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Vibration and Static Analysis of Functionally Graded Porous Plates

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Abstract

This research deals with free vibration and static bending of a simply supported functionally graded (FG) plate with the porosity effect. Material properties of the plate which are related to its change are position-dependent. Governing equations of the FG plate are obtained by using the Hamilton's principle within first-order shear deformation plate theory. In solving the problem, the Navier solution is also used. In this study, the effect of the porosity and material distribution parameters on the static and vibration responses of the FG plate is presented and discussed.

Keywords: Functionally Graded Plate, Porosity; Static Analysis; Vibration Analysis.

1. Introduction

Functionally graded materials (FGMs) are a new generation of composites where the volume fraction of the FGM constituents varies gradually, giving a non-uniform microstructure with continuously graded macro properties such as elasticity modulus, density, heat conductivity, etc.. FGMs have many practical applications, such as biomedical sectors, aircrafts, and space vehicles. In the last few decades, much more attention has been given to the mechanical behavior of FG Plates ([1]-[28]).

In recent years, some studies about the porosity effect in the FG structures have been published in the literature; Wattanasakulpong and Ungbhakorn [29] conducted the vibration analysis of porous FG beams. Mechab et al. [30, 31] examined the free vibration analysis of a FG nano-plate resting on elastic foundations with the porosity effect. Şimşek and Aydın [32] examined the forced vibration of FG microplates with porosity effects based on the modified couple stress theory. Jahwari and Naguib [33] investigated FG viscoelastic porous plates with a higher order plate theory and a statistical based model of cellular distribution. Vibration characteristics of FG beams with porosity effect and various thermal loadings are investigated by [34-35].

In this study, the free vibration and the static bending analysis of a simply supported FG plate are studied with porosity effect by using Navier solution within the first-order shear deformation plate theory. The effects of the porosity and material distribution parameters on the static and vibration responses of the FG plate are investigated. The linear/ nonlinear analysis of buckling and the vibration of FG graphane beams reinforced by the porous nana-composite are investigated by Chen et al. [36] and Kitipornchai et al. [37].

2. Theory and Formulations

Consider a simply supported rectangular FG porous plate with thickness h in X3 direction. The lengths of LX and LY in X1 and X2 directions are shown in Figure 1, respectively. The FG plate is subjected to a uniformly distributed transverse load (q). The FG plate considered in numerical examples is made of Aluminum and Zirconia. The bottom surface of the FG plate is Aluminum and the top surface is Zirconia.

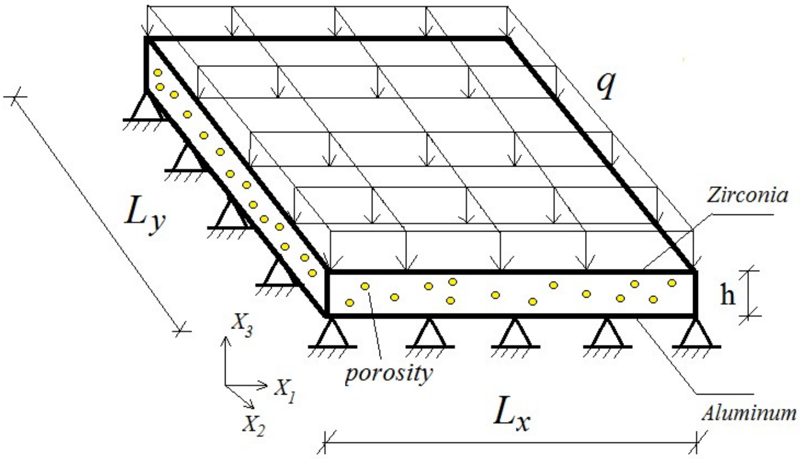

Fig. 1. A simply supported FG plate subjected to the uniform distributed load with porosity.

The effective material properties of the FG plate, P, such as the Young's modulus E, the Poisson's ratio ν, the mass density ρ and the shear modulus G vary continuously in the thickness direction (X3 axis) according to a power-law function as follows:

$$P(X_3)=(P_T-P_B)\left(\frac{X_3}{h}+\frac{1}{2}\right)^k+P_B-(P_T+P_B)\frac{a}{2} \tag{1}$$

Where PT and PB are the material properties of the top surface and the bottom surface materials of the plate, k is the non-negative power-law exponent which determines the material variation profile using the thickness of the plate. It is clear from Eq. (1) that when X3=-h/2, P=PB, and when X3=h/2, P=PT. According to Eq. (1), when k=0 (full bottom material) or k=∞ (full top material) the material of the plate is homogeneous.

