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Research Article

Transient Natural Convection in an Enclosure with Variable Thermal Expansion Coefficient and Nanofluid Properties

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Abstract. Transient natural convection is numerically investigated in an enclosure using variable thermal conductivity, viscosity, and the thermal expansion coefficient of Al₂O₃-water nanofluid. The study has been conducted for a wide range of Rayleigh numbers ($10^3 \leq Ra \leq 10^6$), concentrations of nanoparticles ($0\% \leq \phi \leq 7\%$), the enclosure aspect ratio ($AR = 1$), and temperature differences between the cold and hot walls ($\Delta T = 30$). Transient parameters such as development time and time-average Nusselt number along the cold wall are also presented as a non-dimensional form. Increasing the Rayleigh number shortens the non-dimensional time of the initializing stage. By increasing the volume fraction of nanoparticles, the flow development time shows different behaviors for various Rayleigh numbers. The non-dimensional development time decreases by enhancing the concentration of nanoparticles.

Keywords: Nanofluid, Natural convection, Variable property, Transient natural convection.

1. Introduction

The steady state natural convection in an enclosure with nanofluid has been investigated with variable property as well as variable density in previous studies [1]. Despite the great efforts devoted to investigating the steady state heat transfer characteristics [2-13], there have been few studies on the transient natural convection heat transfer in such geometries. This phenomenon is studied numerically for pure fluid [14] and for nanofluid by Yu et al. in a rectangular enclosure [15] and in a horizontal concentric annulus [16] filled with nanofluid. They showed that at constant Rayleigh numbers, the time-average Nusselt number gradually decreases as the volume fraction of nanoparticles increases. A numerical analysis of the unsteady state natural convection heat transfer in an isosceles triangular cavity filled with Al₂O₃-water nanofluid was carried out with non-uniform hot temperature on the bottom wall of the cavity [17]. Alsabery et al. [18] numerically investigated the transient natural convection heat transfer in a nanoliquid-saturated porous oblique cavity using the finite difference method. The steady and unsteady natural convection heat transfer in a nanofluid-filled square enclosure cavity with various heat source configurations at the bottom was studied by Nguyen et al. [19].

In previous numerical studies, various models are used for representation of the nanofluid properties, and these models have become more precise over time. In general, adding a little amount of nanoparticles to the base fluid leads to an increase in the average Nusselt number, but the average Nusselt number is deteriorated by increasing nanoparticles concentration. The previous study [1] showed that by utilizing a more precise model which describes nanofluid property, the numerical results are similar to the experimental works. As a result, it can be concluded that the actual Nusselt number diminishes by increasing nanoparticle concentration.

The transient natural convection solution as a non-dimensional case with variable thermal expansion coefficient has not



been considered in any previous numerical studies for a better assessment. This study investigates the transient natural convection of Al_2O_3 -water nanofluid with variable properties and thermal expansion coefficient in a rectangular cavity.

2. Mathematical formulation

Fig. 1 shows the schematic diagram of a two-dimensional square cavity that is filled with water- Al_2O_3 nanofluid. The height and width of the cavity are noted by H and W , respectively, and the aspect ratio (AR) is defined as W/H . The nanofluid is considered single phase and homogenous. The diameter of nanoparticles is considered to be 47 nanometers. Table 1 presents the thermophysical properties of the base fluid (water) and nanoparticles at $T=20^\circ\text{C}$. It is assumed that the nanofluid flow is homogenous, laminar, and incompressible. Both the viscosity and thermal conductivity of the nanofluid are considered variable.

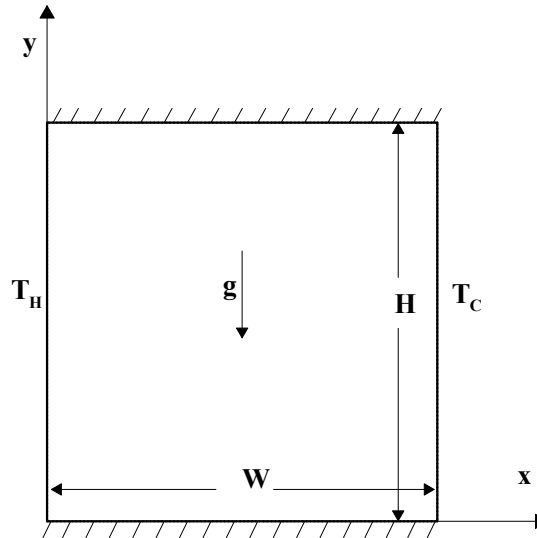


Fig. 1. Schematic diagram of the physical model for the enclosure

Table 1. Physical properties of pure water and Al_2O_3 nanoparticles

	ρ (kg m^{-3})	C_p ($\text{J kg}^{-1}\text{K}^{-1}$)	k ($\text{Wm}^{-1}\text{K}^{-1}$)	β (K^{-1})
Pure water	998.2	4179	0.603	2.064×10^{-4}
Al_2O_3	3970	765	25	8.5×10^{-6}

