^{c^2}}I^c_2\right)+I^t_1\right]\ddot{v}^t_0+\left[\left(\frac{t^{t^2}}{16}I^c_0+\frac{t^{t^2}}{4t^c}I^c_1+\frac{t^{t^2}I^c_2}{4t^{c^2}}\right)+I^t_2\right]\ddot{\theta}^t_y$$
$$+\left[\frac{t^tt^{c^2}}{16}I^c_0+\frac{t^tt^c}{8}I^c_1-\frac{t^t}{4}I^c_2-\frac{t^t}{2t^c}I^c_3\right]\ddot{v}^c_2+\left[\frac{t^tt^{c^2}}{16}I^c_1+\frac{t^tt^c}{8}I^c_2-\frac{t^t}{4}I^c_3-\frac{t^t}{2t^c}I^c_4\right]\ddot{v}^c_3 \quad (16)\text{e}$$
$$+\left[-\frac{t^t}{8}I^c_0+\frac{t^t}{2t^{c^2}}I^c_2\right]\ddot{v}^b_0+\left[-\frac{t^tt^b}{16}I^c_0+\frac{t^tt^b}{4t^{c^2}}I^c_2\right]\ddot{\theta}^b_y$$

$$-2\{S^c_{zx}\}=\left[-\frac{t^{c^2}}{8}I^c_0-\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2+\frac{1}{t^c}I^c_3\right]\ddot{u}^t_0+\left[\frac{t^{c^2}t^t}{16}I^c_0+\frac{t^ct^t}{8}I^c_1-\frac{t^t}{4}I^c_2-\frac{t^t}{2t^c}I^c_3\right]\ddot{\theta}^t_x$$
$$+\left[\frac{t^{c^4}}{16}I^c_0-\frac{t^{c^2}}{2}I^c_2+I^c_4\right]\ddot{u}^c_2+\left[\frac{t^{c^4}}{16}I^c_1-\frac{t^{c^2}}{2}I^c_3+I^c_5\right]\ddot{u}^c_3 \quad (16)\text{f}$$
$$+\left[-\frac{t^{c^2}}{8}I^c_0+\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2-\frac{1}{t^c}I^c_3\right]\ddot{u}^b_0+\left[-\frac{t^{c^2}t^b}{16}I^c_0+\frac{t^ct^b}{8}I^c_1+\frac{t^b}{4}I^c_2-\frac{t^b}{2t^c}I^c_3\right]\ddot{\theta}^b_x$$

$$-2\{S^c_{zy}\}=\left[-\frac{t^{c^2}}{8}I^c_0-\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2+\frac{1}{t^c}I^c_3\right]\ddot{v}^t_0+\left[\frac{t^{c^2}t^t}{16}I^c_0+\frac{t^ct^t}{8}I^c_1-\frac{t^t}{4}I^c_2-\frac{t^t}{2t^c}I^c_3\right]\ddot{\theta}^t_y$$
$$+\left[\frac{t^{c^4}}{16}I^c_0-\frac{t^{c^2}}{2}I^c_2+I^c_4\right]\ddot{v}^c_2+\left[\frac{t^{c^4}}{16}I^c_1-\frac{t^{c^2}}{2}I^c_3+I^c_5\right]\ddot{v}^c_3 \quad (16)\text{g}$$
$$+\left[-\frac{t^{c^2}}{8}I^c_0+\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2-\frac{1}{t^c}I^c_3\right]\ddot{v}^b_0+\left[-\frac{t^{c^2}t^b}{16}I^c_0+\frac{t^ct^b}{8}I^c_1+\frac{t^b}{4}I^c_2-\frac{t^b}{2t^c}I^c_3\right]\ddot{\theta}^b_y$$

$$-2\{M^c_z\}-\left(\frac{t^{c^2}}{4}\right)\{Q^c_{xz,x}\}+\{Q^{*c}_{xz,x}\}-\left(\frac{t^{c^2}}{4}\right)\{Q^c_{yZ,y}\}+\{Q^{*c}_{yz,y}\}$$
$$=\left[-\frac{t^{c^2}}{8}I^c_0-\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2+\frac{1}{t^c}I^c_3\right]\ddot{w}^t+\left[\frac{t^{c^4}}{16}I^c_0-\frac{t^{c^2}}{2}I^c_2+I^c_4\right]\ddot{w}^c_2 \quad (16)\text{h}$$
$$+\left[\frac{t^{c^4}}{16}I^c_1-\frac{t^{c^2}}{2}I^c_3+I^c_5\right]\ddot{w}^c_3+\left[-\frac{t^{c^2}}{8}I^c_0+\frac{t^c}{4}I^c_1+\frac{1}{2}I^c_2-\frac{1}{t^c}I^c_3\right]\ddot{w}^b$$

