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Research Article

# Multi-boiling Heat Transfer Analysis of a Convective Straight Fin with Temperature-Dependent Thermal Properties and Internal Heat Generation

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**Abstract.** In this study, by using the finite volume method, the heat transfer in a convective straight fin with temperature-dependent thermal properties and an internal heat generation under multi-boiling heat transfer modes are analyzed. In this regard, the local heat transfer coefficient is considered to vary within a power-law function of temperature. In the present study, the coexistence of all the boiling modes is taken into consideration. The developed heat transfer models and the corresponding numerical solutions are used to investigate the effects of various thermo-geometric parameters on the thermal performance of the longitudinal rectangular fin. The results shows that the fin temperature distribution, the total heat transfer, and the fin efficiency are significantly affected by the thermo-geometric parameters of the fin and the internal heat generation within the fin. The obtained results can provide a platform for improvements in the design of the fin in the heat transfer equipment.

**Keywords:** Multi-boiling heat transfer; Convective straight fin; Finite volume method; Temperature-dependent properties; Internal heat generation.

## 1. Introduction

The quest for the high-performance heat transfer components with progressively lower weights, volume, costs or accommodating shapes has greatly increased the use of extended surfaces to enhance the heat dissipation from hot primary surfaces. In the design and construction of various types of the heat-transfer equipment and components such as air conditioners, refrigerators, super-heaters, automobiles, power plants, heat exchangers, convectional furnaces, economizers, gas turbines, chemical processing equipment, oil carrying pipelines, computer processors, electrical chips etc., the extended surfaces are used to implement the flow of heat between a source and a sink. In practice, various types of fins with different geometries are used, but due to simplicity of their design and ease of construction and manufacturing process, the rectangular fins are widely applied in the heat transfer equipment. Moreover, in an ordinary fin problem, the thermal properties of the fin and the surrounding medium (the thermal conductivity and the heat transfer coefficient) are assumed to be constant. However, if a high temperature difference exists within the fin, especially between the tip and the base of the fin, the thermal conductivity and the heat transfer coefficient are not constant but temperature-dependent. Therefore, during the fin analysis, the effects of the temperature-dependent thermal properties must be taken into consideration. In order to carry out such analysis and also for many other engineering applications, the thermal conductivity may be modeled by the power law and by the linear dependency on the temperature while the heat transfer coefficient may be expressed as the power law for which the exponents represent different phenomena as reported by Khani and Aziz [1], Ndlovu and Moitsheki [2]. Such dependency of the thermal conductivity and the heat transfer coefficient on the temperature makes the problem highly non-linear and difficult to solve analytically. It is also very realistic to utilize the temperature-dependent internal heat generation in the fins in electric-current carrying conductors, nuclear rods or any other heat generating components of thermal systems.

