



Shape and geometrical parameter effects of a bimorph piezoelectric beam on energy harvesting performance

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Abstract

In this paper, the shape influence of piezoelectric beams including triangle, trapezoid, rectangle, inverted trapezoid, convex parabola, concave parabola, and comb-shaped (a combination of two triangular beams with a connector of 4 mm length) are addressed and analyzed by FEM. The analysis is performed for a bimorph piezoelectric beam. The analyzed parameters include the beam length, thickness and width of the piezoelectric layer. The study is performed using COMSOL Multiphysics software for all seven shapes. The results show that due to the mechanical properties of the beams, the natural frequency of the triangular beam is more for all considered parameters. In addition, as the width of the beam end increases, the natural frequency reduces, too. Since natural frequency is inversely related to electric power, the inverted trapezoidal beam has the highest electric power and the triangular beam has the lowest one.

Keywords: Piezoelectric beam, Bimorph, Geometry, Finite element.

1. Introduction

In recent decades, energy consumption in electronic devices, such as wireless sensor networks, has constantly declined. Despite current significant progress in the field to increase the energy contained in the storage systems (batteries), the limitation in the volume, weight and longevity of the systems is still considered as their major weakness. There are some techniques, such as radio waves that are extremely harmful to human health when a large number of nodes are required. The third method is attractive as the sensor life is highly dependent on the fatigue of components. In this technique, in fact, the sensor does not lose its charge and efficiency, unless the sensor components are damaged. In this method, known as energy harvesting, environmental energy sources such as wind energy, solar energy, acoustic waves, vibrations, etc. In addition to converting into electric energy are used. In this paper, converting vibrational energy in the piezoelectric cantilever beams into the electric energy was taken into consideration to achieve the desired outcomes. Three mechanisms were considered to convert vibrational energy into electricity: electrostatic [1, 2], electromagnetic [3] and piezoelectric [4]. Today, piezoelectric materials are more important than the two other mechanisms due to their higher electromechanical coupling and power density. Among energy harvesters, the bridges, diaphragms and cantilever beams can be noted. The cantilever beams are used more frequently due to their higher productivity as well as lower natural frequency. Among the studies, whether analytical or numerical, done in this field, the following could be mentioned. White et al. [5] examined two single-layer and multi-layer piezoelectric structures using the finite element method and ANSYS software. Liu and Lia [6] evaluated the static and dynamic performance of a bimorph piezoelectric beam using finite element method. Kumar and

Sharma [7] explored the performance of various piezoelectric materials for energy harvesting by simulation using first-order shear deformation theory (FSDT). Reddy et al. [8] examined the effect of trapezoidal cavity in the cantilever beam with a piezoelectric layer using the Euler-Bernoulli beam theory. Mateu and Moll [9] presented an analytical model to harvest the maximum energy from a piezoelectric beam inside the shoe heels. Lu et al. too [10] presented an analytical method based on electrical resistance and its effects on the amount of energy harvested from the piezoelectric harvesters. In the present study, due to the importance of the geometry of piezoelectric beams in harvesting the energy, the seven forms of bimorph beams, i.e. the triangular, trapezoidal, rectangular, inverted trapezoidal, convex parabolic, concave parabolic, and comb-shaped beams were analyzed. The considered outputs consist of natural frequency, capacitance, electric power, electric voltage and current. All outputs are considered with the optimal resistance.

2. Theoretical foundations

In this section, the analysis of a cantilever piezoelectric beam model is to be discussed. To do this, the analytical method of equivalent mechanical mass-spring-damper system was used to obtain the relations for electric power as well as optimal resistance. It should be noted that optimal resistance is the one that can be achieved for maximum power. Finally, the COMSOL finite element software is discussed to achieve desired outputs. Piezoelectricity is a behavior integrated with the electrical behavior of material:

$$D = \varepsilon E \quad (1)$$

Here, D is electric displacement, ε is permeability coefficient and E is the electric field, and according to Hooke's Law:

$$S = sT \quad (2)$$

In the above formula, S represents the strain, s the effective strain and T the load. Combining the two relations presented above yields [11]:

$$\{S\} = [s^E]\{T\} + [d]^T\{E\} \quad (3)$$

$$\{D\} = [d]\{T\} + [\varepsilon^T]\{E\} \quad (4)$$

By combining these two relations, an equation is derived, referred to as the combined strain – load equation. In the above equation, S represents strain, s^E strain on the constant electric field, T stress, and d is the piezoelectric constant, E represents the electric field, D denotes displacement of the electric field and ε^T is the dielectric coefficient.

