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Pole placement algorithm for control of civil structures subjected to earthquake excitation

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Abstract

In this paper the control algorithm for controlled civil structures subjected to earthquake excitation is thoroughly investigated. The structural control consists of monitoring and analyzing the incoming excitation signal and the response of structure, applying the control algorithm for the calculation of the required control action, and, finally, implementing this action to the structure. The objective of this work is the control of structures by means of the pole placement algorithm, in order to improve their response against earthquake actions. The pole placement or pole assignment algorithm is a well-known, classical control algorithm that estimates the feedback matrix so that the system will have poles (eigenvalues) that are pre-decided by the designer. Successful application of the algorithm requires judicious placement of the closed-loop eigenvalues from the part of the designer. The pole placement algorithm was widely applied to control mechanical systems. In this paper, a modification in the mathematical background of the algorithm in order to be suitable for civil fixed structures is primarily presented. The proposed approach is demonstrated by numerical simulations for the control of both single and multi-degree of freedom systems subjected to seismic actions. Numerical results have shown that the control algorithm is efficient in reducing the response of building structures, with small amount of required control forces.

Keywords: Structural control, Pole placement, Structural Dynamics, Earthquake Engineering.

1. Introduction

This Guide has been prepared for authors of papers submitted to JACM (Journal of Applied and Computational Mechanics). Prospective authors are invited to submit papers that fit within the scope of the journal. Structural control is one strategy that when is applied to the structures safety and structural functionality against natural hazards are enhanced. Structural control can be grouped into the following broad areas: passive, active, semi-active, and hybrid control. Main subcategories of passive control systems are the base isolation and the passive energy dissipation systems. In general, such systems are characterized by frequency shift and their capability to enhance energy dissipation in the structural system, in which they are installed. Active, semi active, and hybrid control systems are a natural evolution of passive control technologies. The combination of active, semi-active and passive systems in order to protect the structures against seismic loads has received considerable attention in the last few decades. The control devices are part of an integrated system, with real time processing controllers (control algorithms) and sensors, all installed to the structure. They act simultaneously with the excitation to provide enhanced structural behavior for improved service and safety.

Over the past few decades various control algorithms and control devices have been developed, modified and investigated by various groups of researchers. The work of Yao, Yang, Soong, Housner, Spencer, Symans, Kobori, Lu, Kurata, Renzi, Reigles [1-20], is representative. One of the most suitable algorithms for controlling the structure is the pole placement algorithm. Pole placement algorithms have been studied extensively in the general control literature Sage, Kwakernaak, Brogan, Ogata, Kwon, Kautsky, Laub, Leipholz [21-29]. The application of the algorithm in structural control can be found in the work of Martin, Wang, Meirovotch, Soong, Utku, and Preumont [30-34]. In Pnevmatikos and Gantes [35] a pole placement algorithm where the poles of the structure are estimated based on the characteristics of the incoming earthquake is proposed.

In this paper a modification of the mathematical background of pole placement algorithm in order to apply in any civil structure is presented. The application of the algorithm to systems that are mechanisms and are used in mechanical structures is well known. However in civil structures such as buildings, bridges which are statically determinant or indeterminate structures the application of the algorithm should be modified.

2. Mathematical formulation for controlled civil structures excited by earthquake motion

A large number of physical systems described by linear or non-linear differential equations of order n. These equations can either be formulated directly with the classical form as below

$$\ddot{\mathbf{U}} = \mathbf{g}(\mathbf{U}, \dot{\mathbf{U}}, \mathbf{F}, \mathrm{t}) \tag{1}$$

Or to transform in state space:

$$\dot{\mathbf{X}} = \mathbf{h}(\mathbf{X}, \mathbf{F}, \mathrm{t}) \tag{2}$$

Where $\mathbf{U}, \dot{\mathbf{U}}$ is the displacement and the velocity of system, **X** is the state space vector of systems and **F** is the control force. If the above equation linearized around the function point of system or on the equilibrium point then the equation of time variant system will be obtained and is shown below.

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{A}(\mathrm{t})\mathbf{X} + \mathbf{B}_{\mathrm{f}}(\mathrm{t})\mathbf{F} \\ \mathbf{Y} &= \mathbf{C}(\mathrm{t})\mathbf{X} + \mathbf{D}(\mathrm{t})\mathbf{F} \end{aligned} \tag{3}$$

If the matrixes **A**, $\mathbf{B}_{\mathrm{f}}$, **C**, **D** are not depending on time, then the system is called time invariant or autonomous system and will be:

$$\begin{aligned} \dot{\mathbf{X}} &= \mathbf{A}\mathbf{X} + \mathbf{B}_{\mathrm{f}}\mathbf{F} \\ \mathbf{Y} &= \mathbf{C}\mathbf{X} + \mathbf{D}\mathbf{F} \end{aligned} \tag{4}$$

In civil structures when they are simulated as lumped mass and stiffness structures, with n dynamic degrees of freedom $\mathbf{u}_i$, the equation of motion subjected to earthquake excitation a_g is given by equation:

$$\mathbf{M}\ddot{\mathbf{U}}(\mathrm{t}) + \mathbf{C}\dot{\mathbf{U}}(\mathrm{t}) + \mathbf{K}\mathbf{U}(\mathrm{t}) = -\mathbf{M}\mathbf{E}a_g(\mathrm{t}) + \mathbf{B}_{\mathrm{p}}\mathbf{P} \tag{5}$$

where, **M**, **C**, **K** denote the mass, damping and stiffness matrices of the structure, respectively, **E**, is the location matrix for the earthquake a_g.

