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Research Article

Damage Identification in a Multi-DOF System under Uncertainties Using Optimization Algorithms

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Abstract. In this study, four optimization algorithms are applied to identify the damage in a multi-DOF dynamical system composed of masses, springs, and dampers. The damage is introduced artificially by choosing a lower value for the stiffness. The applied algorithms include Nelder-Mead Simplex, BFGS Quasi-Newton, interior point, and sequential quadratic programming - SQP. In addition, some different strategies to identify damage are applied. First, a deterministic analysis is performed to identify the damage; i.e, the values of the stiffnesses. Then, random forces are considered, and finally, the stiffness values are considered uncertain and the different strategies are compared.

Keywords: Damage identification; Optimization; Uncertainties.

1. Introduction

In recent years, the main industries that use components and mechanical structures in their production line (automotive, aeronautics, building construction, naval, nuclear, oil and gas, etc.) have increased their investments in research and technological development in order to obtain efficient methods to analyze the integrity of structures, prevent catastrophes and/or accidents occurrence, ensure people's lives, and avoid economic losses. Structural integrity monitoring consists of detecting failures in initial states, intervening in their propagation, and consequently preventing any accidents occurrence (causing the stopping or damaging to the structure). This current and important line of research (Hall, 1999 [3]) is called Structural Health Monitoring (SHM).

The failures in mechanical components lead to damage to the mechanical structure. The damage is defined as changes introduced into a system that negatively affect your current or future performance. The significance of damage can be established by comparing two states of the system: the current state and the initial (or undamaged) state. In mechanical vibration area, the system modeling is characterized by the knowledge of the geometry of the structures, the properties of the materials, the initial and boundary conditions, and the force terms that excite the structure. The output (response) of the model, such as displacement, accelerations, vibration modes, etc. depends on the physical properties of the structures. Changes in these properties can be caused by cracks, loosening of connections, or any other damages and may be detectable through modal properties.

Based on the mechanical performance, the process of structural damage identification can be classified into five levels (Ritter, 1993 [9] and Worden et al., 2004 [12]):

1. Detection: the presence of any damage is verified.
2. Location: it determines where the damage is located.

3. Classification: it determines the type of damage.
4. Extension: it gives an estimate of the extension and severity of the damage.
5. Prognosis: it attempts to predict the remaining life of the structure.

According to Ooijevaar (2014) [8], the majority of works found in the literature meet the first two levels of performance (detection and location of damages). Recently, some authors have investigated damage identification in structural dynamics considering a probabilistic approach. For instance, Cadini et al. (2017) [17] applied particle filtering-based model identification of the crack length in aeronautical structures. Yin et al. (2017) [18] investigated damage identification of structural connections using the Bayesian framework. Varmazyar et al. (2016) [19] developed a wavelet-based Bayesian damage identification technique and applied it to a beam structure. Uncertainties related to noise measurements and model error were taken into account.

In order to enrich and improve the studies related to the damage identification in mechanical systems, the present study aims to contribute through a stochastic approach to this class of problems. The main contribution of this paper is to consider a damage identification process related to a stochastic model for the stiffness parameters. In addition, four optimization techniques are compared for solving the identification problem. Up to the authors' knowledge, this is the first time that different strategies of damage identification that considers probabilistic modeling are compared: (1) random forces, (2) random noise in the measurements, and (3) random parameters, where the mean and covariance are the design variables. In addition, as mentioned previously, different optimization strategies are analyzed.

In section 2, the methodology is presented, where optimization techniques, the deterministic problem, and the stochastic problem are depicted. Section 3 shows the application that will be analyzed. The numerical results are discussed in section 4, and the concluding remarks are made in the last section of the article.

2. Methodology

In this section, the methodology used for the damage identification in mechanical structures from the study of dynamics of linear systems is presented. In short, the methodology consists of using optimization techniques to identify stiffness parameters of a mechanical system acted by a given force. Such an analysis is conducted in the frequency domain and the uncertainties in the model and response are considered. Aiming for a better understanding of the reader, this section is divided into three subsections. The first two subsections address the theoretical foundations with the main concepts used in the base of this methodology, namely optimization techniques and uncertainties analysis. The last section is responsible for compiling all the knowledge presented in the previous sections and structuring the methodology itself.

2.1 Optimization Techniques

The optimization techniques used in the damage identification analyzes is presented in this section. In all techniques, the following stopping criteria are used: (1) maximum iteration number: 1000, and (2) objective function's tolerance: 1×10^{-6}.