One of the analytical methods for obtaining the equations related to the power extracted from the beam vibration is the equivalent spring-mass method. The purpose of this section is to use analytical equations and relationship between such parameters as electric power, natural frequency and so on. These relations are crucial in analyzing the obtained results. One of the analytical methods for obtaining the equations related to the power extracted from the beam vibration is the equivalent spring-mass method. In this method, it is necessary that the system be examined in the resonance mode in order to achieve maximum extracted power.

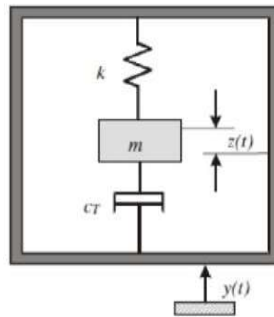


Fig. 1. Linear spring-mass system [12]

Here, m signifies mass, k the coefficient of spring stiffness and C_T the damping coefficient. In fact, C_T is the damping coefficient through which some energy is dissipated. The function $y(t)$ is defined here as an external sinusoidal vibration as follows [12]:

$$y(t) = Y \sin(\omega t) \quad (5)$$

In the spring-mass system, it is assumed that the mass of the vibration source is significantly larger than the mass inside the system. As indicated in Fig. 1, $z(t)$ represents the seismic mass displacement. The differential equation related to the motion of system is defined by:

$$m\ddot{z} + c\dot{z}(t) + kz(t) = -m\ddot{y} \quad (6)$$

Mass displacement can be obtained from the following equation:

$$z(t) = \frac{\omega^2}{\sqrt{\left(\frac{k}{m} - \omega^2\right)^2 + \left(\frac{c_T}{m}\right)^2}} Y \sin(\omega t - \varphi) \quad (7)$$

Here, φ represents the phase angle that is equal to:

$$\varphi = \tan^{-1} \left(\frac{C_T \omega}{(K - \omega^2 m)} \right) \quad (8)$$

Assuming the excitation frequency is equal to the natural frequency of the system, the maximum energy can be extracted from the system [13]:

$$Fr = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m_e}} \quad (9)$$

According to the above statements, the dissipated power at the presence of the damper is defined as follows [14]:

$$P_d = \frac{m\zeta_T Y^2 \left(\frac{\omega}{\omega_n}\right)^3 \omega^3}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta_T \left(\frac{\omega}{\omega_n}\right)\right]^2} \quad (10)$$

where ζ_T is damping ratio, Y is displacement amplitude, and ω is the excitation frequency. In case $\omega = \omega_n$, Eq. (10) can be rewritten as follows:

$$P_d = \frac{m Y^2 \omega_n^3}{4\zeta_T} \quad (11)$$

Also, to observe the acceleration term in Eq. (11), it can be defined as follows:

$$P_d = \frac{m N^2}{4\omega_n \zeta_T} \quad (12)$$

The optimum resistance is also obtained using analytical relations [15]:

$$R_{opt} = \frac{1}{\omega C_b} \frac{2\zeta}{\sqrt{4\zeta^2 + k_{31}^4}} \quad (13)$$

Capacitance is generally obtained according to Eq. (14):

$$C = K\epsilon \frac{A}{q} \quad (14)$$

In this formula, K is the dielectric coefficient, ϵ is the permittivity coefficient, A represents the cross-sectional area and q is the thickness of the piezoelectric layer. The polarization direction of piezoelectric layers is also incredibly important; in general, depending on the direction of polarity, the capacitors can be used in the circuit as either serial or parallel. In this paper, the capacitors are considered as parallel.

3. Modeling of cantilever piezoelectric beams in COMSOL software

To obtain the desired outputs, the basic step is to create the cantilever piezoelectric beam model. For this purpose, a cantilever piezoelectric beam was first modeled using the Geometry command. It is important to use the large mass method (LMM) in modeling the piezoelectric cantilever beams. In this method, instead of fixing the beam, a mass much larger than the mass of free end of the beam is added to one end of the beam that is supposed to be fixed, so that the mass could be at least 5 or 6 times the mass added to the free end. Finally, to make the cantilever beam vibrate, force is applied to the large mass in the beginning of the beam. Considering the large mass in the beginning of the beam, it is important to note that the mass should be considered as rigid and to do so, using the Rigid Domain command, the object model is considered as rigid. After modeling the cantilever beam, it is the time to define the material of each of components of the cantilever piezoelectric beam. To do so, using the Materials command, the specifications related to brass alloy, PZT-5H and steel were considered for the metal part of the beam, piezoelectric

material and the large mass in the beginning of the beam, respectively. These specifications are listed in Tables 1 and 2.