In the state space approach the above eq. (5) can be written as follows:

$$\dot{\mathbf{X}}(\mathrm{t}) = \mathbf{A}\mathbf{X}(\mathrm{t}) + \mathbf{B}_g a_g(\mathrm{t}) \tag{6}$$

The matrixes **X**, **A**, $\mathbf{B}_g$,

$$\mathbf{X} = \begin{bmatrix} \mathbf{U} \\ \dot{\mathbf{U}} \end{bmatrix}_{2nx1}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}_{2nx2n}, \quad \mathbf{B}_g = \begin{bmatrix} \mathbf{0} \\ -\mathbf{E} \end{bmatrix}_{2nx1} \tag{7}$$

The equation of motion of controlled structure the subjected to earthquake excitation ag is given by eq. (8):

$$\mathbf{M}\ddot{\mathbf{U}}(\mathrm{t}) + \mathbf{C}\dot{\mathbf{U}}(\mathrm{t}) + \mathbf{K}\mathbf{U}(\mathrm{t}) = -\mathbf{M}\mathbf{E}a_g(\mathrm{t}) + \mathbf{E}_{\mathrm{f}} sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) + \mathbf{B}_{\mathrm{p}}\mathbf{P} \tag{8}$$

The above equation is similar to eq. (5), just the term of control force, **F**, is added. $\mathbf{E}_{\mathrm{f}}$ is the location matrix for the control forces on the structure, and sat**F** is the saturated control force matrix, which is applied to the structure with time delay, t_{d}, and is given by:

$$sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) = \begin{cases} \mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}), & \mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) < \mathbf{F}_{\mathrm{allowable}} \\ \mathbf{F}_{\mathrm{allowable}}, & \mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) \geq \mathbf{F}_{\mathrm{allowable}} \end{cases} \tag{9}$$

Fallowable is the maximum capacity of the control device. In the state space approach the above eq. (8) can be written as follows:

$$\begin{aligned} \dot{\mathbf{X}}(\mathrm{t}) &= \mathbf{A}\mathbf{X}(\mathrm{t}) + \mathbf{B}_g a_g(\mathrm{t}) + \mathbf{B}_f sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) \\ \mathbf{Y}(\mathrm{t}) &= \mathbf{C}\mathbf{X}(\mathrm{t}) + \mathbf{D}_{\mathrm{f}} sat\mathbf{F}(\mathrm{t}\text{-}\mathrm{t}_{\mathrm{d}}) + \mathbf{D}_{\mathrm{g}} a_{\mathrm{g}}(\mathrm{t}) + \mathbf{v} \end{aligned} \tag{10}$$

The matrixes **X**, **A**, $\mathbf{B}_{\mathrm{g}}$, are the same like in eq. (7), $\mathbf{B}_{\mathrm{f}}$ are given by:

$$\mathbf{B}_f = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{E}_f \end{bmatrix}_{2nx1} \tag{11}$$

The matrixes **Y**, **C**, $\mathbf{D}_{\mathrm{f}}$, $\mathbf{D}_{\mathrm{g}}$, and **v** are the output states, the output matrix, the feed forward control force matrix, the excitation matrix and the noise matrix, respectively. We consider the case where the output variables are the same as the states of the system and there is no application of the control forces to the output variables, so the matrixes **C**, **D** are the identity and zero matrix, respectively. The noise matrix depends on the sensor that is used to measure the response of the system.

It is assumed that the control force, **F**, is determined by linear state feedback:

$$\mathbf{F} = -\mathbf{G}_1\mathbf{U} - \mathbf{G}_2\dot{\mathbf{U}} = -\begin{bmatrix}\mathbf{G}_1 & \mathbf{G}_2\end{bmatrix}\begin{bmatrix}\mathbf{U}\\ \dot{\mathbf{U}}\end{bmatrix} = -\mathbf{G}\mathbf{X} \tag{12}$$

G, is the gain matrix, which will be calculated according to the desired poles of the controlled system. Replacing the force **F** into eq. (8) or (10), the controlled system can be described by:

$$\mathbf{M}\ddot{\mathbf{U}}(\mathrm{t}) + \left(\mathbf{E}_f\mathbf{G}_2 + \mathbf{C}\right)\dot{\mathbf{U}}(\mathrm{t}) + \left(\mathbf{E}_f\mathbf{G}_1 + \mathbf{K}\right)\mathbf{U}(\mathrm{t}) = -\mathbf{M}\mathbf{E}a_g(\mathrm{t}) \tag{13}$$

$$\dot{\mathbf{X}} = \left(\mathbf{A} - \mathbf{B}_{\mathrm{f}}\mathbf{G}\right)\mathbf{X} + \mathbf{B}_{\mathrm{g}}\mathrm{a}_{\mathrm{g}} \tag{14}$$

From the above equation it is clear that control of structures causes the change of their stiffness or damping and, consequently, their dynamic characteristics, in a direct or indirect way, depending on the device we use. With a suitable calculation of the sub-matrixes $\mathbf{G}_1$ and $\mathbf{G}_2$ the damping and the stiffness of the system are chance reducing thus the response of the system and satisfying the design specifications.

The eq. (10) can be solved by the technique of delay differential equation, Shampine and Thompson [36], or one can use the following transformation, which is described by Cai et al. [37]:

$$\mathbf{Z}(\mathrm{t}) = \mathbf{X}(\mathrm{t}) + \int e^{-\mathbf{A}(\eta+\mathrm{t}_d)}\mathbf{B}_f\mathbf{F}(\mathrm{t}+\eta)d\eta \tag{15}$$

Then:

$$\dot{\mathbf{Z}}(\mathrm{t}) = \mathbf{A}\mathbf{Z}(\mathrm{t}) + \mathbf{B}_g a_g(\mathrm{t}) + \mathbf{B}(\mathbf{A})\mathbf{F}(\mathrm{t})$$
$$\mathbf{B}(\mathbf{A}) = e^{-\mathbf{A}\mathrm{t}_d}\mathbf{B}_f \tag{16}$$

The eq. (10) can be solved numerically, one simple way is to use the Simulink toolbox of matlab software, and such a numerical model is shown in Fig. 1a.

In Fig. 1b an actual numerical model simulated in Simulink toolbox is shown. In this model the response of controlled and uncontrolled system is calculated.