2.1.1 Nelder-Mead Simplex

The Nelder-Mead Simplex algorithm, published for the first time in 1965, is an extremely popular direct search method for the minimization of non-linear and unrestricted multidimensional problems (Lagarias et al., 1998 [5]). This method seeks to minimize a nonlinear scalar function of *n* real variables using only function values without any information about the derivative (explicit or implicit). At each step, the method maintains a nondegenerate simplex - a geometric figure in $\Re^n$ of nonzero volume, which is the convex envelope of *n+1* vertices (a triangle formed by three non-collinear points, in $\Re^3$ a tetrahedron formed by four non-coplanar points, and so on). Four scalar parameters are specified to define the Nelder-Mead method completely. They are the coefficients including reflection (ρ); Expansion (χ); Contraction (γ), and shrinkage (σ). According to the original work of the authors of the method (Nelder et al., 1965 [6]), these parameters present the relations as shown in Eq. (1):

$$\rho > 0,\ \chi > 1,\ \chi > \rho,\ 0 < \gamma < 1,\ 0 < \sigma < 1 \tag{1}$$

In the present work, the following values are used for the parameters (usually found in the literature): $\rho = 1,\ \chi = 2,\ \gamma = 1/2\ \ \sigma = 1/2$. Therefore, for the *k*-th iteration, a nondegenerate *simplex* Δ_k consisting of *n*+1 vertices is obtained. These vertices $x^{(k),i}$ are ordered in the form $x^{(k),1},\dots,x^{(k),n+1}$ such that $f_1^{(k)} \le f_2^{(k)} \le \dots \le f_{n+1}^{(k)}$ and $f^{(k),i} = f(x^{(k),i})$. At the end of this *k*-th iteration, a new nondegenerate *simplex* Δ_{k+1} (consisting of new *n*+1 vertices) is obtained such that $\Delta_{k+1} \ne \Delta_k$. A flow chart for the Nelder-Mead algorithm can be found in Hicken et al., (2012) [4].

2.1.2 BFGS Quasi-Newton

Among the methods that use information from the gradient of the objective function, the most usual are the Quasi-Newton methods. These methods construct curvature information at each iteration to formulate a quadratic problem, as in Eq. (2).

$$\min_{\mathbf{x}}\left(\frac{1}{2}\mathbf{x}^T\mathbf{H}\mathbf{x} + \mathbf{c}^T\mathbf{x} + \mathbf{b}\right) \tag{2}$$

where the Hessian matrix H is a symmetric positive definite matrix, c is a constant vector, and b is a constant. The optimal solution to this problem occurs when the gradient vector (taken as objective function) becomes zero:

$$\nabla \mathbf{f}(\mathbf{x}^*) = \mathbf{H}\mathbf{x}^* + \mathbf{c} = \mathbf{0} \tag{3}$$

In this way, the optimal solution is given by $\mathbf{x}^* = \mathbf{H}^{-1}\mathbf{c}$. Traditional Newton methods (as opposed to quasi-Newton methods) calculate **H** directly and proceed in a downward direction to find the minimum after a number of iterations. Calculating **H** numerically involves a large number of numerical operations. In this way, Quasi-Newton methods avoid this by using the observed behavior of $\mathbf{f}(\mathbf{x})$ and $\nabla\mathbf{f}(\mathbf{x})$ to construct curvature information to approximate **H** using an appropriate update technique. Several updating techniques of the Hessian matrix are developed. However, the Broyden-Fletcher-Goldfarb-Shanno algorithm (BFGS) is used because it is considered an effective method for general purposes. The update of the Hessian matrix by the BFGS algorithm is as follows:

$$\mathbf{H}_{k+1} = \mathbf{H}_k + \frac{\mathbf{q}_k \mathbf{q}_k^{\ T}}{\mathbf{q}_k^{\ T} \mathbf{s}_k} - \frac{\mathbf{H}_k \mathbf{s}_k \mathbf{s}_k^{\ T} \mathbf{H}_k^{\ T}}{\mathbf{s}_k^{\ T} \mathbf{H}_k \mathbf{s}_k} \tag{4a}$$

where

$$\mathbf{s}_k = \mathbf{x}_{k+1} - \mathbf{x}_k \quad \text{and} \quad \mathbf{q}_k = \nabla\mathbf{f}(\mathbf{x}_{k+1}) - \nabla\mathbf{f}(\mathbf{x}_k) \tag{4b}$$

At each iteration k, a basic line search procedure (Fletcher, 2000[7]) is performed in the direction $\mathbf{d}_k = -\mathbf{H}_{k+1}^{-1}\nabla\mathbf{f}(\mathbf{x}_k)$. And, therefore, for a given α to be determined, the new value for the iteration $\mathbf{x}_{k+1}$ and $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha\mathbf{d}_k$.

2.1.3 Sequential Quadratic Programming - SQP

In constrained optimization problems, the overall goal is to transform the problem into a simpler sub-problem that can be solved fast, and used as the basis of an iterative process. One of the methods used for this type of simplification uses the Karush-Khun-Tuker (KKT) equations. The KKT equations are necessary conditions for optimality in a constrained optimization problem. If the optimization problem is said convex, then, the KKT equations are both necessary and sufficient for an overall solution of the problem. The solution of the KKT equations is the starting point for many nonlinear programming algorithms. The quasi-Newton methods guarantee super linear convergence by accumulating second order information with respect to the KKT equations using a quasi-Newton update procedure. These methods are commonly referred to as Sequential Quadratic Programming (SQP) methods, since a QP sub-problem is solved in each main interaction. The SQP process is widely used in solving nonlinear problems in several areas, as it is one of the methods that offers the best efficiency and precision.