Table 1. Properties of brass, tungsten and steel

	Brass [16]	Steel
Density (ρ)	8800 $\frac{Kg}{m^3}$	7850 $\frac{Kg}{m^3}$
Poisson's ratio (ν)	0.34	0.33
Modulus of elasticity (E)	110 Gpa	200 GPa

Table 2. Properties of PZT-5H [17]

ρ	7500	$\frac{Kg}{m^3}$
d_{15}	741	$\times 10^{-1} \frac{C}{N}$
d_{31}	-274	
d_{33}	593	
K_{11}	3130	-
K_{22}	3130	
K_{33}	3400	
S_{11}	16.5	$\times 10^{-12} \frac{m^2}{N}$
S_{12}	-4.78	
S_{13}	-8.45	
S_{33}	20.7	
S_{44}	43.5	

Fig. 2 shows a bimorph cantilever piezoelectric beam. Table 3 shows the characteristics of the beam dimensions in the base status.

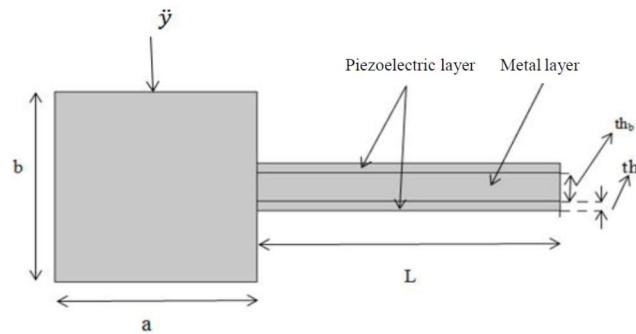
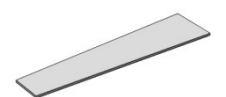


Fig. 2. Bimorph cantilever piezoelectric beam in base mode

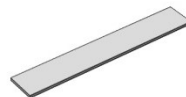
Table 3. Geometric dimensions of beam in base mode

L	W	th _b	th _p	B
3 cm	0.5 cm	0.02 cm	0.01 cm	1.5 cm

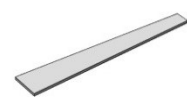
The geometry of piezoelectric beams was modeled and analyzed with seven shapes: triangle, trapezoid, rectangle, inverted trapezoid, convex parabola, concave parabola, and comb-shaped, as shown in the figures below.



a) Inverted trapezoid



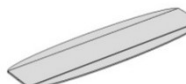
b) Rectangle



c) Trapezoid



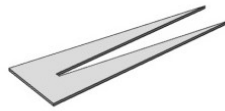
d) Triangle



e) Convex parabola



f) Concave parabola



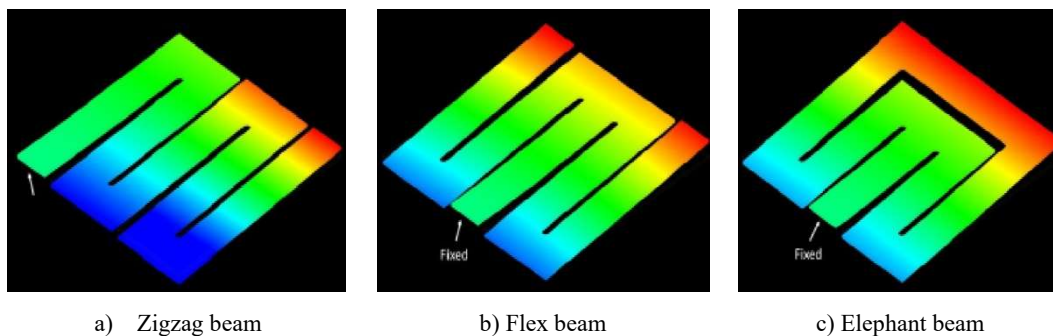
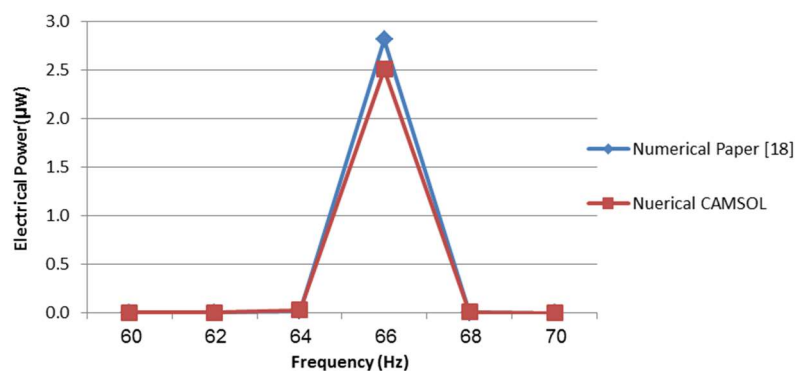
g) Comb-shaped

