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Research Article

Thermal Analysis of Convective-Radiative Fin with Temperature-Dependent Thermal Conductivity Using Chebychev Spectral Collocation Method

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Abstract. In this paper, the Chebychev spectral collocation method is applied for the thermal analysis of convective-radiative straight fins with the temperature-dependent thermal conductivity. The developed heat transfer model was used to analyse the thermal performance, establish the optimum thermal design parameters, and also, investigate the effects of thermo-geometric parameters and thermal conductivity (nonlinear) parameters on the thermal performance of the fin. The results of this study reveal that the rate of heat transfer from the fin increases as the convective, radioactive, and magnetic parameters increase. This study establishes good agreement between the obtained results using Chebychev spectral collocation method and the results obtained using Runge-Kutta method along with shooting, homotopy perturbation, and adomian decomposition methods.

Keywords: Thermal analysis; Convective-radiative fin; Chebychev spectral collocation method; Temperature-dependent thermal conductivity.

1. Introduction

Thermal analysis of fins in a convective and radiative environment has been a subject of research for the past few decades. In the quest of investigating the thermal performance of the fins, different numerical and analytical methods have been developed to provide solutions to the nonlinear equations which are developed using different techniques. Aziz and Enamul-Huq [1] and Aziz [2] employed the regular perturbation expansion to study a pure convection fin with the temperature-dependent thermal conductivity and a uniform internal heat generation in the fin. Campo and Spaulding [3] used the method of successive approximation to predict the thermal behaviour of uniform circumferential fins. Chiu and Chen [4] and Arslanturk [5] adopted the Adomian Decomposition Method (ADM) to obtain the temperature distribution in a pure convection fin with variable thermal conductivity. For the same problem, Ganji [6] applied the homotopy perturbation method which was originally proposed by He [7]. Chowdhury and Hashim [8] adopted the Adomian decomposition method to evaluate the temperature distribution of straight rectangular fins with the temperature-dependent surface flux for all possible types of heat transfer. Rajabi [9] utilized the homotopy perturbation method (HPM) to calculate the efficiency of straight fins with the temperature-dependent thermal conductivity. Mustapha [10] adopted the homotopy analysis method (HAM) to find the efficiency of straight fins with the temperature-dependent thermal conductivity. Besides, Coskun and Atay [11] utilized the variational iteration method (VIM) to analyse convective straight and radial fins with the temperature-dependent thermal conductivity while Languri et al. [12] applied both variation iteration and homotopy perturbation methods to evaluate the efficiency of straight fins with the temperature-dependent thermal conductivity. Sobamowo [13] applied the Galerkin's method

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of weighted residual to analyse the thermal performance of longitudinal fins with temperature-dependent properties and internal heat generation. Atay and Coskum [14] employed variation iteration and finite element methods to carry out a comparative analysis of power-law-fin type problems. Domairry and Fazeli [15] used the Homotopy analysis method to determine the efficiency of straight fins with the temperature-dependent thermal conductivity. Hosseini et al. [16] applied the homotopy analysis method to provide an approximate but accurate solution of heat transfer in the fins with the temperature-dependent internal heat generation and thermal conductivity. Joneidi et al. [17], Moradi and Ahmadikia [18], Moradi [19], Mosayebidorcheh et al. [20], Ghasemi et al. [21], Sandri et al. [22], and Ganji and Dogonchi [23] presented an analytical solution for the fins with a temperature-dependent thermal coefficient using the differential transform method (DTM).

Approximate analytical methods, as applied by past researchers, solve the differential equations without linearization, discretization or approximation, linearization restrictive assumptions or perturbation, complexity of expansion of derivatives and computation of derivatives symbolically. However, the search for a particular value that will satisfy the second boundary condition or the determination of auxiliary parameters necessitates the use of a software and such could result in additional computational cost in the generation of a solution to the problem. Besides, most of the approximate methods give accurate predictions only when the nonlinearities are weak or the values of the fin thermo-geometric parameter are small, and they fail to predict accurate solutions for strong nonlinear models. Also, the methods often involved a complex mathematical analysis leading to the analytic expression involving a large number terms and when they are routinely implemented, they can sometimes lead to erroneous results [13, 24]. Moreover, in practice, approximate analytical solutions with large number of terms are not convenient for use by designers and engineers. Also, variational methods including Ritz and Rayleigh-Ritz methods sometimes provide powerful results, such as upper and lower bounds on quantities of interest, but require more mathematical manipulations than weighted residual method and are not applicable to all problems, and therefore, they reveal the lack of generality. Inevitably, simple yet accurate expressions are required for the thermal analysis of fins.