In the main iterations of the SQP method an approximation of the Hessian **H** of the Lagrangean L is obtained using a quasi-Newton update method. Hessian is then used to generate a QP sub-problem whose solution serves to form a search direction in a one-dimensional search procedure. This methodology used in the algorithm was initially proposed by (Wilson, 1963 [11]) and interpreted by (Beale, 1967 [1]). In this way, Eq. (5) is obtained as:

$$\min_{\mathbf{x}} f(x) \text{ subject to } g_i(x) \le 0 \text{ and } g_i(x) = 0 \tag{5}$$

It is solved for an iteration k as follows:

- 1st Step

A constant $x^{(k)}$ is fixed and the associated quadratic programming problem, Eq. (6), is solved for the direction d.

$$\min_{\mathbf{d}} \frac{1}{2}\mathbf{d}^T\mathbf{S}\mathbf{d} + \nabla f(\mathbf{x}_k)\mathbf{d} \quad \text{subject to} \quad \nabla g^T(\mathbf{x}_k)\mathbf{d} + g(\mathbf{x}) \le 0 \quad \text{and} \quad \nabla h^T(\mathbf{x}_k)\mathbf{d} + h(\mathbf{x}) = 0 \tag{6}$$

- 2nd Step

Update $\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k\mathbf{d}$ and return to step 1.

2.1.4 Interior Point

The approach used by this method to solve constrained minimization problems is to solve a sequence of approximate minimization problems. For each μ>0, the approximate problem is given by Eq. (7):

$$\min_{\mathbf{x},s} f_\mu(x,s) = \min_{\mathbf{x},s} f(x,s) \text{-} \mu\sum_i \ln(s_i) \text{ subject to } h(x) = 0 \text{ and } g(x) + s = 0 \tag{7}$$

For the approximate problem, there are as many s_i variables as the inequality constraints g. The s_i are limited to being positive to keep $\ln(s_i)$ limited. Since μ decreases to zero, the minimum of f_μ must approach the minimum of f. The logarithmic term added is called the barrier function. This method is described by Byrd et al. (1999) [2]. The approximate problem is a sequence of problems with equality constraints. To solve the approximate problem, the algorithm uses a direct step in (x, s). This step attempts to solve the KKT equations for the approximate problem by a linear approximation. This is also called Newton's step. In this way, the direct step (Δx, Δs) to be given in each iteration is determined by solving the following system of equations, Eq. (8):

$$\begin{bmatrix} H & 0 & J_h^T & J_g^T \\ 0 & S\Lambda & 0 & -S \\ J_h & 0 & I & 0 \\ J_g & -S & 0 & I \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta s \\ -\Delta y \\ -\Delta \lambda \end{bmatrix} = - \begin{bmatrix} \nabla f - J_h^T y - J_g^T \lambda \\ S\lambda - \mu e \\ h \\ g + s \end{bmatrix} \tag{8}$$

where,

- H denotes the Hessian of the Lagrangean of f;
- $J_g \equiv$ Jacobian of the function g; $J_h \equiv$ Jacobian of the function h;
- $S = diag(s)$; $\Lambda = diag(\lambda)$;
- λ , $y \equiv$ Lagrange multipliers associated with constraints g, h, respectively;
- $e \equiv$ vector of 1's of the same size of g;

2.2 Uncertainty Analysis

In the present work, two classes of results are presented referring to two strategies to identify parameters (Castello & Ritto 2016 [10]): deterministic and stochastic. In short, the deterministic strategy does not involve probabilistic models, while the stochastic strategy considers some probabilistic models (e.g. parameter and/or measurement noises are modeled as random variables). Considering the deterministic strategy, a point estimator, which provides a single numerical value for the parameter to be estimated p^* is constructed. In this case, the level of knowledge about p is encoded in a single value p^*. In addition, it should be noted that this estimator is a random variable since for each set of experimental data, a different value is observed for p^*. Although a point estimator is a random variable, this strategy is called deterministic identification because there are no probabilistic models involved. For example, given a set of experimental data $\mathbf{x}_{\exp}$ and a predictive model $\mathbf{x}(p)$, which depends on a parameter p, the estimation of the value of this parameter by minimizing the metric $(\mathbf{x}_{\exp} - \mathbf{x}(p))^2$ is possible. That is, the search for the value of p that makes the response of the model $\mathbf{x}(p)$ as close as possible to the available experimental data $\mathbf{x}_{\exp}$ is conducted. In this case, there is no hypothesis related to probability distributions. In the stochastic strategy, the parameters of interest and measurement noises are modeled as random variables and they are assigned to probabilistic models. Regarding the experimental data, the parameters of the probabilistic models (mean, variance, ...) can be identified from point estimators. In this case, not the probability distribution but only the values of its parameters that are estimated change.