The research on the temperature-dependent thermal conductivity and the heat transfer coefficient has been ongoing in the literature. Over the past few decades, the solutions for the highly non-linear differential equations have been proposed by using different techniques. Aziz and Enamul-Huq [3] and Aziz [4] applied the regular perturbation expansion to study a pure convection fin with the temperature-dependent thermal conductivity. A few years later, Campo and Spaulding [5] predicted the thermal behavior of uniform circumferential fins by using the successive approximation method. Chiu and Chen [6] and Arslanturk [7] adopted the Adomian Decomposition Method (ADM) to examine the temperature distribution in a pure convective fin with a variable thermal conductivity. The same problem was solved by Ganji [8] by applying the homotopy perturbation method which was originally proposed by He [9]. In the same year, Chowdhury and Hashim [10] applied the adomian decomposition method to evaluate the temperature distribution of straight rectangular fins with the temperature-dependent surface flux and for all possible types of the heat transfer, while in the next year, Rajabi [11] applied the homotopy perturbation method (HPM) to calculate the efficiency of straight fins with the temperature-dependent thermal conductivity. Moreover, a year later, Mustapha [12] adopted the homotopy analysis method (HAM) to examine the efficiency of straight fins with the temperature-dependent thermal conductivity. Meanwhile, Coskun et al. [13] utilized the variational iteration method (VIM) to analyze convective straight and radial fins with the temperature-dependent thermal conductivity. On the other hand, Languri et al. [14] applied both variation iteration and homotopy perturbation methods to evaluate the efficiency of straight fins with the temperature-dependent thermal conductivity while Coskun and Atay [15] applied the variational iteration method to analyze the efficiency of convective straight fins with the temperature-dependent thermal conductivity. Besides, Atay and Coskum [16] utilized the variation iteration and the finite element methods to carry out the comparative analysis of the power-law fin problems. Domairry and Fazeli [17] used the homotopy analysis method to determine the efficiency of straight fins with the temperature-dependent thermal conductivity. Chowdhury et al.[18] investigated a rectangular fin with the power-law surface heat flux and made a comparative assessment of results by using HAM, HPM, and ADM. Khani et al. [19] used the adomian decomposition method (ADM) to provide a series of solutions for a fin problem with a temperature-dependent thermal conductivity while Moitsheki et al. [20] applied the Lie symmetry analysis to provide exact solutions for a fin problem with a power-law temperature-dependent thermal conductivity. In another study, Hosseini et al. [21] applied the homotopy analysis method to conduct an approximate but accurate solution for the heat transfer in a fin with the temperature-dependent internal heat generation and the thermal conductivity. Joneidi et al. [22], Moradi and Ahmadikia [23], Mosayebidorcheh et al. [24], Ghasemi et al. [25], and Ganjiand Dogonchi [26] applied another approach to solve the fin problem: The application of the differential transform method (DTM) to solve differential equations without linearization, discretization or approximation, linearization restrictive assumptions or perturbation, the complexity of expansion of derivatives and the symbolic computation of derivatives. However, the search for arbitrary values that would satisfy the second boundary condition necessitated the use of the Maple or Mathematica software which could result in an additional computational cost in the generation of solutions for the problem. This drawback was not merely peculiar to DTM. Other approximate analytical methods such as HPM, HAM, ADM and VIM also required an additional computational cost and time for the determination of such auxiliary parameters in their implementation procedures. Moreover, most of the approximate analytical methods made accurate predictions only when the nonlinearities were weak, and therefore, they failed to predict strong nonlinear models accurately. On the other hand, methods often involved a complex mathematical analysis which led to analytic expressions including a large number of terms. When methods including HPM, HAM, ADM and VIM were routinely implemented, they might sometimes lead to erroneous results as observed by Fernandez [27], Aziz and Bouaziz [28]. In practice, approximate analytical solutions with a large number of terms were not convenient for designers and engineers to use. Although, some numerical methods such as the finite difference method (FDM) and the finite element methods (FEM) were adopted to analyze the heat transfer in fins, the FDM and FEM did not enforce the conservation principle in its original form as did the finite volume method (FVM). However, there are a few studies in literature on the application of finite volume methods for the heat transfer analysis in the fin. The inherent advantages, the wide range of applications and the high level of accuracy of the method may justify the corresponding method for the given problem. From the industrial point of view, the finite volume method is known as a robust and cheap method for the discretization of conservation laws. This method is preferred to other numerical methods because it enforces the conservation on each cell, and therefore, ensures that the both local and global conservations are guaranteed no matter how coarse the mesh is. Moreover, its preference to other numerical methods is due to the fact that boundary conditions can be applied non-invasively. This is true because the values of the conserved variables are located within the volume element and not at the nodes or surfaces. The FVM is a geometrically flexible method and like the finite element method, it enjoys an advantage in the memory use and speed for large problems as well as in higher speed flows and source term dominated flows. This method is particularly influential in coarse non-uniform grids and in calculations where the mesh moves toward track interfaces or shocks. It can handle Neumann boundary condition as readily as the Dirichlet boundary condition. Therefore, in the present study, the finite volume method was applied to analyze the heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and the internal heat generation under multi-boiling heat transfer modes such as convection, nucleate boiling, transition boiling, laminar film boiling or condensation and radiation.

## 2. Problem formulation

Consider a straight fin with the assumed length, $L$, that is exposed on both faces to a convective environment at temperature, $T_\infty$, and with the heat transfer co-efficient, $h$, and the internal heat generation, $q$, as shown in Fig. 1. The heat flow in the fin and its temperatures remain constant over time while the temperature of the medium surrounding the fin and also the temperature of the fin base are uniform. There is no contact resistance where the base of the fin joins the prime surface. Moreover, compared with its height and length, the fin thickness is small, therefore, the temperature gradients across the fin thickness and the heat transfer from the edges of the fin may be neglected. The dimension x pertains to the height coordinate which has its origin at the fin base and has a positive orientation from fin base to fin tip, and therefore, it could be stated that:

*the rate of the heat conduction into the element at x = the rate of the heat conduction from the element at x+dx + the rate of the heat convection from the element + the rate of the heat internal generation in the element* (1)

Mathematically, the thermal energy balance could be expressed as shown in Eq. (2).

$$q_x = q_{x+dx} + q_{conv.} + q_{\text{int.}} \tag{2}$$

i.e.

$$q_x - q_{x+dx} = q_{conv.} + q_{\text{int.}} \tag{3}$$

which is equivalent to the following equation.

$$q_x - \left( q_x + \frac{\delta q}{\delta x} dx \right) = h(T)P(T - T_c)dx + q_{\text{int.}}(T)dx \tag{4}$$

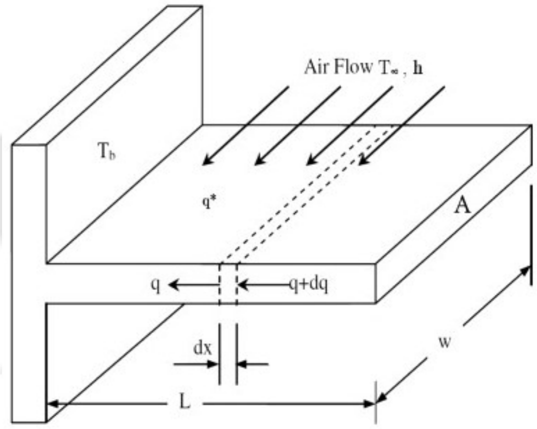



**Fig. 1.** The geometry of the straight rectangular fin

As $dx \to 0$, Eq. (4) reduces to Eq. (5) as follows:

$$-\frac{dq}{dx} = h(T)P(T - T_c) + q_{\text{int.}}(T) \tag{5}$$

According to Fourier's law of the heat conduction,

$$q = -k(T)A_{cr}\frac{dT}{dx} \tag{6}$$

By substituting Eq. (6) into Eq. (5), Eq. (7) was obtained as follows:

$$\frac{d}{dx}\left( k(T)A_{cr}\frac{dT}{dx} \right) = h(T)P(T - T_c) + q_{\text{int.}}(T) \tag{7}$$

