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Research Paper

Viscous Dissipation Impact on Free Convection Flow of Cu-water Nanofluid in a Circular Enclosure with Porosity Considering Internal Heat Source

A.S. Dogonchi[1], Ali J. Chamkha[2,3], Seyyed Masoud Seyyedi[4], M. Hashemi-Tilehnoee[5], D.D. Ganji[6]

[1] Department of Mechanical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran, sattar.dogonchi@yahoo.com,

[2] Mechanical Engineering Department, Prince Sultan Endowment for Energy and Environment, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia, achamkha@pmu.edu.sa,

[3] RAK Research and Innovation Center, American University of Ras Al Khaimah, Ras Al Khaimah, United Arab Emirates, achamkha@pmu.edu.sa,

[4] Department of Mechanical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran, s.masoud_seyedi@yahoo.com,

[5] Department of Mechanical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran, mehdi.hashemi.t@gmail.com,

[6] Mechanical Engineering Department, Babol Noshirvani University of Technology, Babol, Iran, mirgang@nit.ac.ir

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Corresponding author: A.S. Dogonchi, sattar.dogonchi@yahoo.com

Abstract. In this work, free convection of Cu-water nanofluid in an enclosure by considering internally heat generated in the porous circular cavity and the impacts of viscous dissipation are numerically evaluated by control volume finite element method (CVFEM). The outer and inner sides of the circular porous enclosure are maintained at a fixed temperature while insulating the other two walls. The impacts of diverse effective parameters including the Rayleigh number, viscous dissipation, and nanofluid concentration on features of heat transfer and fluid flow are examined. Moreover, a new correlation for the average Nusselt number is developed according to the study's active parameters. It can be deduced by the results that the maximum value of the temperature is proportional to the viscous dissipation parameter.

Keywords: Free convection of nanofluid; Porous circular enclosure; Viscous dissipation; CVFEM; Nusselt number.

1. Introduction

One of the basic problems in improving the heat transfer coefficient in industrial systems is fluid's low thermal conductivity. Researchers have suggested many methods for solving this difficulty. One of the contemporary methods is adding metal based Nano-particles to the original fluid. This combination is known as "Nanofluid". Because the thermal conductivity of many metals is higher than liquids, so Nanofluids have a thermal conductivity greater than basic fluids. Currently, various investigations have been done on the problems of heat transfer and fluid flow using normal fluid and nanofluid [1-29]. Hayat et al. [1] perused the flow behavior of the couple stressed nanomaterial stuck between two walls in a parallel position. Ellahi et al. [2] investigated a nanofluid 2D heat transfer flow with the existence of mixed convection. The results show that the velocity of nanofluid descends by ascending volume fraction of nanoparticle and fractal dimensions. Zhang et al. [3] examined the nanofluid thin film flow and its heat transfer resulting from an expanding sheet undergoing velocity slip, different concentration, and magnetic field impacts. Dogonchi and Ganji [4] studied the transient squeezing flow of a nanofluid among surfaces by considering the influence of thermal radiation. They declared that Nusselt number is a rising function of the heat source factor. Bhatti and Rashidi [5] studied entropy generation on nanofluid flow approaching a stagnation point stream along a plane which is penetrable and stretchable. The results reveal that fluid's velocity grows as a result of the higher porosity

