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Research Paper

Experimental and Numerical Investigations on the Effect of Rectangular Openings' Aspect Ratio on Outflow Discharge

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Abstract. Up to now, a few formulas have been suggested by scholars for the amount of discharge from openings, however, the effect of opening's geometry on the amount of discharge has not addressed thoroughly. In this study, to assess the effect of rectangular openings' aspect ratio on the discharge amount, experimental and numerical investigations have been conducted on the discharge amount from rectangular openings at the bottom of tanks. In the experimental part of the study different water depths have been considered and the amounts of discharge have been measured for openings with identical area, but different aspect ratios. In the numerical part of the study the test results have been compared to those obtained from finite-volume-based numerical simulation. The experimental and numerical results are in good agreement, and both show that there is a trend of increase in the amount of discharge with increase of the opening's aspect ratio. The amount of this increase is from 13% to 21% for hydraulic head varying between 0.3 to 0.6 meters. On this basis, the conventional orifice formula for calculation of the rectangular opening discharge needs modification.

Keywords: Rectangular opening, Aspect ratio, Leakage discharge, Finite volume method, Orifice formula.

1. Introduction

Water supplies are vulnerable to earthquakes. Previous earthquakes, such as the 1906 San Francisco [1-4], the 1971 San Fernando [5-7], and the 1994 Northridge [8-11] earthquakes, have confirmed the earthquake-induced damage and the vulnerability of the water supply networks. Earthquake damage to water supply systems may disrupt residential, commercial, and industrial activities; impair fire-fighting capacities; and prolong local community recovery in the aftermath of earthquakes. Therefore, it is an important issue to model the seismic performance of water supply systems in a way adequate from different points of view, e.g. community restoration [12].

A key parameter is the leakage from each component of the network. The leaking area depends on pipe material, joints characteristics, mechanism of the seismic damage destruction [12], aging of the networks, and the small earthquakes in the past. Different crack types happening because of earthquakes in different types of pipes are introduced by [12]. Longitudinal cracks (Fig. 1) are addressed as the generally most important type of crack. These cracks generally occur in pipes with large diameters, and are caused by internal pressure, water pressure surges, water hammer or some type of weakening of the pipe wall from

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either corrosion or manufacturing defects [13] and seismic activities. Metal pipes, including cast iron, ductile iron, and riveted steel, are vulnerable to longitudinal cracking [12]**.** An important issue which is not considered in modeling of water supply networks, especially when studied against earthquakes, is accurate calculation of discharge from cracks [12]. In estimation of water loss in damaged networks, the shape of the breakage should be considered carefully [14-16]. Considering these as a motivation; the authors reviewed the literature for generalization of the Torricelli formula, i.e.

$$Q=A.\,(2g.H)^{0.5} \tag{1}$$

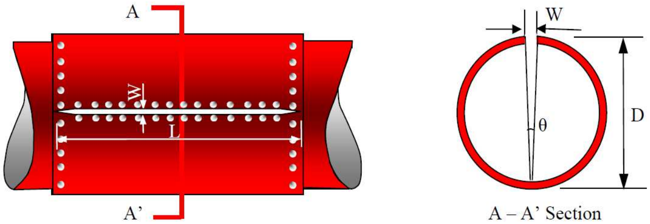

Fig. 1. Shape of longitudinal crack [12]

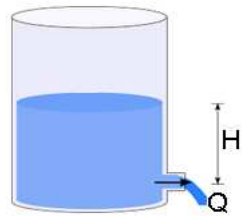

Fig. 2. Schematic flow under consideration in the Torricelli's formula

(A: opening area; H: Water head; g: Ground acceleration; Q: Opening discharge).Torricelli's formula, broadly accepted as the orifice formula, was proposed in 1644 by Evangelista Torricelli [17]. Equation (1) calculates the discharge from the typical tank displayed in Fig. 2. In the Torricelli's formula, some factors such as position of the orifice, thickness sharpness and shape of the opening were not considered. So, many works have been conducted to enhance Torricelli's formula, by considering

$$Q = CH^{\alpha} \tag{2}$$

instead of Eq. (1) and trying to set the appropriate values of C and α**,** generally known as discharge coefficient and exponent number [16, 18-22]. One of the earliest works has been done by Michelotti [24] who proposed different values for the discharge coefficient in different situations, e.g. in Eq. (2), C=0.62, $\alpha = 0.5$ and $C = 0.625$, $\alpha = 0.5$ for circular and square orifices respectively.