In the above process the critical point is how to estimate the control force or the feedback matrix, **G**, in such a way, that the civil structure will have the desirable response. In other words how the feedback matrix, **G**, will be calculated in order to achieve the desired dynamic characteristics for the controlled structure so that resonance with the incoming signal is avoided and the response of the system is reduced further by adding equivalent damping and/or stiffness.

In the application of pole placement one should assume the desired location for the poles (eigenvalues) of the controlled system, and then continue with the calculation of the feedback matrix **G**. As stated for example by Soong [8], the successful application of the algorithm requires judicious placement of the closed loop eigenvalues. The mathematical procedure of calculating the feedback matrix **G** based on the desired eigenvalues of controlled system is presented next.

3. Pole placement algorithm for civil structures excited by earthquake motion

The eigenperiods $\mathbf{T}_i$, eigenfrequencies $\mathbf{f}_i$, eigenmodes $\mathbf{\Phi}_i$ and the damping coefficient ζ_i, for each mode are obtained solving the classical eigenvalue problem:

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]_{n\mathrm{x}n}\mathbf{\Phi} = 0 \Rightarrow \left|\mathbf{K} - \omega^2\mathbf{M}\right| = 0 \Rightarrow \begin{cases}\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_n\\ \omega_1, \omega_2, \ldots\omega_n\end{cases} \Rightarrow$$
$$\mathrm{T}_i = \frac{2\pi}{\omega_i}, \quad \mathrm{f}_i = \frac{\omega_i}{2\pi}, \ \mathrm{i} = 0, \ldots, \mathrm{n}-1 \tag{17}$$

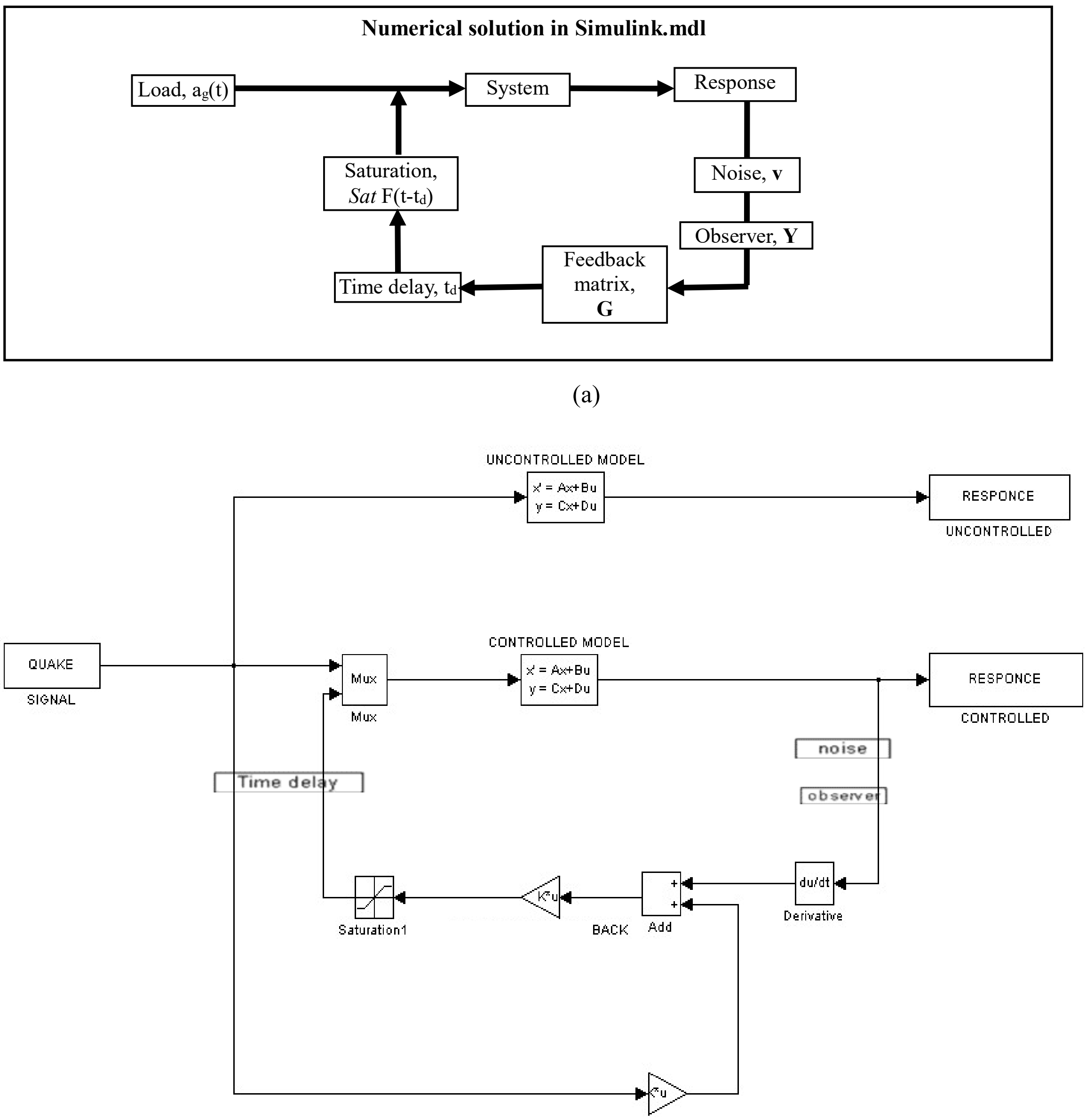
$$\mathbf{C}_n = \mathbf{\Phi}_n^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}_n, \quad \mathbf{M}_n = \mathbf{\Phi}_n^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}_n, \quad \zeta_n = 2\mathbf{C}_n\mathbf{M}_n\omega_n \tag{18}$$

The eigenvalues or poles of the system which are the roots of the characteristic equation of the differential eq. (5) for the under-damping systems (ζ<1), are given as:

$$\lambda_i = -2\pi \mathrm{f}_i\zeta_i \pm j2\pi \mathrm{f}_i\sqrt{1-\zeta_i^2} \tag{19}$$

If the state space approach is used then eigenvalues or poles of the system are the eigenvalues of matrix **A**:

$$det\left[\lambda\mathbf{I} - \mathbf{A}\right] = 0 \rightarrow \lambda_i = \alpha_i \pm \beta_i = -2\pi \mathrm{f}_i\zeta_i \pm j2\pi \mathrm{f}_i\sqrt{1-\zeta_i^2} \tag{20}$$



(b)

Fig. 1. The schematically numerical model (a), and the actual numerical model in Simulink toolbox in MAT LAB software, (b).