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parameter and magnetic field impact. In addition, the number of entropy generation increases in both Reynold's number and the Brinkman number. Dogonchi and Ganji [6] examined Nanofluid flow between unparalleled plates exposed to a magnetic field. The results confirmed that an increase in Schmidt number increases the value of the Nusselt number. Incompressible micropolar Nanofluid flow over a moving plane subjected to the Brownian motion impact and magnetic field has been explored by Hsiao [7]. He reported that as Prandtl number ascends the temperature descends. Rashidi et al. [8] analyzed Nanofluid flow and its related heat transfer along an expanding plane. They proved that temperature distribution along the plane is proportional to the magnetic parameter. Mixed convection of nanofluid over an expanding plane subjected to electrical and magnetic fields was analyzed by Hsiao [9]. In another work, viscous dissipation and radiative impacts on Maxwell fluid over an expanding plane subjected to electrical and magnetic fields were analyzed by Hsiao [10]. Hsiao [11] analyzed magneto-hydrodynamic Carreau nanofluid over an expanding plane with radiation and electrical field impacts. Viscous Dissipation impact on the forced convection of nanofluid inside a micro-channel was analytically analyzed by Hung [12]. He discovered that microchannel's thermal performance is overrated when viscous dissipation is inactive. Viscous dissipation and bidispersivity impacts on the thermal performance of forced convection flow in a porous channel were explored by Wang and Li [13]. Mehmood and Ali [14] analytically explored the influence of viscous dissipation on heat transfer and fluid flow among two planes. They discovered that when viscous dissipation is active the Prandtl number leads makes temperature ascend. Dogonchi et al. [15] studied MHD nanofluid's unsteady squeezing heat transfer and flow between parallel plates with an infinite dimension with considering the impact of thermal radiation. The outcomes portray that the Nusselt number and temperature profile are in direct relationship with the radiation parameter. Dogonchi et al. [16] analyzed the influence of thermal radiation on the Go-water flow between two parallel flat plates. They found a clear relationship between the Nusselt number, temperature profile and the solid volume fraction. Conversely, they discovered a reverse relationship between the radiation parameter and the Nusselt number/temperature profile. Dib et al. [17] studied squeezing nanofluid during unsteady flow between two walls.

Recently, the free/natural convection flow of normal fluid and nanofluid in enclosures turns into a subject of interest to various researchers [30-49] owing to the breadth of applications. Rashad et al. [30] evaluated internal heat generation and the impact of the magnetic field on the free convection fluid flow in a rectangular cavity filled with nanoparticles. The results illustrate that as the Hartmann number or the solid volume fraction rises, the average Nusselt number diminishes while the reverse is true for augmentation of the magnetic field vector. In addition, they deduced that increases in the Hartmann number cause abatement of the stream function's maximum value; however, it augments the maximum temperature. Soleimani et al. [31] explored heat transfer of natural convection within a semi-annulus cavity that contains nanofluid. They observed that there is a choice turn angle wherein average the Nusselt number is optimal for each Rayleigh number. Natural convection of fluid flow within a wavy porous cavity in consideration of thermal dispersion's impact was analyzed by Sheremet et al. [32]. Hatami et al. [33] examined the natural convection of Nanofluid in a circular wavy cavity. They detected that Lewis number's influence on Nusselt number is only meaningful in numbers with a higher buoyancy ratio, while with lower buoyancy ratio numbers, its effect is approximately negligible. Sheremet et al. [34] studied nanofluid flow and natural convective heat transfer within a wavy porous cavity. Their results demonstrate the existence of an efficient local heat source influence on the nanofluid heat transfer rate and flow. Sheremet et al. [35] explored entropy generation in a square nanofluid filled enclosure in the presence of fluctuating temperature distribution adjacent to the left wall. Findings indicate that a rise in the temperature distribution amplitude adjacent to the left wall and a rise in the wave number cause an ascendant in average entropy generation.

The main objective of this study is to investigate the free convection of a Nanofluid in a permeable circular enclosure by considering the internal heat generation and viscous dissipation effects utilizing the CVFEM. In the results section, the impact of some parameters like Rayleigh number, Nanofluid's volume fraction, and viscous dissipation parameter are considered in the calculation, as well.

2. Problem Description and Basic Equations

In this section, Nanofluid free convection in a porous circular enclosure considering internal heat generation and viscous dissipation impacts is studied (Fig. 1). The circular walls, both internal and external are kept at a steady temperature, T_0, while the other two walls have been insulated. Using Boussinesq approximation, the governing equations including momentum, continuity, and energy equations for the current study can be defined as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$-\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}=-\frac{gK\beta_{nf}}{\nu_{nf}}\frac{\partial T}{\partial x} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{q_0'''}{\left(\rho C_p\right)_{nf}}+\frac{\mu_{nf}}{K\left(\rho C_p\right)_{nf}}\left(u^2+v^2\right) \tag{3}$$

Here v and u are the fluid velocities, and K stands for porous medium's permeability. Also, T is the temperature, and g indicates gravitational acceleration.