Considering the above-mentioned longitudinal cracks, the most interest and attention in this study is concentrated on the effect of rectangular cracks' aspect ratios on the outflow discharge from openings at the bottom of water tanks. In spite of the thorough literature reviewed by the authors, no specific achievements could be found. Only, the results from experiments on shape of the various openings with the same area of 127 mm^2 and different aspect ratios are reported by [22] from the experimental data originally produced by Bovey in 1909 [15] and reproduced later in 1976 [23]. Without talking about the discharge, these results indicate that the circular orifice has the lowest discharge coefficient followed by square, short rectangular and long rectangular.

Considering the small existing knowledge on the effect of openings' aspect ratio on the discharge, this paper is dedicated to resolve the question for a simple case. Figure 3 displays the case under study, where the orifice has a rectangular shape with different aspect ratios and located at the center bottom of the tank. With the objective to study the effect of the openings' aspect ratio on the discharge regardless of the C and α in Eq. (2), some experiments are reported after this introduction. The study is continued with the report of the corresponding numerical study. Later the experimental and numerical observations are compared and discussed for the reasons of the observations. The paper is eventually concluded with a brief set of the achievements, and a perspective of the future works.

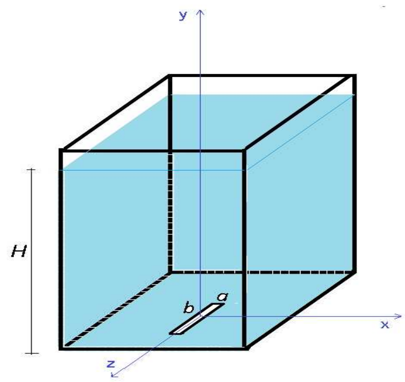

Fig. 3. The schematic of the tank used in the paper

2. Experimental Study

2.1. Set-up of the tests

As the first part of the study, water discharge was measured by using four tanks of different heights, made of Plexiglas [25], with square bases having small rectangular openings at the centers of their bases. The temperature of the water was about 20^{o}C. For each tank, four different values were considered for the opening lengths, such that the areas of the openings were all constant equal to 1 cm^2, while the aspect ratios changed from 1 to 64, as shown in Figure 4. Water tanks with 30×30 cm square base, heights (H) of 30, 40, 50 and 60 cm, were used leading to the sixteen experiments given in Table 1.

