



The effect of small scale on the vibrational behavior of single-walled carbon nanotubes with a moving nanoparticle

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Received February 06 2017; Revised April 02 2017; Accepted for publication April 06 2017.
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Abstract

In this paper, free and forced vibration of simply-supported Single-walled carbon nanotube is investigated under the moving nanoparticle by considering nonlocal cylindrical shell model. To validate the theoretical results, modal analysis of nanotube is conducted using ANSYS commercial software. Excellent agreement is exhibited between the results of two different methods. Furthermore, the dynamic response of SWCNT under moving nanoparticle is also studied. It is assumed that the nanoparticle travels along the center of nanotube with constant velocity and the van der Waals force between CNT and particle is taken into account. The dynamic response of the SWCNT under the influence of C₆₀ particle obtained using dynamic Green's function and modal expansion. The obtained results show that the nonlocal scale effect decreases the natural frequency and dynamic displacement of the CNT.

Keywords: SWCNT, Nonlocal scale effects, Moving nanoparticle, Cylindrical shell, Dynamic and vibration.

1. Introduction

The discovery of carbon nanotubes by Iijima [1] has given rise to speculation on many new potential Nano devices. owing to the unique mechanical properties of carbon nanotubes, such as high strength, low weight and flexibility [2], both multi-walled and single-walled carbon nanotubes promise many new applications in Micro/Nano mechanical systems [3-6]. However, owing to a lack of theoretical understanding of their precise behaviour and also their behaviour when they interact with their environment, there remain many fundamental challenges incorporating carbon nanotubes into a system. Single-walled carbon nanotubes divided into 3 major categories; chiral, zigzag and armchair. The difference between these types of nanotube is in the rolling angle of Graphene sheet. Armchair and chiral have metallic properties while zigzag has semi-conductive ones [7]. The application of the CNTs in transistors, sensors, displays, composites, and medical devices is the evidence of the importance of the CNTs in industry. Drug delivery is a rapidly growing area that is now taking the advantage of nanotube technology. In recent years, several ways of attaching small molecules such as anti-cancer drugs, with covalent and noncovalent bond have been presented. Also, theoretical models for drug delivery using CNTs are presented to deliver drug molecules to the target cells [8]. Nanotube attached to DNA plays an important role as a biosensor for detecting Nucleic acids [9]. There are several methods to investigate the mechanical behaviour of nanotubes. Experimental methods, molecular dynamics and continuum models are the ways of investigating the CNT's behaviour. The experimental methods are usually for determination of the physical and mechanical properties of nanotube, such as length, density, Young's modulus and etc. AFM, TEM and SEM are the microscopes that are used in this method. In molecular dynamics, all the atoms of carbon simulated by nodes and the covalent band between them simulated by elements in FE software. In spite of high accuracy of this method, it's too expensive and takes too much time. So, researchers tend to use continuum models for simulating the nanotube. Beam and shell models are usual methods used for analysing carbon nanotubes. Nonlocal theory presented by Eringen [10] can be added to these models to account for the mechanical

behaviour in nanoscale. Zhang and Xie [11] investigated the vibrational behaviour of carbon nanotubes by using nonlocal beam models. They showed that the small-scale parameter had a significant effect on the predicted natural frequency in which by increasing the nonlocal parameter, the natural frequency decreases. Wang and Vardan [12] studied the free vibration of nanotubes by nonlocal Euler-Bernoulli beam theory and nonlocal Timoshenko beam model. Wang et al. [13] investigated the vibration of nanotube by using nonlocal Timoshenko beam model. Their results demonstrated that, decreasing in the value of the natural frequency due to the increase of the nonlocal parameter is more significant for the higher modes. Ansari and Sahmani [14] investigated the nonlocal Euler-Bernoulli, Timoshenko and Reddy beams for free vibration of the nanotubes and compared the corresponding natural frequencies of three methods for different boundary conditions.