Further simplification of Eq. (7) gives the governing differential equation for the fin as given by

$$\frac{d}{dx}\left[ k(T)\frac{dT}{dx} \right] - \frac{h(T)}{A_{cr}}P(T - T_\infty) + q_{\text{int.}}(T) = 0 \tag{8}$$

where the boundary conditions are

$$\begin{aligned} x &= 0, \quad T = T_b \\ x &= L, \quad \frac{dT}{dx} = 0 \end{aligned} \tag{9}$$

For many engineering applications, the thermal conductivity and the coefficient of the heat transfer are temperature-dependent. Therefore, the temperature-dependent thermal properties and the internal heat generation are given respectively by:

$$k(T) = k_a[1 + \lambda(T - T_\infty)] \tag{10}$$

$$h(T) = h_b \left( \frac{T - T_\infty}{T_w - T_\infty} \right)^n \tag{11}$$

$$q_{\text{int}}(T) = q_a [1 + \psi(T - T_\infty)] \tag{12}$$

The constant $n$ (the multi-boiling heat transfer mode constant) ranges from -6.6 and 5. However, in most practical applications it ranges from -3 and 3 [30]. The exponent n represents (1) the laminar film boiling or condensation when $n= -1/4$, (2) the laminar natural convection when $n= 1/4$, (3) the turbulent natural convection when $n=1/3$, (4) the nucleate boiling when $n=2$, (5) the radiation when $n=3$, while $n=0$ implies a constant heat transfer coefficient [30, 31, 32]. By substituting Eqs. (10-12) into Eq. (8), we have:

$$\frac{d}{dx}\left[ k_a [1 + \lambda(T - T_\infty)] \frac{dT}{dx} \right] - \frac{h_b P (T - T_\infty)^{n+1}}{A_{cr} (T_w - T_\infty)^n} + q_a [1 + \psi(T - T_\infty)] = 0 \tag{13}$$

In order to non-dimensionalize Eq. (13), the following dimensionless parameters are introduced:

$$\begin{gathered} X = \frac{x}{b}, \quad \theta = \frac{T - T_\infty}{T_b - T_\infty}, \quad H = \frac{h}{h_b}, \quad K = \frac{k}{k_a}, \quad M^2 = \frac{P h_b L^2}{A_{cr} k_a}, \\ Q = \frac{q_a A_{cr}}{h_b P (T_b - T_\infty)}, \quad \gamma = \psi(T_b - T_\infty), \quad \beta = \lambda(T_b - T_\infty) \end{gathered} \tag{14}$$

The dimensionless governing differential Eq. (14) and the boundary conditions lead to the following equation:

$$\frac{d}{dX}\left[ (K(\theta)] \frac{d\theta}{dX} \right] - M^2 \theta^{n+1} + M^2 Q (1 + \gamma\theta) = 0 \tag{15}$$

While the boundary conditions are:

$$\begin{gathered} X = 0, \quad \theta = 1 \\ X = 1, \quad \frac{d\theta}{dX} = 0 \end{gathered} \tag{16}$$

where

$$K(\theta) = 1 + \beta\theta$$

## 3. Method of Solution

The above-mentioned non-linear Eq. (15) permits the generation of any closed-form solutions. Therefore, the recourse has to be made to the approximation analytical method, the semi-numerical method or the numerical method to find a solution. In this study, the finite volume method is used which divides the domain into a finite number of non-overlapping cells or control volumes (Fig. 2) over which conservation of function (the dependent variable) is enforced in a discrete sense. It is possible to start the discretization process with a direct statement of conservation on the control volume. By integrating the governing equation with the control volume, the finite volume formulation of the fin equation is obtained as:

$$\int_w^e \left[ \frac{d}{dX} [K(\theta)] - M^2 \theta^{n+1} + M^2 Q + M^2 Q \gamma \theta \right] dV = 0 \tag{17}$$

Since $V=AdX$, Eq. (17) could be expressed as:

$$\int_\omega^e \frac{d}{dX}\left[ K(\theta) \frac{d\theta}{dX} \right] A\, dX - M^2 \int_\omega^e \theta^{n+1} A\, dX + M^2 \gamma Q \int_\omega^e \theta A\, dX + M^2 Q \int_\omega^e A\, dX = 0 \tag{18}$$

Since the area A of the fin is constant, Eq. (18) reduces to:

$$\int_\omega^e \frac{d}{dX}\left[ K(\theta) \frac{d\theta}{dX} \right] dX - M^2 \int_\omega^e \theta^{n+1} dX + M^2 \gamma Q \int_\omega^e \theta\, dX + M^2 Q \int_\omega^e dX = 0 \tag{19}$$

As shown in Fig. 2, in order to derive the discretization equation, the grid point cluster is used. The $P$ point represents the main point where the temperature is to be determined. The east and the west neighbours of the main grid point are represented by $E$ and $W$, respectively. The narrow line depicts the face of the control volume while e and w letters denote these faces. The thickness in the y and z direction is assumed to be unity for the corresponding one-dimensional problem. This implies that the temperature variation is only in the x-direction and grid points are uniformly distributed only in this direction.