For the transient natural convection; the continuity, momentum, and energy equations in a two-dimensional cavity can be written in non-dimensional forms as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{\rho} \frac{\partial P}{\partial X} + \frac{\mu_{f_0}}{\rho_{nf} \alpha_{f_0}} \left(2 \frac{\partial}{\partial X} \left(\mu \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\mu \frac{\partial U}{\partial Y} \right) + \frac{\partial}{\partial Y} \left(\mu \frac{\partial V}{\partial X} \right) \right) \quad (2)$$

$$\begin{aligned} \frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = & -\frac{1}{\rho} \frac{\partial P}{\partial Y} + \frac{\mu_{f_0}}{\rho_{nf} \alpha_{f_0}} \left(2 \frac{\partial}{\partial Y} \left(\mu \frac{\partial V}{\partial Y} \right) + \frac{\partial}{\partial X} \left(\mu \frac{\partial V}{\partial X} \right) + \frac{\partial}{\partial X} \left(\mu \frac{\partial U}{\partial Y} \right) \right) \\ & + \frac{\rho_{f_0}}{\rho_{nf}} ((1-\phi)f(\beta_r) + \phi \frac{\rho_{np} \beta_{np}}{\rho_{f_0} \beta_{f_0}}) Ra Pr \theta \end{aligned} \quad (3)$$

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial}{\partial X} \left(\alpha \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\alpha \frac{\partial \theta}{\partial Y} \right) \quad (4)$$

In the above equations, the following non-dimensional parameters are used:

$$\begin{aligned} X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{uH}{\alpha_{f_0}}, \quad V = \frac{vH}{\alpha_{f_0}}, \quad P = \frac{\bar{p}H^2}{\rho_{nf} \alpha_{f_0}^2}, \quad \tau = \frac{\alpha_{f_0} t}{H^2} \\ \theta = \frac{T - T_c}{T_h - T_c}, \quad Ra = \frac{g \beta_{f_0} H^3 (T_h - T_c)}{g_{f_0} \alpha_{f_0}}, \quad Pr = \frac{g_{f_0}}{\alpha_{f_0}}, \quad \mu = \frac{\mu_{nf}}{\mu_{f_0}}, \quad \alpha = \frac{\alpha_{nf}}{\alpha_{f_0}} \end{aligned} \quad (5)$$

Other parameters and properties were defined or obtained in the previous study [1]. The actual Rayleigh number is also defined as:

$$Ra_{act} = Ra_f = \frac{g \beta H^3 (T_h - T_c)}{9\alpha} \quad (6)$$

The boundary conditions used to solve Eqs. (1) to (4) are as follows:

On the walls: $U=V=0$

On the horizontal adiabatic walls: $\frac{\partial \theta}{\partial Y} = 0$ (7)

On the heated walls: at $\tau = 0 \rightarrow \theta = 0$, $\tau > 0 \rightarrow \theta = 1$

On the cold walls: $\theta = 0$

The local and average Nusselt numbers for the vertical heated or cooled wall are given by:

$$Nu = -\frac{k_{eff}}{k_{f_0}} \left(\frac{\partial \theta}{\partial X} \right)_{on \text{ the heated or cooled wall}} \quad (8)$$

$$\overline{Nu} = \int_{on \text{ the heated or cooled wall}} NudY \quad (9)$$

The overall heat transfer during the course of flow development is presented by the time-averaged Nusselt number of the cold wall, which is defined as:

$$\langle \overline{Nu} \rangle = \frac{1}{\tau_{total}} \int_0^{\tau_{total}} \overline{Nu} d\tau \quad (10)$$