According to the pole placement algorithm the feedback matrix or gain matrix, **G**, is calculated based on desired eigenvalues of the controlled system.

$$\dot{\mathbf{X}} = \left(\mathbf{A} - \mathbf{B}_f\mathbf{G}\right)\mathbf{X} + \mathbf{B}_g\mathbf{a}_g + \mathbf{B}_p\mathbf{P} \rightarrow \dot{\mathbf{X}} = \bar{\mathbf{A}}\mathbf{X} + \mathbf{B}_g\mathbf{a}_g + \mathbf{B}_p\mathbf{P}$$
$$\bar{\mathbf{A}} = \mathbf{A} - \mathbf{B}_f\mathbf{G} \tag{21}$$

The new eigenvalues or poles, λ_{ci}, of the controlled system will be :

$$\lambda_{ci} = -2\pi f_{ci}\zeta_{ci} \pm j2\pi f_{ci}\sqrt{1-\zeta_{ci}^2} \rightarrow \lambda_{ci} = \alpha_{ci} \pm \beta_{ci} \tag{22}$$

And will satisfy the following:

$$\det\left(\left[\lambda_c\mathbf{I} + \mathbf{B}_f\mathbf{G} - \mathbf{A}\right]\right) = 0 \tag{23}$$

The desire location of the controlled structure eigenbalues, λ_{ci}, will also satisfy the following:

$$\left(\lambda-\lambda_{c1}\right)\left(\lambda-\lambda_{c2}\right)\ldots\left(\lambda-\lambda_{c2n}\right) = 0 \;\Leftrightarrow\; \lambda^{2n} + a_{1,d}\lambda^{2n-1} + \ldots + a_{2n-1,d}\lambda + a_{2n,d} = 0 \tag{24}$$

In order to find the feedback matrix first the system becomes in canonical form thus define a transformation matrix

T:

$$T = SW \tag{25}$$

Where **S** is the controllability matrix

$$S = [B \vdots AB \vdots \ldots \vdots A^{n-1}B] \tag{26}$$

The necessary ant sufficient condition to apply the pole placement algorithm is that the system should be complete state controllable thus

$$\mathrm{rank}[S] = 2n \tag{27}$$

And the matrix, **W**, is

$$W = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{28}$$

where, a_i are the coefficients of characteristic polynomial of the uncontrolled system.

$$det\left[\lambda \mathbf{I} - \mathbf{A}\right] = 0 \rightarrow \lambda_i = \alpha_i \pm \beta_i = -2\pi f_i \zeta_i \pm j2\pi f_i \sqrt{1-\zeta_i^2} \tag{29}$$

$$det\left[\lambda \mathbf{I} - \mathbf{A}\right] = 0 \rightarrow \lambda^{2n} + a_1 \lambda^{2n-1} + a_2 \lambda^{2n-2} + \ldots + a_{2n} = 0 \tag{30}$$

The new state vector $\hat{\mathbf{X}}$

$$\mathbf{X} = \mathbf{T}\hat{\mathbf{X}} \tag{31}$$

Since the system is complete controllable, the inverse of matrix, **T,** exists and the transformation system in canonical form is:

$$\dot{\hat{\mathbf{X}}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}\hat{\mathbf{X}} + \mathbf{T}^{-1}\mathbf{B}_f\mathbf{F} + \mathbf{T}^{-1}\mathbf{B}_g\mathbf{a}_g + \mathbf{T}^{-1}\mathbf{B}_p\mathbf{P} \tag{32}$$

Replacing the control force

$$\mathbf{F} = -\mathbf{G}\mathbf{X}(t) = -\mathbf{G}\mathbf{T}\hat{\mathbf{X}} \tag{33}$$

is obtained:

$$\dot{\hat{\mathbf{X}}} = \left(\mathbf{T}^{-1}\mathbf{A}\mathbf{T} - \mathbf{T}^{-1}\mathbf{B}_f\mathbf{G}\mathbf{T}\right)\hat{\mathbf{X}} + \mathbf{T}^{-1}\mathbf{B}_g\mathbf{a}_g + \mathbf{T}^{-1}\mathbf{B}_p\mathbf{P} \tag{34}$$

or

$$\dot{\hat{\mathbf{X}}} = \left(\hat{\mathbf{A}} - \hat{\mathbf{B}}_f\hat{\mathbf{G}}\right)\hat{\mathbf{X}} + \mathbf{T}^{-1}\mathbf{B}_g\mathbf{a}_g + \mathbf{T}^{-1}\mathbf{B}_p\mathbf{P} \tag{35}$$

where:

$$\hat{A} = T^{-1}AT = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_{2n} & -a_{2n-1} & \cdots & -a_2 & -a_1 \end{bmatrix} \tag{36}$$

In case where the control force is applied in one point into the structure (one specific location)

$$\hat{B}_f = T^{-1}B_f = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{2n\times 1} \tag{37}$$

$$\hat{\mathbf{G}} = \mathbf{GT} = \left[\delta_{2n} \quad \delta_{2n-1} \quad \dots \quad \delta_1\right] \tag{38}$$

The desired eigenvalues of the controlled system are λ_{ci}. The characteristic equation of the controlled system is given by eq. (24). The characteristic equation of the controlled system is also given as:

$$det\left[\boldsymbol{\lambda}\mathbf{I} - \mathbf{T}^{-1}\mathbf{AT} - \mathrm{T}^{-1}\mathrm{B_f GT}\right] = 0 \rightarrow \boldsymbol{\lambda}^{2n} + \left(\mathbf{a_1} + \delta_1\right)\boldsymbol{\lambda}^{2n-1} + \left(\mathbf{a_2} + \delta_2\right)\boldsymbol{\lambda}^{2n-2} + \dots + \left(\mathbf{a}_{2n} + \delta_{2n}\right) = 0 \tag{39}$$