The Chebychev spectral collocation method (CSCM) is a relatively new numerical method with a high level of accuracy and it has been widely applied in computational fluid dynamics, electrodynamics and magnetohydrodynamics [25-40]. Despite the high accuracy and efficiency of this method, it has not been significantly applied to nonlinear flow problems, therefore, in this study, the Chebychev spectral collocation method is applied to analyse these kind of problems. To the best of the authors' knowledge, the analysis of heat transfer in convective-radiative fins subjected to temperature-dependent properties usin the Chebychev spectral collocation method has not be carried out. Therefore, in the present study, the Chebychev spectral collocation method (CSCM) is used to develop approximate analytical solutions for heat transfer in convective-radiative fins with the temperature-dependent thermal conductivity.

2. Problem Formulation

Consider a convective-radiative straight fin with the temperature-dependent thermal conductivity $k(T)$, length L and thickness δ, that is exposed on both faces to a convective environment at temperature T_a and with heat transfer co-efficient h as shown in Fig.1. Assuming that the heat flow in the fin and its temperatures remain constant over time, the temperature of the medium surrounding the fin along with the fin base temperature are uniform. Moreover, there is no contact resistance where the base of the fin joins the prime surface, and also, compared with its width and length, the fin thickness is small. Therefore, temperature gradients across the fin thickness and heat transfer from the edges of the fin may be neglected. The dimension x pertains to the length coordinate which has its origin at the tip of the fin and has a positive orientation from the fin tip to the fin base. Considering the model assumptions, the governing differential equation for the problem is shown in equation (1).

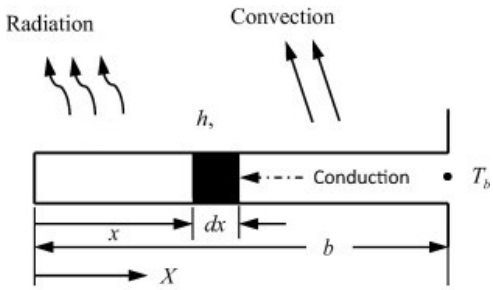

Fig. 1. Schematic of the convective-radiative longitudinal fin

Based on the assumptions, we developed the governing equation as:

$$\frac{d}{dx}\left(k_a\left[1+\lambda\left(T-T_\infty\right)\right]A_{cr}\frac{dT}{dx}+\frac{4\sigma A_{cr}}{3\beta_R}\frac{dT^4}{dx}\right)=hP(T-T_\infty)+\sigma\varepsilon P(T^4-T_\infty^4)dx \tag{1}$$

Further simplification of Eq. (2) gives the governing differential equation for the fin as:

$$\frac{d}{dx}\left[\left[1+\lambda\left(T-T_\infty\right)\right]\frac{dT}{dx}\right]+\frac{4\sigma}{3\beta_R k_a}\frac{d}{dx}\left(\frac{dT^4}{dx}\right)-\frac{h}{k_a t}(T-T_\infty)-\frac{\sigma\varepsilon}{k_a t}(T^4-T_\infty^4)=0 \tag{2}$$

Where the boundary conditions are:

$$x = 0, \quad \frac{dT}{dx} = 0$$
$$x = b, \quad T = T_b \tag{3}$$

But

$$\frac{J_c \times J_c}{\sigma} = \sigma B_o^2 u^2 \tag{4}$$

By substituting Eq. (4) into Eq. (2), and taking the magnetic term as a linear function of temperature, we arrived at:

$$\frac{d}{dx}\left[\left[1+\lambda(T-T_\infty)\right]\frac{dT}{dx}\right] + \frac{4\sigma}{3\beta_R k_a}\frac{d}{dx}\left(\frac{dT^4}{dx}\right) - \frac{h}{k_a t}(T-T_\infty) - \frac{\sigma\varepsilon}{k_a t}(T^4 - T_\infty^4) = 0 \tag{5}$$