There are many studies and investigations exploring the free vibration of carbon nanotubes. However, the dynamic behaviour of the CNTs is still an important issue that researchers have paid less attention to it. Lee and Chang [15] studied the dynamic response of the nanotube in the presence of moving nanoparticle. They used nonlocal Euler-Bernoulli beam theory for simulating the behaviour of nanotube. For nanoparticle, C60 molecule was chosen to move along the nanotube. They investigated the effects of nonlocal theory on the dynamic response of nanotubes and showed that by increasing the nonlocal parameter, the maximum of the dynamic deflection decreases. Kiani and Mehri [16] compared the dynamic response of a nanotube under moving mass by assuming different beam models. Euler-Bernoulli, Timoshenko and high order beam theories were investigated in their paper. Their results showed that the effects of shear deformation theory have significant influence on the dynamic response of a nanotube. Pouryousefi et al. [17] investigated the influence of moving mass along a nanotube on the mechanical response of nanotube by considering nonlocal Rayleigh and Timoshenko beam models. They showed that for nanotube with a small length, and for higher nonlocal values, Timoshenko and Rayleigh beams have different results; while, for bigger values of nanotube length, the results of two models are approximately the same. In spite of the convenience of beam models for their simplifications and assumptions, they may have meaningful errors in the case of 3 dimensional problems. Shell models, instead, are suitable for modelling and analysing of nanotubes to capture more accurate results in 3 dimensions.

In this paper, the nonlocal cylindrical shell method with first order shear deformation effects is employed to investigate the dynamic response of the nanotube under the influence of moving nanoparticle. To achieve the dynamic response, Green's function and modal expansion method are used to obtain the time history response of the nanotube. The nonlocal theory effects also applied to the stiffness equations. The nanotube is simply supported and the initial conditions are set to be zero. The effects of nonlocal parameter, environment stiffness and velocity of the nanoparticle on nanotube's dynamic response are investigated in this paper.

2. Methodology

Before investigating the equilibrium equation of cylindrical shell, the nonlocal theory must be presented.

2.1 Nonlocal theory of elasticity

The constitutive equation of classical elasticity is an algebraic relationship between stress and strain tensors while Eringen nonlocal elasticity includes spatial integrals which indicate the average effect of strain of all points of the body to the stress tensor at the given point [10]. Since the spatial integrals in constitutive equations are mathematically difficult to solve, they can be converted to the equal differential constitutive equations under certain conditions. The nonlocal constitutive stress-strain relation for an elastic cylindrical shell can be simplified as [10]:

$$(1 - (e_0 a)^2 \nabla^2) \sigma = \varepsilon E \quad (1)$$

Where $e_0 a$ is nonlocal parameter that depends on the atomic properties of the sample, ∇^2 is Laplacian operator, σ is tension, ε is the strain tensor of the whole body and E is the Young's modulus.

2.2 Cylindrical Shell equations of motion

According to applied coordinate system to cylindrical shell shown in figure 1, the displacement in the x , θ and z direction are;

$$\begin{aligned} u_x(x, \theta, z, t) &= u(x, \theta, t) + z\psi_x(x, \theta, t) \\ u_\theta(x, \theta, z, t) &= v(x, \theta, t) + z\psi_\theta(x, \theta, t) \\ u_z(x, \theta, z, t) &= w(x, \theta, t) \end{aligned} \quad (2)$$

Where, u , v and w are the reference surface displacement in the x , θ and z direction. ψ_x and ψ_θ are also the rotation of the middle surface around the x and θ direction.

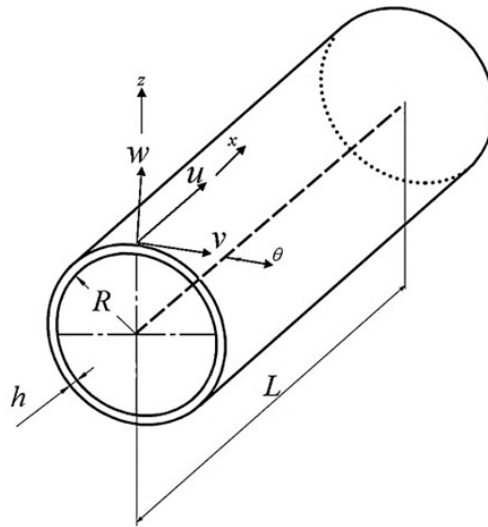


Fig. 1. Applied coordinate system to cylindrical shell.