3. Numerical approach

The finite volume method [20, 21] is used to solve the governing Eqs. (1) - (4) with the corresponding boundary conditions given in Eq. (7). The study was validated in a steady state condition [1]. Both second-order implicit and explicit schemes are adopted to deal with the discretization of temporal terms in the governing equations. The SIMPLE algorithm is utilized to solve the coupling between velocity and pressure. The convection-diffusion terms are discretized by the Hybrid scheme, and the system is numerically modeled in FORTRAN. The solution procedure is iterated in every step until the following two convergence criteria are satisfied:

$$Max \left(\chi_{i,j}^{new} - \chi_{i,j}^{old} \right) p 10^{-7} \quad (11)$$

$$\sum_{i,j} \left(\frac{\chi_{i,j}^{new} - \chi_{i,j}^{old}}{\chi_{i,j}^{new}} \right) p 10^{-6} \quad (12)$$

where χ in Eqs. (11) and (12) is U , V , θ , and the residual mass of the grid control volume. To ensure the grid and time independency of the numerical solutions, different grids and time steps have been tested for the case of water ($\phi=0$) at $Ra=10^5$ as listed in Table 2 for both implicit and explicit schemes. To accomplish this, eight combinations of the spatial resolution and dimensionless time-step are considered, and the time-average Nusselt number is calculated along the cold sidewall over a process of $\tau_{total}=1.2$. The relative deviations of the measure are also given with reference to that of the case with the finest spatial (120×120) and dimensionless time step (2.0×10^{-6}) resolutions. The difference between the spatial 100×100 and 120×120 is negligible, so 100×100 grid is used in this study. When the grid density is 100×100 or more, the solution with an explicit scheme is unstable for the dimensionless time steps greater than 8.0×10^{-6} . Additionally, if an implicit method is used, the process of computer running takes more time. However, solutions with explicit and implicit schemes are similar as shown in Fig. 2. This figure compares the time-average Nusselt number between explicit and implicit methods for $Ra=10^5$ and pure water. Therefore, a rectangular enclosure with the spatial (100×100) and temporal (4.0×10^{-6}) resolutions is chosen for all the cases in this study.

Table 2. Grid and time independency ($Ra=10^5$, $\phi=0$, $\tau=1.2$)

Number of cells	Time-step interval(s)	Time-averaged Nusselt number (Explicit)	Deviation (%) (Explicit)	Time-averaged Nusselt number (Implicit)	Deviation (%) (Implicit)
60×60	4E-6	6.0779	1.65	6.0753	1.62
80×80	4E-6	6.0347	0.92	6.0338	0.91
100×100	4E-6	6.0086	0.49	6.0081	0.48
120×120	4E-6	5.9903	0.18	5.9895	0.18
100×100	8E-6	-	-	6.0400	1.01
100×100	4E-6	6.0086	0.49	6.0081	0.48
100×100	2E-6	5.9841	0.08	5.9820	0.04
120×120	2E-6	5.9795	0.000	-	-

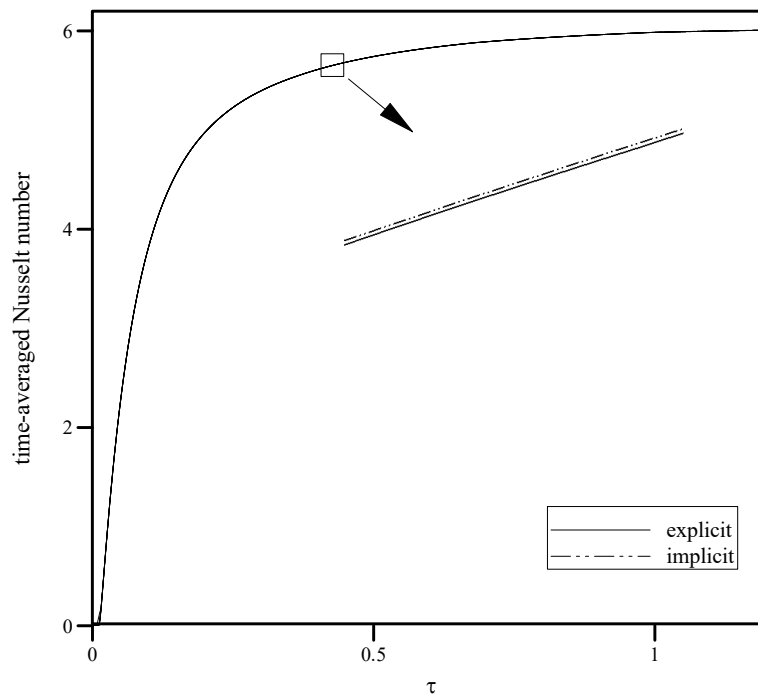


Fig. 2. Comparison between explicit and implicit schemes for time-average Nusselt number.