Fig. 3. A schematic diagram of the piezoelectric beams

4. Results

In this section, the obtained results by COMSOL finite element software will be reviewed. As evident in the research title, geometrical parameters such as beam length, thickness and width of the piezoelectric layer have already been studied in seven rectangular, triangular, trapezoidal, inverted trapezoidal, convex and concave parabolic beams, and finally a comb-shaped beam that is a combination of two triangular beams with a 4 mm long connector. It is notable that a constant acceleration of 9.8 m/s^2 was applied to the beam base. The outputs examined in this article included natural frequency, electric power, electric voltage, current passing through the piezoelectric layer, and capacitance.

In order to validate the works done in COMSOL software, the output powers of three cantilever beams were compared for the zigzag, Flex and Elephant shape in the presence of a piezoelectric layer. All geometrical figures are displayed in fig. 4. It should be noted that the extracted power is considered in optimal resistance. By comparing the results obtained by the COMSOL software with the results in [18], the accuracy of the modeling in the COMSOL software was confirmed.

**Fig. 4.** A schematic diagram of the zigzag, flex and elephant beam**Fig. 5.** Variations of power with respect to natural frequency in zigzag beam

4.1 Effect of piezoelectric beam length

4.1.1 Natural frequency

The figure below shows the first natural frequencies in the range of considered lengths for the seven shapes in the bimorph mode. Fig. 8 indicates that as length increases, due to changes in the mechanical properties of piezoelectric beam, there is a downward trend in the natural frequency for each of the seven shapes. The chart shows that the natural frequencies of the triangular and inverted trapezoidal beam have the highest and lowest values, respectively. According to Eq. (12) and the inverse relationship between electric power and natural frequency, in case the acceleration amplitude is constant, it is expected that as the beam length increases, the extracted power displays an upward trend. As evident in the chart of electric power, by increasing the length of piezoelectric beam, the extracted

power keeps rising. Among the analyzed shapes, the inverted trapezoid has the highest power and the triangular beam has the lowest one.