The normal and shear strains of the middle surface and the curvatures (k_{ij}) are given by;

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{x\theta} \\ k_x \\ k_\theta \\ k_{x\theta} \\ \gamma_{xz} \\ \gamma_{\theta z} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{1}{R} & 0 & 0 \\ \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\partial}{\partial x} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R} \frac{\partial}{\partial \theta} \\ 0 & 0 & 0 & \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial x} & 1 & 0 \\ 0 & -\frac{1}{R} & \frac{1}{R} \frac{\partial}{\partial \theta} & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ \psi_x \\ \psi_\theta \end{bmatrix} \quad (3)$$

Where, R is the nanotube radius. Normal, shear and moment force relations by the effects of nonlocal theory are described as follows:

$$\begin{bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \\ Q_{xx} \\ Q_{\theta\theta} \end{bmatrix} - (e_0 a)^2 \nabla^2 \begin{bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \\ Q_{xx} \\ Q_{\theta\theta} \end{bmatrix} = \begin{bmatrix} \frac{Eh}{1-\nu^2} \frac{\partial}{\partial x} & \frac{\nu Eh}{1-\nu^2} \frac{\partial}{R \partial \theta} & \frac{\nu Eh}{1-\nu^2} \frac{1}{R} & 0 & 0 \\ \frac{\nu Eh}{1-\nu^2} \frac{\partial}{\partial x} & \frac{Eh}{1-\nu^2} \frac{\partial}{R \partial \theta} & \frac{Eh}{1-\nu^2} \frac{1}{R} & 0 & 0 \\ Gh \frac{\partial}{R \partial \theta} & Gh \frac{\partial}{\partial x} & 0 & 0 & 0 \\ 0 & 0 & 0 & D \frac{\partial}{\partial x} & D \frac{\nu}{R} \frac{\partial}{\partial \theta} \\ 0 & 0 & 0 & D \nu \frac{\partial}{\partial x} & D \frac{1}{R} \frac{\partial}{\partial \theta} \\ 0 & 0 & 0 & \frac{D(1-\nu)}{2} \frac{1}{R} \frac{\partial}{\partial \theta} & \frac{D(1-\nu)}{2} \frac{\partial}{\partial x} \\ 0 & 0 & Gh \frac{\partial}{\partial x} & Gh & 0 \\ 0 & -\frac{Gh}{R} & \frac{Gh}{R} \frac{\partial}{\partial \theta} & 0 & Gh \end{bmatrix} \begin{bmatrix} u \\ v \\ w \\ \psi_x \\ \psi_\theta \end{bmatrix} \quad (4)$$

where;

$$\begin{bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ Q_{xx} \\ Q_{\theta\theta} \end{bmatrix} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \\ \sigma_{xz} \\ \sigma_{\theta z} \end{bmatrix} dz, \quad \begin{bmatrix} M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{bmatrix} = \int_{-\frac{h}{2}}^{+\frac{h}{2}} \begin{bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \end{bmatrix} z dz \quad (5)$$

And also, h is the thickness of the nanotube, D and G denote the bending and shear rigidity of the shell, and ν is the Poisson ratio.

According to the above equations, the equilibrium equations of nonlocal cylindrical shell with the consideration of first order shear deformation effects are;

$$\begin{aligned} (1-(e_0a)^2 \nabla^2) \left(-R \frac{\partial N_{xx}}{\partial x} - \frac{\partial N_{x\theta}}{\partial \theta} + R \rho h \ddot{u} \right) &= (1-(e_0a)^2 \nabla^2) R q_x \\ (1-(e_0a)^2 \nabla^2) \left(-R \frac{\partial N_{x\theta}}{\partial x} - \frac{\partial N_{\theta\theta}}{\partial \theta} - k' Q_{\theta\theta} + R \rho h \ddot{v} \right) &= (1-(e_0a)^2 \nabla^2) R q_\theta \\ (1-(e_0a)^2 \nabla^2) \left(-R k' \frac{\partial Q_{xx}}{\partial x} - k' G h \frac{\partial Q_{\theta\theta}}{\partial \theta} + N_{\theta\theta} + R \rho h \ddot{w} \right) &= (1-(e_0a)^2 \nabla^2) R q_z \\ (1-(e_0a)^2 \nabla^2) \left(R \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{\theta x}}{\partial \theta} - k' Q_{xx} - \frac{R \rho h^3}{12} \ddot{\psi}_x \right) &= 0 \\ (1-(e_0a)^2 \nabla^2) \left(R \frac{\partial M_{x\theta}}{\partial x} + \frac{\partial M_{\theta\theta}}{\partial \theta} - k' Q_{\theta\theta} - \frac{R \rho h^3}{12} \ddot{\psi}_\theta \right) &= 0 \end{aligned} \quad (6)$$