4. Results and discussion

The ranges of the Rayleigh numbers value, the volume fractions of nanoparticles, and the aspect ratios used in this work are $Ra=10^3$ - 10^6 , $0 \leq \phi \leq 7\%$, and $AR = 1$, respectively. The temperature of the enclosure's right wall is fixed at the reference temperature, i.e., 20°C , whereas the difference between the hot and cold walls is 30°C . The Prandtl number at the reference temperature is calculated as 6.93. Furthermore, in the transient study, the computations are advanced up to 1.2, which is considered to be adequate to capture the transient features.

The Nusselt number along the cold wall is plotted in Fig. 3 for $Ra=10^3$ - 10^6 and $\phi = 0, 1\%$, and 4% assuming ΔT to be 30°C . By comparing Fig. 3 with those in relevant papers [15] it is revealed that the average Nusselt number increases more than 40% for all the Rayleigh numbers because of the impact of thermal expansion coefficient [1]. By increasing the Rayleigh number, the initializing stage (zero-value parts on the curves) lasts for a shorter non-dimensional time. Through the developing process, the average Nusselt number of the nanofluid with 4% nanoparticles concentration is higher than that of pure water for the case of $Ra=10^3$, but at the end of this stage, its value becomes less than that of the pure fluid. The developing time is also decreased by the Rayleigh number increment.

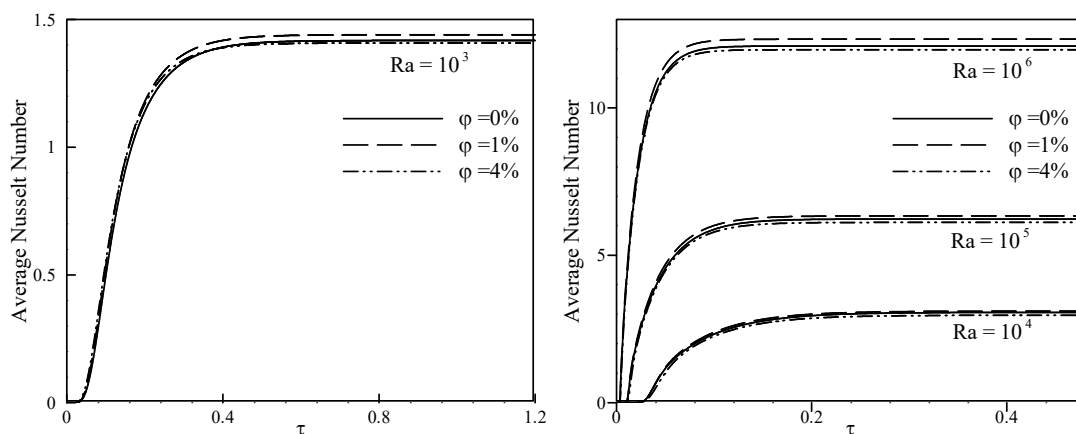


Fig. 3. Development of average Nusselt number along cooled wall for $Ra=10^3$ (left) and $Ra=10^4$, 10^5 and 10^6 (right)

In this study, the flow development time is defined as the time during which the value of the average Nusselt number along the cooled wall becomes larger than 99.9% of its value at steady state condition. The non-dimensional developing time is depicted as a function of the actual Rayleigh number in Fig. 4. For $Ra=10^3$, the non-dimensional development time (τ_{dev}) is decreased as the nanoparticles concentration is enhanced. For example, τ_{dev} for a nanofluid with $\phi = 7\%$ is approximately 10% less than that of pure water. The non-dimensional development time also decreases as the Rayleigh number augments as mentioned earlier. For $Ra \geq 10^4$, τ_{dev} decreases for the case of nanofluid with volume fractions in the range of 1% -3%, whereas above 4% a contrariwise trend is observed.

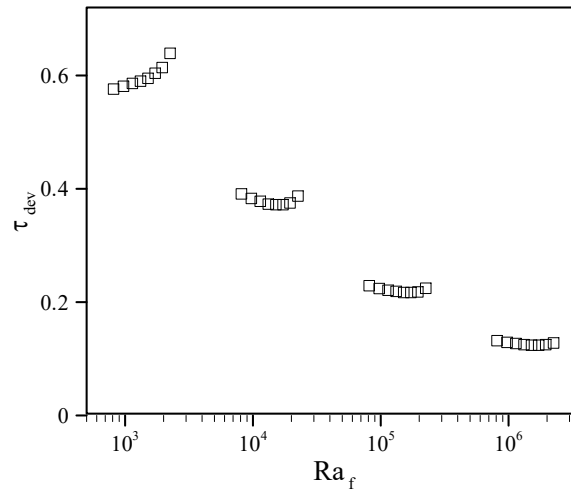


Fig. 4. Flow development time (non-dimensional) as a function of fluid Rayleigh number