Two porosity models (even and uneven) are used in the porosity effect which was proposed by Wattanasakulpong and Ungbhakorn [29] for the power law distribution. In the even model, the porosity spreads uniformly though height direction and in the uneven model, the porosity spreads functionally though height direction. The distribution of the even and uneven porosity models is shown in Fig. 2.

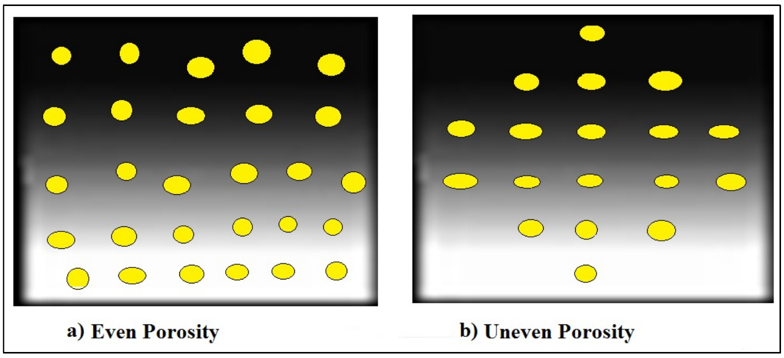

Fig.2. Porosity models for the FG material.

According to the power law distribution, the effective material property for the even porosity can be expressed as follows:

$$P(X_3)=(P_T-P_B)\left(\frac{X_3}{h}+\frac{1}{2}\right)^k+P_B-(P_T+P_B)\frac{a}{2} \tag{2}$$

Where *a* (*a*<<1) is the volume fraction of porosities. When *a*=0, the plate becomes a perfect FGM. For uneven porosity distribution and according to the power law distribution, the effective material property can be expressed as follows:

$$P(X_3)=(P_T-P_B)\left(\frac{X_3}{h}+\frac{1}{2}\right)^k+P_B-(P_T+P_B)\frac{a}{2}\left(1-\frac{2|X_3|}{h}\right) \tag{3}$$

According to the first-order shear deformation plate theory, the axial and the displacement fields are expressed as:

$$u(x_1,x_2,x_3,t)=u_0(x_1,x_2,t)+x_3\phi_{x_1}(x_1,x_2,t) \tag{4}$$

$$v(x_1,x_2,x_3,t)=v_0(x_1,x_2,t)+x_3\phi_{x_2}(x_1,x_2,t) \tag{5}$$

$$w(x_1,x_2,x_3,t)=w_0(x_1,x_2,t) \tag{6}$$

where u, v, w are X_1, X_2 and X_3 components of the displacements, respectively, ϕ_{x_1} and ϕ_{x_2} are rotations of transverse normal on the middle plane around X_1 and X_2 directions, and t indicates time. By using equations (4-6), the strain- displacement relation can be obtained as follows:

$$\varepsilon_{x_1}=\frac{\partial u}{\partial x_1}=\frac{\partial u_0(x_1,x_2,t)}{\partial x_1}+x_3\frac{\partial \phi_{x_1}(x_1,x_2,t)}{\partial x_1} \tag{7a}$$

$$\varepsilon_{x_2}=\frac{\partial v}{\partial x_2}=\frac{\partial v_0(x_1,x_2,t)}{\partial x_2}+x_3\frac{\partial \phi_{x_2}(x_1,x_2,t)}{\partial x_2} \tag{7b}$$

$$\varepsilon_{x_1x_2}=\frac{1}{2}\left(\frac{\partial u_0(x_1,x_2,t)}{\partial x_2}+\frac{\partial v_0(x_1,x_2,t)}{\partial x_1}+\frac{\partial \phi_{x_2}(x_1,x_2,t)}{\partial x_2}+\frac{\partial \phi_{x_2}(x_1,x_2,t)}{\partial x_1}\right) \tag{7c}$$

The constitutive equations of the FG plate are as follows:

$$\sigma_{ij}=\frac{E(X_3)}{(1-v(X_3)^2)}\left[v(X_3)\varepsilon_{kl}\delta ij+(1-v(X_3))\varepsilon ij\right] \tag{8}$$

The elastic strain energy (U_i) and the kinetic energy (T) of the porous FG plate are expressed as follows:

$$U_i=\frac{1}{2}\int_V \sigma ij\varepsilon ijdV \tag{9a}$$

$$T=\frac{1}{2}\int_V \rho(X_3)\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dV \tag{9b}$$