$$\left(\frac{t^{c^2}}{4}\right)\{Q^c_{zx}\}-3\{Q^{*c}_{zx}\}$$
$$=\left[-\frac{t^{c^2}}{8}I^c_1-\frac{t^c}{4}I^c_2+\frac{1}{2}I^c_3+\frac{1}{t^c}I^c_4\right]\ddot{u}^t_0+\left[\frac{t^{c^2}t^t}{16}I^c_1+\frac{t^ct^t}{8}I^c_2-\frac{t^t}{4}I^c_3-\frac{t^t}{2t^c}I^c_4\right]\ddot{\theta}^t_x$$
$$+\left[\frac{t^{c^4}}{16}I^c_1-\frac{t^{c^2}}{2}I^c_3+I^c_5\right]\ddot{u}^c_2+\left[\frac{t^{c^4}}{16}I^c_2-\frac{t^{c^2}}{2}I^c_4+I^c_6\right]\ddot{u}^c_3 \quad (16)\text{i}$$
$$+\left[-\frac{t^{c^2}}{8}I^c_1+\frac{t^c}{4}I^c_2+\frac{1}{2}I^c_3-\frac{1}{t^c}I^c_4\right]\ddot{u}^b_0+\left[-\frac{t^{c^2}t^b}{16}I^c_1+\frac{t^ct^b}{8}I^c_2+\frac{t^b}{4}I^c_3-\frac{t^b}{2t^c}I^c_4\right]\ddot{\theta}^b_x$$

$$\left(\frac{t^{c2}}{4}\right)\{Q_{zy}^{c}\}-3\{Q_{zy}^{*c}\}=\left[-\frac{t^{c2}}{8}I_1^c-\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c+\frac{1}{t^c}I_4^c\right]\ddot{v}_0^t+\left[\frac{t^{c2}t^t}{16}I_1^c+\frac{t^ct^t}{8}I_2^c-\frac{t^t}{4}I_3^c-\frac{t^t}{2t^c}I_4^c\right]\ddot{\theta}_y^t+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{v}_2^c+\left[\frac{t^{c4}}{16}I_2^c-\frac{t^{c2}}{2}I_4^c+I_6^c\right]\ddot{v}_3^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{v}_0^b+\left[-\frac{t^{c2}t^b}{16}I_1^c+\frac{t^ct^b}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{\theta}_y^b \tag{16j}$$

$$\left(\frac{t^{c2}}{4}\right)\{N_z^c\}-3\{N_z^{*c}\}-\left(\frac{t^{c2}}{4}\right)\{S_{yz,y}^c\}+\{S_{yz,y}^{*c}\}-\left(\frac{t^{c2}}{4}\right)\{S_{xz,x}^c\}+\{S_{xz,x}^{*c}\}=\left[-\frac{t^{c2}}{8}I_1^c-\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c+\frac{1}{t^c}I_4^c\right]\ddot{w}^t+\left[\frac{t^{c4}}{16}I_1^c-\frac{t^{c2}}{2}I_3^c+I_5^c\right]\ddot{w}_2^c+\left[\frac{t^{c4}}{16}I_2^c-\frac{t^{c2}}{2}I_4^c+I_6^c\right]\ddot{w}_3^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{w}^b \tag{16k}$$

$$\left(\frac{1}{t^c}\right)\{Q_{zx}^c\}+\{N_{x,x}^b\}+\{N_{yx,y}^b\}=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{u}_0^t+\left[-\frac{t^t}{8}I_0^c+\frac{t^t}{2t^{c2}}I_2^c\right]\ddot{\theta}_x^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{u}_2^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{u}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{u}_0^b+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{\theta}_x^b \tag{16l}$$