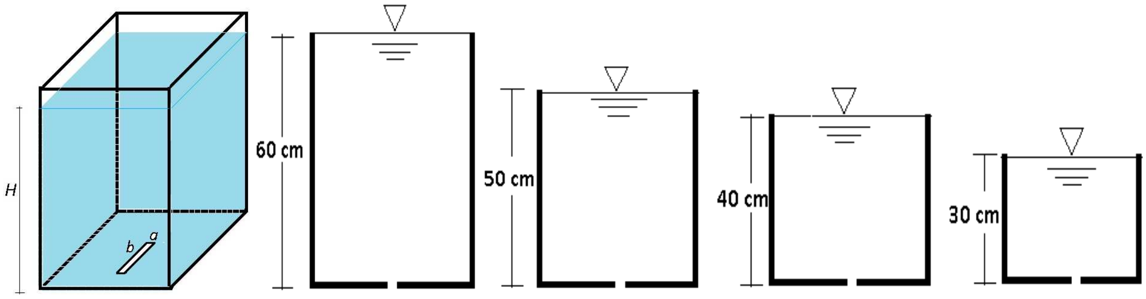

Fig. 4. The tanks with various water heads used in the tests

Table 1. Details of the experiments

Experiment No.	*H* (cm)	Opening's length, *a* (cm)	Opening's width, *b* (cm)	Aspect Ratio *(a/b)*
1	30	1.000	1.000	1
2	30	2.000	0.500	4
3	30	4.000	0.250	16
4	30	8.000	0.125	64
5	40	1.000	1.000	1
6	40	2.000	0.500	4
7	40	4.000	0.250	16
8	40	8.000	0.125	64
9	50	1.000	1.000	1
10	50	2.000	0.500	4
11	50	4.000	0.250	16
12	50	8.000	0.125	64
13	60	1.000	1.000	1
14	60	2.000	0.500	4
15	60	4.000	0.250	16
16	60	8.000	0.125	64

In order to maintain the precision of the openings geometry, specifically regarding the area and the aspect ratio, the openings were created by laser cut with tolerance of 0.1 mm. It should be notified that because of the relatively low pressures on the one hand, and relatively stiff bottom plate of the tank, on the other, deformation of the opening has been negligible. Figure 5 schematically shows the set-up used in the discharge measurement tests. The necessity of the adjustable inlet valve can be explained by the fact that depending on the experiment, a specific amount of inlet is essential to guarantee the constant water head.

As it is seen in Figure 5, two tanks are used in each test, a main tank with an opening in its bottom and a volume measuring tank with 5 cc measurement precision. The capacity of the second tank is set with attention to the procedure of the set described in the next section equal to 2000 cc.

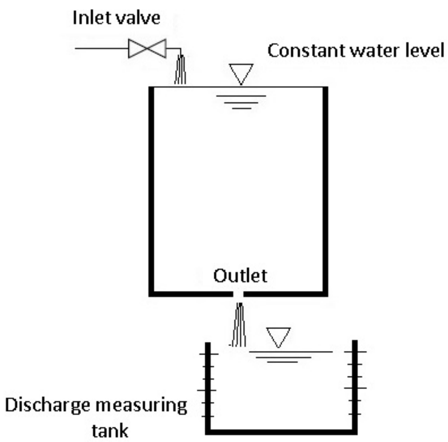

Fig. 5. Schematic representation of the discharge measurement tests

2.2. Procedure of the tests

In each experiment, first the inlet valve was set such that the water level became stable, creating a steady state flow in the opening. Then the main part of the experiment started using the measuring tank and a chronometer for measuring the outlets volume and the corresponding time duration. In order to make more confidence on the results, each experiment was repeated five times (see [26, 27]) to use the average discharge value as the outlet discharge based on recommendation of [26, 27] reference.

2.3. Test Results

Considering the explanations in the previous sections, the test results are reported in Tables 2, and the discharges corresponding to the experiments, Q, are computed using the following formula:

$$Q = \frac{1}{5}\left(\sum_{i=1}^{5}\frac{V_i}{t_i}\right) \tag{3}$$

where the discharge values of $V_{i=1,2,3,4,5}$ and the corresponding durations, $t_{i=1,2,3,4,5}$ are presented in Table 2, and the results computed by Eq. (3) are reported in Table 3.

Table 2. Results of leakage discharge test

Experiment No.	H (cm)	Aspect ratio	Amounts of measured discharge in cc (upper number) and discharge duration in seconds (lower number) for the 5 test repetitions				
			1	2	3	4	5
1	30	1	1780 10.05	1730 9.79	1820 10.29	1670 9.41	1780 10.12
2	30	4	1750 9.89	1700 9.48	1810 10.16	1800 10.13	1720 9.73
3	30	16	1960 10.03	1900 9.83	1910 9.87	9.79 1900	9.77 1880
4	30	64	1840 8.99	1860 9.17	1890 9.21	1960 9.57	1850 9.10
5	40	1	2040 10.16	1960 9.73	1940 9.66	1970 9.76	1970 9.77
6	40	4	2010 10.05	1900 9.49	1900 9.44	1960 9.79	2320 11.32
7	40	16	2210 9.90	2210 10.06	2240 10.09	2160 9.76	2170 9.92
8	40	64	2040 8.56	1890 7.91	1770 7.37	1840 7.66	1860 7.84
9	50	1	2260 9.86	2200 9.66	2250 9.86	2180 9.60	2210 9.52
10	50	4	2000 9.00	2220 9.78	2400 10.65	2230 9.76	2260 9.95
11	50	16	2570 10.51	2420 9.91	2500 10.27	2470 9.97	2380 9.74
12	50	64	2660 10.00	2640 9.97	2550 9.56	2560 9.83	2570 9.73
13	60	1	2260 9.55	2520 10.21	2440 9.98	2460 9.91	2400 9.82
14	60	4	2660 10.69	2420 9.81	2430 9.90	2380 9.70	2550 10.31
15	60	16	2630 9.91	2640 9.95	2560 9.64	2700 10.21	2640 9.91
16	60	64	2870 9.83	2790 9.61	2860 9.95	2660 8.99	2680 9.09