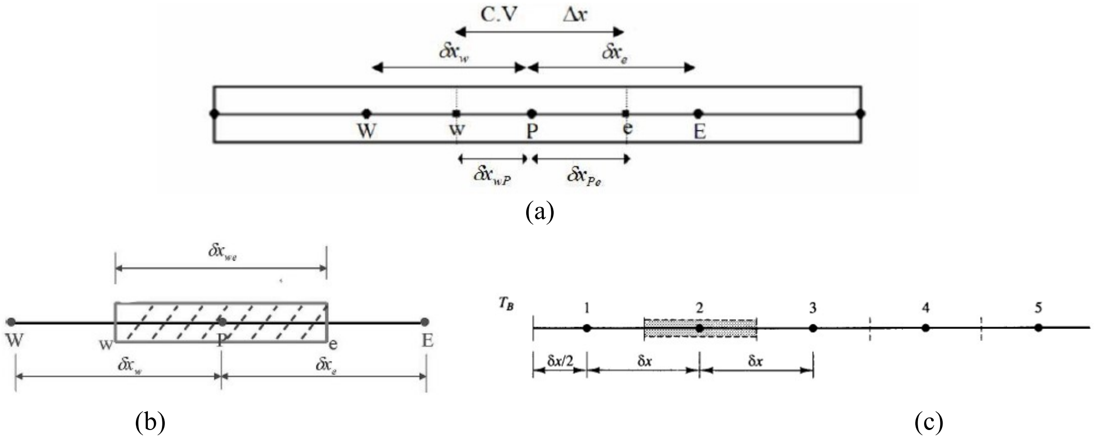



**Fig. 2.** (a) The grid point cluster for the one-dimensional problem (b) Arrangement of control volumes (c) The grid used for the one-dimensional problem

Therefore, the integration of Eq. (18) with Eq. (19) leads to the following equation:

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2\theta^{n+1}\delta X+M^2\gamma Q\theta\delta X+M^2Q\delta X=0 \tag{20}$$

The finite volume discretization of Eq. (20) for the first nodal point in Fig. 2 is expressed as:

$$\left[K(\theta)\right]_e\left(\frac{\theta_E-\theta_P}{\delta X}\right)-\left[K(\theta)\right]_w\left(\frac{\theta_P-\theta_B}{\frac{\delta X}{2}}\right)-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{21}$$

By collecting the equivalent terms, the following equation is obtained:

$$M^2\theta_P^{n+1}\delta X+\left[\frac{\left[K(\theta)\right]_e}{\delta X}+\frac{2\left[K(\theta)\right]_w}{\delta X}-M^2Q\gamma\delta X\right]\theta_P=\frac{\left[K(\theta)\right]_e}{\delta X}\theta_E+\frac{2\left[K(\theta)\right]_B}{\delta X}\theta_B+M^2Q\delta X \tag{22}$$

where

$$\left[K(\theta)\right]_e=1+\beta\theta_E$$

$$\left[K(\theta)\right]_B=1+\beta\theta_B$$

$$\left[K(\theta)\right]_w=1+\beta\theta_W$$

Therefore, Eq. (22) changes to the following equation:

$$M^2\theta_P^{n+1}\delta X+\left[\frac{(1+\beta\theta_E)}{\delta X}+\frac{2(1+\beta\theta_W)}{\delta X}-M^2Q\gamma\delta X\right]\theta_P=\frac{(1+\beta\theta_E)}{\delta X}\theta_E+\frac{2(1+\beta\theta_W)}{\delta X}\theta_B+M^2QdX \tag{23}$$

By using the nodal number as shown in Fig. 2, the following equation is obtained:

$$M^2\theta_1^{n+1}(\delta X)^2+\left[(1+\beta\theta_2)+2(1+\beta\theta_B)-M^2Q\gamma(\delta X)^2\right]\theta_1-(1+\beta\theta_2)\theta_2=2(1+\beta\theta_2)\theta_B+M^2Q(\delta X)^2 \tag{24}$$

For the middle nodal points the equation is:

$$\int_\omega^e\frac{d}{dX}\left[K(\theta)\frac{d\theta}{dX}\right]AdX-M^2\int_\omega^e\theta^{n+1}AdX+M^2\gamma Q\int_\omega^e\theta AdX+M^2Q\int_\omega^e AdX=0 \tag{25}$$

Similarly;

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{26}$$

The finite volume discretization of Eq. (19) for the middle nodal points in Fig. 2 is expressed as

$$\left[K(\theta)\right]_e\left[\frac{\theta_E-\theta_P}{\delta X}\right]-\left[K(\theta)\right]_w\left[\frac{\theta_P-\theta_W=}{\delta X}\right]-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{27}$$

