J. Appl. Comput. Mech., 5(1) (2019) 141-149
DOI: 10.22055/jacm.2018.24953.1222

ISSN: 2383-4536
jacm.scu.ac.ir

M&MoCS
Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Generalized 2-Unknown's HSDT to Study Isotropic and Orthotropic Composite Plates

L.M. Cuba[1], R.A. Arciniega[2], J.L. Mantari[3]

[1] Department of Civil Engineering, Universidad Peruana de Ciencias Aplicadas (UPC)
Surco, Lima, Peru, pccilcub@upc.edu.pe

[2] Department of Civil Engineering, Universidad Peruana de Ciencias Aplicadas (UPC)
Surco, Lima, Peru, roman.arciniega@upc.edu.pe

[3] Faculty of Mechanical Engineering, Universidad de Ingeniería y Tecnología (UTEC)
Barranco, Lima, Peru, jmantari@utec.edu.pe

Received February 13 2018; Revised May 15 2018; Accepted for publication June 17 2018.
Corresponding author: J.L. Mantari, jmantari@utec.edu.pe

Abstract. The present study introduces a generalized 2-unknown's higher order shear deformation theory (HSDT) for isotropic and orthotropic plates. The well-known Shimpi's two-unknown's HSDT is reproduced as a special case. Reddy's shear strain shape function (SSSF) is also adapted to the present generalized theory. The results show that both Shimpi and the adapted Reddy' HSDT are essentially the same, i.e., both present the same static results. This is due to the fact that both theories use polynomial SSSFs. This study presents a new optimized cotangential SSSF. The generalized governing equation obtained from the principle of virtual displacement is solved via the Navier closed-form solution. Results show that transverse shear stresses can be improved substantially when non-polynomial SSSFs are utilized. Finally, this theory is attractive and has the potential to study other mechanical problems such as bending in nanoplates due to its reduced number of unknown's variables.

Keywords: Layered structures; Plates; Elasticity; Analytical modeling.

1. Introduction

Natural and artificial composite materials are present everywhere. Industrial classical composite structures such as the single skin and sandwich-structured composites are common in many industries. Composite materials are attractive and demanding nowadays due to their increased reliability, fatigue resistance, and more importantly their high performance applications (weight reduction), and thereby such metarials are increasing the speed of vehicles with more efficient power plants (lower fuel consumption). Moreover, the launch of the first commercial airplane (see Boing 787 and Airbus A350) along with the representative amount of composite structures usage are speeding up the uses of composite materials in other industries. Composite structures are kinds of multilayered structures that exhibit a different mechanical behavior of metals. Therefore, the need for a clear understanding of the mechanics of the material through experimental and numerical studies is of vital importance.

From the analytical point of view, several plate theories were developed. For example the Classical Plate Theory and First order Shear Deformation Theory (CPT, FSDT) are relevant ones. Some disadvantages of them are as follows: the first one has limitation to estimate transverse shear stresses; the second one is not capable to correctly model transverse shear stresses. Fortunately, the HSDT can overcome such limitations and improve the results from moderate thick to very thick plates. Nowadays, there exist several HSDTs developed on the top of two variational statements: Principle of virtual displacement (PVD) and Reissner mixed variational theorem (RMVT). However, very few theories that consider limited number of

Published online: January 11 2019

unknown variables exist, i.e. computational effective.

In this sense, a remarkable study on isotropic and layered structures (plates) by Shimpi [1] and Shimpi and Patel [2, 3] is introduced. Based on this interesting work, Mechab et al. [4] studied the static behavior of advanced composites by using 4-unknown plate theory. Then, Abdelaziz et al. [5] studied the static analysis of advanced composite sandwich plates with the same plate theory. Houari et al. [6] and Hamidi [7] analyzed the thermoelastic behavior of advanced composite plates by the 4-unknown plate theory. Recently, Mechab et al. [8] studied the static and dynamic analysis of advanced composite plates with a hyperbolic shear strain shape function (SSSF).

Thai and Kim [9] developed a 5-unknown trigonometric plate theory (TPT) with thickness stretching effect having a good accuracy regarding its counterpart, the TPT, with 6-unknown. Based on this work and previous experience on non-polynomial HSDTs, Mantari and Guedes Soares [10-12] developed an optimized TPT with an stretching effect (5 and 6-unknown) having improved results compared with the 5 and 6-unknown quasi-3D trigonometric plate theories (TPT) [9][13]. Recently, Zenkour [14] developed an interesting HSDT with 4-unknown and a thickness stretching effect. Based on non-polynomial HSDT, Mantari and Guedes Soares [15-17] developed an optimized 4-unknown's quasi-3D HSDT having improved results. The elastic foundation of FGPs modeled as a two-parameter Pasternak foundation on the bases of a 4-unknown shear deformation theory was introduced by Thai and Choi [44]. Overall, several new and optimized shear strain shape functions adopted by shear deformation theories did not include the HSDT of two-unknown's variables. The present paper attempts to fill this gap.

Applications of the HSDT can be used to solve mechanical problems in nonlinearities of material or/and geometry. For example, readers may found nonlinear thermo-stability of classical and advanced composites (Refs. [45, 46]) an extension of this work to the nonlinear thermal stability of the composite plate with embedded and through-the-width delamination under a uniform temperature rise (Ref. [47]). Moreover, the nonlinear thermal instability of moving shape memory alloy (SMA) sandwich plates subjected to a constant moving speed was investigated by using the HSDT in Ref. [48]. Further studies on deformation theories and nonlinearities can be found in Refs. [49, 50].