At this point, it is worth mentioning that in the present work the identification process is performed using the same computational model that was used in previous research to generate the synthetic data required for identification (data generation was performed using the Monte Carlo method). This is known in the literature as an inverse crime and often generates extremely optimistic estimates. However, it was decided to work in this way to simplify the understanding of the methodology studied.

2.3 Mathematical Model of the Dynamical System

The set of the equations of motion referring to a generic problem of linear dynamics of mechanical structures, written in the time domain, is given by Eq. (9):

$$[M]\{\ddot{x}\}(t) + [C]\{\dot{x}\}(t) + [K]\{x\}(t) = \{f\}(t) \tag{9}$$

where $x(t) \equiv$ position vector, $x \in \Re^n$; $[M] \equiv$ mass matrix; $[C] \equiv$ damping matrix; $[K] \equiv$ stiffness matrix, in which $[M]$, $[C]$, and $[K]$ are n by n real matrices. A lumped parameter system with 6 degrees of freedom is considered in the analysis, as shown in Fig. 3. At this point, it is worth noting that the stiffness matrix is the only "mathematical entity" representative of the model that contains the parameter information to be determined in the damage identification process. By way of illustration, the "format" of this "mathematical entity" applied to the model problem discussed further in this paper is presented in Eq. (10):

$$[K] = [K](k_1, k_2, \ldots, k_n) = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \vdots \\ 0 & -k_3 & \ddots & -k_{n-1} \\ 0 & \ldots & -k_{n-1} & k_{n-1} + k_n \end{bmatrix} \tag{10}$$

Given the system of differential equations that model the dynamic behavior of the system, the Fourier transform is applied to it in order to work in the frequency domain. In this way, Eq. (9) is rewritten in the frequency domain, as shown in Eq. (11):

$$-\omega^2 [M]\{X\}(\omega) + i\,\omega [C]\{X\}(\omega) + [K]\{X\}(\omega) = \{\hat{f}\}(\omega) \tag{11}$$

In this way,

$$\{X\}(\omega)=\left(-\omega^2[M]+i\,\omega[C]+[K]\right)^{-1}\{\hat{f}\}(\omega) \tag{12}$$

At this point, it is valid again to emphasize the dependence of the stiffness matrix $[K]$ with the parameters of interest, according to Eq. (10). For simplicity, the set of parameters (k_1, k_2, …, k_n,) as a vector $\vec{k}$ are rewritten as follows:

$$\{X\}\left(\omega,\vec{k}\right)=\left(-\omega^2[M]+i\,\omega[C]+[K]\left(\vec{k}\right)\right)^{-1}\{\hat{f}\}(\omega) \tag{13}$$

Once the response function of the system is determined in the frequency domain, and considering that the same function generates the experimental results, four analyses are carried out that make the basis of the methodology of this work. Before explaining such analyses, it is worth mentioning that the system response in the frequency domain is a vector $X(\omega)\in\Re^n$. The importance of this observation is that in real physical systems, it is not always possible to collect all types of information, either due to the complexity of the system (difficulty of access, physical limitations, etc.) or the unavailability of sensors and measuring equipment. Therefore, since the mechanical system is studied under a discrete approach (presenting n degrees of freedom), only three components of the vector $X(\omega)\in\Re^n$ are used in the analyses. Therefore, in the following details of the performed analyses, the vector $X(\omega)$ is adopted as being the one that contains the components of the observed vector $X(\omega)$.

2.3.1 Deterministic Analysis

This analysis is presented only to illustrate the efficiency and robustness of each optimization method as well as to compare the computational time required in each of the methods. No uncertainty is considered in this analysis. The damage identification process is summarized in obtaining a set of parameters $\mathbf{k}^*=(k_1^*,k_2^*,...,k_n^*)$ from an optimization problem involving the comparison of a model and the experimental responses, and compare such a set with a set of nominal parameters of a healthy structure $\mathbf{k}_{nom}$. In this way, the metric used in the optimization problem is related to the *least mean square method*, that is, given a set of experimental data of a damaged structure, it is desired to obtain a set of parameters $\mathbf{k}^*$.

$$\mathbf{k}^*=\arg\min_{\vec{k}}\left\|X_{\exp}(\omega)-X(\omega,\mathbf{k})\right\|^2 \tag{14}$$