Eq. (27) could be further expressed as:

$$\frac{(1+\beta\theta_E)(\theta_E-\theta_P)}{\delta X}-\frac{(1+\beta\theta_W)(\theta_P-\theta_W)}{\delta X}-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{28}$$

Collection of equivalent terms in Eq. (28) leads to:

$$M^2\theta_P^{n+1}(\delta X)^2+\left[(1+\beta\theta_E)+(1+\beta\theta_W)-M^2Q\gamma(\delta X)^2\right]\theta_P-(1+\beta\theta_E)\theta_E-(1+\beta\theta_W)\theta_W=M^2Q(\delta X)^2 \tag{29}$$

By using the interior nodal numbers, the following equation is obtained:

$$M^2\theta_i^{n+1}(\delta X)^2+\left[(1+\beta\theta_{i+1})+(1+\beta\theta_{i-1})+M^2(1-QY)(\delta X)^2\right]\theta_i-(1+\beta\theta_{i+1})\theta_{i+1}-(1+\beta\theta_{i-1})\theta_{i-1}=M^2Q(\delta X)^2 \tag{30}$$

Where *i= 2, 3, 4,…, N-1. N is the number of numbered points on the one-dimensional mesh*. For the last node (N), the equation is obtained as follows:

$$\int_\omega^e\frac{d}{dX}\left[K(\theta)\frac{d\theta}{dX}\right]A\,dX-M^2\int_\omega^e\theta^{n+1}A\,dX+M^2\gamma Q\int_\omega^e\theta A\,dX+M^2Q\int_\omega^e A\,dX=0 \tag{31}$$

The integration of Eq. (30) with Eq. (31) leads to the following equation:

$$\left[K(\theta)\frac{d\theta}{dX}\right]_e-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{32}$$

By using the second boundary condition $X=1,\quad \frac{d\theta}{dX}=0$, the following equation is obtained:

$$\cancel{\left[K(\theta)\frac{d\theta}{dX}\right]_e}-\left[K(\theta)\frac{d\theta}{dX}\right]_\omega-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{33}$$

By applying the second boundary condition to Eq. (32) as shown in Eq. (33), and after finite volume discretization, Eq. (32) reduces to:

$$\frac{-(1+\beta\theta_w)(\theta_P-\theta_W)}{(\delta X/2)}-M^2\theta_P^{n+1}\delta X+M^2\gamma Q\theta_P\delta X+M^2Q\delta X=0 \tag{34}$$

Rearranging the terms in Eq. (34) leads to Eq. (35) as follows:

$$M^2\theta_P^{n+1}(\delta X)^2+\left[2(1+\beta\theta_W)-M^2Q\gamma(\delta X)^2\right]\theta_P-2(1+\beta\theta_W)\theta_W=M^2Q(\delta X)^2 \tag{35}$$

Therefore, for the last node, the following equation is obtained:

$$M^2\theta_N^{n+1}(\delta X)^2+\left[2(1+\beta\theta_{N-1})-M^2Q\gamma(\delta X)^2\right]\theta_N-2(1+\beta\theta_{N-1})\theta_{N-1}=M^2Q(\delta X)^2 \tag{36}$$

The non-linear systems of Eq. (24) along with all the equations introduced in Eqs. (30) and (36) are solved by applying the MATLAB software using ***fsolve***.

## 4. Fin parameter for thermal performance indication

The performance indication parameter for the fin such as the efficiency of the fin is analyzed.

### *4.1 Fin efficiency*

The amount of heat dissipated from the entire fin is obtained by using Newton's law of cooling as follows:

$$Q_f=\int_0^1 Ph(T)(T-T_\infty)dX \tag{37}$$

Moreover, if the fin base temperature is kept constant throughout the fin, the maximum dissipated heat is obtained as follows:

$$Q_{\max}=Ph_bL(T_b-T_\infty) \tag{38}$$

The fin efficiency is defined as the ratio of the heat transfer rate of the fin to the rate that would be if the entire fin were at the base temperature. The following equation is obtained:

$$\eta = \frac{Q_f}{Q_{\max}} = \frac{\int_0^L Ph_b (T - T_\infty)^{n+1} dx}{Ph_b L (T_b - T_\infty)} \tag{39}$$

Therefore, the fin efficiency in dimensionless variables is given by:

$$\eta = \int_0^1 \theta^{n+1} dX \tag{40}$$

The discretized form of Eq. (40) is written as:

$$\eta = \sum_{i=1}^{N} \theta_i^{n+1} \tag{41}$$

It is very important to point out that the thermo-geometric parameter or the fin performance factor, M, could be written in terms of Biot number, Bi, and the aspect ratio, $a_r$, as shown in Eq. (41).