The supposed case in this study is a situation of small temperature difference existing within the material during the heat flow. This difference necessitated the use of temperature-invariant physical and thermal properties of the fin. Besides, it is established that under such a scenario, the term T^4can be expressed as a linear function of temperature. Therefore, we have:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

By substituting Eq. (6) into Eq. (7), we arrived at:

$$\frac{d}{dx}\left[\left[1+\lambda(T-T_\infty)\right]\frac{dT}{dx}\right] + \frac{16\sigma}{3\beta_R k_a}\frac{d^2T}{dx^2} - \frac{h}{k_a t}(T-T_\infty) - \frac{4\sigma\varepsilon P T_\infty^3}{k_a t}(T-T_\infty) = 0 \tag{7}$$

By introducing the following dimensionless parameters of Eq. (8) into Eq. (9),

$$X = \frac{x}{b}, \quad \theta = \frac{T-T_\infty}{T_b - T_\infty} \qquad \beta = \lambda\left(T_b - T_\infty\right) \qquad M^2 = \frac{pbh}{A_b k_a} \qquad R_d = \frac{4\sigma_{st} T_\infty^3}{3\beta_R k_a} \qquad N_r = \frac{4\sigma_{st}\varepsilon b T_\infty^3}{k_a} \tag{8}$$

we arrived at the dimensionless form of the governing Eq. (7) as:

$$\left(1+4R_d\right)\frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta - N_r\theta = 0 \tag{9}$$

Therefore,

$$\frac{d^2\theta}{dX^2} + \frac{\beta}{\left(1+4R_d\right)}\theta\frac{d^2\theta}{dX^2} + \frac{\beta}{\left(1+4R_d\right)}\left(\frac{d\theta}{dX}\right)^2 - \frac{M^2}{\left(1+4R_d\right)}\theta - \frac{N_r}{\left(1+4R_d\right)}\theta = 0 \tag{10}$$

which is the same as:

$$\frac{d^2\theta}{dX^2} + \beta^*\theta\frac{d^2\theta}{dX^2} + \beta^*\left(\frac{d\theta}{dX}\right)^2 - \left(M^*\right)^2\theta - N_r^*\theta = 0 \tag{11}$$

where

$$\beta^* = \frac{\beta}{\left(1+4R_d\right)}, \quad \left(M^*\right)^2 = \frac{M^2}{\left(1+4R_d\right)} \quad N_r^* = \frac{N_r}{\left(1+4R_d\right)} \tag{12}$$

and the dimensionless boundary conditions are:

$$X = 0, \quad \frac{d\theta}{dX} = 0$$
$$X = 1, \quad \theta = 1 \tag{13}$$

3. Solution Procedure

The Chebyshev collocation spectral method is accomplished through, starting with Chebyshev approximation for the approximate solution and generating approximations for the higher-order derivatives through successive differentiation of the approximate solution. Looking for an approximate solution, which is a global Chebyshev polynomial of degree *N* defined on the interval [-1, 1], the interval is discretized by using collocation points to define the Chebyshev nodes in [-1, 1], namely:

$$x_j = cos\left(\frac{j\pi}{N}\right), \quad j = 0,1,2,...N \tag{14}$$

The derivatives of the functions at the collocation points are given by:

$$f^n\left(x_j\right) = \sum_{j=0}^{N} d_{kj}^n f\left(x_j\right), \quad n = 1,2. \tag{15}$$

where d_{kj}^{n} represents the differential matrix of order n which are given by:

$$d_{kj}^{1} = \frac{4\gamma_j}{N} \sum_{n=0,l=0}^{N} \sum_{n+l=odd}^{n-1} \frac{n\gamma_n}{c_l} T_l^{\,n}\left(x_k\right) T_n\left(x_j\right), \quad k, j = 0,1,...N, \tag{16}$$

$$d_{kj}^{2} = \frac{2\gamma_j}{N} \sum_{n=0,l=0}^{N} \sum_{n+l=even}^{n-1} \frac{n\gamma_n\left(n^2 - l^2\right)}{c_l} T_l^{\,n}\left(x_k\right) T_n\left(x_j\right), \quad k, j = 0,1,...N, \tag{17}$$

where $T_n\left(x_j\right)$ are the Chebyshev polynomial and coefficients γ_j and c_l are defined as:

$$\gamma_j = \begin{cases} \frac{1}{2} & j = 0, \text{ or } N \\ 1 & j = 1,2,... N-1 \end{cases} \tag{18a}$$

$$c_l = \begin{cases} 2 & l = 0, \text{ or } N \\ 1 & l = 1,2,... N-1 \end{cases} \tag{18b}$$