This study presents a generalized two-unknown HSDT in which polynomial and non-polynomial SSSFs can be arbitrarily introduced, and, if desired, the theory can be optimized for further improvement of the bending mechanical problems. Moreover, an unavailable optimized cotangential SSSF within the generalized theory is introduced. Results show that the transverse shear stresses can be improved substantially with the optimization. The present theory is excellent for isotropic plates and some types of orthotropic materials. Additionally, it has the potential to study nanosheets.

2. Analytical modelling

The displacement field, satisfying the conditions of transverse shear stresses (and hence strains) vanishing at a point (x, y, $\pm h/2$) on the outer (top) and inner (bottom) surfaces of the plate, is given as follows:

$$\begin{aligned} \bar{u} &= z\left[y^{**}\frac{\partial w}{\partial x} + q^{**}\frac{\partial \theta}{\partial x}\right] + f(z)\frac{\partial \theta}{\partial x}, \\ \bar{v} &= z\left[y^{**}\frac{\partial w}{\partial y} + q^{**}\frac{\partial \theta}{\partial y}\right] + f(z)\frac{\partial \theta}{\partial y}, \\ \bar{w} &= w + \theta. \end{aligned} \tag{1a-c}$$

where $w(x,y)$ and $\theta\,(x\,,\,y)$ are the two unknown displacement functions of the middle surface of the plate, $f(z)$ allows to model the in-plane displacements using the plate thickness, whilst y^{**}, q^{*}, y^{*} and $q^{**}=y^{*}+q^{*}$ are constant values depending of the selected SSSF obtained following the same strategy presented in Reddy and Liu [18] (see Table 1 for details). By considering the kinematic and constitutive relations in the linear regimen, the principle of virtual works can be formulated as in Mantari and Guedes Soares [17].

Table 1. SSSFs of classical and new theories.

SSSFs	$f(z)$	m	y^{*}	q^{*}	y^{**}	q^{**}
Polynomial Shimpi	$-5z^3/3h^2$	NA	$5/4$	-1	-1	$-1/4$
Polynomial Reddy	z^3	NA	$-3h^2/4$	-1	-1	$-1-3h^2/4$
Non-polynomial	$arc\tan(mz/h)$ [!]	$1/4$	$-4m/\left[h(m^2+4)\right]$	-1	-1	$-1-4m/\left[h(m^2+4)\right]$
	$arc\tan(mz/h)$ [!]	$-11/2$	$-4m/\left[h(m^2+4)\right]$	-1	-1	$-1-4m/\left[h(m^2+4)\right]$

[!] Obtained through optimization procedure mentioned in the manuscript (Opt1, m=1/4; Opt2, m=-11/2).

2.1. The principle of virtual works

Considering the static version of the principle of virtual work, the following expressions can be obtained:

$$0=\left[\int_{-h/2}^{h/2}\left\{\iint_{\Omega}\left[\sigma_{xx}^{(k)}\delta\varepsilon_{xx}+\sigma_{yy}^{(k)}\delta\varepsilon_{yy}+\sigma_{yz}^{(k)}\delta\varepsilon_{yz}+\sigma_{xz}^{(k)}\delta\varepsilon_{xz}+\sigma_{xy}^{(k)}\delta\varepsilon_{xy}\right]dx\,dy\right\}dz\right]-\left[\int_{\Omega}q\,\delta\bar{w}\,dx\,dy\right] \tag{2}$$

where $\varepsilon^{(k)}$ or $\sigma^{(k)}$ are the stress and the strain vectors of the k^{th} layer and q is the distributed transverse load. Figure 1 show the layer and laminate coordinate system.

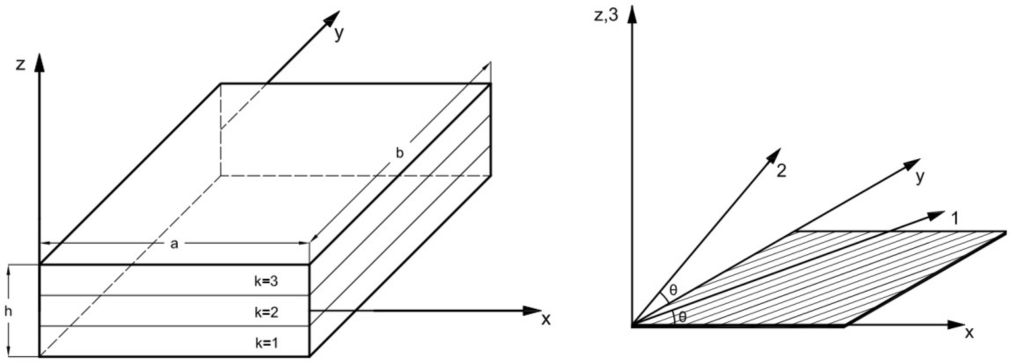

Fig. 1. Laminate and layer coordinate system.