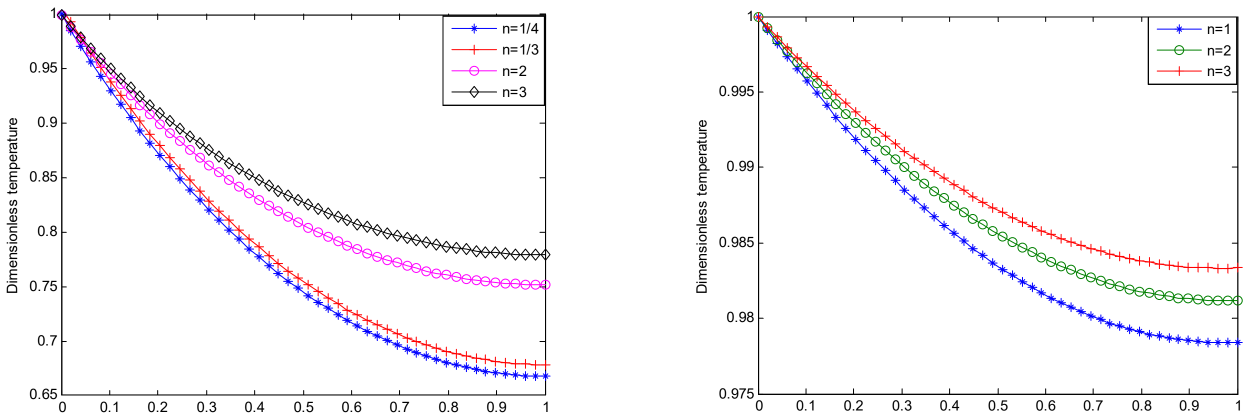
$$M^2 = \frac{Ph_b L^2}{A_c k_a} = \frac{(2L)h_b L^2}{(L\delta)k_a} = \frac{2h_b \delta L^2}{\delta^2 k_a} = \frac{2h_b \delta}{k_a}\left(\frac{L}{\delta}\right)^2 = 2Bia_r^2$$

$$Where \quad Bi = \frac{h_b \delta}{k_a}, a_r = \frac{L}{\delta} \tag{42}$$

According to Eq. (42), it implies that $M = a_r \sqrt{2Bi}$ .

## 5. Results and Discussion

Figs. 3a and 3b depict the effects of the boiling condition parameter on the dimensionless temperature of the fin. The dimensionless temperature distribution falls monotonically along the fin length for all the various boiling condition parameters, n. The lower the boiling condition parameter, the more the heat convected from the fin through its length and the more thermal energy is efficiently transferred into the environment through the fin length.




(a) (b)

**Fig. 3.** a) Dimensionless temperature distribution in the fin when M=1, β=1, Q=0.8, γ=0.5 b) M=1, β=0, Q=0, γ=0

Fig. 4 show the variation of the dimensionless temperature with the dimensionless length and also the effect of the thermo-geometric parameter on the straight fin with an insulated tip. According to the above-mentioned figure, as the thermo-geometric parameter increases, the rate of the heat transfer (the convective heat transfer) through the fin increases as well, while the temperature in the fin drops faster and becomes steeper which reflects high rates of the base-heat flow. It can be inferred from the results that the ratio of the convective heat transfer to the conductive heat transfer at the base of the fin ($h_b/k_b$) has a great effect on the temperature distribution, the rate of heat transfer at the base of the fin, and the efficiency and effectiveness of the fin. As $h_b$ increases (or $k_b$ decreases), the ratio of $h_b$ to $k_b$ increases at the base of the fin as well, and consequently, the temperature along the fin, especially at the tip of the fin, decreases: the tip end temperature decreases as M increases. The profile has steepest temperature gradient at $M=1.0$, but this much higher value of $M$, which has been obtained from the lower value of the thermal conductivity, produces a lower heat-transfer rate. This shows that the thermal performance or the efficiency of the fin is verified at low values of the thermo-geometric parameter since the aim (the highly effective use of the fin) is to minimize the temperature decrease along the fin length, whereas the best possible scenario is when $T=T_b$ everywhere.