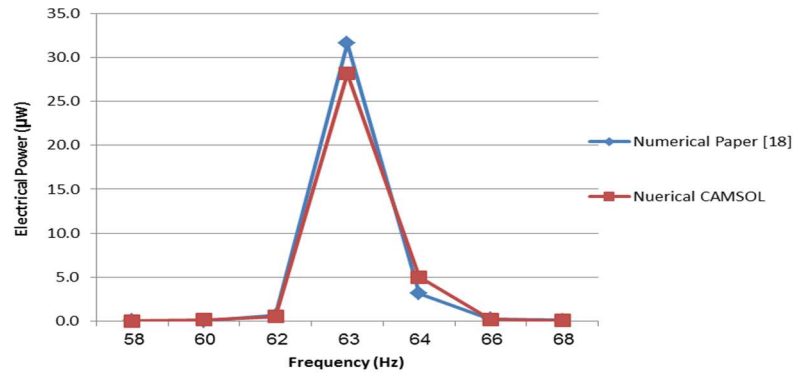


Fig. 6. Variations of power with respect to natural frequency in Flex beam

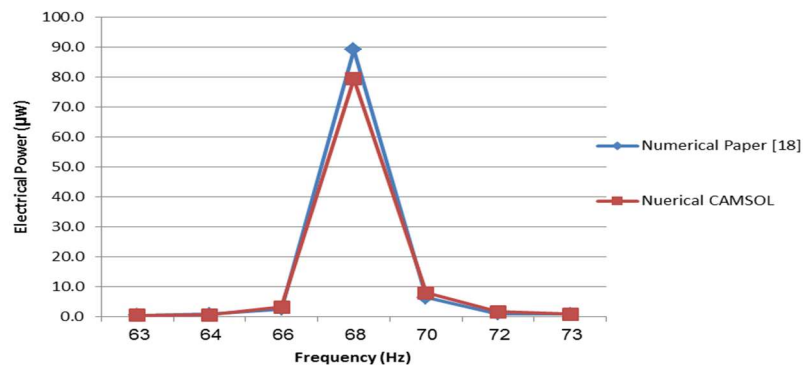


Fig. 7. Variations of power with respect to natural frequency in Elephant beam

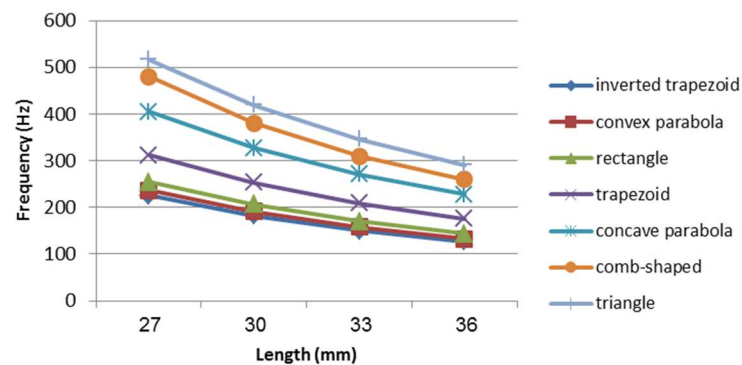


Fig. 8. Variation of natural frequency in terms of length

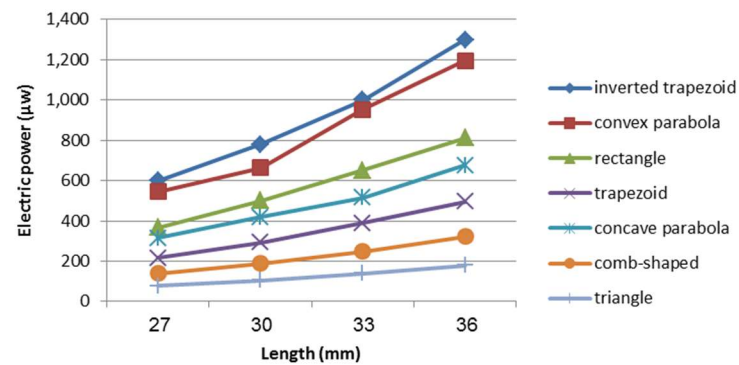


Fig. 9. Variation of electric power in terms of length

4.1.2 Capacitance

Based on Eq. (14), by increasing the beam length, the area covered by the electrodes on the piezoelectric layer

increases, which in turn causes the capacitance to rise, as shown in the Figure below.

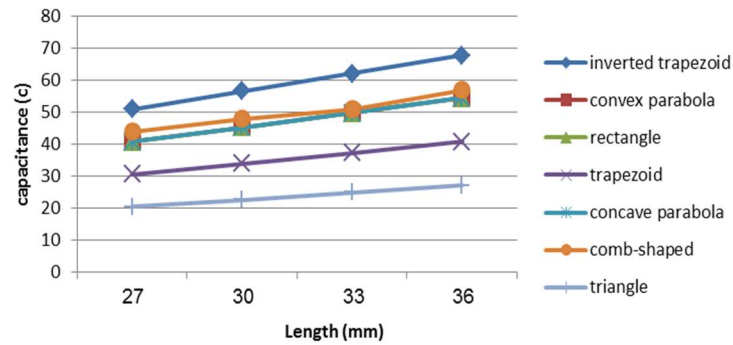


Fig. 10. Variation of capacitance in terms of length

4.1.3 Electrical voltage

Fig. 11 depicts how the voltage changes with length; as can be seen, by increasing the length of the beam, the resulting voltage will follow an upward trend, because for constant optimal resistance (and according to the direct relationship as $P = \frac{V^2}{R}$ between power and voltage), by increasing the output power, the voltage at both ends of the resistance certainly increases.