In this section, the time-average Nusselt number is expressed as the overall heat transfer along the cold wall from the beginning ($\tau = 0$) until the flow development time ($\tau = \tau_{dev}$). The time-average Nusselt number is shown in Fig. 5 as a function of the actual Rayleigh number. The time-average Nusselt number enhances as the Rayleigh number increases. For all the Rayleigh numbers, the time-average Nusselt number for the fluid with variable thermal expansion coefficient is 20% more than that of the fluid with constant thermal expansion coefficient (comparing with relevant papers [15]). The time-average Nusselt number is in its utmost value for the nanofluid with 1% nanoparticles concentration for all the ranges of Rayleigh numbers. The correlation equation for the time-average Nusselt number with a coefficient of determination of 0.9921 is given by:

$$\langle Nu \rangle = 0.1578 Ra^{0.306} \quad (13)$$

This correlation is valid within the range of $Ra = 10^3 - 10^6$, $0 \leq \phi \leq 7\%$.

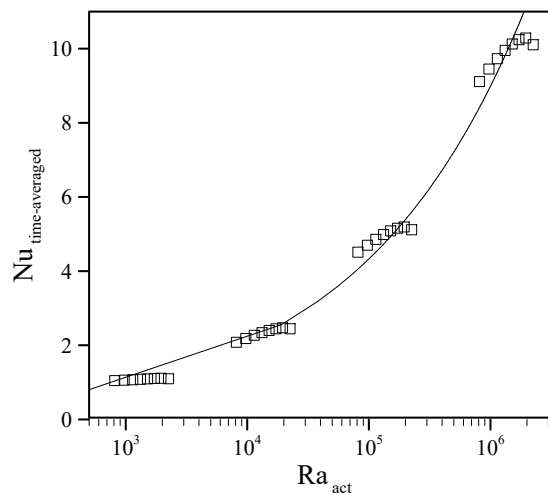


Fig. 5. Time-average Nusselt number along cooled wall as a function of Rayleigh number

5. Conclusion

The transient natural convection heat transfer of Al_2O_3 -water nanofluid in an enclosure with vertical hot and cold walls is investigated numerically by means of explicit and implicit methods. Considering the impact of fluid temperature on the thermal expansion coefficient causes totally different patterns in the natural convection heat transfer by increasing the nanoparticle concentration. The difference between the results of the implicit method and the explicit scheme is negligible.

Non-dimensional transient procedures are calculated along the cold surface. As the Rayleigh number is increased, the natural convection along the cold wall takes shorter non-dimensional time to initialize and develop. For all the ranges of the Rayleigh number, if the variable thermal expansion coefficient be considered, the developed Nusselt number and the time-average Nusselt number will increase significantly compared with the one with the constant thermal expansion coefficient. The flow development time is diminished by increasing the volume fraction of nanoparticles for $Ra = 10^3$; however, for $Ra \geq 10^4$, addition of nanoparticles to the base fluid slightly increases this parameter. For all the ranges of Rayleigh numbers and when ΔT is 30 °C, adding 1% volume fraction of nanoparticles to pure water has more positive effect on the time-average Nusselt number.

Nomenclature

C_p	specific heat, J/(kgK)	P	dimensionless pressure
d_f	molecule water diameter	Pr	Prandtl number
d_p	nanoparticles diameter	R_{np}	radius of nanoparticles, m
g	gravitational acceleration, m/s ²	Ra	Rayleigh number
k	thermal conductivity, W/(mK)	t	time, s
k_b	Boltzmann constant	T	temperature, K or °C
L	length of the cavity, m	T_c	cooled wall temperature, K or °C
Nu	local Nusselt number on the heated or cooled wall	T_h	heated wall temperature, K or °C
$\overline{Nu}$	average Nusselt number	T_{fr}	freezing point temperature, K
$\langle \overline{Nu} \rangle$	time-average Nusselt number	u, v	velocity components in x, y directions, m/s
p	fluid pressure, Pa	U, V	dimensionless velocity components
$\overline{p}$	modified pressure	X, Y	dimensionless coordinates

Greek symbols

α	thermal diffusivity, m ² /s	τ	dimensionless time
β	thermal expansion coefficient, 1/K	τ_{dev}	development dimensionless time
ϕ	solid volume fraction	θ	dimensionless temperature
μ	dynamic viscosity, Ns/m ²	ρ	density, kg/m ³
ν	kinematic viscosity, m ² /s		

Subscripts

act	actual	f_0	base fluid
eff	effective	nf	nanofluid
f	fluid	np	nanoparticle

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