Table 3. Calculated results by Eq. (3)

Experiment No.	H (cm)	Aspect Ratio	Discharge (cc/s)
1	30	1	176.80
2	30	4	177.77
3	30	16	193.75
4	30	64	204.17
5	40	1	201.30
6	40	4	201.44
7	40	16	220.99
8	40	64	238.94
9	50	1	226.87
10	50	4	228.09
11	50	16	244.84
12	50	64	264.41
13	60	1	244.19
14	60	4	246.78
15	60	16	265.42
16	60	64	291.97

By looking at the results presented in Tables 2 and 3, one can realize that by increase in the value of pressure, the discharge value also increases as expected. However, interesting point here is that the amount of discharge increases with increase in the value of aspect ratio for the rectangular opening. This seems to be a new result, which has also been investigated numerically, as explained in the next section.

3. Numerical Study

3.1. Modeling

In this section first the modeling process of the tanks by using a powerful computer program is explained, and then the numerical results obtained by the computer program are presented in comparison with those obtained from the tests. The numerical model considered in this section was a tank with 30×30 square base, as used in the tests, with variable height, and a rectangular opening with various aspect ratios in the center of the base. Using this model, the steady state behaviors, in the 16 cases addressed in the previous section, were studied. Some assumptions are reasonably considered in the numerical method. The walls and base were modeled with no roughness, no elasticity, no thickness, also no-slip condition. In inlet and outlet of the tanks, pressure was considered to be equal to the pressure of atmosphere. The density (ρ) and dynamic viscosity (μ) of the fluid were considered, 998.2 kg/m^3 and 0.00102 kg/m/sec, respectively.

The governing equations in fluid mechanics are the continuity and Navier-Stokes equations [27]. The difficulty arises from the fact that the conservation of mass and momentum are coupled. Besides, in many cases, because of the nonlinearity, in practice the analytical solutions are not available specifically in real cases. To analyze the behavior approximately, the CFD (Computational Fluid Dynamics) method utilized in this paper is the finite volume method [30]. In this method, the governing Navier-Stokes equations are discretized through finite volumes.

For more efficient analysis, the fluid was modeled using structured meshing [30], and as essential, in the vicinity of the opening (at the bottom center of the tank), the meshing was finer, as shown in Figure 6. Convergence and mesh independence [30] were checked as displayed in Figure 7. The solution was repeated until the relative residuals in the governing equations were reduced to 10^{-7}. Figure 8 displays the convergence of discharge with respect to the number of elements in the finite volume analysis corresponding to the fifth experiment (see Tables 2). For achieving the desired accuracy it was tried to reduce the elements' sizes successively by almost halving them in each new analysis. In view of the convergence plot depicted in Figure 8 and the definition below:

$$D_i = \left| Q_i - Q_{i-1} \right| \tag{4}$$

where D_i is the discharge difference between two successive meshing, i.e. pseudo error of discharge [32, 33], and Q_{i-1} and Q_i are respectively the amount of discharge in the two successive cases.