The Hamilton's principle of the problem is

$$\partial\int_0^t\left[T-U_i\right]dt=0 \tag{10}$$

After using the Hamilton's principle, the governing equations of the FG porous plate can be obtained as follows:

$$\frac{\partial N_{X_1}}{\partial_{X_1}}+\frac{\partial N_{X_1X_2}}{\partial_{X_2}}=J_1\frac{\partial^2 u_0}{\partial t^2}+J_2\frac{\partial^2 \phi_{X_1}}{\partial t^2} \tag{11a}$$

$$\frac{\partial N_{X_2}}{\partial_{X_2}}+\frac{\partial N_{X_1X_2}}{\partial_{X_1}}=J_1\frac{\partial^2 u_0}{\partial t^2}+J_2\frac{\partial^2 \phi_{X_2}}{\partial t^2} \tag{11b}$$

$$\frac{\partial M_{X_1}}{\partial_{X_1}}+\frac{\partial M_{X_1X_2}}{\partial_{X_2}}-V_{X_1}=J_3\frac{\partial^2 \phi_{X_1}}{\partial t^2}+J_2\frac{\partial^2 u_0}{\partial t^2} \tag{11c}$$

$$\frac{\partial M_{X_2}}{\partial_{X_2}}+\frac{\partial M_{X_1X_2}}{\partial_{X_1}}-V_{X_2}=J_3\frac{\partial^2\phi_{X_2}}{\partial t^2}+J_2\frac{\partial^2 v_0}{\partial t^2} \tag{11d}$$

$$\frac{\partial V_{X_1}}{\partial_{X_1}}+\frac{\partial V_{X_2}}{\partial_{X_2}}+q=J_1\frac{\partial^2 w_0}{\partial t^2} \tag{11e}$$

where $N_{X_1}, N_{X_2}, N_{X_1X_2}, M_{X_1}, M_{X_2}, M_{X_1X_2}, V_{X_1}, V_{X_2}$ are the stress resultants defined as follows:

$$\begin{Bmatrix} N_{X_1}\\ N_{X_2}\\ M_{X_1}\\ M_{X_2}\end{Bmatrix}=\left(\int_{-0.5h}^{0.5h}\begin{pmatrix} Q_{11} & Q_{12} & X_3Q_{11} & X_3Q_{12}\\ Q_{12} & Q_{22} & X_3Q_{12} & X_3Q_{22}\\ X_3Q_{11} & X_3Q_{12} & X_3^{\,2}Q_{11} & X_3^{\,2}Q_{12}\\ X_3Q_{12} & X_3Q_{22} & X_3^{\,2}Q_{12} & X_3^{\,2}Q_{22}\end{pmatrix}dX_3\right)\begin{Bmatrix}\dfrac{\partial u_0}{\partial X_1}\\ \dfrac{\partial v_0}{\partial X_2}\\ \dfrac{\partial\phi X_1}{\partial X_1}\\ \dfrac{\partial\phi X_1}{\partial X_2}\end{Bmatrix} \tag{12a}$$

$$\begin{Bmatrix} N_{X_1X_2}\\ M_{X_1X_2}\end{Bmatrix}=\left(\int_{-0.5h}^{0.5h}\begin{matrix} Q_{44} & Q_{44} & X_3Q_{44} & X_3Q_{44}\\ X_3Q_{44} & X_3Q_{44} & X_3^{\,2}Q_{44} & X_3^{\,2}Q_{44}\end{matrix}\;dX_3\right)\begin{Bmatrix}\dfrac{\partial u_0}{\partial X_1}\\ \dfrac{\partial v_0}{\partial X_2}\\ \dfrac{\partial\phi X_1}{\partial X_1}\\ \dfrac{\partial\phi X_1}{\partial X_2}\end{Bmatrix} \tag{12b}$$

$$V_{X_1}=\int_{-0.5h}^{0.5h}\left(\phi_{X_1}+\frac{\partial w_0}{\partial_{X_1}}\right)Q_{66}dX_3,\quad V_{X_2}=\int_{-0.5h}^{0.5h}\left(\phi_{X_2}+\frac{\partial w_0}{\partial_{X_2}}\right)Q_{55}dX_3 \tag{12c}$$

where $Q_{11}, Q_{12}, Q_{22}, Q_{44}, Q_{55}, \text{and}\, Q_{66}$ are the elastic coefficients which are presented as follows:

$$Q_{11}=Q_{22}=\frac{E(X_3)}{\left(1-v(X_3)^2\right)},\quad Q_{12}=vQ_{11},\quad Q_{44}=Q_{55}=Q_{66}=\frac{E(X_3)}{\left(1+v(X_3)\right)} \tag{13}$$