Equating the corresponding coefficients of eqs. (38) and (24) we get

$$\begin{aligned} &\mathbf{a_1} + \delta_1 = a_{1,d} \\ &\mathbf{a_2} + \delta_2 = a_{2,d} \\ &\vdots \\ &\mathbf{a_{2n}} + \delta_{2n} = a_{2n,d} \end{aligned} \tag{40}$$

From the eq. (39) the δ_i's are calculated and the feedback matrix G is obtained as follows

$$\mathbf{G} = \hat{\mathbf{G}}\mathbf{T}^{-1} = \left[\delta_{2n}\ \delta_{2n-1} \cdots \delta_1\right]\mathrm{T}^{-1} = \left[a_{2n,d} - a_{2n} \vdots a_{2n-1,d} - a_{2n-1} \vdots \dots \vdots a_{1,d} - a_1\right]\mathrm{T}^{-1} \tag{41}$$

From the above equation comparing with the eq. (12) the first, n, values of matrix, G, above correspond to the submatrix, $\mathbf{G}_1$, and influence the stiffness and the rest values from, n+1, potation until, 2n correspond to the submatrix, $\mathbf{G}_2$, and influence the damping, (as can be seen by eq. (13)). This allows the designer with suitable choice of the controlled eigenvalues to focus more to the influence of stiffness and consequently, reduce the displacement of the system or to give more priority to the influence of damping and decrease the velocity and acceleration of the system.

4. Numerical examples and discussions

The above procedure has been applied to control a single-story, three-story and five-story building. The three buildings are subjected to kalamata, 1986, earthquake. The earthquake excitation is shown in Fig. 2.

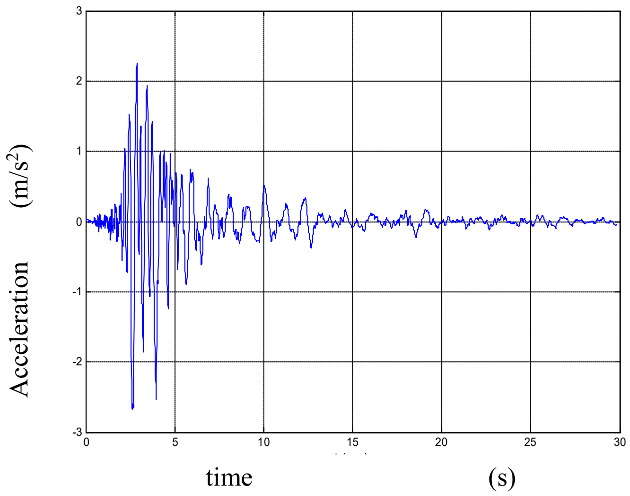
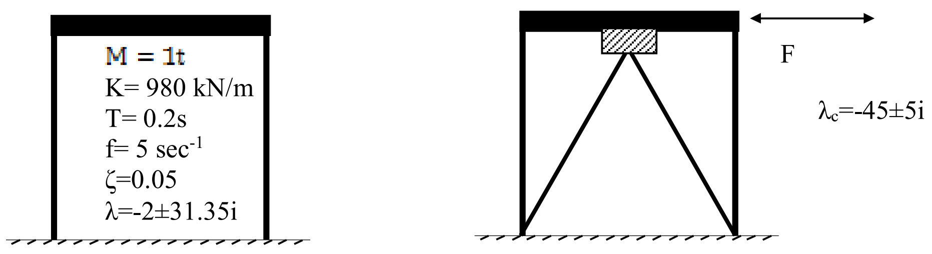


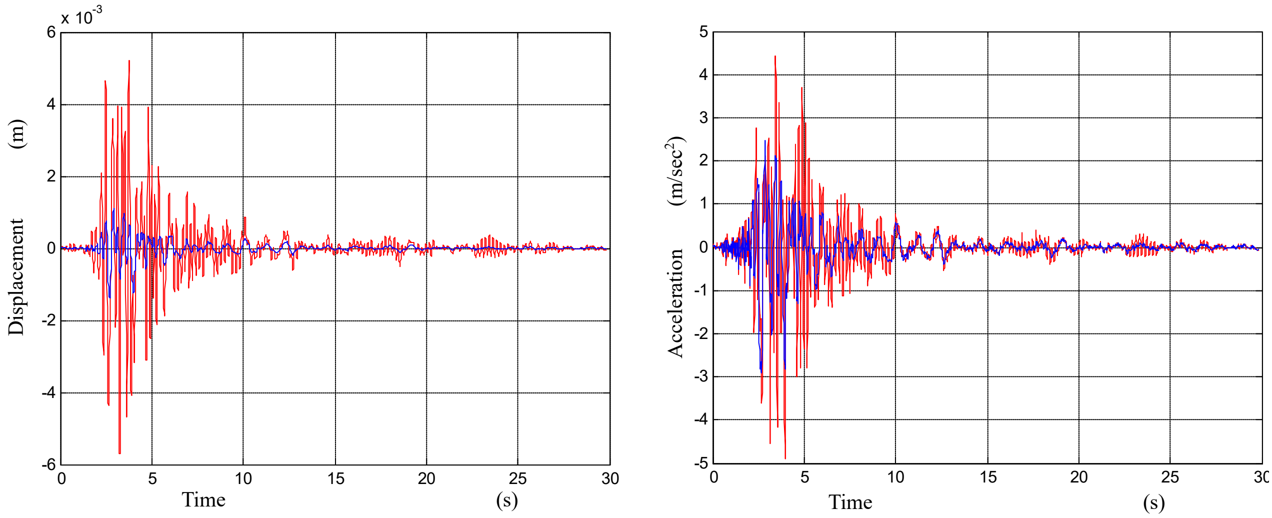
Fig. 2. Time history of kalamata, 1986, earthquake excitation.