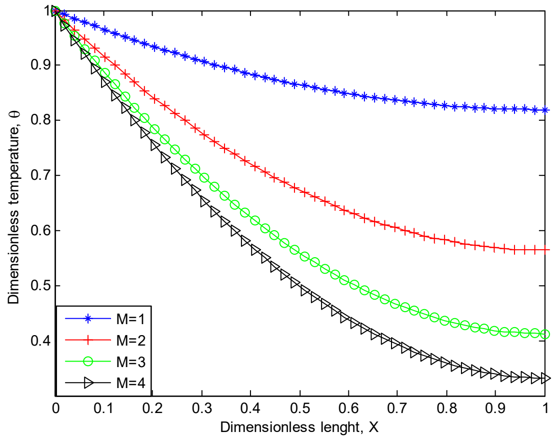



**Fig. 4.** Dimensionless temperature distribution in the fin when β=0.5, Q=0.4, γ=0.6

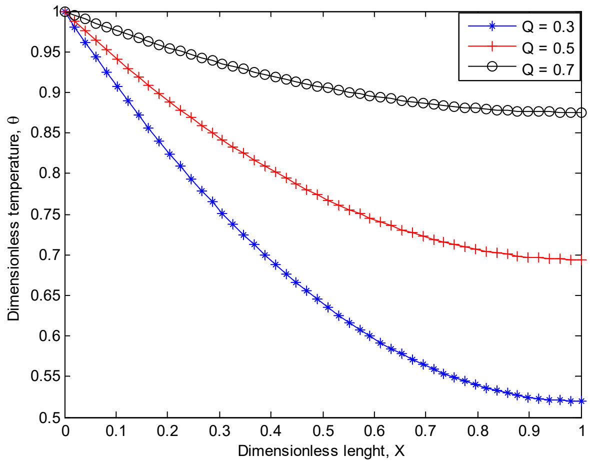



**Fig. 5.** Dimensionless temperature distribution in the fin parameters when β=0.2, M=2, γ=0.2

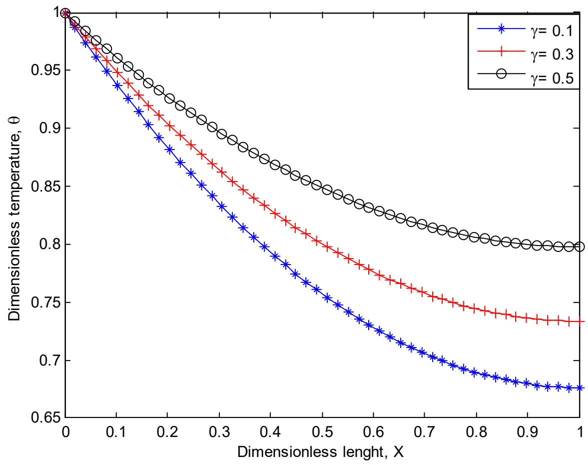



**Fig. 6.** Dimensionless temperature distribution in the fin parameters when M=2, β=0.3, Q=0.3

The effect of the internal heat generation parameter on the temperature distribution is depicted in Figs. 5 and 6 while Fig. 7 shows the effects of the internal heat generation on the fin thermal performance for different thermo-geometric parameters. According to the figures, as the internal heat generation parameter increases, the temperature gradient of the fins decreases. The reason is that, as the rate of the internal heat generation within the fin increases, the thermal performance of the fin decreases. However, the figures show that the dimensionless temperature gradient of the fin length increases as the thermo-geometric parameter increases.

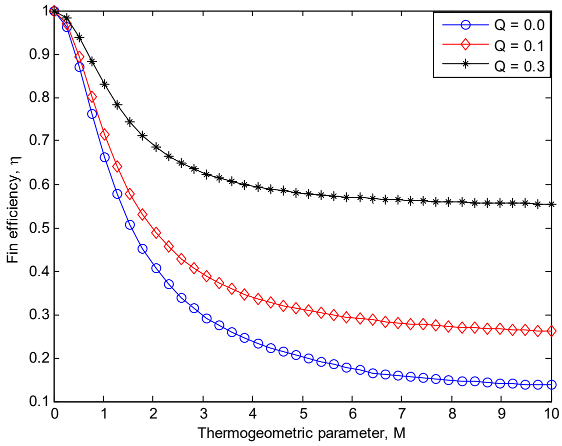
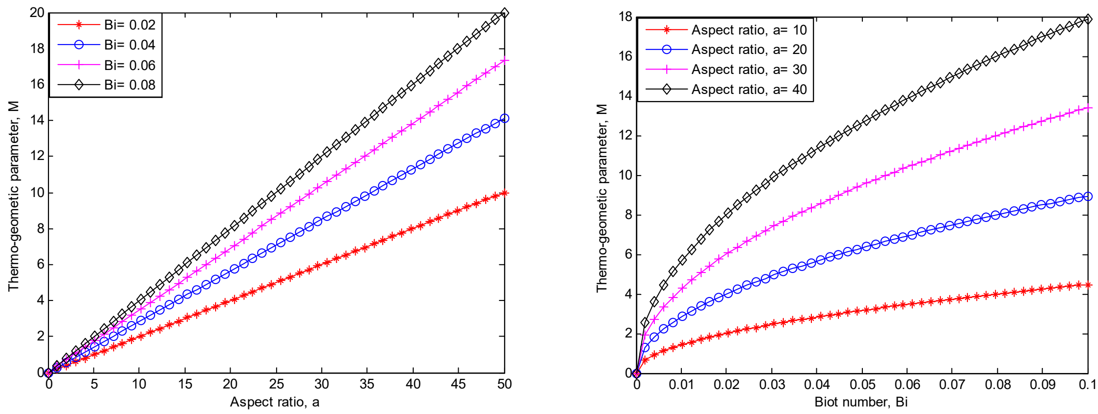