Where q_i is the external force in the i th direction, and k' is the shear correction factor which equals to $5/6$ [14]. Also Laplacian operator can be defined as $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$. Finally, the following equations of motion are achieved for circular cylindrical shell considering first order shear deformation effects.

$$\begin{aligned} \frac{Eh}{1-\nu^2} \frac{\partial^2 u}{\partial x^2} + \left(\frac{1}{R} \right)^2 \frac{Eh}{2(1+\nu)} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{R} \left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} \right) \frac{\partial^2 v}{\partial x \partial \theta} \\ + \frac{1}{R} \frac{Eh\nu}{1-\nu^2} \frac{\partial w}{\partial x} - (1-(e_0a)^2 \nabla^2) \rho h \ddot{u} = (1-(e_0a)^2 \nabla^2) q_x \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{1}{R} \left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)} \right) \frac{\partial^2 u}{\partial x \partial \theta} + \left(\frac{Eh}{2(1+\nu)} \right) \frac{\partial^2 v}{\partial x^2} - \left(\frac{1}{R} \right)^2 \frac{Eh}{1-\nu^2} \frac{\partial^2 v}{\partial \theta^2} \\ - k' \frac{Gh}{R^2} v - \left(\frac{1}{R} \right)^2 \left(\frac{Eh}{1-\nu^2} + k' Gh \right) \frac{\partial w}{\partial \theta} + k' \frac{Gh}{R} \psi_\theta - (1-(e_0a)^2 \nabla^2) \rho h \ddot{v} = (1-(e_0a)^2 \nabla^2) q_\theta \end{aligned} \quad (8)$$

$$- \frac{1}{R} \frac{Eh\nu}{1-\nu^2} \frac{\partial u}{\partial x} + \left(\frac{1}{R} \right)^2 \left(\frac{Eh}{1-\nu^2} + k' Gh \right) \frac{\partial v}{\partial \theta} + k' Gh \frac{\partial^2 w}{\partial x^2} - \left(\frac{1}{R} \right)^2 k' Gh \frac{\partial^2 w}{\partial \theta^2} \quad (9)$$

$$\begin{aligned} - \left(\frac{1}{R} \right)^2 \frac{Eh}{1-\nu^2} w + k' Gh \frac{\partial \psi_x}{\partial x} + k' \frac{Gh}{R} \frac{\partial \psi_\theta}{\partial \theta} - (1-(e_0a)^2 \nabla^2) \rho h \ddot{w} = (1-(e_0a)^2 \nabla^2) q_z \\ - k' Gh \frac{\partial w}{\partial x} + D \frac{\partial^2 \psi_x}{\partial x^2} - k' Gh \psi_x + \left(\frac{1}{R} \right)^2 \frac{(1-\nu)D}{2} \frac{\partial^2 \psi_x}{\partial \theta^2} \end{aligned} \quad (10)$$

$$\begin{aligned} + \frac{1}{R} \left(\nu D + \frac{(1-\nu)D}{2} \right) \frac{\partial^2 \psi_\theta}{\partial x \partial \theta} - (1-(e_0a)^2 \nabla^2) \frac{\rho h^3}{12} \ddot{\psi}_x = 0 \\ k' \frac{Gh}{R} v + k' \frac{Gh}{R} \frac{\partial w}{\partial \theta} + \frac{1}{R} \left(\nu D + \frac{(1-\nu)D}{2} \right) \frac{\partial^2 \psi_x}{\partial x \partial \theta} - k' Gh \psi_\theta \\ + \frac{(1-\nu)D}{2} \frac{\partial^2 \psi_\theta}{\partial x^2} + \left(\frac{1}{R} \right)^2 D \frac{\partial^2 \psi_\theta}{\partial \theta^2} - (1-(e_0a)^2 \nabla^2) \frac{\rho h^3}{12} \ddot{\psi}_\theta = 0 \end{aligned} \quad (11)$$