The dynamic characteristics (eigenperiod and damping ratio) as the eigenvalues of the uncontrolled and controlled system for single-story building are shown in Fig. 3. The response of the controlled system in terms of the relative displacement and acceleration is shown in Fig. 4. While the control force needed to drive the controlled system to have the above eigenvalues λc is shown in Fig. 5.



(a) (b)

Fig. 3. The eigenvalues of the uncontrolled (a), and controlled system, (b).



(a) (b)

Fig. 4. The relative displacement (a), and acceleration, (b) of the uncontrolled and controlled single degree of freedom system subjected to Kalamata earthquake.

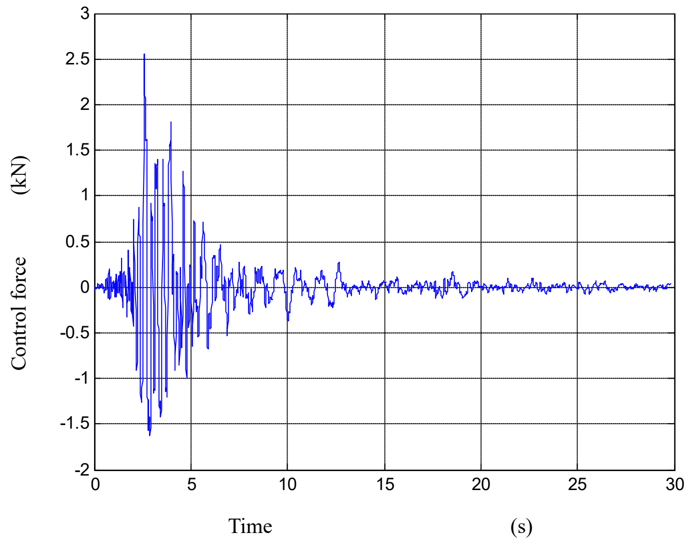


Fig. 5. The control force applied to the single degree of freedom system subjected to Kalamata earthquake.

From the results above it is shown that both displacement and acceleration of controlled system are reduced compared to the uncontrolled system. The displacement is reduced one order of magnitude while the acceleration is about 40%. The maximum control force is in order of 2.5 kN which represents the 25% of the mass of the structure. Furthermore, this control force is achievable with the current control devices such as MR Dampers or hydraulic control actuator.

Furthermore, parametric analysis have been performed for different location of pole of the controlled structure in order to find the parametric variation of the control force in conjunction with the norm of the pole of controlled structure. The results of theses analysis are shown in Fig. 6. It can be seen that as the pole move away of the origin of complex plane the control force is reduced. There is a limit, (in our case when the norm becomes equal to 40) that beyond this point the reduction of control force is negligible.

The dynamic characteristics (eigenperiod and damping ratio) as the eigenvalues of the uncontrolled and controlled system for three-story building are shown in Fig. 7.

The response of the controlled system in terms of the relative displacement and acceleration of third floor is shown in Fig. 8. While the control forces needed to drive the controlled system to have the above eigenvalues λ_c is shown in Fig. 9.

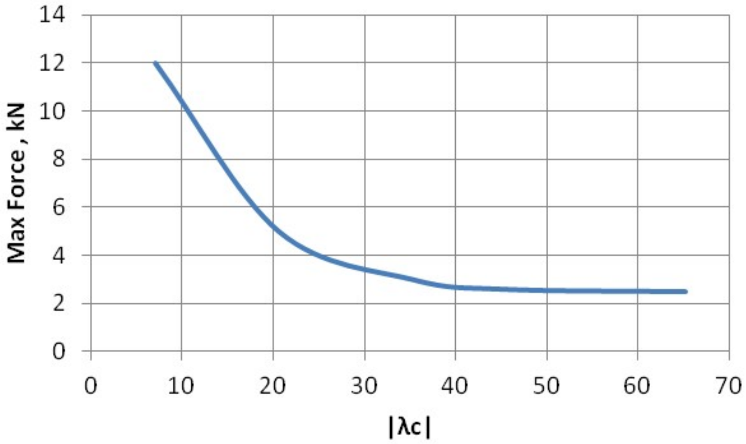
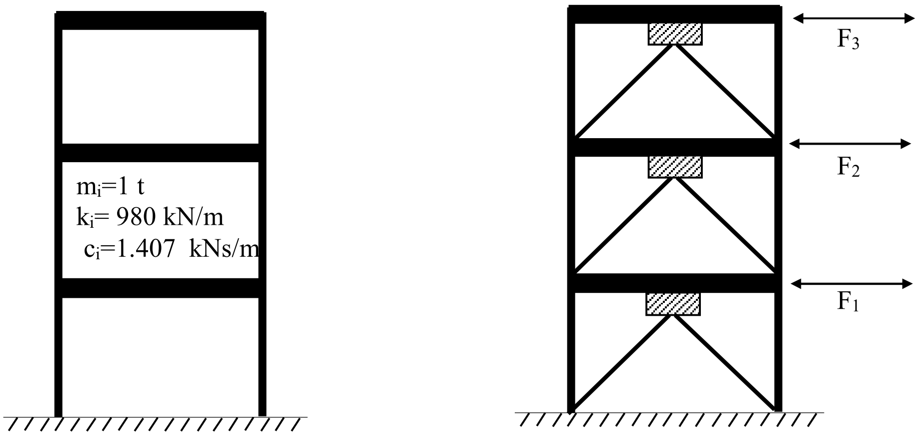


Fig. 6. Parametric variation of the control force in conjunction with the norm of the pole of controlled structure