2.2. Plate governing equations

Using the generalized kinematic and constitutive relations, applying integration of parts and collecting the coefficients of δw and $\delta\theta$, the equations of motion are obtained as follows:

$$\delta w:\ y^{**}(\frac{\partial^2 M_1}{\partial x^2}+\frac{\partial^2 M_2}{\partial y^2}+2\frac{\partial^2 M_6}{\partial x\partial y})=q\ ,$$

$$\delta\theta:\ q^{**}(\frac{\partial^2 M_1}{\partial x^2}+\frac{\partial^2 M_2}{\partial y^2}+2\frac{\partial^2 M_6}{\partial x\,\partial y})+\frac{\partial^2 P_1}{\partial x^2}+\frac{\partial^2 P_2}{\partial y^2}+2\frac{\partial^2 P_6}{\partial x\,\partial y}-q^{**}(\frac{\partial N_4}{\partial y}+\frac{\partial N_5}{\partial x})-(\frac{\partial K_4}{\partial y}+\frac{\partial K_5}{\partial x})=-q^{*}q \qquad \text{(3a-b)}$$

where N_i, M_i, P_i, and K_i are the resultants of the following integrations:

$$(M_i, P_i)=\int_{-h/2}^{+h/2}\sigma_i\,(1,z,f(z))dz\ ,\qquad \text{(i=1,2,6)}$$

$$N_i=\int_{-h/2}^{+h/2}\sigma_i\,dz\ ,\qquad \text{(i=4,5)} \qquad \text{(4a-c)}$$

$$K_i=\int_{-h/2}^{+h/2}\sigma_i\,f'(z)dz\ ,\qquad \text{(i=4,5)}$$

Details of the formulation presented above along with the methodology used to solve the simple supported advanced composites, and the optimization procedure used in this study can be found in Refs. [15-17, 52-54]. Readers are invited to consult these papers for further details.

3. Solution procedure

Exact solutions of the partial differential equations (3a-b) on the arbitrary domain and for general boundary conditions are easy to be found. General boundary conditions require solution strategies involving deeper mathematical formulations [19-20]. Results based on HSDT using such strategies are available in the literature for laminated shells [21-26]. However, in the case of simply supported plates, the above-mentioned constitutive equations can be solved in a simpler manner. The drawback of the present adopted solution is that the lamination scheme must be cross-ply [0º/90º...]$_s$ type and symmetric.

Solution functions of the partial differential Equations (3a-b) of a cross-ply plate for simply supported boundary conditions are assumed as follows:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(\alpha x)\sin(\beta y),\quad 0\le x\le a;\ \ 0\le y\le b$$

$$\theta(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\theta_{mn}\sin(\alpha x)\sin(\beta y),\quad 0\le x\le a;\ \ 0\le y\le b \qquad \text{(5a-b)}$$

where

$$\alpha=\frac{m\pi}{a},\quad \beta=\frac{n\pi}{b}, \qquad (6)$$

while m=n=1 and m=n=101 are the terms of harmonic series for sinusoidal and uniform distributed load, respectively. Substituting Eqs. (5a-b) into Eqs. (4a-b), the following equations are obtained:

$$K_{ij} d_j = F_j \qquad (i,j = 1,......,2) \ \text{and}\ (K_{ij} = K_{ji}), \tag{7}$$

Elements of K_{ij} in Eq. (7) can be derived as follows (Refs. [15-17]):

$$\{d_j\}^T = \{W_{mn} \quad \theta_{mn} \quad \},$$
$$\{F_j\}^T = \{Q_{mn} \quad -q^*.Q_{mn} \quad \}, \tag{8a-b}$$

where Q_{mn} is the coefficient in the double Fourier expansion of the transverse load,

$$q(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Q_{mn} \sin(\alpha x)\sin(\beta y) \tag{9}$$

4. Numerical Results and Discussion

The bending analyses of isotropic and orthotropic plates are presented in this section. Shimpi's HSDT and a HSDT containing strong similarities with the one developed by Reddy and Liu [18] are also introduced in the context of this generalized formulation. In addition, a non-polynomial SSSF is utilized and optimized following the recommendation in Ref. [15] and implemented in the present generalized formulation (see Fig. 2). Three different case problems are presented. Figure 2 shows that it is possible to achieve excellent solutions for transverse shear stresses without sacrificing much accuracy in deflection and normal stresses by selecting the proper argument of the shear strain shape function. This is something that should be further studied. Results show that the generalized theory with cotangential SSSF performs well for isotropic and orthotropic materials, but for some orthotropic materials they are just acceptable. The reason behind that fact can be difficult to understand in one glance, but it may be related to the reduced number of unknown's variables and the strong dependency on the shear strain shape function in order to model six strains and six stresses. This could be really difficult to handle just for one function while a couple of them could make a remarkable difference. Consequently, the *case dependent problem* may be generalized. Overall, the transverse shear stress results can be substantially improved after running optimization procedures.