3. Free vibration of CNT

To obtain the impact of nonlocal scale on the vibrational behavior of SWCNT, at first, free vibration of single-walled carbon nanotube is investigated. Soedel [18] showed that for simply supported boundary condition, the following answers can be assumed by Fourier transform;

$$\begin{aligned}
 u(x, \theta, t) &= U \cos\left(\frac{m\pi x}{L}\right) \cos(n\theta) e^{i\omega t} \\
 v(x, \theta, t) &= V \sin\left(\frac{m\pi x}{L}\right) \sin(n\theta) e^{i\omega t} \\
 w(x, \theta, t) &= W \sin\left(\frac{m\pi x}{L}\right) \cos(n\theta) e^{i\omega t} \\
 \psi_x(x, \theta, t) &= \Psi_x \cos\left(\frac{m\pi x}{L}\right) \cos(n\theta) e^{i\omega t} \\
 \psi_\theta(x, \theta, t) &= \Psi_\theta \sin\left(\frac{m\pi x}{L}\right) \sin(n\theta) e^{i\omega t}
 \end{aligned} \tag{12}$$

Where L is the nanotube length, m is the axial half wave number, n is the circumferential mode number and ω is the natural frequency. The above-mentioned answers should satisfy the simply supported boundary conditions, note that for this type of boundary conditions we have;

$$\begin{aligned}
 v(0, \theta, t) &= w(0, \theta, t) = M_{xx}(0, \theta, t) = N_{xx}(0, \theta, t) = \psi_\theta(0, \theta, t) \\
 v(L, \theta, t) &= w(L, \theta, t) = M_{xx}(L, \theta, t) = N_{xx}(L, \theta, t) = \psi_\theta(L, \theta, t)
 \end{aligned} \tag{13}$$

Finally, by assuming equations (13) and substituting equation (7) in equation (6) and solving the eigenvalue problem, the natural frequency of SWCNT will be obtained. Noted that, for cylindrical shell equations by considering the first order shear deformation effects, there will be five distinct frequencies for every m and n combination. To validate the accuracy of this method, modal analysis of SWCNT is conducted using ANSYS commercial software. In this research, Shell181 element is used to account for the shear deformation effects. It should be noted that ANSYS employs the classical method in its analysis and therefore we have to put the nonlocal parameter equal to 0 in cylindrical shell equations. Figure 2 shows the type of boundary conditions at each end of nanotube in FEM software.

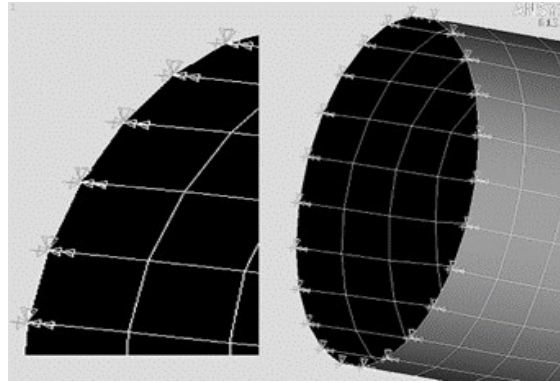


Fig. 2. The type of boundary conditions at each end of nanotube

4. Dynamic response of SWCNT under moving nanoparticle

After obtaining the natural frequency, the dynamic response of the SWCNT under moving mass is investigated. To this end, Green's function is used to achieve the time history response of the nanotube. In addition, in order to obtain the appropriate results, modal expansion effects are included in the equations. Modal expansion is the effects of other mode shapes in the displacement of cylindrical shell.

In this study, the C60 fullerene is taken into consideration as the moving mass. Studies show that a C60 molecule located on the axis of a nanotube and a short distance away will be sucked into the nanotube and spontaneously begin oscillatory motion due to the van der Waals interaction between nanotube and the fullerene molecule [19]. It's also assumed that the moving mass with mass m travels along the centerline of nanotube with a constant velocity of v . The van der Waals interaction is modeled by spring type forces acts in the z direction. The nanotube lies on an elastic foundation with uniform stiffness along the nanotube. The SWCNT is simply supported at its both ends and its initial

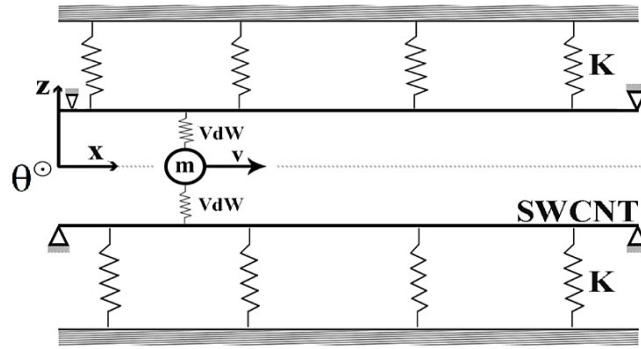


Fig. 3. Schematic representation of the current problem.