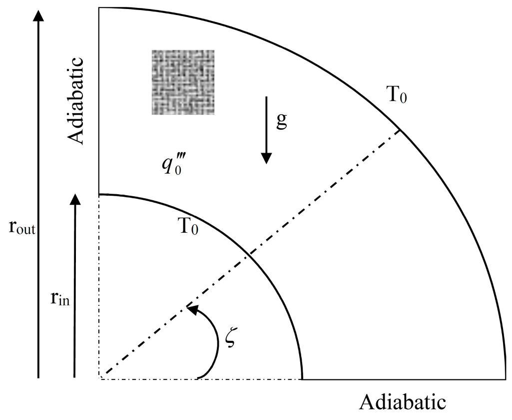

Fig. 1. Physical model and the coordinate system

Further, ρ_{nf}, $(\rho Cp)_{nf}$, β_{nf}, μ_{nf} and k_{nf} portray effective density, thermal expansion coefficient, heat capacitance, thermal conductivity of nanofluid and effective dynamic viscosity, respectively, which are demonstrated as follows:

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \tag{4}$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{5}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{6}$$

$$\frac{k_{nf}}{k_f} = \frac{2k_f + k_s - 2\phi(k_f - k_s)}{2k_f + k_s + \phi(k_f - k_s)} \tag{7}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}. \tag{8}$$

Table 1 presents the thermal-physical specification of nanofluid.

Table 1. Thermo-physical properties of water and the nanoparticles

#	$\rho(kg/m^3)$	$C_p(J/kg\ K)$	$k\ (W/m\ K)$
Copper (Cu)	8933	385	401
Pure water	997.1	4179	0.613

The following dimensionless variables are described:

$$X = \frac{x}{L},\quad Y = \frac{y}{L},\quad U = \frac{uL}{\alpha_f},\quad V = \frac{vL}{\alpha_f},\quad \theta = \frac{T - T_0}{\Delta T},\quad \Delta T = q_0'''L^2/k_f \tag{9}$$

here, $L=r_{out}-r_{in}$. On the other hand, the stream function is as follows:

$$V = -\frac{\partial\Psi}{\partial X},\quad U = \frac{\partial\Psi}{\partial Y} \tag{10}$$

By applying the defined dimensionless variables in Eq. (9), the dimensionless form of the governing equations is written as:

$$\frac{\partial^2\Psi}{\partial Y^2} + \frac{\partial^2\Psi}{\partial X^2} = -\frac{\mu_f}{\mu_{nf}}\frac{(\rho\beta)_{nf}}{(\rho\beta)_f}Ra\frac{\partial\theta}{\partial X} \tag{11}$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X} - \frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y} = \frac{(\rho C_p)_f}{(\rho C_p)_{nf}}\left(\frac{k_{nf}}{k_f}\left(\frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}\right) + 1 + \frac{\mu_{nf}}{\mu_f}\varepsilon\left[\left(\frac{\partial\Psi}{\partial X}\right)^2 + \left(\frac{\partial\Psi}{\partial Y}\right)^2\right]\right) \tag{12}$$

subject to the boundary conditions:

$$\begin{aligned} &\theta = 0 && \text{on the inner wall} \\ &\theta = 0 && \text{on the outer wall} \\ &\frac{\partial\theta}{\partial n} = 0 && \text{on the two other adiabatic walls} \\ &\Psi = 0 && \text{on all walls} \end{aligned} \tag{13}$$

where $Ra = gK\beta_f\Delta TL/\alpha_f\nu_f$ is the Rayleigh number and $\varepsilon = \mu_f\alpha_f/\Delta TK(\rho C_p)_f$ is the viscous dissipation parameter.

The average and local Nusselt number along the inner wall can be described as:

$$Nu_{loc.} = \frac{k_{nf}}{k_f}\frac{\partial \theta}{\partial n} \tag{14}$$

$$Nu_{ave.} = \frac{1}{\pi/2}\int_0^{\pi/2} Nu_{loc.} d\zeta \tag{15}$$

here n depicts the direction normal to the inner wall.

3. Mathematical Solution

Applying the boundary conditions, the governing equations are solved using CVFEM [26]. The mesh independency studied by the average Nusselt number with various mesh sizes. Table 2 represents that the grid of 71×141 should be considered for the study. In addition, this calculation has been substantiated by comparing our FORTRAN code outcomes with other works [30, 49]. They are in great agreement as they have been reported in Table 3.

Table 2. Impact of grid size on the average Nusselt number (Nu_{avg}) for $Ra = 1000$, $\phi = 0.04$ and $\varepsilon = 0.002$.