With regard to pseudo-error and the equivalence between convergence and pseudo-convergence [32-34], Figure 8 displays the provided first order proper convergence of the computation [35]. Meanwhile, and in view of the region displaying proper convergence in a better way (see Figure 8), the value of 212 cc indicated in Figure 7, is an appropriate selection for the discharge, also reasonable from the point of view of computational cost. With attention to this appropriateness, the sizes of the elements and their distribution in tanks have been considered as the basis of the selections in the other fifteen cases in Table 3.

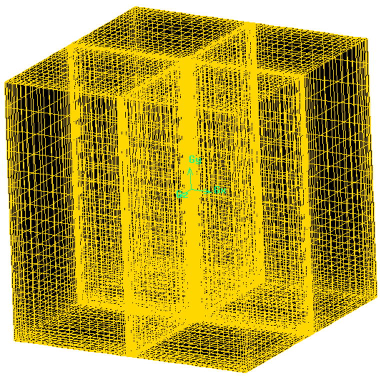

Fig. 6. Meshing the liquid model

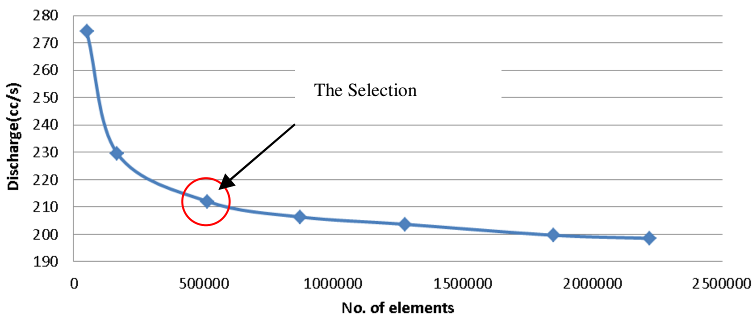

Fig. 7. Changes of discharge with respect to the number of elements in the fifth case in Table 4

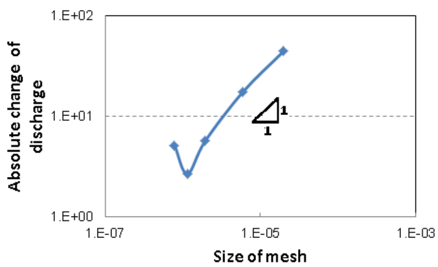

Fig. 8. Pseudo-convergence plot for the numerical study carried out for the fifth case in Table 2

3.2. Numerical results and their comparison with experimental results

Considering the details explained in Section 3.1, the analyses were carried out leading to the results reported in the fourth column of Table 4, compared with the experimental results presented in the fifth column of the table.

Table 4. Comparison of experimental and numerical results

Experiment No.	H (cm)	Aspect Ratio	Numerical Discharge (cc/s)	Experimental Discharge. (cc/s)	Difference (%)
1	30	1	183.70	176.80	-3.90
2	30	4	189.20	177.77	-6.47
3	30	16	201.30	193.75	-3.90
4	30	64	206.70	204.17	-1.24
5	40	1	212.03	201.30	-5.41
6	40	4	212.70	201.44	-5.59
7	40	16	226.03	220.99	-2.28
8	40	64	244.28	238.94	-2.23
9	50	1	237.20	226.87	-4.91
10	50	4	244.90	228.09	-7.31
11	50	16	258.68	244.84	-5.65
12	50	64	284.10	264.41	-7.45
13	60	1	252.60	244.19	-3.44
14	60	4	260.40	246.78	-5.52
15	60	16	276.75	265.42	-4.27
16	60	64	311.54	291.97	-6.70

The small relative differences between the experimental and numerical results, shown in the last column of Table 4, evidence the results good accuracy. The abovementioned small differences can be explained by the fact that some physical features in the experiments, e.g. the elasticity of the tank, were ignored in the numerical analyses. It should be also noted that the main aim of this paper, has been to study the changes of discharge with respect to opening's aspect ratio, regardless of the values of the discharge coefficient and exponent term in Eq. (2). Comparison between the experimental and numerical results of discharge for the openings with different aspect ratios is shown in Figure 9.