In equation 11, J_i is defined as follows:

$$J_i=\int_{-0.5h}^{0.5h}\rho(X_3)^i\,dX_3,\; i=1,2,3 \tag{14}$$

To solve the the problem, the Navier solution is used for the simply supported rectangular porous FG plate. In the Navier solution, the displacement fields and boundary conditions of the plate are expressed as follows:

$$u_0(x_1,x_2,t)=\sum_{n=1}^{N}\sum_{m=1}^{N}A_{mn}\cos(\mathrm{k}\mathrm{X}_1)\sin(l\,\mathrm{X}_2)e^{-i\beta t} \tag{15a}$$

$$v_0(x_1,x_2,t)=\sum_{n=1}^{N}\sum_{m=1}^{N}B_{mn}(\mathrm{t})\sin(\mathrm{k}\mathrm{X}_1)\cos(l\,\mathrm{X}_2)e^{-i\beta t} \tag{15b}$$

$$w_0(x_1,x_2,t)=\sum_{n=1}^{N}\sum_{m=1}^{N}C_{mn}(\mathrm{t})\sin(\mathrm{k}\mathrm{X}_1)\cos(l\,\mathrm{X}_2)e^{-i\beta t} \tag{15c}$$

$$\varphi_{X_1}(x_1,x_2,t)=\sum_{n=1}^{N}\sum_{m=1}^{N}D_{mn}(\mathrm{t})\cos(\mathrm{k}\mathrm{X}_1)\sin(l\,\mathrm{X}_2)e^{-i\beta t} \tag{15d}$$

$$\varphi_{X_2}(x_1,x_2,t)=\sum_{n=1}^{N}\sum_{m=1}^{N}E_{mn}(\mathrm{t})\sin(\mathrm{kX}_1)\cos(l\,\mathrm{X}_2)e^{-i\beta t} \tag{15e}$$

$$v_0=0,w_0=0,\varphi_{X_2}=0,M_{X_1}=0,N_{X_1}=0,at,x_1=0,L_X \tag{16a}$$

$$u_0=0,w_0=0,\varphi_{X_2}=0,M_{X_2}=0,N_{X_2}=0,at,x_2=0,L_Y \tag{16b}$$

where $A_{mn},B_{mn},C_{mn},D_{mn},E_{mn}$ are displacement coefficients, $k=m\pi/L_X, l=n\pi/L_Y,\beta$ is the natural frequency and $i=\sqrt{-1}$.

In the static bending problem, time and its derivatives are zero in the governing equations. In the Navier solution, the uniform distributed load q_0 is defined as follows:

$$q_0=\sum_{n=1}^{N}\sum_{m=1}^{N}q_{mn}(\mathrm{t})\sin(\mathrm{kX}_1)\sin(l\,\mathrm{X}_2) \tag{17}$$

where

$$q_{mn}=\int_0^{L_X}\int_0^{L_Y}\frac{4q}{L_XL_Y}\sin(\mathrm{kX}_1)\sin(l\,\mathrm{X}_2)\,\mathrm{dX}_1\,dX_2 \tag{18}$$

By substituting Eqs. (15) and Eqs. (17-18) into Eqs. (11), and then using the matrix procedure, the algebraic equations of the static problem can be expressed as follows:

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \\ D_{mn} \\ E_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ q_{mn} \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

where

$$K_{11}=k^2\int_{-0.5h}^{0.5h}Q_{11}dX_3+l^2\int_{-0.5h}^{0.5h}Q_{44}dX_3 \tag{20a}$$

$$K_{12}=K_{21}=kl\int_{-0.5h}^{0.5h}Q_{12}dX_3+kl\int_{-0.5h}^{0.5h}Q_{44}dX_3 \tag{20b}$$

$$K_{13}=K_{31}=0 \tag{20c}$$

$$K_{14}=K_{41}=k^2\int_{-0.5h}^{0.5h}X_3Q_{11}dX_3+l^2\int_{-0.5h}^{0.5h}X_3Q_{44}dX_3 \tag{20d}$$

$$K_{15}=K_{51}=kl\int_{-0.5h}^{0.5h}X_3Q_{12}dX_3+kl\int_{-0.5h}^{0.5h}X_3Q_{44}dX_3 \tag{20e}$$

$$K_{22}=l^2\int_{-0.5h}^{0.5h}Q_{22}dX_3+k^2\int_{-0.5h}^{0.5h}Q_{44}dX_3 \tag{20f}$$

$$K_{23}=K_{32}=0 \tag{20g}$$

$$K_{24}=K_{42}=kl\int_{-0.5h}^{0.5h}X_3Q_{12}dX_3+kl\int_{-0.5h}^{0.5h}X_3Q_{44}dX_3 \tag{20i}$$