Grid dimension (Radial direction × Angular direction)	Nu_{avg}
31×61	0.554083
51×101	0.650828
71×141	0.709070
91×181	0.748339
111×221	0.776722

Table 3. Comparison between present results and other works for Ψ_{max} and θ_{max} at $Ra = 1000$

Ra	Present study	Rashad et al. [30]	Grosan et al. [49]
Ψ_{max}	3.52031	3.5132	3.5338
θ_{max}	0.096156	0.0985	0.097359

4. Results and Discussion

We have accomplished a computative investigation in the current study in order to investigate the impacts of active parameters, e.g. the Rayleigh number ($Ra = 500\text{-}1000$), viscous dissipation parameter ($\varepsilon = 0.000\text{-}0.002$), and volume fraction of a nanofluid (ϕ=0%, 2% and 4%) on a nanofluid's free convection in the permeable circular enclosure considering internal heat generation and viscous dissipation impacts.

The influences of Rayleigh number and viscous dissipation parameter on streamlines and isotherms are demonstrated in Figs. 2 and 3. The maximum values of the stream function point to the convective flow's strength. It means that the performance of natural convection is an arising function of the maximum value of the stream function. In addition, ascendant in the Rayleigh number and viscous dissipation parameter due to increment maximum value of stream function. Based on these reasons, the convective flow strength ascends with an enhancement in Rayleigh number and viscous dissipation parameter. For higher values of Ra, the impacts of viscous dissipation parameter on the strength of the convective flow is more pronounced. From the streamline pattern, it can be also concluded that by increasing Ra and ε the flow moves downward along the internal and external circular walls.