$$\left(\frac{1}{t^c}\right)\{Q_{zy}^c\}+\{N_{y,y}^b\}+(B^b)\{N_{xy,x}^b\}=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{v}_0^t+\left[-\frac{t^t}{8}I_0^c+\frac{t^t}{2t^{c2}}I_2^c\right]\ddot{\theta}_y^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{v}_2^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{v}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{v}_0^b+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{\theta}_y^b \tag{16m}$$

$$\left(\frac{1}{t^c}\right)\{N_z^c\}+\left(\frac{1}{2}\right)\{Q_{xz,x}^c\}-\left(\frac{1}{t^c}\right)\{S_{xz,x}^c\}+\left(\frac{1}{2}\right)\{Q_{yz,y}^c\}-\left(\frac{1}{t^c}\right)\{S_{yz,y}^c\}+(k_s)\{Q_{xz,x}^b\}+(k_s)\{Q_{yz,y}^b\}=\left[\frac{1}{4}I_0^c-\frac{1}{t^{c2}}I_2^c\right]\ddot{w}^t+\left[-\frac{t^{c2}}{8}I_0^c+\frac{t^c}{4}I_1^c+\frac{1}{2}I_2^c-\frac{1}{t^c}I_3^c\right]\ddot{w}_2^c+\left[-\frac{t^{c2}}{8}I_1^c+\frac{t^c}{4}I_2^c+\frac{1}{2}I_3^c-\frac{1}{t^c}I_4^c\right]\ddot{w}_3^c+\left[\left(\frac{1}{4}I_0^c-\frac{1}{t^c}I_1^c+\frac{1}{t^{c2}}I_2^c\right)+I_0^b\right]\ddot{w}^b \tag{16n}$$

$$\left(\frac{t^b}{2t^c}\right)\{Q_{zx}^c\}+\{M_{x,x}^b\}+\{M_{yx,y}^b\}-k_s\{Q_{zx}^b\}=\left[\frac{t^b}{8}I_0^c-\frac{t^b}{2t^{c2}}I_2^c\right]\ddot{u}_0^t+\left[-\frac{t^tt^b}{16}I_0^c+\frac{t^tt^b}{4t^{c2}}I_2^c\right]\ddot{\theta}_x^t+\left[-\frac{t^bt^{c2}}{16}I_0^c+\frac{t^bt^c}{8}I_1^c+\frac{t^b}{4}I_2^c-\frac{t^b}{2t^c}I_3^c\right]\ddot{u}_2^c+\left[-\frac{t^bt^{c2}}{16}I_1^c+\frac{t^bt^c}{8}I_2^c+\frac{t^b}{4}I_3^c-\frac{t^b}{2t^c}I_4^c\right]\ddot{u}_3^c+\left[\left(\frac{t^b}{8}I_0^c-\frac{t^b}{2t^c}I_1^c+\frac{t^b}{2t^{c2}}I_2^c\right)+I_1^b\right]\ddot{u}_0^b+\left[\left(\frac{t^{b2}}{16}I_0^c-\frac{t^{b2}}{4t^c}I_1^c+\frac{t^{b2}I_2^c}{4t^{c2}}\right)+I_2^b\right]\ddot{\theta}_x^b \tag{16o}$$

$$\left(\frac{t^b}{2t^c}\right)\{Q^c_{zy}\}+\{M^b_{y,y}\}+\{M^b_{xy,x}\}-k_s\{Q^b_{zy}\}$$
$$=\left[\frac{t^b}{8}I^c_0-\frac{t^b}{2t^{c2}}I^c_2\right]\ddot{v}^t_0+\left[-\frac{t^t t^b}{16}I^c_0+\frac{t^t t^b}{4t^{c2}}I^c_2\right]\ddot{\theta}^t_y$$
$$+\left[-\frac{t^b t^{c2}}{16}I^c_0+\frac{t^b t^c}{8}I^c_1+\frac{t^b}{4}I^c_2-\frac{t^b}{2t^c}I^c_3\right]\ddot{v}^c_2$$
$$+\left[-\frac{t^b t^{c2}}{16}I^c_1+\frac{t^b t^c}{8}I^c_2+\frac{t^b}{4}I^c_3-\frac{t^b}{2t^c}I^c_4\right]\ddot{v}^c_3$$
$$+\left[\left(\frac{t^b}{8}I^c_0-\frac{t^b}{2t^c}I^c_1+\frac{t^b}{2t^{c2}}I^c_2\right)+I^b_1\right]\ddot{v}^b_0+\left[\left(\frac{t^{b2}}{16}I^c_0-\frac{t^{b2}}{4t^c}I^c_1+\frac{t^{b2}I^c_2}{4t^{c2}}\right)+I^b_2\right]\ddot{\theta}^b_y \quad (16)p$$

Applying DQM [13], the following motion equations can be derived:

$$\{[K]-\omega^2[M]\}\{d\}=0 \quad (17)$$

where $[K]$ and $[M]$ are the stiffness and the mass matrices, respectively. Moreover, $\omega$ is the frequency of the structure. The eigenvalue of Eq. (17) can be solved for various boundary conditions to obtain the fundamental frequency.