Therefore, the objective function is $\left\|X_{\exp}(\omega)-X(\omega,\mathbf{k})\right\|^2$ and the design variables are the stiffness parameters $\mathbf{k}$. The aim is to find out the stiffness values and associate a damage to it. At this point, a concept for damage, which depends on $\mathbf{k}$, is defined for each stiffness element i:

$$D_i=1-\frac{k_i}{k_{iNOM}} \tag{15}$$

2.3.2 Stochastic Analysis - Uncertainty on Force

This analysis could be considered identical to the previous one except for the concern that the estimated value for the force is uncertain. In this way, uncertainty is considered to be an additive term following a Normal distribution. In this stage, $X(\omega,\mathbf{k})$ is random, and consequently, $\mathbf{k}^*$ is random too. Monte Carlo simulations are performed using Eq. (14), and the mean and standard deviations are computed for $\mathbf{k}^*$ and D_i. The goal of this analysis is to evaluate the sensitivity of the optimization methods to variations of uncertain nature in the model. For each i-th random force $\{F\}^{(i)}$, a random response $X^{(i)}(\omega)$ is obtained. Therefore, the objective function for each i-th Monte Carlo simulation is $\left\|X_{\exp}(\omega)-X^{(i)}\left(\omega,\mathbf{k}^{(i)}\right)\right\|^2$ and the design variables are the stiffness parameters, which are computed for each i-th Monte Carlo simulation $\mathbf{k}^{(i)}$. The aim is to find out the probability distribution of stiffness values and associate a damage to it.

2.3.3. Stochastic Analysis - Uncertainty on Response

Analogously to the previous section, the vector $X(\omega,\mathbf{k})$ is random. The difference is that the uncertainty is modeled directly in this term and a random Gaussian noise is added to the response. Monte Carlo simulations are used in order to determine statistics for the parameters $\mathbf{k}^*$ and D_i. A random noise to the response is added, which leads to $X^{(i)}(\omega)$. Therefore, the objective function for each i-th random simulation is $\left\|X_{\exp}(\omega)-X^{(i)}\left(\omega,\mathbf{k}^{(i)}\right)\right\|^2$ and the design variables are the stiffness parameters, which are computed for each i-th Monte Carlo simulation $\mathbf{k}^{(i)}$. The aim is to find out the probability distribution of stiffness values and associate a damage to it.

2.3.4 Stochastic Analysis - Uncertainties on Parameters

This analysis can be considered the most important part of the present work. It consists of considering a stochastic model for the variables to be determined in the optimization process related to the damage identification. Vector $\mathbf{k}$ is modeled by a random vector $\mathbf{K}$, which follows an assumed probabilistic distribution, with mean and covariance matrix $\theta = (\mu, [\Sigma])$. The central idea of this analysis is to determine the optimal parameters of the probabilistic distribution $\theta^* = (\mu^*, [\Sigma^*])$, that best fit the stochastic model. For this analysis, only one experimental output is considered. The optimization problem associated with the damage identification is as follows:

$$\theta^* = \arg\min_{\theta} \alpha E\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\} + (1-\alpha)\sigma\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\} \tag{16}$$

Where α is a parameter for weighting between the information regarding the mean $E\{.\}$ and the standard deviation $\sigma\{.\}$. That is, for each iteration k of the optimization process, Monte Carlo simulations are performed generating the random vector $\mathbf{K}$ with parameters $\theta_k = (\mu_k, [\Sigma_k])$ and thus several samples of the mean square difference are collected. Using these samples, the mean and standard deviation values are calculated; then, it proceeds to the next iteration in order to minimize the weighting between these values. At the end of the analysis, the best values (in the sense of optimization) are obtained $\theta^* = (\mu^*, [\Sigma^*])$. Applications of optimization under uncertainties can be found in Lopez (2010) [14] and Ritto et al. (2011) [15], and Ritto et al. (2010) [16]. For each Monte Carlo sample, 500 simulations, the mean $E\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\}$ and the standard deviation $\sigma\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\}$ are approximated. Then, the objective function is computed as $\alpha E\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\} + (1-\alpha)\sigma\left\{\| X_{\exp}(\omega) - X(\omega,\theta) \|^2\right\}$ and the design variables are the mean stiffness and its covariance $\theta_k = (\mu_k, [\Sigma_k])$. The aim is to find out $\mu^*, [\Sigma^*]$ and associate a damage to it.

3. Application – Linear Dynamical System

The analyzed mechanical system consists of a set of mass elements interconnected in series by elastic springs and dissipating elements. An illustration for *n* interconnected masses (and consequently *n* degrees of freedom - DOF) is visualized in Fig. 2.

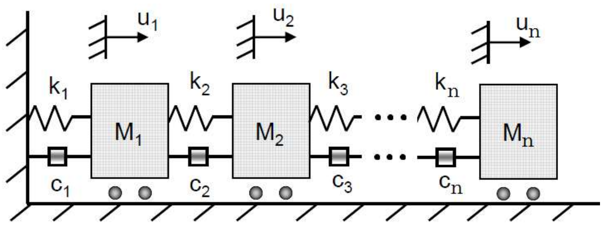

Fig. 2. Linear n-DOF mechanical system.