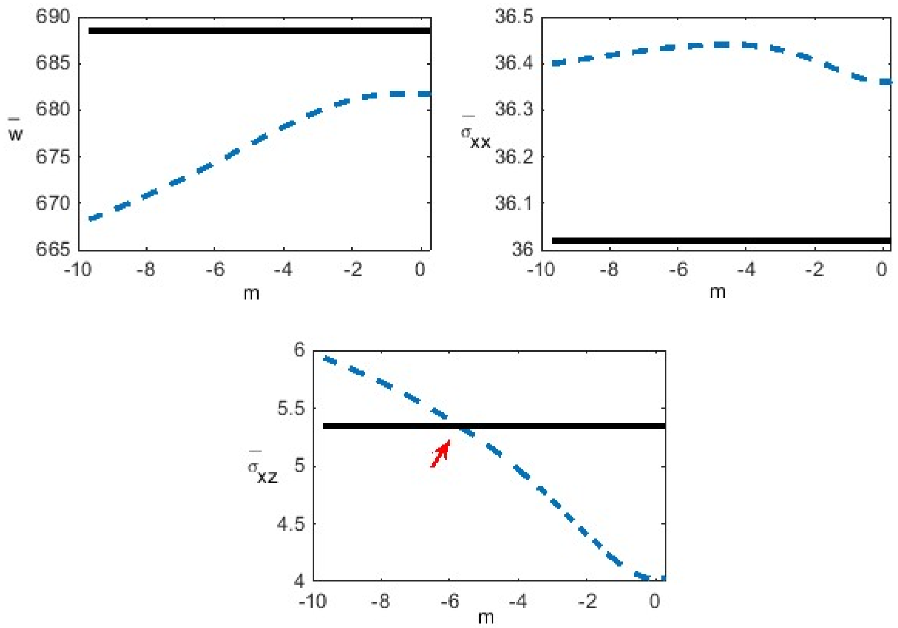

Fig. 2. Optimal values of *m* for the bending response of the new cotangential SSSF.

4.1. Case Problem 1: Isotropic plate

A moderate thick to thick isotropic plate is considered in this section (E=1, ν=0.3). It is worth mentioning that the present generalized theory contains strong similarities with the CPT; therefore further comparisons with the CPT and the FSDT (Reissner) are performed. Since the theory is capable of reproducing Shimpi's HSDT [2] results by using the SSSF and related constant parameters shown in Table 1, further comparisons are performed between the following parameters : the present optimized cotangential HSDT (see Fig. 2); the HSDT developed by Shimpi and Patel [2]; the adapted Reddy HSDT with two-unknown variables introduced in this study (see Table 1, where the non-polynomial function with two optimization parameters #1 and #2 is also introduced); the CPT, the FSDT, and the exact elasticity solution [27]. Table 2 indicates that the present HSDT with the selected parameter corresponding to the optimization #2 (Opt2) shows the best accuracy and a good agreement with the other theories.

Table 2. Normalized central deflection of simply-supported isotropic rectangular plate subjected to uniformly distributed transverse load.

b/a	a/h	Exact [27]	FSDT [27]	CPT [27]	Shimpi [2]	Present: Polynomial Reddy	Present: Non-polynomial Opt1	Present: Non-polynomial Opt2
	20	6855.0	6852.9	6806.5	6861.1	6861	6861	6853
2	10	437.5	437.0	425.4	439.1	439	439	437
	7.14	116.9	116.7	110.7	117.7	118	118	117
	20	2761.3	2760.0	2729.9	2765.3	2765	2765	2760
1	10	178.5	178.1	170.6	179.5	179	179	178
	7.14	48.4	48.2	44.4	48.9	49	49	48
	20	437.5	437.0	425.4	439.1	439	439	437
0.5	10	29.6	29.5	26.6	30.0	30	30	29
	7.14	8.5	8.4	6.9	8.7	9	9	8
%Average error.			0.2	6.0	0.8	0.8	0.8	0.3

$\bar{\bar{w}} = \bar{w}\, G / hq$, $G = E / (2 + 2\nu)$

4.2. Case Problem 2: Orthotropic plate

A simply supported orthotropic rectangular plate subjected to the uniform distributed load is considered. Several thickness ratios a/h, and aspect ratios *b/a* are considered. The orthotropic properties are given in Eq. (10). For this case problem, the 3D elasticity solution was introduced by Srinivas and Rao [28].

$$\bar{Q} = \begin{bmatrix} 0.999781 & 0.231192 & 0 & 0 & 0 \\ 0.231192 & 0.524886 & 0 & 0 & 0 \\ 0 & 0 & 0.262931 & 0 & 0 \\ 0 & 0 & 0 & 0.266810 & 0 \\ 0 & 0 & 0 & 0 & 0.159914 \end{bmatrix} \tag{10}$$

The present results are compared with the following theories: Srinivas and Rao [28]; the HSDT developed by Shimpi and Patel [2]; the original theory by Reddy [29]; the adapted Reddy HSDT with two-unknown introduced in this study (see Table 1); the CPT, and the FSDT. The normalized quantities used in Tables 3-5 are defined in Eqs. (11a-c).

$$\bar{\bar{w}} = \bar{w}\left(\frac{a}{2},\frac{a}{2},0\right)\frac{Q_{11}}{hq},\quad \bar{\sigma}^{1}_{xx} = \sigma^{1}_{xx}\left(\frac{a}{2},\frac{b}{2},\frac{h}{2}\right)\frac{1}{q},\quad \bar{\tau}^{1}_{xz} = \tau^{2}_{xz}\left(0,\frac{b}{2},0\right)\frac{1}{q}. \tag{11a-c}$$

No substantial difference between the theories can be found in Tables 3 and 4. The SSSFs and their parameter of optimization, *m* (see Table 1), yield basically the same results. However, in the case of a higher modular ratio (top or bottom/middle ratio of mechanical properties) as in Table 5, the transverse shear stresses are substantially different and the second parameter of optimization, *m*=-11/2, appears to be the best choice (see Fig. 2). Consequently, it can be concluded that transverse shear stresses results can be improved when optimized non-polynomial SSSFs are implemented in the present generalized formulation.