As described above, the Chebyshev polynomials are defined on the finite interval [-1, 1]. Therefore, to apply the Chebyshev spectral method to Eq. (11), a suitable linear transformation of the physical domain [-1, 1] to the Chebyshev computational domain [-1,1] is made. We sample the unknown function w at the Chebyshev points to obtain the data vector $w = \left[w\left(x_o\right), w\left(x_1\right), w\left(x_2\right), ...w\left(x_N\right)\right]^T$. The next step is to find a Chebyshev polynomial P of degree N that interpolates the data (i.e., $P(x_j) = w_j$, $j = 0,1,...N$) and obtains the spectral derivative vector w by differentiating P and evaluating at the grid points (i.e., $w_j^{'} = P'(x_j) = w_j$, $j = 0,1,...N$). The result is transformation of the nonlinear differential equation into system nonlinear algebraic equations, which are solved by the Newton's iterative method starting with an initial guess. A suitable transformation to map the physical domain [0, 1] to a computational domain [-1, 1] is made to facilitate the computations.

$$\frac{d^2\theta}{dX^2} + \beta^*\theta\frac{d^2\theta}{dX^2} + \beta^*\left(\frac{d\theta}{dX}\right)^2 - \left[(M^*)^2 + N_r^*\right]\theta = 0 \tag{19}$$

where the boundary conditions are:

$$\tilde{\theta}'(-1) = 0, \quad \tilde{\theta}(1) = 1 \tag{20}$$

After applying CSCM and using Eq. (19), the governing equation and boundary conditions are transformed into a system of nonlinear algebraic equation as:

$$\sum_{j=0}^{N} d_{kj}^{2}\tilde{\theta}\left(X_j\right) + \beta^*\sum_{j=0}^{N}\tilde{\theta}\left(X_j\right)d_{kj}^{2}\tilde{\theta}\left(X_j\right) + \beta^*\left\{\sum_{j=0}^{N} d_{kj}^{1}\tilde{\theta}\left(x_j\right)\right\}^2 - \left[(M^*)^2 + N_r^*\right]\tilde{\theta}\left(x_j\right) = 0 \tag{21}$$

where the boundary conditions are:

$$\sum_{j=0}^{N} d_{kj}^{1}\tilde{\theta}\left(x_j\right) = 0, \quad \tilde{\theta}\left(x_j\right) = 1 \tag{22}$$

The above-mentioned system of the nonlinear algebraic equation is solved using Newton's method to determine the temperature distribution in the fin. In order to verify the solution by the Chebyshev collocation spectral method, Eq. (11) with the boundary conditions is solved using Runge-Kutta method with shooting method and *ode45* in *MATLAB.*

4. Result and Discussion

Figs. 2 and 3 depict the effect of the thermogeometric parameter on the fin. According to the figures, as the thermogeometric parameter increases, the rate of heat transfer through the fin increases while the temperature in the fin drops faster. It can be inferred from the results that the ratio of convective heat transfer to conductive heat transfer has much effect on the temperature distribution, the rate of heat transfer at the base of the fin, and the efficiency of the fin. As h increases (or k_b decreases), the ratio of h/k_b increases at the base of the fin and consequently, the temperature along the fin, especially at the tip of the fin, decreases; i.e. the tip end temperature decrease as M increases. This shows that the thermal performance or efficiency of the fin is favoured at low values of the thermos-geometric parameter [13].