## 4. Results and discussion

In this section, the effects of the face sheet-to-core stiffness ratio, the boundary conditions, and the core-to-face sheet thickness ratio are shown on the frequency of the sandwich plate. The orthotropic material properties of the face sheets and the core are given in Table 1 [14].

**Table 1.** Material property of the face sheets and the core [14]

| Material Properties | Face Sheets | Core |
|---|---|---|
| (0/90/core/0/90) | $E_1=131\,GPa, E_2=E_3=10.34\,GPa$<br>$G_{12}=G_{13}=6.895\,GPa, G_{13}=6.205\,GPa$<br>$\nu_{12}=\nu_{13}=0.22, \nu_{23}=0.49, \rho=1627\,kg/m^3$ | $E_1=E_2=E_3=0.00689\,GPa$<br>$G_{12}=G_{13}=G_{23}=3.45\,GPa$<br>$\nu=0, \rho=94.195\,kg/m^3$ |

The effect of different boundary conditions on the frequency of the sandwich plate versus the core-to-face sheets thickness ratio (*c/d*) is shown in Fig. 1. As can be seen, by increasing the core-to-face sheets thickness ratio, the frequency of the structure increases due to an increase in the stiffness of the system. It is also concluded that the CCCC plate leads to higher frequency. However, the effect of different boundary conditions on the frequency of the structure follows the below order:

CCCC>CSCS>CFCF>SSSS>SFSF>CFFF

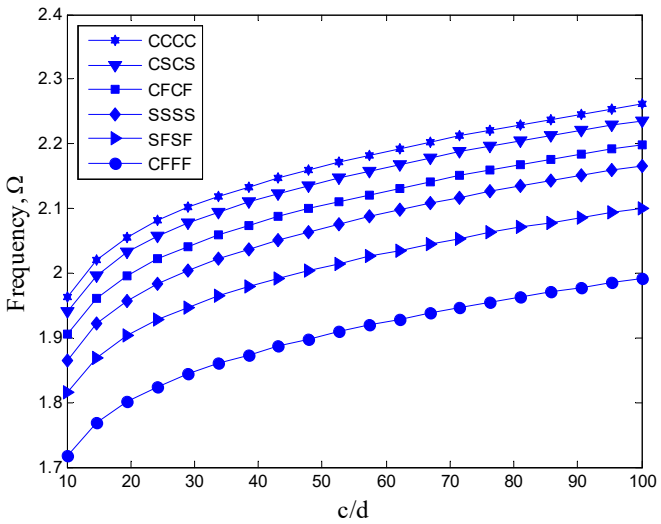


**Fig. 1.** The effects of the boundary conditions and the thickness ratio of core-to-face sheets on the frequency of the sandwich plate

Fig. 2 illustrates the effect of face sheet-to-core modulus ratio and the boundary conditions on the frequency of the sandwich plates. It can be found that by increasing the face sheet-to-core modulus ratio, the stiffness of the structure increases, and consequently, the frequency of the system enhances. Furthermore, the frequency of the CCCC and CFFF sandwich plates predicted the higher and lower frequency, respectively.