According to the mentioned assumptions, the external force applied on the nanotube is only in the z direction and equals to

$$q_x = q_\theta = 0, \quad q_z = (mg - K_{vdw}w)\delta(x - vt) - Kw \quad (14)$$

where K_{vdw} is the van der Waals stiffness constant, mg is the weight of the moving mass, δ is the Dirac delta function and K is the foundation stiffness. According to Soedel [18] assumptions, the transverse dynamic response of a simply supported cylindrical shell, using Green's function and modal expansion with zero initial condition is expressed as;

$$w(x, \theta, t) = \left[\frac{F}{\rho h} \frac{1}{\omega N_k} W \sin\left(\frac{m\pi x}{L}\right) \cos(n\theta) \right] \times I_k \quad (15)$$

Where ρ is the cylindrical shell density, ω is the natural frequency and h is the thickness of the shell. In addition, the parameters F and N_k are described as;

$$\begin{aligned} F &= (1 - (e_0 a)^2 \nabla^2) (mg - K_{vdw}w) = \\ &= \left\{ mg - [1 + (e_0 a)^2 \left(\left(\frac{n\pi x}{L} \right)^2 + \left(\frac{m}{R} \right)^2 \right)] K_{vdw}w \right\} \\ N_k &= \frac{RL\pi}{2} (U^2 + V^2 + W^2 + \Psi_x^2 + \Psi_\theta^2) \end{aligned} \quad (16)$$

Where, the nonlocal theory parameter is applied to the right side of governing equation and without the damping effect, I_k is given by

$$I_k = \int_0^t \sin \omega(t - t^*) \sin \frac{m\pi v}{L} t^* dt^* \quad (17)$$

In equation (17), t^* is the time in which the moving mass is at the position of x^* . The dimensionless parameter α is the speed ratio with respect to the critical velocity. Critical velocity is the speed at which the moving nanoparticle will excite the SWCNT at its fundamental resonance frequency and given by

$$v_{cr} = \frac{L\omega}{n\pi} \quad (18)$$

5. Results and Discussion

In this paper a (10,10) single-walled carbon nanotube is considered for free and forced vibrational behavior of CNT with 0.68 nm radius, Young's modulus of 1 TPa and length of 40 nm and 35nm thickness. Moreover, C60 molecule is modeled as the nanoparticle with the mass of 1.196×10^{-24} kg and the van der Waals interaction between nanotube and C60 fullerene is assumed to be 3.24 eV [19].

Table 1 shows the comparison between the results of the asymptotic study and those obtained by FEM modal analysis. As shown in Table 1, there exists a good agreement between the obtained results of two different methods. It is noted that, ANSYS software analyses the problems with the consideration of classical theory of elasticity and we must set the nonlocal parameter equal to zero.

Table 1. The natural frequency of CNT from the asymptotic analysis and FEM model.

Length	20 (nm)	15 (nm)	10 (nm)	5 (nm)
<i>Current study</i>	0.038760	0.068101	0.14842	0.51604
<i>ANSYS</i>	0.038809	0.068221	0.14888	0.52085
<i>Error</i>	0.12%	0.17%	0.3%	0.92%

According to the tabulated results in Table 1, one can observe that by increasing the length of the nanotube, the natural frequency of the structure decreases. Generally speaking, it is inferred that the stiffness to mass ratio decreases by increasing the length of nanotube, and therefore the natural frequency of nanotube decreases. To better show the influence of nanotube length on the natural frequency of CNT, Fig. 4 exhibits the variation of natural frequency versus the length of nanotube. In this figure, the nonlocal scale effect is set to 0.87 nm and wave numbers are $n=m=1$. As can be seen, in the smaller values, the trend of reduction in the slope of the natural frequency is much greater than those for higher values.