For the present study, a system with six degrees of freedom is considered. In addition, harmonic forces are applied in the second and fourth mass elements, with magnitudes 5*N* and 8*N,* respectively. In this way, the representation of the system is illustrated in Fig. 3.

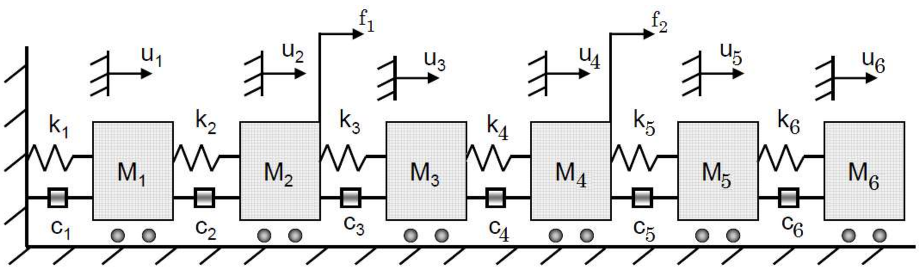

Fig. 3. Mechanical system analyzed with 6 DOF.

The simulated damage in the mechanical system is a 25% reduction in the stiffness of spring 1 and a 65% in the stiffness of spring 5. As explained before, in section 2.3, only three components of the vector $X(\omega) \in \Re^n$ are used in the analyses. The authors have made some simulations considering some set of permutations among the degrees of freedom indexes. As no difference was observed in these analyses, three displacements are chosen arbitrarily to observe; then, the vector $X(\omega)$ is constituted by the displacements of masses 1, 5 and 6.

$$X(\omega) = \left[X_1(\omega), X_2(\omega), X_3(\omega), X_4(\omega), X_5(\omega), X_6(\omega)\right]^T \tag{17}$$

The nominal values the parameters include masses (20, 12, 24, 18, 20, 15) *kg*, and damping coefficients (25, 24, 23, 26, 20, 21) *N.s/m*, (40, 28, 13, 31, 23, 24) *N/m*. For the analyses including uncertainties, the following values were used: $\sigma_{\exp} = 10\%$. For the stochastic analyses with uncertainties in the parameters, it was assumed that the stiffness coefficients are independent from the probabilistic model: Gamma and Truncated Gaussian (the parameters must be positive).

4. Numerical Results

The numerical results are presented in this section. All analyses are made using Matlab R2015a and a computer with the following systems characteristics: *AMD Quad Core A10-4600M with Radeon(tm) HD Graphics 2.30 GHz, 8GB RAM, Windows 7 64 bits*. In the results presented below, the following nomenclature are used for the optimization methods:

- Method 1 – Nelder-Mead Simplex;
- Method 2 – BFGS Quasi-Newton;
- Method 3 – Interior Point;
- Method 4 – Sequential Quadratic Programming - SQP;

4.1 Deterministic Analysis

Figure 4a shows healthy and damaged structures responses. Only the first component of the $X(\omega)$ is shown. It was observed that the two peaks at the natural frequencies are shifted to the left for the damage structure. After identifying the stiffness parameters, with the four methods, the reconstruction of the damage structure response is possible.

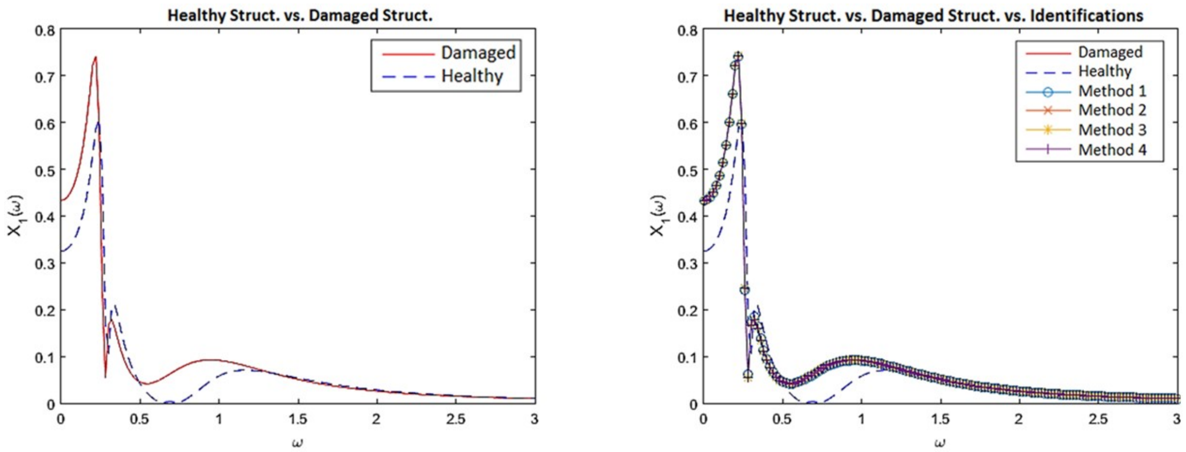

Fig. 4. Healthy structure response vs. Damaged structure response.