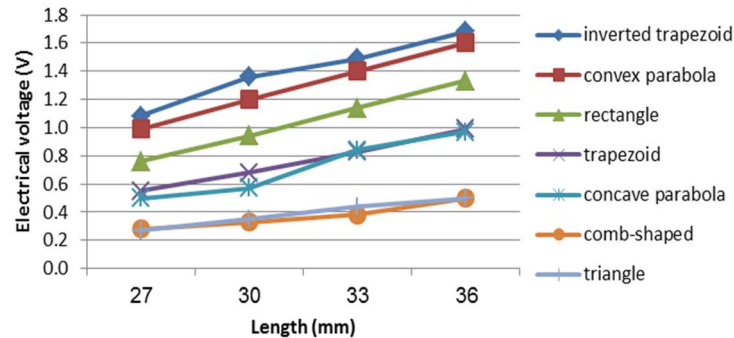


Fig. 11. Variation of electrical voltage in terms of length

4.2 Effect of thickness of piezoelectric beam

4.2.1 Natural frequency

The first figure of this section depicts the variation of natural frequency of system, in case the thickness of piezoelectric layer varies from 0.1 to about 0.2. It should be noted that all dimensions of beam are in the base status, except for thickness that is variable. As can be seen in Fig. 12, as the thickness of the piezoelectric layer increases, the obtained natural frequency keeps rising. It can also be observed that the natural frequency of the triangular shape has the highest value, and the more the ratio between the width of the free end of beam to that of the fixed end, the less the natural frequency obtained. In the next Figures, the impact of these trends of natural frequency on the capacitance, extracted power and electrical voltage will be further explored.

4.2.2 Capacitance

Since the value of capacitance is only related to the piezoelectric layer, according to Eq. (14) for the capacitance, it can be found that by increasing the thickness of the piezoelectric layer, the capacitance decreases. This is shown in Fig. 13. As can be seen, the capacitance has the highest value in the inverted trapezoid, and the lowest one in the triangular beam. Because of higher surface area, the comb-shaped beam has a higher capacitance than other shapes. As the dimension of the piezoelectric layer is identical for the rectangular, concave and convex parabolic beams, they have equal capacitance.

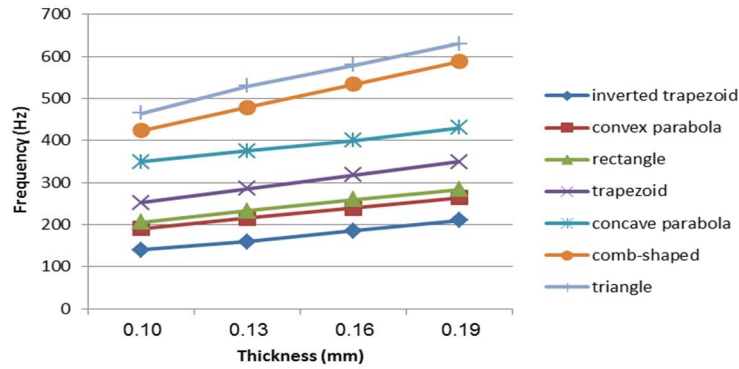


Fig. 12. Variation of natural frequency in terms of thickness

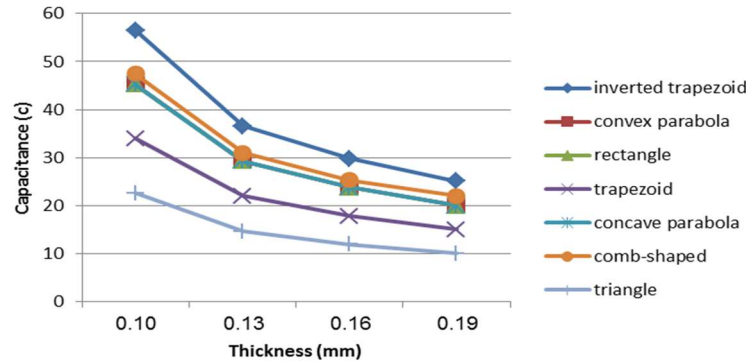


Fig. 13. Variation of capacitance in terms of thickness

4.2.3 Electric power and voltage

As evident in Fig. 14, the extracted power has a downward trend. The electric power decreases because the natural frequency of the system increases with the thickness of piezoelectric layer. In addition, the value of capacitance has declined, and based on Eq. (13) proposed for the optimal resistance, due to the increase in frequency and the decrease in the capacitance at the same time, the optimum resistance remains almost unchanged. According to $P = V \times I$, by reducing the electric power and the current passing through the piezoelectric layer, the electrical voltage is fixed for each of the seven shapes (Fig. 14). The electrical voltage has the highest value for the convex parabolic and inverted trapezoidal shapes, respectively.