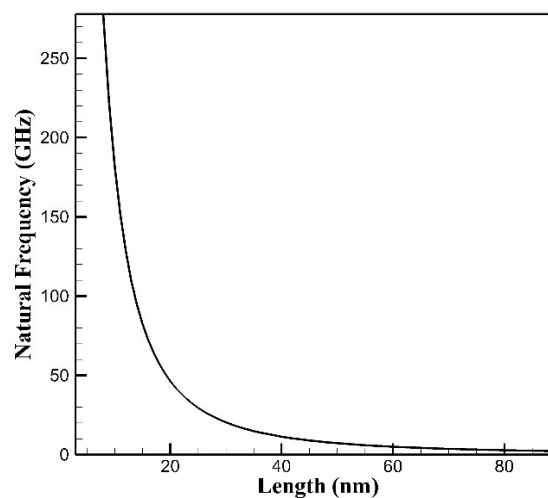
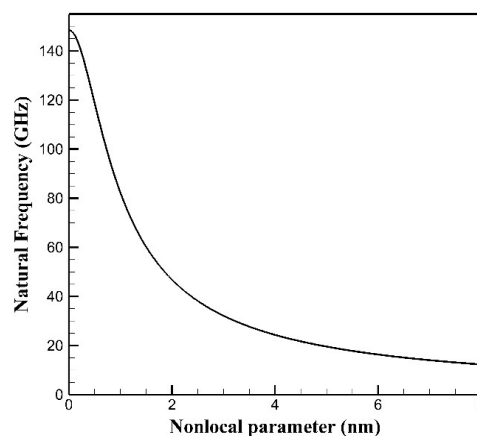
**Fig. 4.** Natural frequency of SWCNT versus the lengths of nanotube.

Figure 5 represents the effect of nonlocal parameter on the natural frequency of the nanotube. As can be observed, by increasing the nonlocal scale effects, the natural frequency of the nanotube decreases and thereby one can conclude that the frequency predicted by the nonlocal model is always less than those by the classical method.

**Fig. 5.** Natural frequency of SWCNT versus the nonlocal parameter.

According to figure 5, it is concluded that by increasing the nonlocal parameter, the value of natural frequency is less sensitive to the nano-scale parameter. Figure 6 shows the variation of natural frequency of nanotube versus the non-dimensional parameter L/R for different values of nonlocal parameter. When this ratio is small, the difference between the results of nonlocal theory and classical one is meaningful. However, by increasing the length to radius ratio, the results predicted by nonlocal theory get closer to the classical ones.

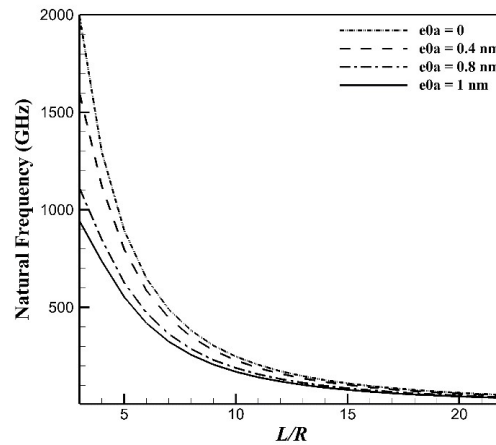


Fig. 6. Effects of nonlocal parameter and length of nanotube on the natural frequency.

To investigate the time history response of CNT in the presence of moving nanoparticle, the dimensionless displacement of the nanotube (the ratio of the nanotube deflection under moving mass with respect to the static one) is plotted. One notes that the static deflection is the static transverse displacement at the mid-span of the SWCNT owing to the nanoparticle weight. Moreover, the parameter T is the travel time of the moving nanoparticle through the SWCNT. Figure 7 shows the dynamic response of the nanotube for different nonlocal parameters. As can be seen from the diagram, the maximum dynamic displacement is not located at the mid-span of the SWCNT. This is due to the effect of Green's function on the displacement of the nanotube. It should be noted that the location of the maximum dynamic displacement of nanotube depends on the velocity of moving nanoparticle. In addition, one can observe that the dimensionless transverse displacement of nanotube decreases by increasing the small scale parameter.