Regarding the four methods, the identified damages are presented in the following Table 1, and the computational costs are shown in Table 2. Methods 2, 3, and 4 performed better, which means they that are able to correctly predict the damage imposed to the structure, and also the computational cost is small. Method 1, the Nelder-Mead, produce the worse response. Possibly, it will perform better if its internal parameters are adjusted.

Table 1. Results - Damage Identification

Damage	Method 1	Method 2	Method 3	Method 4
Element 1	**24,78%**	**25%**	**25%**	**25%**
Element 2	**5,96%**	0%	0%	0%
Element 3	**0,95%**	0%	0%	0%
Element 4	**-15,84%**	0%	0%	0%
Element 5	**64,31%**	**65%**	**65%**	**65%**
Element 6	**-1,23%**	0%	0%	0%

Table 2. Computational cost (in seconds)

Method 1	Method 2	Method 3	Method 4
14.57	6.60	4.14	5.81

4.2 Stochastic Analysis - Uncertainty on the Force Amplitude

The sensitivity of the model of a healthy structure to the variations imposed on the force is very small. All the algorithms converge to the solution of a healthy structure, that is, no damage is identified in a healthy structure subject to uncertainties in the force imposed to the studied mechanical system. The identified damages are presented in Table 3, and the computational costs are shown in Table 4. All the methods are able to identify the correct damage with low standard deviation in the stiffness coefficients. One thousand Monte Carlo simulations are sufficient to achieve convergence for all the cases analyzed in this paper.

Table 3. Damages - Uncertainty on force

Element		Method 1	Method 2	Method 3	Method 4
Elem. 1	Mean	**24,98%**	**25,00%**	**25,00%**	**25,00%**
	Std.Dev.	0,15%	0,00%	0,00%	0,00%
Elem. 2	Mean	**0,43%**	0,00%	0,00%	0,00%
	Std.Dev.	2,16%	0,00%	0,00%	0,00%
Elem. 3	Mean	**0,04%**	0,00%	0,00%	0,00%
	Std.Dev.	0,40%	0,00%	0,00%	0,00%
Elem. 4	Mean	**-1,27%**	0,00%	0,00%	0,00%
	Std.Dev.	6,54%	0,00%	0,00%	0,00%
Elem. 5	Mean	**64,95%**	**65,00%**	**65,00%**	**65,00%**
	Std.Dev.	0,32%	0,00%	0,00%	0,00%
Elem. 6	Mean	**-0,18%**	0,00%	0,00%	0,00%
	Std.Dev.	0,86%	0,00%	0,00%	0,00%

Table 4. Computational costs (in seconds)

Method 1	Method 2	Method 3	Method 4
2966	1278	1268	1103

4.3 Stochastic Analysis – Uncertainty on the Response

4.3.1. Healthy Structure

The results found for the identified damages in a healthy structure are presented in Table 5, and the computational costs are shown in table 6. These results show the sensitivity of the methods to uncertainties in the measurements of the experimental responses. A standard deviation lower than 10% is observed, and a damage of about 6% might be identified, even though the healthy structure is under analysis.

Table 5. Damages in healthy structure - Uncertainty on response

Element		Method 1	Method 2	Method 3	Method 4
Elem. 1	Mean	-1,64%	1,24%	0,06%	0,06%
	Std.Dev.	4,01%	3,89%	3,88%	3,88%
Elem. 2	Mean	0,37%	-4,00%	1,08%	1,08%
	Std.Dev.	6,17%	6,38%	6,19%	6,19%
Elem. 3	Mean	-3,25%	-4,16%	-3,48%	-3,48%
	Std.Dev.	9,71%	10,07%	9,99%	9,99%
Elem. 4	Mean	5,79%	2,33%	1,90%	1,90%
	Std.Dev.	6,25%	4,51%	5,46%	5,46%
Elem. 5	Mean	-3,78%	-1,50%	-6,13%	-6,13%
	Std.Dev.	7,56%	7,06%	7,61%	7,61%
Elem. 6	Mean	1,12%	2,75%	2,39%	2,39%
	Std.Dev.	6,36%	6,32%	7,96%	7,96%

Table 6. Computational costs (in seconds)

Method 1	Method 2	Method 3	Method 4
2272	1445	1400	1278

4.3.2. Damage Structure

Table 7 shows the results for the identification result for the damage structure, and Table 8 shows its computational costs. It is noticed again that the methods are able to predict damage close to the reference damage imposed, although a shift up to 8% is observed, and a sensitivity up to 10% (standard deviation) is found.