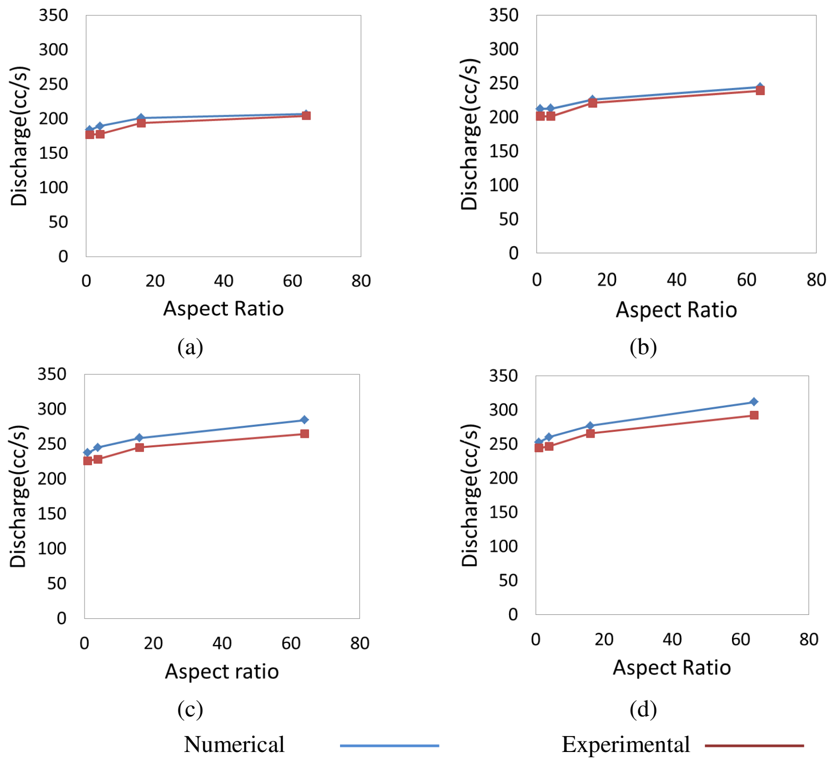

Fig. 9. Comparison of experimental and numerical results of leakage discharge of openings with various aspect ratios for different heads (H):
(a) $H = 30$ cm ; (b) $H = 40$ cm ; (c) $H = 50$ cm , and (d) $H = 60$ cm

The trends shown in Figure 9 display the good consistence between experimental and numerical results. These trends are discussed in detail in the continuation of this paper. To find out if the surface tension and current regime has any effect on the observed trends of variation of the leakage discharge for different values of opening's aspect ratio, the values of Weber and Reynolds numbers were calculated in each case. These two numbers are calculated by [17]:

$$We = \frac{\rho v^2 L}{\sigma} \tag{5}$$

$$Re = \frac{\rho v D}{\mu} \tag{6}$$

In Eq. (5) and (6), v stands for the velocity of the flow, L implies the length of the flow, σ represents the surface tension, and D is the hydraulic diameter. As an example, in the fifth case (see Table 5), the above mentioned two numbers can be computed as follow:

$$\begin{aligned} We &= \frac{\rho v^2 L}{\sigma} = \frac{998.2 \times 2.0667^2 \times 0.002}{0.0179} = 119 \\ Re &= \frac{\rho v D}{\mu} = \frac{998.2 \times 2.0667 \times 0.01}{0.00102} = 20225 \end{aligned} \tag{7}$$

These values were obtained for all cases of experiments and numerical calculations, based on the average of velocities obtained from experiments and numerical computations. Table 5 presents these results.

Table 5. Weber and Reynolds numbers for 16 experiments

No.	H (cm)	Aspect Ratio	Weber Number	Reynolds Number
1	30	1	90	17640
2	30	4	93	15961
3	30	16	108	9097
4	30	64	117	4949
5	40	1	119	20225
6	40	4	119	18013
7	40	16	139	10293
8	40	64	162	5820
9	50	1	149	22708
10	50	4	155	20572
11	50	16	176	11594
12	50	64	209	6607
13	60	1	171	24309
14	60	4	179	22060
15	60	16	204	12484
16	60	64	253	7269

As it is observed in Table 5, the Weber number is far above 1 in all cases, which shows that the surface tension force does not have significant effect in the problem at hand [17]. Also in all cases, the Reynolds number is far above 2000, which means that the regime is not laminar. On this basis, and regarding the results presented in Table 4, it can be stated that the increase of discharge value with the opening's aspect ratio is valid specifically for transient and turbulent regimes.