Table 3. Normalized central deflection of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

b/a	a/h	Exact [28]	Reddy [29]	FSDT [27]	CPT [27]	Shimpi [2]	Present: Polynomial Reddy	Present: Non-polynomial Opt1	Present: Non-polynomial Opt2
	20	21542	21542	21542	21201	21514	21514	21514	21465
2	10	1409	1409	1408	1325	1402	1402	1402	1390
	7.14	387	388	387	345	384	384	384	378
	20	10443	10450	10442	10246	10413	10413	10413	10388
1	10	689	690	688	640	682	682	682	675
	7.14	191	192	191	167	188	188	188	184
	20	2049	2051	2048	1988	2043	2043	2043	2034
0.5	10	139	140	139	124	138	138	138	136
	7.14	40	40	40	32	39	39	39	38
%Average error.			0.2	0.0	8.0	0.8	0.8	0.8	1.9

Table 4. Normalized in-plane normal stresses of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

b/a	a/h	Exact [28]	Reddy [29]	FSDT [27]	CPT [27]	Shimpi [2]	Present Polynomial Reddy	Present Non-polynomial Opt1	Present Non-polynomial Opt2
2	20	262.67	262.60	262.07	262.26	262.78	262.76	262.76	262.91
	10	65.98	65.95	65.38	65.56	66.07	66.07	66.07	66.22
	7.14	33.86	33.84	33.27	33.45	33.96	33.96	33.96	34.11
1	20	144.31	144.30	143.87	144.39	144.68	144.65	144.65	144.73
	10	36.02	36.01	35.58	36.10	36.36	36.36	36.36	36.44
	7.14	18.35	18.34	17.91	18.42	18.68	18.68	18.68	18.76
0.5	20	40.66	40.67	40.48	40.86	40.98	40.97	40.97	41.00
	10	10.03	10.05	9.85	10.22	10.33	10.33	10.33	10.36
	7.14	5.04	5.07	4.86	5.21	5.32	5.32	5.32	5.36
%Average error.			0.1	1.4	0.9	1.4	1.4	1.4	1.7

Table 5. Normalized transverse shear stresses of simply-supported orthotropic rectangular plate subjected to uniformly distributed transverse load.

b/a	a/h	Exact [28]	Reddy [29]	FSDT [27]	CPT [27]	Shimpi [2]	Present Polynomial Reddy	Present Non-polynomial Opt1	Present Non-polynomial Opt2
2	20	14.05	13.98	14.11	0.00	12.67	12.75	12.78	17.20
	10	6.93	6.96	7.06	0.00	6.31	6.31	6.32	8.42
	7.14	4.88	4.94	5.04	0.00	4.47	4.47	4.48	5.91
1	20	10.87	10.85	10.86	0.00	8.05	8.17	8.19	10.97
	10	5.34	5.38	5.43	0.00	4.02	4.02	4.03	5.31
	7.14	3.73	3.81	3.87	0.00	2.84	2.84	2.84	3.70
0.5	20	6.24	6.16	6.22	0.00	4.15	4.18	4.19	5.53
	10	2.96	2.89	3.05	0.00	2.03	2.03	2.03	2.61
	7.14	2.00	2.08	2.14	0.00	1.42	1.42	1.42	1.78
Average error (%)			1.4	2.5	100.0	21.7	21.5	21.3	11.3

4.3. Case Problem 3: Cross-ply laminated composites

The response of square cross-ply laminated composites subjected to the bi-sinusoidal load is discussed in this section. Mechanical properties are given in the corresponding tables (Tables 6 & 7) and normalization formulas are provided in Eqs. (12a-c). The results of the present theory are compared with the refined HSDT by Carrera et al. [30] (Table 6) as well as the other analytical and numerical results in Refs. [31-33] (Table 7). It can be noticed that highly accurate results are obtained for a cross-ply 0°/90°/0° (see Table 6). A cross-ply laminated composite 0°/90°/90°/0° is also studied and the results show less accuracy than the FSDT for thick plates while they are still comparable for thin plates (Table 7). In general, the reason could be due to the fact that results for thicker laminate schemes are more difficult to capture for any kinds of the shear deformation theory.

$$\bar{\bar{w}} = \bar{w}(\frac{a}{2},\frac{a}{2},0)\frac{100E_2h^3}{qa^4},\ \bar{\sigma}_{xx} = \sigma_{xx}(\frac{a}{2},\frac{b}{2},\frac{h}{2})\frac{h^2}{qa^2},\quad \bar{\sigma}_{yy} = \sigma_{yy}(\frac{a}{2},\frac{b}{2},\frac{h}{4})\frac{h^2}{qa^2},\ \bar{\tau}_{xz} = \tau_{xz}(0,\frac{b}{2},0)\frac{h}{qa}. \tag{12}$$

Table 6. Normalized central deflection of simply-supported cross-ply plate (0°/90°/0°) subjected to sinusoidal transverse load.