4.3 Effect of width of piezoelectric layer

4.3.1 Natural frequency

As can be seen from Fig. 16, by increasing the width of the piezoelectric layer, the natural frequency of the bimorph beam will increase. Furthermore, comparing the seven shapes revealed that the natural frequency of the system is highest in the triangular shape, and like two previous parameter, it has the lowest value in the inverted trapezoidal beam.

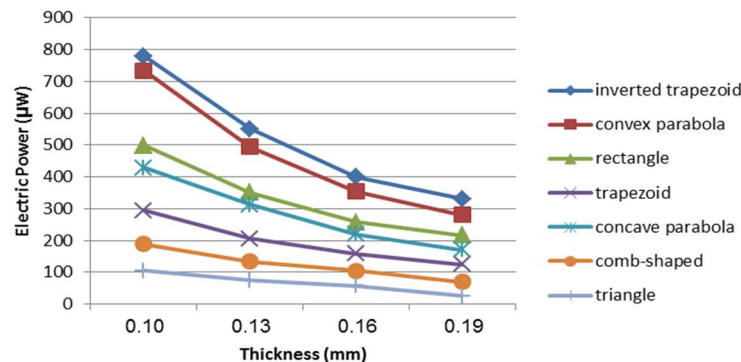


Fig. 14. Variation of electric power in terms of thickness

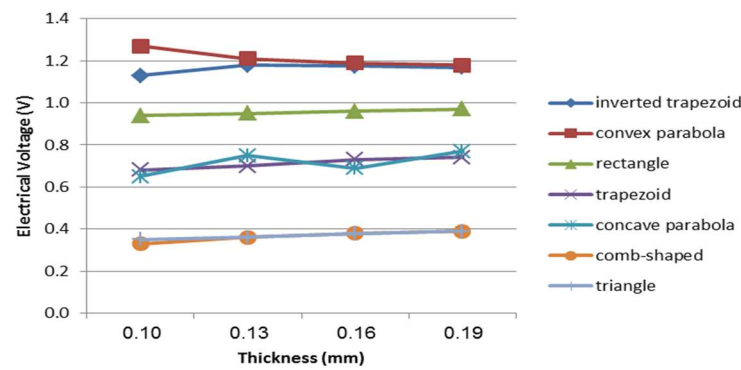


Fig. 15. Variation of electrical voltage in terms of thickness

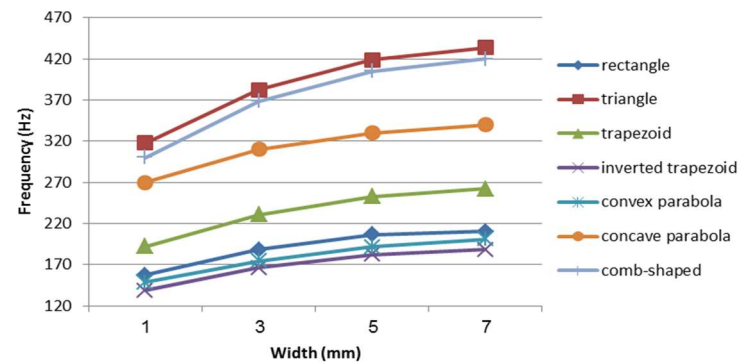


Fig. 16. Variation of natural frequency in terms of width

4.3.2 Electric power

With respect to the electrical power generated from the analyzed cantilever beam, the analytical relation for the equivalent mass-spring damping system is known by four terms, namely, mass, acceleration, mechanical damping and natural frequency of system which have direct and reverse effects on the generated electrical power. In this research, the fixed acceleration was applied to vibrate the beam. In case there was no mass in the free end of the beam, the beam mass was assumed to be negligible, and the damping of the system was also fixed. However, according to the equation for the electrical power, there is a reverse relationship between the natural frequency of the system and the extracted power. Moreover, knowing that the natural frequency increases with the width of the piezoelectric layer, so the generated amount reduces, which can be seen in Fig. 17.