4. The Observed Phenomenon and Its Explanation

Both the numerical and the experimental results show the increase of discharge with respect to the opening's aspect ratio. This result is also reported by Cassa [22] from the experimental data originally produced by Bovey in 1909, [15], and reproduced in study of Brater and King [23]. The numerical study, reported in Section 3, which is consistent with the experimental results reported in Section 2 as well as those reported by Brater and King [23], is for the first time presented in this paper. Furthermore, the explanation below is the first attempt to explain the increase of discharge with respect to the opening's aspect ratio.

It is well-known that the flow becomes more turbulent at the sharp corners of openings [36]. This additional turbulence decreases the fluid velocity at some neighborhood of the corner, resulting in some disturbance in the flow passing from regions far from corners and accordingly decreasing the total amount of discharge. It is reasonable to accept that the decrease depends on the sharpness of the corner and the size of the neighborhood affected by the corner's additional turbulences. The sharpness depends on the geometry of the corner and for the experiments discussed in Section 2 and their corresponding computations reported in Section 3, can be defined in terms of the angle created by the sharpness, e.g. 90 degrees for rectangular openings; the sharpness is more when the angle is smaller and equals zero when there is no angle. Reasonably, the sharpness does not change with the aspect ratio, and there is no influence originating in sharpness on the discharge. Therefore, the difference between the discharges should be because of the sizes of the areas affected by the additional turbulences in the corners. Obviously, and as displayed in Figure 10, the total area affected by the corners turbulences decreases when the aspect ratio increases.

Because of the nonlinear nature of turbulence, in the common areas, i.e. areas affected by the turbulence at two corners (dark areas in Figure 10), more reduction of discharge cannot be expected in these areas. Meanwhile, as apparent in Figures 10 and 11, with the increase of the common areas, the percentage of areas not under the effect of the corners additional turbulences increases. Consequently, in cases with smaller total areas affected by the corners turbulence, i.e. openings with higher aspect ratios, the effect of the corner turbulence in reduction of the discharge is less and the resulting discharge is more. In brief, existence of common sections between the areas under the effect of openings turbulences at different corners causes the discharge to be more in openings with higher aspect ratios. Obviously, these common areas are more when the corners effect is more, and with a specific shape of opening this occurs when the total head is higher; this is in consistence with the observations reported in Table 5. Nevertheless, since as addressed above, the area of the opening not affected by the corner effect increases with the aspect ratio, e.g. see Figure 11, the discharge increases with the increase of the aspect ratio, regardless of the water head, shown in different colors in Figure 11. Obviously, the area of the rectangular opening is an upper-bound for the increase of areas not affected by the corners turbulence, causing the downward curvature of the plots in Figure 11.

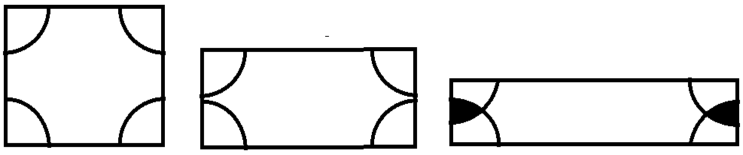

Fig. 10. Schematic representation of the areas under the effect of corners turbulences in rectangular openings with different aspect ratios