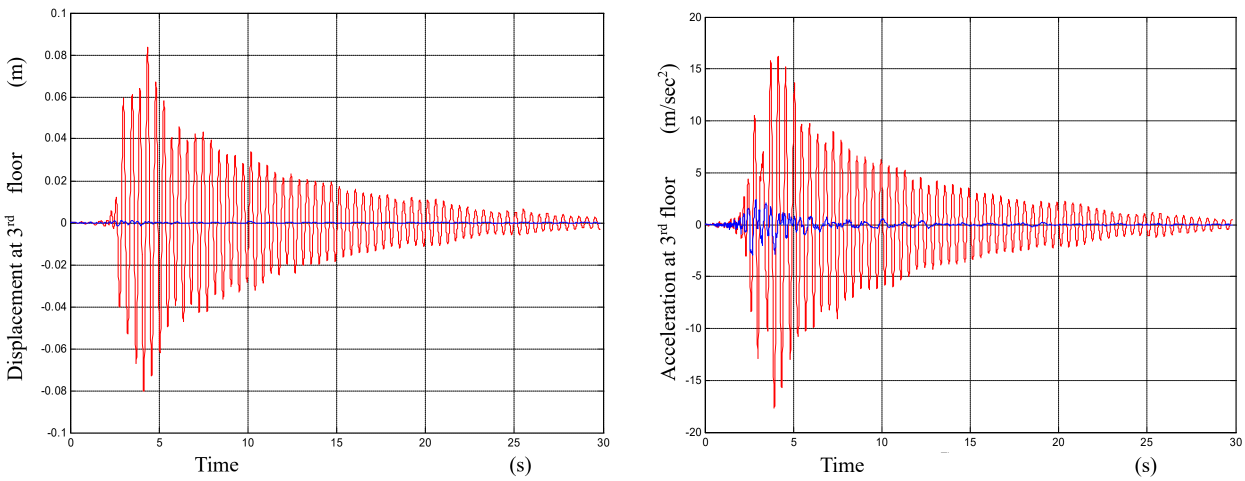


T_i= {0.45, 0.16, 0.12}sec
f_i ={2.217, 6.212, 8.977}sec^{-1}
λ_i={-2.48 ± 56.36 i, -1.093 ± 39.0i, -0.13 ± 13.93 i}

λ_{ci}={-55±10i, -50±10i, -45 ±10i}

(a) (b)

Fig. 7. The eigenvalues of the uncontrolled (a), and controlled system, (b).



(a) (b)

Fig. 8. The relative displacement (a), and acceleration, (b) of the uncontrolled and controlled three-story building subjected to Kalamata earthquake.

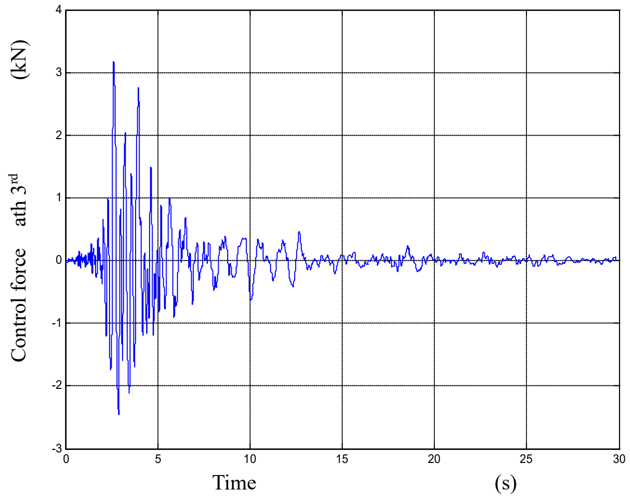


Fig. 9. The control force applied to 3rd floor of the three-story building subjected to Kalamata earthquake.

From the results above it is shown that both displacement and acceleration of controlled system are reduced compared to the uncontrolled system. The displacement is completely compensated while the acceleration is about 90%. Similar results are observed in the response of the rest floors. The maximum control force in each floor is about 4 kN. The maximum force represents the 13% of the mass of the structure and can be applied with the current control devices.

Additionally, a five-story building with analogous properties is analyzed in order to explore their dynamic characteristics. The story mass and stiffness are the same as in three –story building. The properties and the eigenvalues of the uncontrolled and controlled system for five-story building are shown in Fig. 10. The response of the controlled system in terms of the relative displacement and acceleration of fifth floor is shown in Fig. 11. While the control forces needed to drive the controlled system to have the above response is shown in Fig. 12.