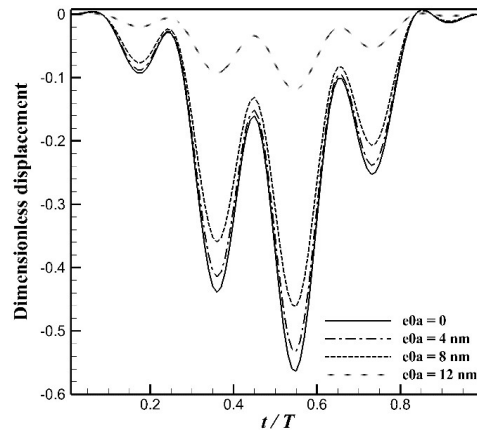


Fig. 7. Dimensionless displacement of the SWCNT under moving nanoparticle for different nonlocal values.

In order to check the integrity of the dynamic results in this section, a comparison is conducted with the results of Lee and Chang [15]. It is noted that they employed the Euler-Bernoulli beam method to model the dynamic behavior of CNTs. One can observe that there exist good agreements between the present results and those reported by Lee and Chang.

Figure 8 shows the dimensionless displacement of the nanotube for some assigned values for the velocity of moving nanoparticle. It is shown that by increasing the velocity, the maximum displacement of the nanotube increases. Also it is clear from this figure that the speed of the moving nanoparticle defines the location of the maximum displacement.

Table 2. Maximum dynamic deflection obtained by different models

Nonlocal parameter	0 nm	4 nm	8 nm
Cylindrical shell method	0.56	0.53	0.46
Euler-Bernoulli beam method [11]	0.54	0.53	0.42

Finally, figure 9 presents the effects of foundation stiffness on the dynamic displacement of nanotube. It is concluded that by increasing the foundation stiffness, the total stiffness of the system increases and consequently the dynamic displacement of nanotube decreases.

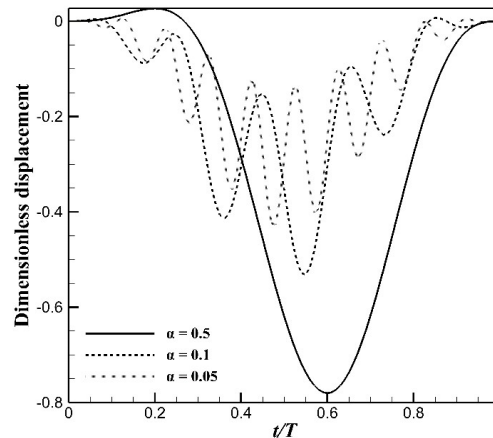


Fig. 8. Dimensionless displacement of the SWCNT under moving nanoparticle for different velocity ratios.

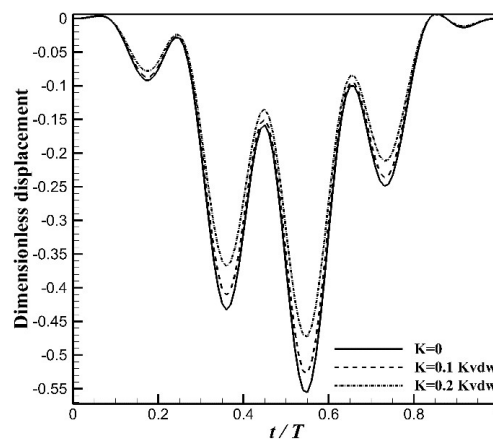


Fig. 9. Dimensionless displacement of the SWCNT under moving nanoparticle for different foundation stiffness.

6. Conclusions

In this paper, the effect of nonlocal elasticity on the free and forced vibrational behavior of single-walled carbon nanotube was investigated. Cylindrical shell model with the consideration of first order shear deformation theory was developed. Furthermore, to check the validity of the obtained results, nanotube was modeled in ANSYS software and by conducting modal analysis, the natural frequencies were obtained. Results of this investigation showed that, by increasing the nonlocal parameter, the natural frequency of nanotube decreases and any increase in the nanotube length decreases the frequency of nanotube.

In the second part, the dynamic response of the single-walled carbon nanotube under moving nanoparticle was investigated. Green's function and modal expansion method were used to obtain the dynamic displacement. By considering C60 molecule as the moving mass, the van der Waals interaction between nanotube and molecule was applied. Finally, the dynamic response of the nanotube under the effect of moving mass was investigated. It was demonstrated that by increasing the nonlocal parameter, the dynamic response of the nanotube decreases.

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