Table 7. Damages in healthy structure - Uncertainty on response

Elements		Method 1	Method 2	Method 3	Method 4
Elem. 1	Mean	24,10%	24,00%	24,95%	24,52%
	Std.Dev.	3,50%	3,50%	3,48%	3,53%
Elem. 2	Mean	1,93%	-1,08%	1,47%	0,33%
	Std.Dev.	5,64%	5,85%	5,15%	5,98%
Elem. 3	Mean	-6,67%	-4,94%	-7,44%	-5,78%
	Std.Dev.	10,38%	10,18%	10,55%	10,02%
Elem. 4	Mean	-6,68%	-0,57%	-8,75%	-0,55%
	Std.Dev.	7,94%	6,66%	7,72%	7,13%
Elem. 5	Mean	62,23%	60,86%	60,93%	60,77%
	Std.Dev.	5,18%	5,47%	5,47%	5,37%
Elem. 6	Mean	-3,16%	-2,53%	-3,92%	-1,19%
	Std.Dev.	8,13%	5,86%	8,82%	7,84%

Table 8. Computational costs (in seconds)

Method 1	Method 2	Method 3	Method 4
3086	1479	1690	1427

4.4 Stochastic Analysis - Uncertainty on Parameter

The central idea of this analysis is to determine the parameters that best describe a given probability distribution prescribed for a set of variables. The results are summarized in Tables 9 to 12 for the two probabilistic models (Gamma and Truncated Gaussian). No difference in the results is observed for these two probabilistic models. The four optimization techniques produce satisfactory results. A shift up to 4% is observed, and a sensitivity up to 3% (standard deviation) is found. This strategy is much more time consuming since Monte Carlo simulations are need at each iteration point of the optimization algorithm. But, it is possible to compute uncertainties directly to the stiffness coefficients, which are used to determine the damage in the structure. In the field, these parameters are not easy to determine and this strategy allows some flexibility for the damage identification.

Table 9. Means and Variances - Gama distribution

Stiffness		Method 1	Method 2	Method 3	Method 4	Nominal with Damage
k_1	μ	32,8	28,4	29,0	29,1	30
	Var.	3,01	2,88	2,59	1,98	
k_2	μ	29,3	28,3	28,3	28,1	28
	Var.	3,31	2,75	2,39	2,12	
k_3	μ	14,1	13,5	13,4	13,4	13
	Var.	3,65	2,55	2,40	2,08	
k_4	μ	33,7	32,9	32,8	32,3	31
	Var.	3,26	3,18	2,79	2,28	
k_5	μ	9,2	8,8	8,6	8,3	8,05
	Var.	3,03	2,78	2,37	1,88	
k_6	μ	25,2	24,7	24,6	24,6	24
	Var.	2,91	2,82	2,48	1,94	

Table 10. Means and Variances - Normal Truncated distribution

Stiffness		Method 1	Method 2	Method 3	Method 4	Nominal with Damage
k_1	μ	32,2	29,2	29,2	29,3	30
	Var.	2,91	2,76	2,32	1,78	
k_2	μ	28,7	28,3	27,8	27,5	28
	Var.	2,87	2,56	2,19	1,96	
k_3	μ	13,8	13,5	12,6	12,4	13
	Var.	2,65	2,51	2,30	1,98	
k_4	μ	32,6	32,1	30,8	30,3	31
	Var.	3,16	2,28	2,19	1,99	
k_5	μ	8,6	8,5	8,2	7,9	8,05
	Var.	2,63	2,44	2,27	1,90	
k_6	μ	24,8	24,6	23,4	22,6	24
	Var.	2,61	2,37	2,18	1,89	

- **Gamma distribution**

Table 11. Computational costs (in hours)

Method 1	Method 2	Method 3	Method 4
12.6	2.6	2.0	2.3

- **Normal Truncated distribution**

Table 12. Computational costs (in hours)

Method 1	Method 2	Method 3	Method 4
12.7	2.7	2.0	2.3

5. Concluding remarks

The contribution of this ongoing work is to produce a material that adds knowledge of optimization techniques and stochastic modeling applied in processes of damage identification in mechanical systems. Therefore, with the results obtained in the previous analyses, a detailed analysis of the results found in each section is carried out. It is observed that all of the optimization techniques are able to satisfactory predict the damage imposed to the structure. Although, it is observed that the Nelder-Mead Simplex method performe worse than the other methods, and it is also more computationally expensive than the other methods applied. It will perform better if its internal parameters are adjusted.It was found that, for the type of force modeled, the system response is little sensitive to uncertainties in the force amplitude. On the other hand, the model is more sensitive to the uncertainties in the responses. Despite the sensitivity of the healthy structure to the uncertainties in the measurement of the experimental response, all techniques achieve a good approximation of the damage in the damaged structure. It should be mentioned that the parameter uncertainty is very expensive computationally. The motivation for the continuity of this work is to look for another technique for the propagation of uncertainties other than the Monte Carlo technique, such as expansion techniques based on the Chaos Polynomials and the method of Stochastic Collocation (Xiu, 2010 [13]).

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