$$K_{25} = K_{52} = l^2 \int_{-0.5h}^{0.5h} X_3 Q_{22} dX_3 + k^2 \int_{-0.5h}^{0.5h} X_3 Q_{44} dX_3 \tag{20j}$$

$$K_{33} = k^2 \int_{-0.5h}^{0.5h} Q_{66} dX_3 + l^2 \int_{-0.5h}^{0.5h} Q_{55} dX_3 \tag{20k}$$

$$K_{34} = K_{43} = k \int_{-0.5h}^{0.5h} Q_{66} dX_3 \tag{20l}$$

$$K_{35} = K_{53} = l \int_{-0.5h}^{0.5h} Q_{55} dX_3 \tag{20m}$$

$$K_{44} = k^2 \int_{-0.5h}^{0.5h} X_3^2 Q_{11} dX_3 + l^2 \int_{-0.5h}^{0.5h} X_3^2 Q_{44} dX_3 + \int_{-0.5h}^{0.5h} X_3^2 Q_{66} dX_3 \tag{20n}$$

$$K_{45} = K_{54} = kl \int_{-0.5h}^{0.5h} X_3^2 Q_{12} dX_3 + kl \int_{-0.5h}^{0.5h} X_3^2 Q_{44} dX_3 \tag{20o}$$

$$K_{55} = l^2 \int_{-0.5h}^{0.5h} X_3^2 Q_{22} dX_3 + k^2 \int_{-0.5h}^{0.5h} X_3^2 Q_{44} dX_3 + \int_{-0.5h}^{0.5h} Q_{55} dX_3 \tag{20p}$$

For the free vibration problem of the FG porous plate, the algebraic equations expressed as follows:

$$\left(\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} - (\beta)^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} \\ M_{21} & M_{22} & M_{23} & M_{24} & M_{25} \\ M_{31} & M_{32} & M_{33} & M_{34} & M_{35} \\ M_{41} & M_{42} & M_{43} & M_{44} & M_{45} \\ M_{51} & M_{52} & M_{53} & M_{54} & M_{55} \end{bmatrix} \right) \begin{Bmatrix} A_{mn} \\ B_{mn} \\ C_{mn} \\ D_{mn} \\ E_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{gathered} M_{11} = J_1, M_{12} = M_{21} = 0, M_{13} = M_{31} = 0, M_{14} = M_{41} = J_2, M_{15} = M_{51} = 0, M_{52} = J_2, M_{33} = J_1, M_{34} = M_{43} = 0, M_{35} = M_{53} = 0, \\ M_{44} = J_3, M_{45} = 0, M_{55} = J_3 \end{gathered} \tag{22}$$

By solving the eigenvalue problem which given in eq. (21), the fundamental frequencies can be determined and the dimensionless quantities can be expressed as follows:

$$w = \frac{w E_T h^3}{q L_{X^4}}, \omega = \beta \sqrt{\frac{\rho_T L_{X^4}}{E_T h^2}} \tag{23}$$

where w is the dimensionless deflection and ω is the dimensionless fundamental frequency.

3. Results And Discussion

In the numerical study, the natural frequencies and the static bending deflections of the simply supported FG plate are calculated and presented in figures for different porosity parameters, porosity models and material distributions. The FG porous plate considered in numerical examples is made of Aluminum (Al; E=70 GPa, ν=0.3, ρ=2702 kg/m3) and Zirconia (E=151GPa, ν=0.3, ρ=3000 kg/m3). The top surface material of the FG plate is Zirconia, the bottom surface material of the FG plate is Aluminum. According to Eq. (2) and Eq. (3), when k=0 and k=∞, the material of the plate gets homogeneous Aluminum and homogeneous Zirconia, respectively. The dimensions of the FG plate in the numerical examples are considered as follows: h = 0.3 m, LX =10h and LY=10h.

Figures 3 and 4 display the effect of the material distribution parameter k on the dimensionless bending deflections and fundamental frequencies of the porous FG plate for different porosity parameters and porosity models, respectively.

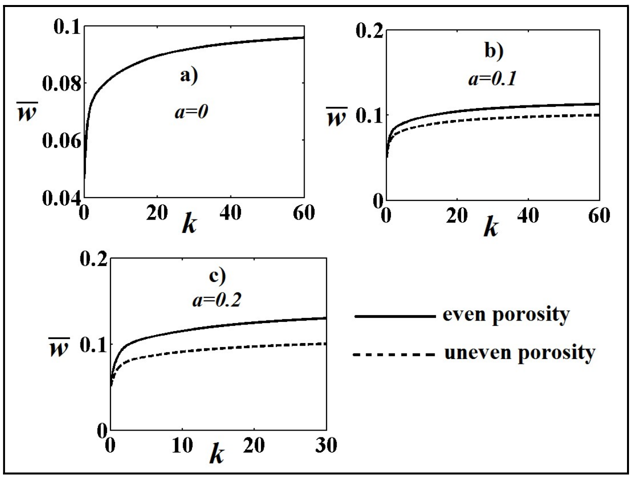

Fig. 3. The relationship between of the dimensionless bending deflections $\overline{w}$ and the material distribution parameter k for different the porosity models for a) $a=0$, b) $a=0.1$, c) $a=0.2$.