Theory	Strain	DOFs	$\bar{w}$ 10	50	100	500	1000
Present non-polynomial Opt 1	$\varepsilon_{zz}=0$	2	0.913	0.778	0.774	0.772	0.772
Present non-polynomial Opt 2	$\varepsilon_{zz}=0$	2	0.913	0.778	0.774	0.772	0.772
Present Polynomial Reddy	$\varepsilon_{zz}=0$	2	0.913	0.778	0.774	0.772	0.772
Analytical LW-4 [30]	$\varepsilon_{zz}\neq 0$	>30	0.981	0.779	0.773	0.771	0.770
MITC4 LW-4 [30]	$\varepsilon_{zz}\neq 0$	>30	0.975	0.774	0.767	0.765	0.763

$E_1 = 132.38GPa,\ E_2 = E_3 = 10.756GPa, G_{23} = 3.606\,GPa, G_{12} = G_{13} = 5.6537GPa,\ \nu_{12} = \nu_{13} = 0.24,\ \nu_{23} = 0.49$

Table 7. Normalized bending results of simply-supported cross-ply plate (0°/90°)$_s$ subjected to sinusoidal transverse load.

a/h	Theory	strain	DOFs	$\bar{w}$	%	$\bar{\sigma}_{xx}$	%	$\bar{\sigma}_{yy}$	%	$\bar{\sigma}_{zz}$	%	Avrg. %
	Elasticity [35]	$\varepsilon_{zz}\neq 0$	-	0.743	-	0.559	-	0.403	-	0.301	-	-
	Present non-polynomial Opt 1	$\varepsilon_{zz}=0$	2	0.604	18.7	0.575	2.8	0.253	37.1	0.136	54.8	28
	Present non-polynomial Opt 2	$\varepsilon_{zz}=0$	2	0.575	22.6	0.585	4.6	0.243	39.7	0.182	39.5	27
10	Present Polynomial Reddy	$\varepsilon_{zz}=0$	2	0.604	18.7	0.575	2.8	0.254	37.1	0.136	54.9	28
	HSDT [32]	$\varepsilon_{zz}=0$	5	0.715	3.8	0.546	2.4	0.389	3.5	0.264	12.3	6
	FSDT [33]	$\varepsilon_{zz}=0$	5	0.663	10.8	0.499	10.8	0.362	10.3	0.167	44.6	19
	RBF [31]	$\varepsilon_{zz}\neq 0$	9	0.733	1.4	0.563	0.7	0.391	3.0	0.332	10.3	4
	Elasticity [27]	$\varepsilon_{zz}\neq 0$	-	0.435	-	0.539	-	0.271	32.8	0.339	-	-
	Present non-polynomial Opt 1	$\varepsilon_{zz}=0$	2	0.433	0.4	0.539	0.0	0.269	0.7	0.137	59.7	15
	Present non-polynomial Opt 2	$\varepsilon_{zz}=0$	2	0.433	0.5	0.539	0.0	0.269	0.7	0.186	45.2	12
100	Present Polynomial Reddy	$\varepsilon_{zz}=0$	2	0.433	0.4	0.539	0.0	0.269	0.7	0.136	59.8	15
	HSDT [32]	$\varepsilon_{zz}=0$	5	0.434	0.1	0.539	0.1	0.271	0.1	0.290	14.5	4
	FSDT [33]	$\varepsilon_{zz}=0$	5	0.434	0.2	0.538	0.1	0.271	0.2	0.178	47.5	12
	RBF [31]	$\varepsilon_{zz}\neq 0$	9	0.431	0.9	0.543	0.8	0.273	0.7	0.377	11.2	3

$E_1 = 25E_2, G_{12} = G_{13} = 0.5E_2, G_{23} = 0.2E_2, \nu_{12} = \nu_{13} = \nu_{23} = 0.25$

4.4. Capability and limitation of this theory

The capability to calculate the bending mechanical response of isotropic plates is demonstrated in this study. However, in case of orthotropic plates and in the case of some laminations schemes, the present theory is satisfactory. But for other lamination schemes, the results are just comparable with existing theories in the thin regimen. The authors believe that this is due to the so-called *case dependent problem*. However, it should be kept in mind that the present theory contains only two unknown variables much less than that of the FSDT or other HSDTs (Tables 6 & 7). It is important to remark that this theory is valid for symmetric laminated composites since the mathematical model for the displacement field presented in this study cannot capture the membrane effect existing in this type of laminate scheme. Therefore, the response of schemes such as cross-ply 0/90 cannot accurately predict the in-plane deformations and stresses.

Overall, this theory has reduced unknown variables and is highly attractive for the modeling of nanosheets or nanoplates, therefore, it deserves further research and consideration. Readers may consult the relevant and recent research work on this topic is provided in Ref. [34]. Moreover, additional information regarding shear deformation theories with reduced number of unknown variables can be found in Refs. [35-44].

5. Conclusions

An unavailable generalized two-unknown higher order shear deformation theory (HSDT) for isotropic and orthotropic plates was presented and discussed. Shimpi's HSDT along with a similar theory to Reddy HSDT with two unknown variables were reproduced as special cases. Results showed that both Shimpi and the adapted Reddy's HSDT are essentially the same. This is due to the fact that both theories use polynomial SSSFs. Moreover, an unavailable optimized cotangential SSSF within the generalized theory was introduced. The generalized governing equation obtained by the principle of virtual displacement was solved via the Navier closed form solution. Results showed that transverse shear stresses can be substantially improved when non-polynomial SSSFs are utilized. Overall, this theory appears to be attractive to nanosheets and nanoplates studies as recently reported in the literature, therefore, the present study deserves attention.

Acknowledgments

This work is dedicated to Professor Shimpi for his remarkable trajectory and outstanding contribution to the shear deformation theory of plates.

Conflict of Interest

The authors declare no conflict of interest.