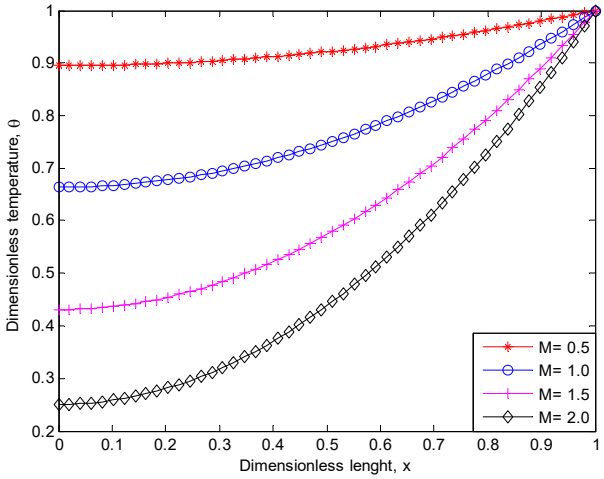

Fig. 2. Effects of thermo-geometric parameter M on the temperature distribution in the fin when β=0.1, N=0

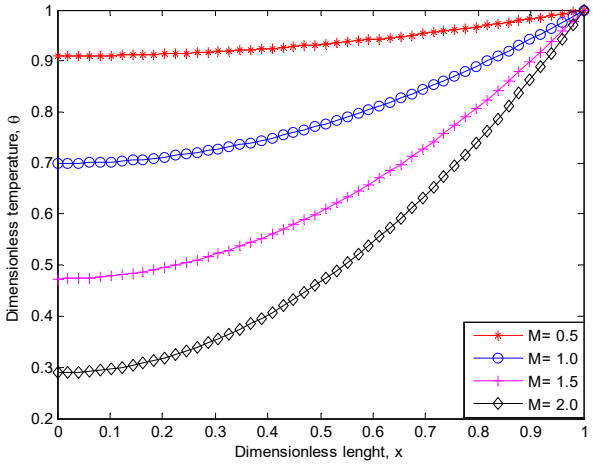

Fig. 3. Effects of thermo-geometric parameter M on the temperature distribution in the fin when β=0.3, N=0

Fig. 5 and 6 show the effects of the thermo-geometric term and the radiation number on the dimensionless temperature distribution or the thermal performance of the fin. In addition, the figures depict the effect of nonlinear parameter or the temperature-dependent thermal conductivity term on the thermal performance of the fin. It can therefore be deduced from the figures that the nonlinear thermal conductivity parameter, the thermo-geometric term and the radiation number have direct and significant effects on the rate of heat transfer at the base of the fin. Comparing Figs. 2-5 shows that the rate of heat transfer increases by the radiation.

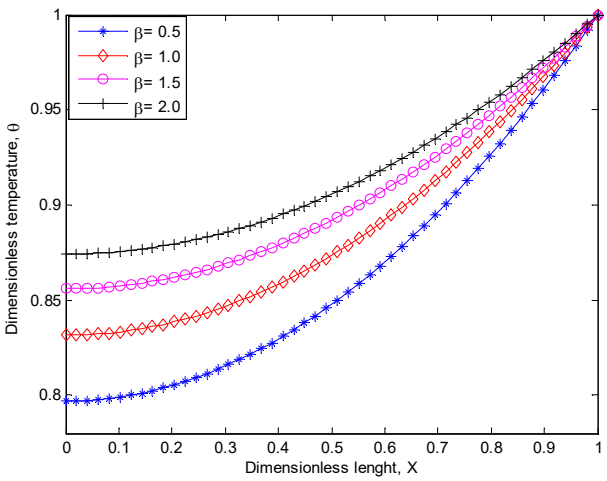

Fig. 5. Effects of non-linear parameter β on the temperature distribution in the fin when M=0.50, N=1.25

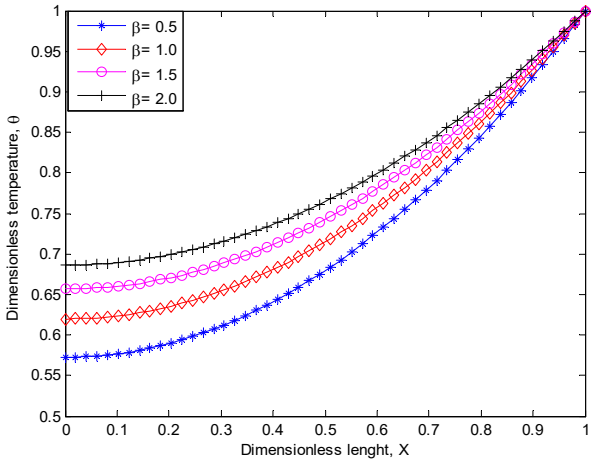

Fig. 6. Effects of non-linear parameter β on the temperature distribution in the fin when M=1.50, N=1.75.