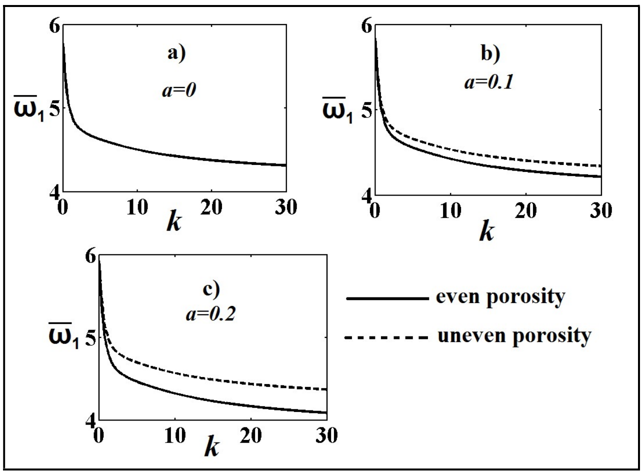

Fig. 4. The relationship between the dimensionless fundamental frequencies $\overline{\omega}_1$ and the material distribution parameter k for different porosity models for a) $a=0$, b) $a=0.1$, c) $a=0.2$.

It can be seen from Figures 3 and 4 that by an increase in the power-law exponent k the dimensionless bending deflections increase and also the dimensionless fundamental frequencies decrease. The reason is that, according to equation (1), the elasticity modulus and bending rigidity of the FG plate decrease with an increase in the k. As can be seen in Figures 3 and 4, by an increase in k parameter, the difference between the even and uneven porosity models increases as well. In small values of k parameter, this difference can be neglected and this shows that the material distribution plays an important role in the porosity effect.
In order to investigate the effect porosity parameters on the bending and vibration characteristics, the dimensionless bending deflections and fundamental frequencies of the porous FG plate versus the porosity parameter a for different porosity models for k=2 were obtained in Figures 5 and 6, respectively.

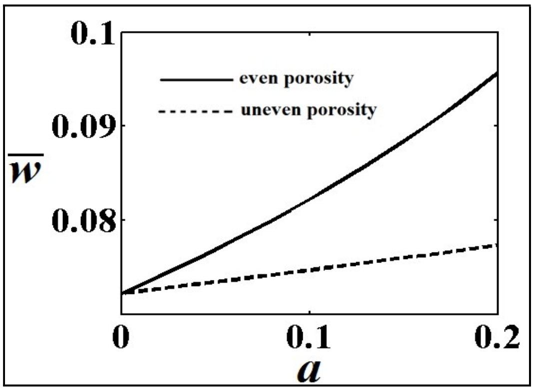

Fig. 5. The effect of porosity parameter (a) and porosity models on the dimensionless bending deflections $\overline{w}$ of the FG porous plate.

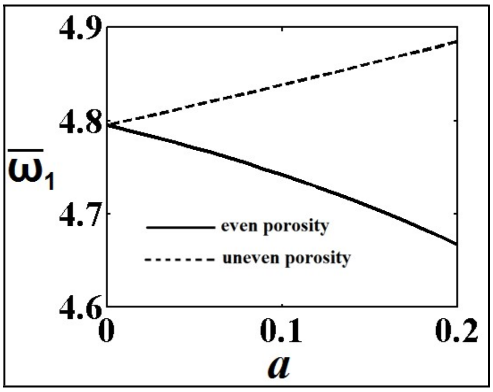

Fig. 6. The effect of porosity parameter (a) and porosity models on the dimensionless fundamental frequencies $\overline{\omega}_1$ of the FG porous plate.

As can be seen in Figure 5, by an increase in the porosity parameter a in both even and uneven porosities, the dimensionless bending deflections of the FG plate also increase. The reason is that, by an increase in the porosity parameter, the strength of the material decreases. In Figure 6, by an increase in the porosity parameter a in the even porosity model, the dimensionless fundamental frequencies decrease. However, by an increase in the porosity parameter a in the uneven porosity model, the dimensionless fundamental frequencies increase. Moreover, it can be seen in Figures 5 and 6 that, by increasing porosity parameter (a), the difference between the even and uneven porosity models increases significantly. Porosity parameters have a very important role in the static and vibration behavior of the FG porous plates.