References

[1] R.P. Shimpi. Refined plate theory and its variants. *AIAA J*, 40(1) (2002) 137-46.
[2] R.P. Shimpi, H.G. Patel. A two variable refined plate theory for orthotropic plate analysis. *Int J Solids Struct*, 43(22) (2006) 6783-99.
[3] R.P. Shimpi, H.G. Patel. Free vibrations of plate using two variable refined plate theory. *J Sound Vib*, 296(4-5) (2006) 979-99.
[4] I. Mechab, H. Ait Atmane, A. Tounsi, H.A. Belhadj, E.A. Adda Bedia, A two variable refined plate theory for bending of functionally graded plates, *Acta Mech Sin*, 26(6) (2010) 941.

[5] H.H. Abdelaziz, H.A. Atmane, I. Mechab, L. Boumia, A. Tounsi, A.B.E. Abbas, Static Analysis of Functionally Graded Sandwich Plates Using an Efficient and Simple Refined Theory, *Chinese Journal of Aeronautics*, 24 (2011) 434-448.

[6] M.S.A. Houari, S. Benyoucef, I. Mechab, A. Tounsi, E.A. Adda bedia, Two variable refined plate theory for thermoelastic bending analysis of functionally graded sandwich plates, *J Therm Stresses*, 34 (2011) 315-34.

[7] A. Hamidi, M. Zidi, M.S.A. Houari, A. Tounsi, A new four variable refined plate theory for bending response of functionally graded sandwich plates under thermomechanical loading, *Composites: Part B*, (2012), DOI:10.1016/j.compositesb.2012.03.021.

[8] B. Mechab, I. Mechab, S. Benaissa, Static and dynamic analysis of functionally graded plates using Four-variable refined plate theory by the new function, *Composites: Part B*, 45 (2013) 748-757.

[9] H.T. Thai, S.E. Kim, A simple quasi-3D sinusoidal shear deformation theory for functionally graded plates. *Composite Structures*, 99 (2013) 172-180.

[10] J.L.Mantari, C Guedes Soares. A trigonometric plate theory with 5-unknowns and stretching effect for advanced composite plates. *Composite Structures*, 107 (2014) 396-405.

[11] J.L.Mantari, C Guedes Soares. Optimized sinusoidal higher order shear deformation theory for the analysis of functionally graded plates and shells. *Composites Part B: Engineering*, 56 (2014) 126-136.

[12] J.L.Mantari, C Guedes Soares. Five-unknowns generalized hybrid-type quasi-3D HSDT for advanced composite plates. *Applied Mathematical Modelling*, 39 (2015) 5598–5615.

[13] A.M. Zenkour, Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate, *Applied Mathematical Modelling*, 77 (2007) 197-214.

[14] A.M. Zenkour. A simple four-unknown refined theory for bending analysis of functionally graded plates. *Applied Mathematical Modelling*, 37(20-21) (2013) 9041-9051.

[15] J.L.Mantari, C Guedes Soares. Four-unknown quasi-3D shear deformation theory for advanced composite plates. *Composite Structures*, 109 (2014) 231-239.

[16] J.L.Mantari, C Guedes Soares. A quasi-3D tangential shear deformation theory with four unknowns for functionally graded plates. *Acta Mech*, 226 (2015) 625-642.

[17] J.L. Mantari. A simple polynomial quasi-3D HSDT with four unknowns to study FGPs. Reddy's HSDT assessment. *Composite Structures*. 137 (2016) 114–120.

[18] J.N. Reddy, C.F. Liu. A higher-order shear deformation theory of laminated elastic shells. *Int J Eng Sci*, 23 (1985) 319–30.

[19] R.A. Chaudhuri. On boundary-discontinuous double Fourier series solution to a system of completely coupled P.D.E.'S. *Int J Eng Sci*, 27(9) (1989) 1005-22.

[20] R.A. Chaudhuri. On the roles of complementary and admissible boundary constraints in Fourier solutions to boundary-value problems of completely coupled rth order P.D.E.'s. *J Sound Vib*, 251 (2002) 261–313.

[21] R.A. Chaudhuri, H.R.H. Kabir. Fourier Solution to higher order theory based laminated shell boundary-value problem. *AIAA J*, 33 (1995) 1681-88.

[22] A.S. Oktem, R.A. Chaudhuri. Levy type analysis of cross-ply plates based on higher-order theory. *Compos Struct*, 78 (2007) 243-53.

[23] A.S. Oktem, R.A. Chaudhuri. Fourier solution to a thick Levy type clamped plate problem. *Compos Struct*, 79 (2007) 481-92.

[24] A.S. Oktem, R.A. Chaudhuri. Fourier analysis of thick cross-ply Levy type clamped doubly-curved panels. *Compos Struct*, 80 (2007) 489-503.

[25] A.S. Oktem, R.A. Chaudhuri. Boundary discontinuous Fourier analysis of thick cross-ply clamped plates. *Compos Struct*, 82 (2008) 539-48.

[26] A.S. Oktem, R.A. Chaudhuri. Effect of in-plane boundary constraints on the response of thick general (unsymmetric) cross-ply plates. *Compos Struct*, 83 (2008) 1-12.

[27] S. Srinivas. *Three dimensional analysis of some plates and laminates and a study of thickness effects*. Ph.D. Thesis, Dept. of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, 1970.

[28] S. Srinivas, A.K. Rao. Bending, vibration and buckling of simply-supported thick orthotropic rectangular plates and laminates. *International Journal of Solids and Structures*, 6(11) (1970) 1463–1481.