Table 1 shows the comparison of previous results in literature using other methods with obtained results of the present study using variation of parameter methods. The table indicates that the variation of parameter methods agrees excellently with both the numerical method (NM) results using Runge-Kutta with shooting method and the results of the Adomian decomposition method (ADM) and the homotopy perturbation method (HPM).

Fig. 7 shows the effects of the radiative parameter N_r on the dimensionless fin efficiency η at different values of the thermal conductivity parameter for the fin. According to the figure, it is established that the numerical values of the fin efficiency decreases with increasing the radiative parameter while the numerical values of the efficiency increases as the non-linear thermal conductivity parameter increases.

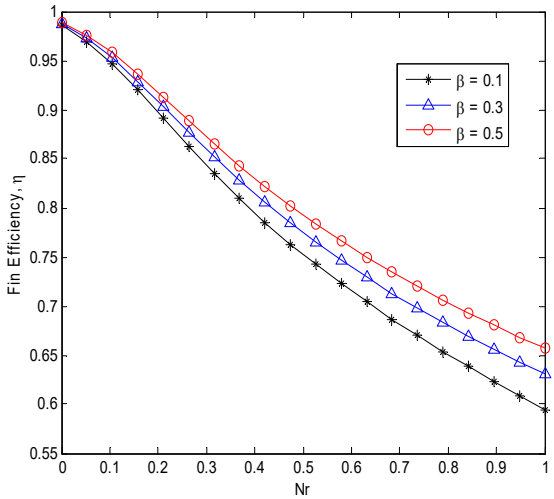

Fig. 7. Effects of radiative parameter N_r on the temperature distribution in the fin.

Table 1. Comparison between different methods and the results of current study

X	NM	ADM [5]	HPM [9]	CSCM
0.0	0.648054	0.648054	0.648054	0.648054
0.1	0.651297	0.651297	0.651297	0.651297
0.2	0.661059	0.661059	0.661059	0.661057
0.3	0.677436	0.677436	0.677436	0.677436
0.4	0.700594	0.700594	0.700594	0.700594
0.5	0.730763	0.730763	0.730763	0.730763
0.6	0.768246	0.768246	0.768246	0.768246
0.7	0.813418	0.813418	0.813418	0.813418
0.8	0.866731	0.866731	0.866731	0.866731
0.9	0.928718	0.928718	0.928718	0.928718
1.0	1.000000	1.000000	1.000000	1.000000

5. Conclusion

In this study, heat transfer in a convective-radiative straight fin with the temperature-dependent thermal conductivity has been investigated using the Chebychev spectral collocation method. The developed heat transfer model was used to analyse the thermal performance, establish optimum thermal design parameters, and also, investigate the effects of thermo-geometric and

thermal conductivity (non-linear) parameters on the thermal performance of the fin. Good agreements were established between the results of Chebychev spectral collocation method and the results obtained using Runge-Kutta method along with shooting method, homotopy perturbation and Adomian decomposition methods.

Nomenclature

a_r	Aspect ratio	M	Dimensionless thermo-geometric fin parameter
b	Length of the fin	m^2	Thermo-geometric fin parameter
A_c	Cross sectional area of the fins	N_r	Radiative parameter
A_p	Profile area of the fins	P	Perimeter of the fin
Bi	Biot number	T	Temperature
h	Heat transfer coefficient	T_∞	Ambient temperature
k	Thermal conductivity of the fin material	T_b	Temperature at the base of the fin
k_a	Thermal conductivity of the fin material at ambient temperature	X	Dimensionless length of the fin
k_b	Thermal conductivity of the fin material	q	Rate of heat transfer
K	Dimensionless thermal conductivity of the fin material	Q_f	Dimensionless heat transfer

Greek Symbols

β	Thermal conductivity parameter or non-linear parameter	θ_b	Dimensionless temperature at the base of the fin
δ	Thickness of the fin, m	η	Efficiency of the fin
θ	Dimensionless temperature	ε	Effectiveness of the fin

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