4. Conclusions

Free vibration and static bending responses of a simply supported FG plate are investigated through the porosity effect and based on the first-order shear deformation theory by using Navier solution. According to power-law distributions and within different porosity models, the material properties of the plate are assumed to vary though the height direction. The effects of material distribution and porosity parameters on the vibration characters and bending deflections of the FG porous plate are presented within the different porosity models. It is observed in the results that material distribution and porosity parameters have very important roles in the static and vibration responses of the FG porous plates. The difference between the even and uneven porosity models increases with an increase in k and a parameters.

References

[1] Reddy, J. N., and Chin, C.D., "Thermomechanical analysis of functionally graded cylinders and plates", Journal of Thermal Stresses, 21(6), pp.593-626, 1998.
[2] Reddy, J. N. "Analysis of functionally graded plates". International Journal for Numerical Methods in Engineering, 47(1-3), pp. 663-684, 2000.
[3] Yanga, J. and Shen, H.S. (2003), "Non-linear analysis of functionally graded plates under transverse and in-plane loads", International Journal of Non-Linear Mechanics, 38(4) pp.467-482, 2003.
[4] Lanhe, W. "Thermal buckling of a simply supported moderately thick rectangular FGM plate", Composite Structures, 64(2), pp.211-218, 2004.
[5] Abrate, S., "Free vibration, buckling, and static deflections of functionally graded plates", Composites Science and Technology. 66(14), pp.2383-2394, 2006.
[6] Chi, S.H. and Chung, Y.L., "Mechanical behavior of functionally graded material plates under transverse load—Part I: Analysis", International Journal of Solids and Structures, 43(13), pp. 3657-3674, 2006
[7] Samsam Shariat, B.A. and Eslami M.R., "Buckling of thick functionally graded plates under mechanical and thermal loads", Composite Structures. 78(3), pp. 433-439, 2007.
[8] Civalek, Ö. "Analysis of thick rectangular plates with symmetric cross-ply laminates based on first-order shear deformation theory", Journal of Composite Materials, 42(26), pp. 2853-2867, 2008.
[9] Zhao, X., Lee, Y.Y. and Liew, K.M., "Mechanical and thermal buckling analysis of functionally graded plates", Composite Structures, 90(2), pp. 161-171, 2009.
[10] Oyekoya, O.O., Mba, D.U. and El-Zafrany, A.M., "Buckling and vibration analysis of functionally graded composite structures using the finite element method", Composite Structures, 89(1), pp. 134-142, 2009.
[11] Zhao, X., Lee, Y. Y. and Liew, K. M., "Free vibration analysis of functionally graded plates using the element-free kp-Ritz method", Journal of sound and Vibration, 319(3), pp. 918-939, 2009.
[12] Mohammadi, M., Saidi, A.R. and Jomehzadeh, E., "Levy solution for buckling analysis of functionally graded rectangular plates", Applied Composite Materials, 17(2), pp. 81-93, 2010.