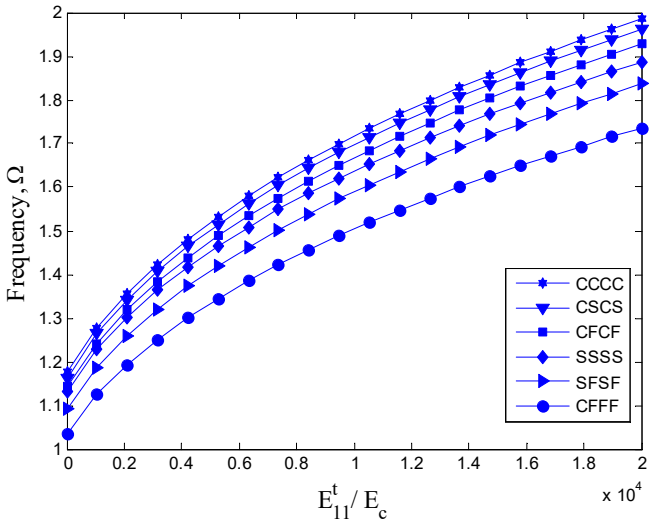


**Fig. 2.** The effects of the boundary conditions and the face sheet-to-core modulus ratio on the frequency of the sandwich plate

The mode shapes related to the CCCC sandwich plates are plotted in Figs. 3 to 6 for vibration of the first to fourth modes, respectively. It can be observed that the boundary conditions of the CCCC sandwich plates are satisfied in these figures.