[28] J.N. Reddy. A refined nonlinear theory of plates with transverse shear deformation. *International Journal of Solids and Structures*, 20(9-10) (1984) 881–896.

[30] E. Carrera, M. Cinefra, P. Nali, MITC technique extended to variable kinematic multilayered plate elements, *Composite Structures*, 92 (2010) 1888–1895.

[31] A. Ferreira, E. Carrera, M. Cinefra, C. Roque, Radial basis functions collocation for the bending and free vibration analysis of laminated plates using the Reissner-Mixed variational theorem, *European Journal of Mechanics - A/Solids*, 39 (2012) 104–112.

[32] J. Reddy, A simple higher order theory for laminated composite plates, *J Appl Mech*, 51 (1984) 745–752.

[33] J. Reddy, W. Chao, A comparison of closed-form and finite-element solutions of thick laminated anisotropic rectangular plates, *Nuclear Engineering and Design*, 64 (1981) 153–167.

[34] M. Sobhy. Hygrothermal vibration of orthotropic double-layered grapheme sheets embedded in an elastic medium using the two-variable plate theory. *Applied Mathematical Modelling,* 40 (2016) 85–99.

[35] I. Senjanović, N Vladimir, M Tomić. On new first-order shear deformation plate theories. *Mechanics Research Communications*, 73 (2016) 31–38.

[36] M. Bouazza, A. Lairedj, N. Benseddiq, S. Khalki. A refined hyperbolic shear deformation theory for thermal buckling analysis of cross-ply laminated plates. *Mechanics Research Communications*, 73 (2016)117–126.
[37] Y.M. Ghugal, A.S. Sayyad. Flexure of thick orthotropic plates by exponential shear deformation theory. *Latin American Journal of Solids and Structures*, 10 (2013) 473-490.
[38] Y. M. Ghugal, A. S. Sayyad. Free vibration analysis of thick orthotropic plates using trigonometric shear deformation theory. *Latin American Journal of Solids and Structures*, 8 (2011) 229-243.
[39] D. Lanc, T.P. Vo, G. Turkalj, J. Lee. Buckling analysis of thin-walled functionally graded sandwich box beams. *Thin-Walled Structures*, 86 (2015) 148–156.
[40] I. Senjanovic, S. Tomasevic, N. Vladimir. An advanced theory of thin-walled girders with application to ship vibrations. *Mar Struct*, 22(3) (2009) 387–437.
[41] I. Senjanovic, I. Catipovic, S. Tomasevic. Coupled flexural and torsional vibrations of ship-like girders. *Thin-Wall Struct*, 45(12) (2007) 1002–21.
[42] A.H. Sofiyev, N. Kuruoglu. Buckling and vibration of shear deformable functionally graded orthotropic cylindrical shells under external pressures. *Thin-Walled Structures*, 78 (2014) 121-130.
[43] A.H. Sofiyev, The effect of elastic foundations on the nonlinear buckling behavior of axially compressed heterogeneous orthotropic truncated conical shells. *Thin-Walled Structures*, 80 (2014) 178-191.
[44] H.T. Thai, D.H. Choi. A refined plate theory for functionally graded plates resting on elastic foundation. *Composites Science and Technology*, 71 (2016) 1850-1858.
[45] H. Asadi, A.H. Akbarzadeh, Q. Wang. Nonlinear thermo-inertial instability of functionally graded shape memory alloy sandwich plates. *Composite Structures*, 120(1) (2015) 496-508.
[46] H. Asadi, M. Eynbeygi, Q. Wang. Nonlinear thermal stability of geometrically imperfect shape memory alloy hybrid laminated composite plates. *Smart Materials and Structures*, 23(7) (2014) 075012.
[47] S.F. Nikrad, H. Asadi. Thermal postbuckling analysis of temperature dependent delaminated composite plates. *Thin-Walled Structures*, 97 (2015) 296-307.
[48] S.F. Nikrad, H. Asadi, A.H. Akbarzadeh, ZT Chen. On thermal instability of delaminated composite plates. *Composite Structures*, 132(1) (2015) 1149-1159.
[49] T.R. Mahapatra, V.R. Kar, S.K. Panda, K. Mehar. Nonlinear thermoelastic deflection of temperature-dependent FGM curved shallow shell under nonlinear thermal loading. *Journal of Thermal Stresses*, 40(9) (2017) 1184-1199.
[50] T.R. Mahapatra, V.R. Kar, S.K. Panda. Large Amplitude Free Vibration Analysis of Laminated Composite Spherical Panel under Hygrothermal Environment. *International Journal of Structural Stability and Dynamics*, 16(3) (2016) 1450105.
[51] T.R. Mahapatra, S.K. Panda, V.R. Kar. Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel, *International Journal of Mechanics and Materials in Design*, 12(2) (2015) 942-959.
[52] J.L. Mantari, E.V. Granados, M.A. Hinostroza, C.G. Soares. Modelling advanced composite plates resting on elastic foundation by using a quasi-3D hybrid type HSDT. *Composite Structures*, 118 (2014) 455-471.
[53] J.L. Mantari. Computational Development of a 4-Unknowns Trigonometric Quasi-3D Shear Deformation Theory to Study Advanced Sandwich Plates and Shells. *International Journal of Applied Mechanics*, 8(4) (2016) 1650049.
[54] J.L. Mantari. General recommendations to develop 4-unknowns quasi-3D HSDTs to study FGMs. *Aerospace Science and Technology*, 58 (2016) 559-570.