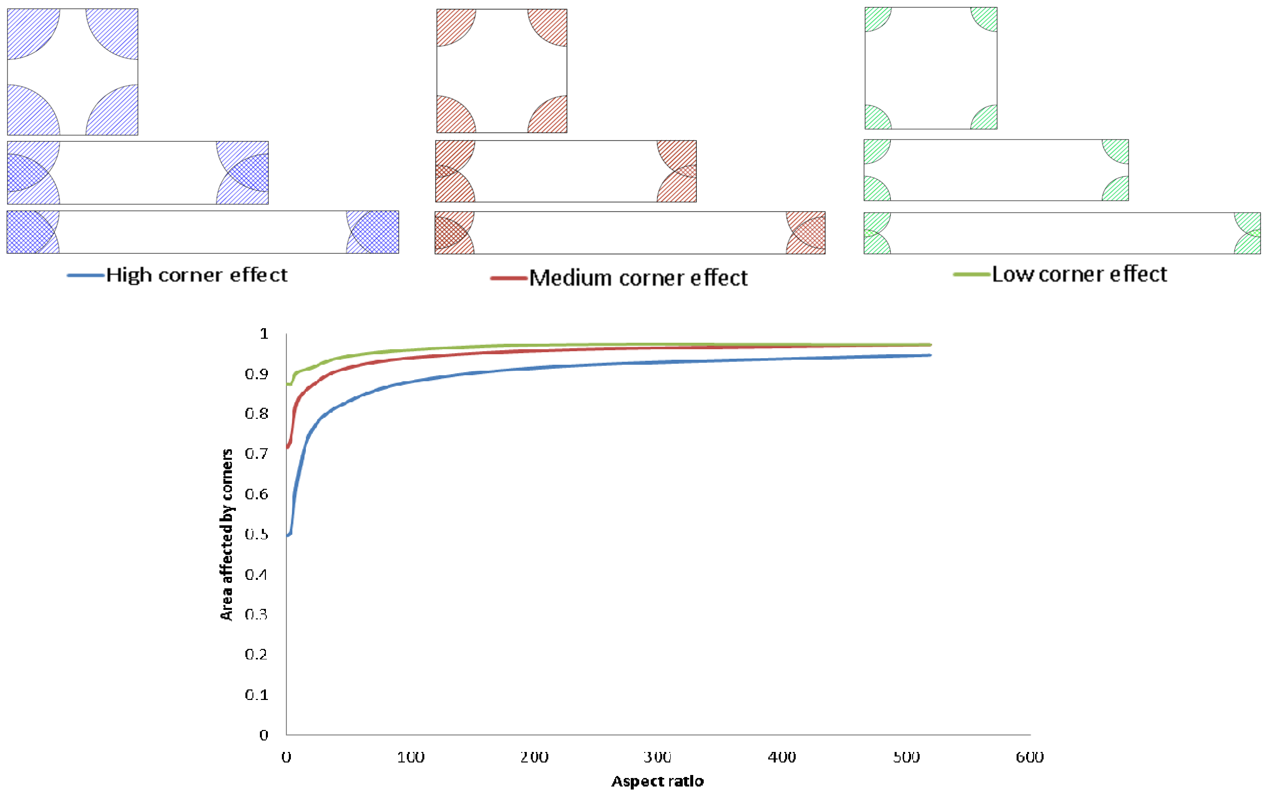

Fig. 11. Typical influence of the opening's aspect ratio on the area of cross-section not affected by the corners

5. Conclusions

Sixteen laboratory tests and the corresponding numerical studies on tanks, each having a rectangular opening at their bottom, were carried out and reported in this paper. The results validate each other and are in consistence with the observations reported in the literature. Specifically, the following conclusions may be pointed out for flow through an opening in the bottom of a tank:

- Increase in the aspect ratio of the opening increases the discharge, regardless of the amount of the water head. This observation seems to be explained for the first time in this paper.
- The above mentioned increase of discharge is explained by the opening's corner effects (i.e. the additional turbulences at the corners) on the velocity and area of the flow through the opening.
- The above two conclusions are valid particularly when the flow regime from the leak is turbulent which occurs in large leaks.

Based on the conclusions above the following issues are recommended for further studies:

- The effect of viscosity on the discharge in openings with various aspect ratios
- Extension of the study to opening with non-rectangular shapes
- Repetition of this study for openings on the wall of tanks, as well as wall of pipes
- Repetition of this study considering deformation of the opening at higher pressures.

Conflict of Interest

The authors declare no conflict of interest.

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