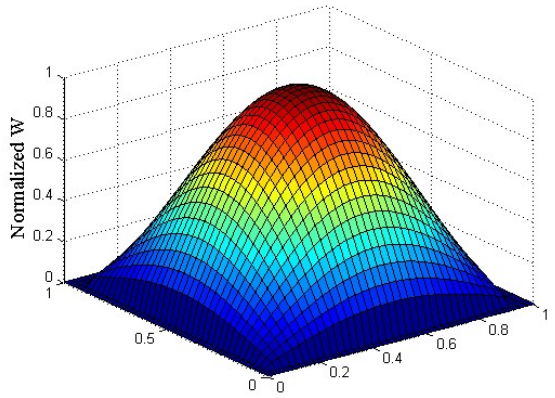


**Fig. 3.** First mode shape of the CCCC sandwich plate

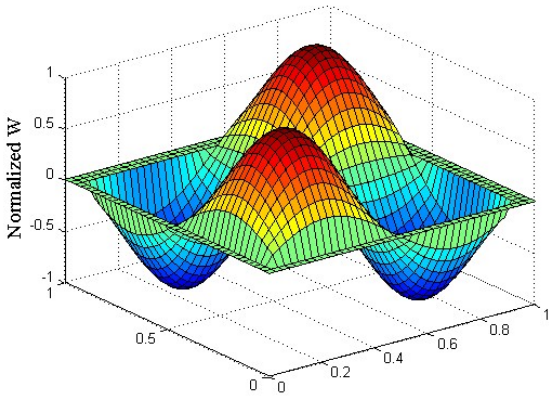


**Fig. 4.** Second mode shape of the CCCC sandwich plate

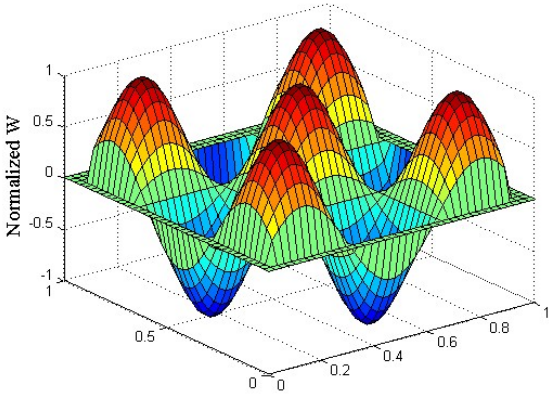


**Fig. 5.** Third mode shape of the CCCC sandwich plate

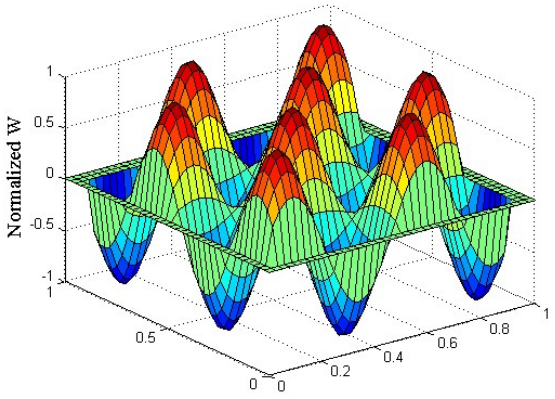


**Fig. 6.** Fourth mode shape of the CCCC sandwich plate

## 5. Conclusion

The free vibration analysis of the orthotropic laminated composite plate using the higher order shear deformation theory and Hamilton's principle with the appropriate kinematic relations of small deformations was presented in this paper. The face sheets were considered as laminated composites which follow the first-order shear deformation theory and the core was considered based on the higher order shear deformation theory of the sandwich structure. Based on DQM, the frequency of the structure was calculated for different boundary conditions. The effects of different parameters such as face sheet-to-core stiffness ratio, boundary conditions, and core-to-face sheet thickness ratio were shown on the frequency of the sandwich plate. The numerical results indicated that by increasing the core-to-face sheets thickness ratio, the frequency of the structure increased. Furthermore, by increasing the face sheet-to-core modulus ratio, the stiffness of the structure increased, and consequently, the frequency of the system enhanced. Additionally, the frequency of the CCCC and CFFF sandwich plates predicted the higher and lower frequency, respectively.

## References

[1] Noor, A.K., Burton, W.S. and Bert, C.W., Computational models for sandwich panels and shells, Appl. Mech. Rev., Vol. 49(3), pp. 155-199, 1996.
[2] Bhimaraddi, A., A higher order theory for free vibration analysis of circular cylindrical shells, Int. J. Solids Struct., Vol. 20(7), pp. 623-630, 1984.
[3] Leissa, A.W. and Chang, J.D., Elastic deformation of thick, laminated composite shells, Compos. Struct., Vol. 35(2), pp. 153-170, 1996.
[4] Khalili, S.M.R., Davar, A. and Malekzadeh Fard, K., Free vibration analysis of homogeneous isotropic circular cylindrical shells based on a new three-dimensional refined higher-order theory, Int. J. Mech. Sci., Vol. 56(1), pp .1-25, 2012.
[5] Thai, C.H., Nguyen-Xuan, H., Nguyen-Thanh, N. and Rabczuk, T., Static, free vibration, and buckling analysis of laminated composite Reissner–Mindlin plates using NURBS-based isogeometric approach, Int. J. Numeric. Meth. Eng., Vol. 91(6), pp. 571-603, 2012.
[6] Valizadeh, N., Natarajan, S., Gonzalez-Estrada, O.A., Rabczuk, T., Quoc Bui, T. and Bordas, S.P.A., NURBS-based finite element analysis of functionally graded plates, pp. Static bending, vibration, buckling and flutter, Compos. Struct., Vol. 99(0), pp.309-326, 2013.
[7] Kapoor, H. and Kapania, R.K., Geometrically nonlinear NURBS isogeometric finite element analysis of laminated composite plates, Compos. Struct., Vol. 94(12), pp.3434-3447, 2012.
[8] Viola, E., Tornabene, F.and Fantuzzi, N., General higher-order shear deformation theories for the free vibration analysis of completely doubly-curved laminated shells and panels, Compos. Struct., Vol. 95(0), pp.639-666, 2013.
[9] Lal, R. and Rani, R., On radially symmetric vibrations of non-uniform annular sandwich plates, Thin-Wall. Struct., Vol. 94, pp. 562–576, 2015.
[10] Nguyen, T.K., Nguyen, V.H., Chau-Dinh, T., Vo, T.P. and Nguyen-Xuan, H., Static and vibration analysis of isotropic and functionally graded sandwich plates using an edge-based MITC3 finite elements, Compos. Part B: Eng., Vol. 107, pp. 162–173, 2016.
[10] Reddy, J., Mechanics of laminated composite plates and shells, theory and application, CRC Press, Boca Raton FL, 2003.
[11] Kheirikhah, M.M., Khalili, S.M.R. and Malekzadeh Fard, K., Biaxial buckling analysis of soft-core composite sandwich plates using improved high-order theory, Europ. J. Mech. - A/Solids, Vol. 31(1), pp. 54-66, 2012.
[12] Kolahchi, R. and Rabani Bidgoli, M., Beygipoor, G. and Fakhar, M.H., A nonlocal nonlinear analysis for buckling in embedded FG-SWCNT-reinforced microplates subjected to magnetic field, J. Mech. Sci. Tech., Vol. 29, pp. 3669-3677, 2015.
[13] Malekzadeh Fard, K., Livani, M. and Gholami, M., Improved high-order bending analysis of double curved sandwich panels subjected to multiple loading conditions, Latin Americ. J. Solids Struct., Vol. 11(9), pp. 1591-1614, 2014.