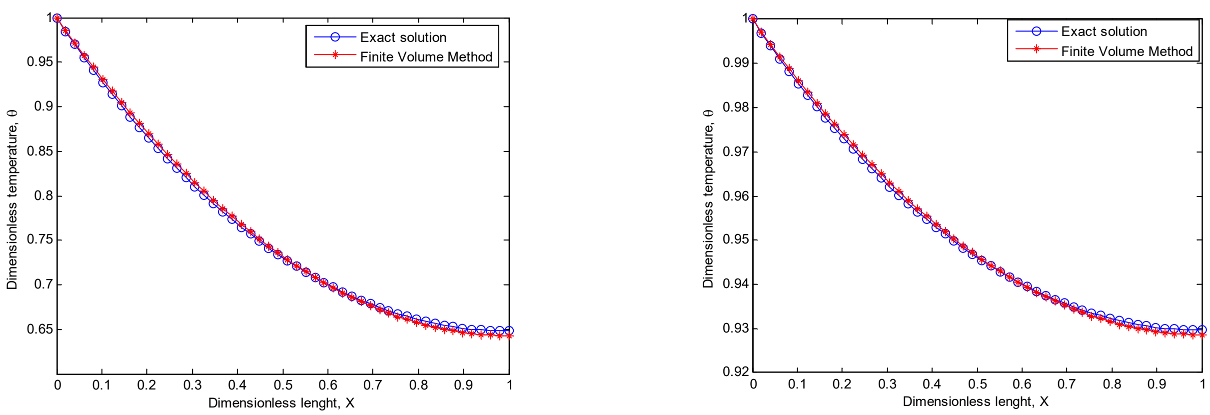

**Fig. 7.** Dimensionless temperature distribution in the fin parameters when β=0.1, γ=0.8




(a) (b)

**Fig. 8.** Effects of Biot number on the thermo-geometric parameter of the fin

The effects of the Biot number and the aspect ratio on the thermo-geometric parameter (the fin performance factor) are shown in Fig. 8. According to the results, the fin performance factor increases as the aspect ratio and the Biot number increase. However, the thermal performance or the efficiency of the fin is verified at low values of the thermo-geometric parameter since the aim (the highly effective use of the fin) is to minimize the temperature decrease along the fin length, whereas the best possible scenario is when $T=T_b$ everywhere. It must be pointed out that equation (40) shows the direct relationship between the thermo-geometric parameter, M, and the Biot number, Bi, which directly depends on the fin length. A low value of M corresponds to a relatively short and thick fin with a poor thermal conductivity and a high value of M implies a long fin or a fin with a low value of the thermal conductivity. Since the thermal performance or the efficiency of the fin is verified at low values of the thermo-geometric fin parameter, very long fins are to be avoided in practice. A compromise is reached for the one-dimensional analysis of fins $0 < Bi < 0.1$. When the Biot number is greater than 0.1, the two-dimensional analysis of the fin is recommended since the one-dimensional analysis predicts unreliable results for such a limit point.




(a) (b)

**Fig. 9.** Dimensionless temperature distribution in the fin parameters when (a) M=1, β=0, Q=0 and (b) M=1.5, Q=0.8

The finite volume method of solution is verified by the exact solution in Figs. 9a and 9b for the linear thermal model of the fin. This assures the predicted results by the finite volume method for the non-linear problems in which no closed-form solution is difficult or impossible to obtain.

## 6. Conclusion

In this study, first the steady-state heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and then the internal heat generation under the multi-boiling heat transfer where the local heat transfer coefficient is considered to vary with a power-law function of temperature were analyzed by using the finite volume method. The solution was verified by the exact solution for the linear problem. The developed heat transfer models were used to investigate the effects of the thermo-geometric and the coefficient of the heat transfer along with the effects of thermal conductivity (non-linear) parameters on the temperature distribution, the heat transfer and the thermal performance of the longitudinal rectangular fin. These results serve as a basis for the comparison of other various methods of the problem analysis. Moreover, they provide a platform for improvement in the design of the fin in the heat transfer equipment such as air–land–space vehicles and their power sources; in chemical, refrigerating, and cryogenic processes; in the electrical and electronic equipment; in the conventional furnaces and gas turbines; in the design of firebox for the generation of the steam power from fossil fuels; in the heat dissipators and waste heat boilers; in the nuclear-fuel modules, steam power plants, automobiles radiators etc.

## Nomenclature

| | | | |
|---|---|---|---|
| $a_r$ | aspect ratio | M | dimensionless thermo-geometric fin parameter |
| A | cross sectional area of the fins, $m^2$ | $m^2$ | thermo-geometric fin parameter $m^{-1}$ |
| Bi | Biot number | n | convective heat transfer power |
| h | heat transfer coefficient, $Wm^{-2}k^{-1}$ | P | perimeter of the fin, m |
| $h_b$ | heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$ | T | Temperature, K |
| H | dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$ | $T_\infty$ | ambient temperature, K |
| j | geometric parameter | $T_b$ | Temperature at the base of the fin, K |
| k | thermal conductivity of the fin material, $Wm^{-1}k^{-1}$ | x | fin axial distance, m |
| $k_b$ | thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$ | X | dimensionless length of the fin |
| K | dimensionless thermal conductivity of the fin material, $Wm^{-1}k^{-1}$ | Q | dimensionless heat transfer |
| L | Length of the fin, m | $q_i$ | the uniform internal heat generation in $W/m^3$ |

## Greek Symbols

| | | | |
|---|---|---|---|
| $\beta$ | thermal conductivity parameter or non-linear parameter | $\theta$ | dimensionless temperature |
| $\delta$ | thickness of the fin, m | $\theta_b$ | dimensionless temperature at the base of the fin |
| $\delta_b$ | fin thickness at its base | $\eta$ | efficiency of the fin |
| $\gamma$ | dimensionless internal heat generation parameter | $\varepsilon$ | effectiveness of the fin |

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