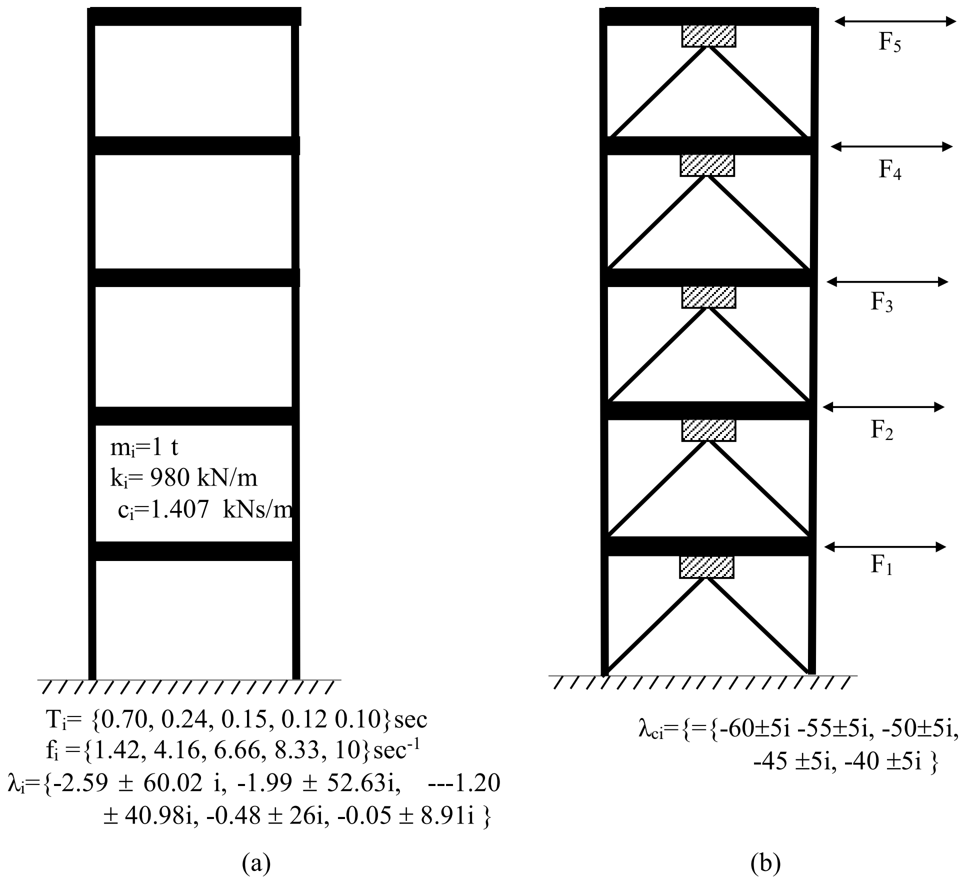


Fig. 10. The eigenvalues of the uncontrolled (a), and controlled system, (b).

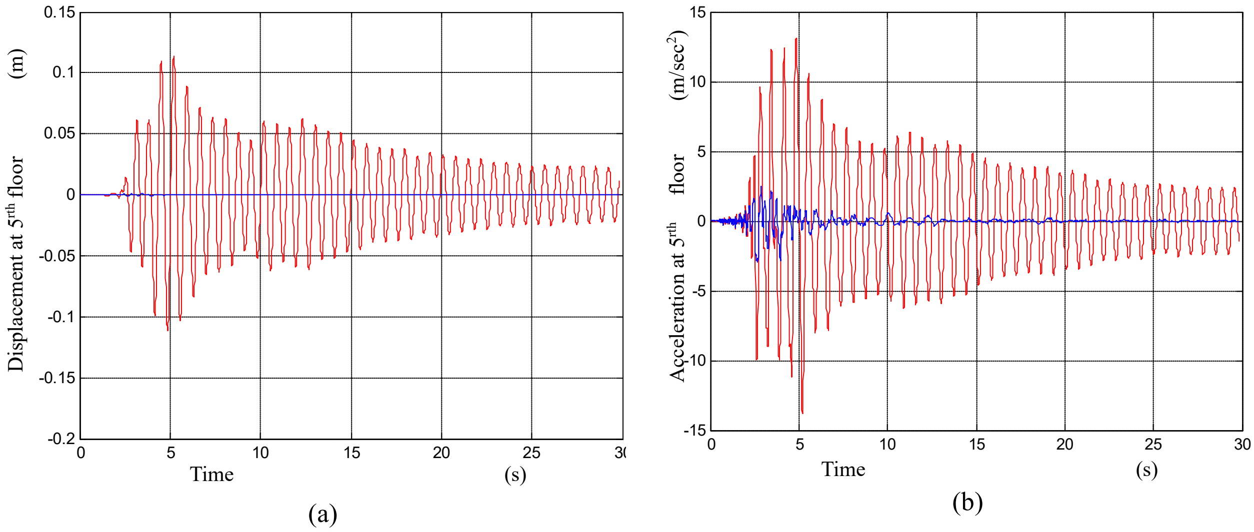


Fig. 11. The relative displacement (a), and acceleration, (b) of the uncontrolled and controlled fith-story building subjected to Kalamata earthquake.

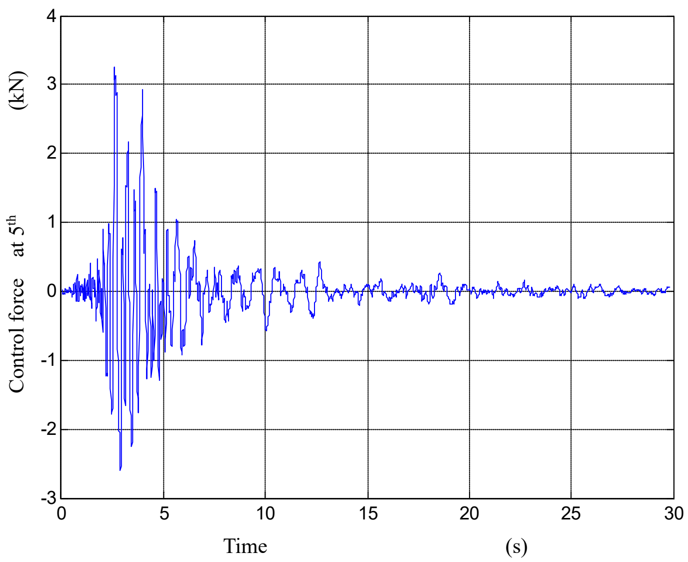


Fig. 12. The control force applied to 3rd floor of the fith-story building subjected to Kalamata earthquake.

For the fifth story building it is shown that both displacement and acceleration of controlled system are reduced compared to the uncontrolled system. The displacement is completely compensated as it was in third story building while the acceleration is reduced about 95%. Similar results are observed in the response of the rest floors. The maximum control force is about 3.1 kN which represents the 6% of the mass of the structure. This maximum force is achievable with the current control devices such as MR Dampers or hydraulic control actuator.

5. Conclusions

A modified pole placement algorithm for control civil structures subjected to earthquake excitation was investigated. The proposed algorithm is suitable to control structures in order to improve their response against earthquake actions. The mathematical background of modified pole placement or pole assignment algorithm was presented. The feedback matrix is obtained based on the eigenvalues of the controlled structure that are pre-decided by the designer. With the proposed algorithm there is a flexibility to choose which part of feedback matrix, the one that affects the stiffness or the other that can affect the damping, in order to influence more the reduction in acceleration or the displacement. The proposed approach is demonstrated by numerical simulations for the control of both single and multi-degree of freedom systems subjected to seismic actions. Base on the numerical results, it is shown that the control algorithm is efficient in reducing the response of building structures, with small amount of required control forces.

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