[13] Fereidoon, A., Asghardokht Seyedmahalle, M. and Mohyeddin, A., “Bending analysis of thin functionally graded plates using generalized differential quadrature method”, Archive of Applied Mechanics, 81(11), pp. 1523-1539, 2011.
[14] Kumar, J.S., Reddy, B.S., Reddy, C.E. and Reddy, K.V.K., “Higher order theory for free vibration analysis of functionally graded material plates”, ARPN J. Eng. Appl. Sci., 6(10), pp. 105-111, 2011.
[15] Jadhav, P.A. and Bajoria, K.M., “Buckling of piezoelectric functionally graded plate subjected to electro-mechanical loading”, Smart Materials and Structures, 21(10), pp. 105005, 2012.
[16] Singh, J. and Shukla, K. K., “Nonlinear flexural analysis of functionally graded plates under different loadings using RBF based meshless method”, Engineering Analysis with Boundary Elements, 36(12), pp. 1819-1827, 2012.
[17] Daouadji, T.H., Tounsi and Adda Bedia, E-A. “Analytical solution for bending analysis of functionally graded plates”, Scientia Iranica, 20(3), pp. 516-523, 2013.
[18] Asemi, K. and Shariyat, M., “Highly accurate nonlinear three-dimensional finite element elasticity approach for biaxial buckling of rectangular anisotropic FGM plates with general orthotropy directions”, Composite Structures. 106, pp. 235-249, 2013.
[19] Czechowski, L. and Kowal-Michalska, K. “Static and dynamic buckling of rectangular functionally graded plates subjected to thermal loading”, Strength of Materials, 45(6), pp. 666-673, 2013.
[20] Civalek, Ö., Korkmaz, A. and Demir,C. “Discrete singular convolution approach for buckling analysis of rectangular Kirchhoff plates subjected to compressive loads on two-opposite edges” Advances in Engineering Software, 41(4), pp. 557-560, 2010.
[21] Tahouneh, V., “Free vibration analysis of thick CGFR annular sector plates resting on elastic foundations”, Structural Engineering and Mechanics, 50(6), pp. 773-796, 2013.
[22] Swaminathan, K., and Naveenkumar, D. T. “Assessment of First Order Computational Model for Free Vibration Analysis of FGM Plates”, International Journal of Scientific and Engineering Research, 4(5), pp. 115-118, 2013.
[23] Jin, G., Su, Z., Ye, T. and Gao, S., “Three-dimensional free vibration analysis of functionally graded annular sector plates with general boundary conditions”, Composites Part B: Engineering, 83, pp. 352-366, 2015.
[24] Akbaş, Ş.D., “Termo-Mechanical Vibration of Functionally Graded Nano Plates and Beams Based on Couple Stress Theory”, 3rd International Conference on Advanced Technology Sciences, Konya/Turkey, 01-03 September, 2016.
[25] Van Long, N., Quoc, T.H. and Tu, T.M., “Bending and free vibration analysis of functionally graded plates using new eight-unknown shear deformation theory by finite-element method”, International Journal of Advanced Structural Engineering, 8(4), pp. 391-399, 2016.
[26] Civalek, Ö. “Free vibration of carbon nanotubes reinforced (CNTR) and functionally graded shells and plates based on FSDT via discrete singular convolution method” Composites Part B: Engineering, 111, pp. 45-59, 2017.
[27] Barati, M.R. and Zenkour, A.M., “Electro-thermoelastic vibration of plates made of porous fuctionally graded piezoelectric materials under various boundary conditions”, Journal of Vibration and Control, doi: 10.1177/1077546316672788, 2016.
[28] Mercan, K., Demir, Ç. And Civalek, Ö. “Vibration analysis of FG cylindrical shells with power-law index using discrete singular convolution technique”Curved and Layered Structures, 3(1), 2016.
[29] Wattanasakulpong, N. and Ungbhakorn, V., “Linear and nonlinear vibration analysis of elastically restrained ends FGM beams with porosities”, Aerospace Science and Technology, 32(1), pp. 111-120, 2014.
[30] Mechab, I., Mechab, B., Benaissa, S., Serier, B., Bouiadjra, B.B., “Free vibration analysis of FGM nanoplate with porosities resting on Winkler Pasternak elastic foundations based on two-variable refined plate theories”, Journal of the Brazilian Society of Mechanical Sciences and Engineering, 38(8), pp. 2193–2211, 2016.
[31] Mechab, B., Mechab, I., Benaissa, S., Ameri, M. and Serier, B., “Probabilistic analysis of effect of the porosities in functionally graded material nanoplate resting on Winkler–Pasternak elastic foundations”, Applied Mathematical Modelling, 40(2), pp. 738-749, 2016.
[32] Şimşek, M. and Aydın, M., “Size-dependent forced vibration of an imperfect functionally graded (FG) microplate with porosities subjected to a moving load using the modified couple stress theory”, Composite Structures, 160, pp. 408-421, 2017.
[33] Al Jahwari, F.and Naguib, H.E., “Analysis and homogenization of functionally graded viscoelastic porous structures with a higher order plate theory and statistical based model of cellular distribution”, Applied Mathematical Modelling, 40(3), pp. 2190-2205, 2016.
[34] Ebrahimi,F. and Jafari, A., “A Higher-Order Thermomechanical Vibration Analysis of Temperature-Dependent FGM Beams with Porosities”, Journal of Engineering, doi:10.1155/2016/9561504, 2016.
[35] Ebrahimi, F., Ghasemi, F. and Salari, E., “Investigating thermal effects on vibration behavior of temperature-dependent compositionally graded Euler beams with porosities”, Meccanica, 51(1), pp. 223-249, 2016.
[36] Chen, D., Yang, J. and Kitipornchai, S., “Nonlinear vibration and postbuckling of functionally graded graphene reinforced porous nanocomposite beams”, Composites Science and Technology, 142, pp. 235-245, 2017.
[37] Kitipornchai, S., Chen, D. and Yang, J., “Free vibration and elastic buckling of functionally graded porous beams reinforced by graphene platelets”, Materials & Design, 116, pp. 656-665, 2017.