4.3.3 Electrical voltage

According to the direct relationship between the voltage and electric power, as power reduces for various widths of the piezoelectric layer, the voltage also decreases. Even though as the optimal resistance can have a significant effect on the generated voltage, which is evident for each width of the piezoelectric layer, this downward trend has been maintained. In Fig. 18, the same downward trend related to electrical voltage can be vividly observed.

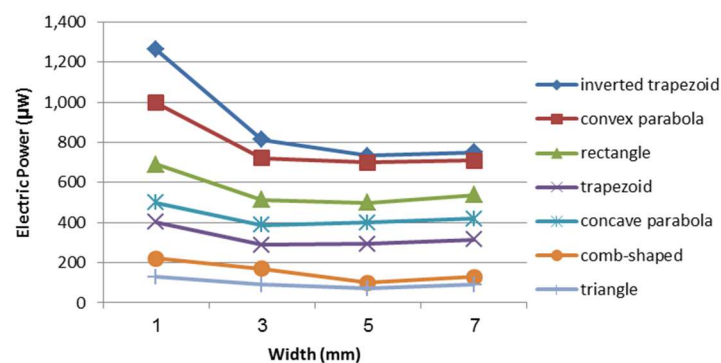


Fig. 17. Variations of electric power in terms of width of piezoelectric layer

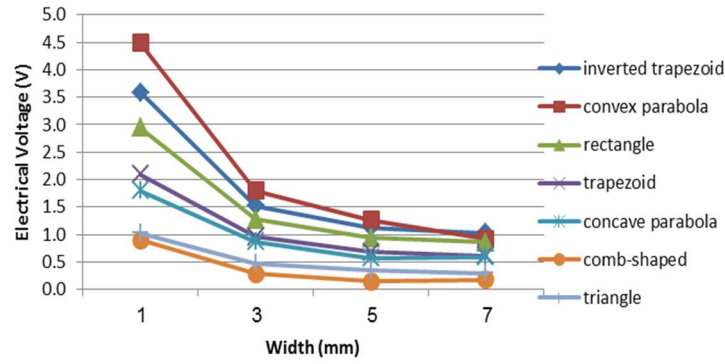


Fig. 18. Variation of electrical voltage in terms of width of piezoelectric layer

5. Conclusions

In this paper, a cantilever piezoelectric beam was analyzed for seven shapes in bimorph mode using the COMSOL Multiphysics finite element software. The results of the study show that due to mechanical properties, in all three parameters of beam length, width and thickness of the piezoelectric layer, the natural frequency of the triangular beam manifest the lowest value and inverted trapezoid the highest one. Since power is inversely related to the frequency, the extracted power for all three parameters has the highest value in the inverted trapezoidal beam. Another interesting point is that by increasing the length parameter, the electrical power displays an increasing trend, and by increasing the width and thickness of the piezoelectric layer, the electrical power shows a decreasing trend for all shapes. The electrical voltage also has the highest value for the two parameters width and thickness of the piezoelectric layer in the convex parabolic beam and the lowest value in the comb-shaped beam. As for the length parameter, the highest and lowest values of voltage are related to the inverted trapezoid and comb-shaped beams, respectively. It is worth mentioning that the amount of electrical power generated for both trapezoidal and concave parabolic shapes is roughly achieved in the same way in all the 3 parameters. With respect to the concave parabolic and trapezoidal beams, the rectangular beam produces more voltage, but this voltage is less than that of inverted trapezoidal and convex parabolic beams.

Nomenclature

A	Cross Section	S	Strain
C	Capacitance [F]	s	Effective strain
C _b	Capacitance of the piezoelectric bender [F]	s ^E	Strain on the constant electric field
D	Electrical displacement	T	Stress [$\frac{N}{m^2}$]
d	Piezoelectric constant	Y	Displacement amplitude
E	Electric field	ε	Permeability coefficient
F _r	Resonant frequency [Hz]	ε^T	Dielectric coefficient
K	Stiffness factor [$\frac{N}{m^3}$]	ν	Poisson's ratio
m	Mass [Kg]	ρ	Density [$\frac{Kg}{m^3}$]
N	Acceleration [$\frac{m}{s^2}$]	φ	Phase angle
P _d	Electrical Power [w]	ω	Excitation frequency [Hz]
q	Thickness of the piezoelectric layer [m]	ω_n	Natural frequency [Hz]
R _{opt}	Optimum resistance [Ω]	ζ_T	Damping ratio

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