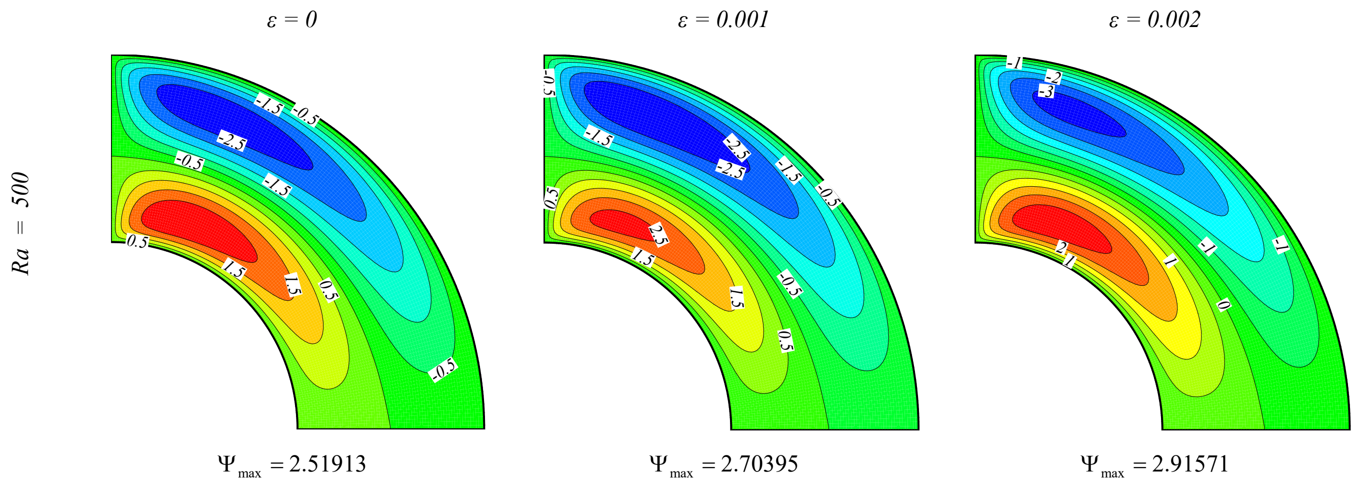

Fig. 2. Streamlines contours for different values of Ra and ε when $\phi = 2\%$

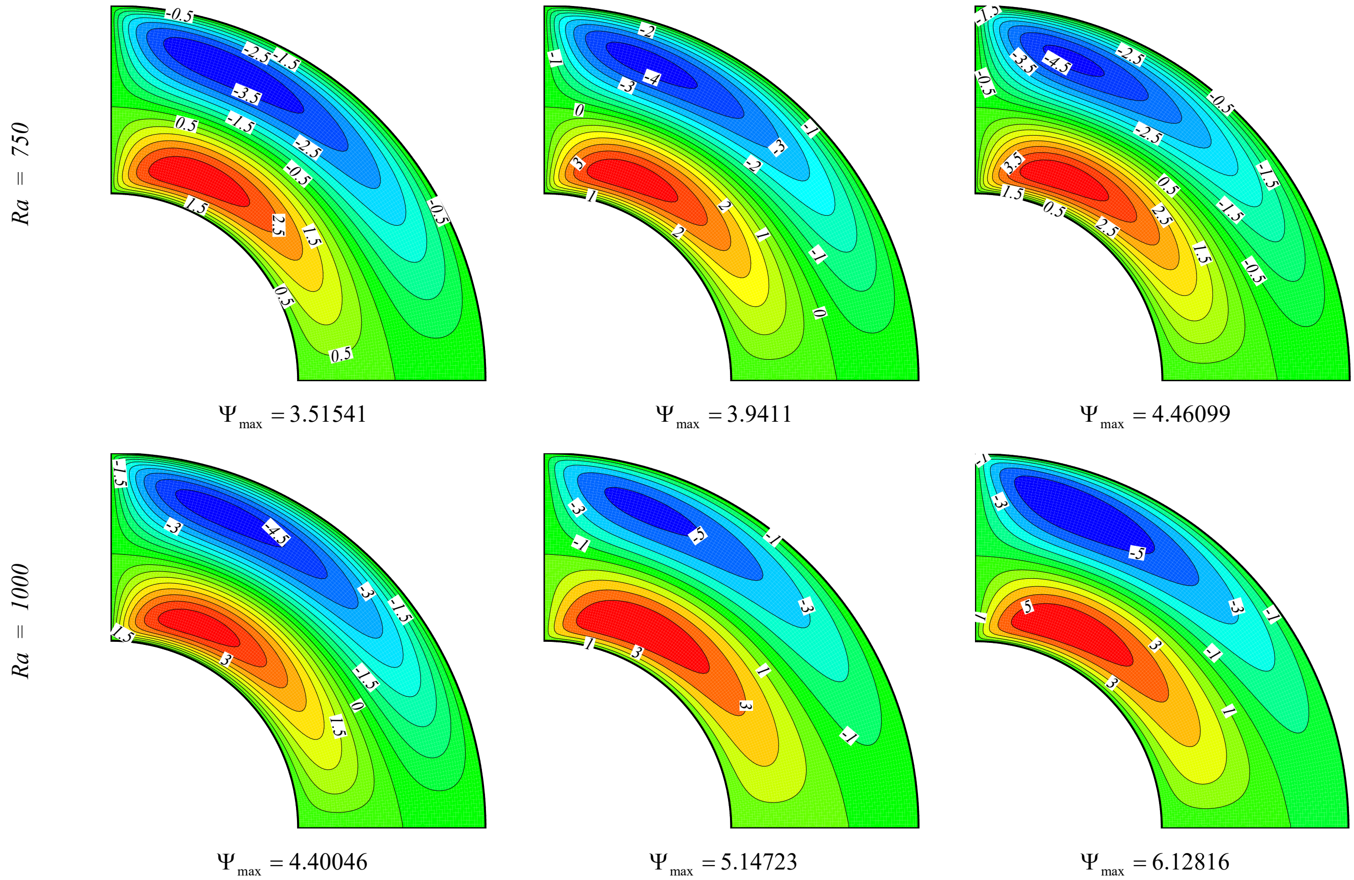

Fig. 2. Continued

According to the Rayleigh number definition, if the viscous forces overcome the buoyancy forces, then the Rayleigh number will be low. Therefore, we conclude that at a low *Ra*, conduction is the primary mode of heat transfer compared to convection, while at a high *Ra* this conclusion is completely reversed, meaning the primary means of heat transfer here is convection. One can observe that at higher values of ε, by raising Rayleigh number, maximum value of the temperature increases. At a low *Ra*, the isotherms are nearly parallel, which demonstrates that the conduction is more notable than the convection. However, at a high *Ra*, the isotherms are distorted, showing convection as the primary means of heat transfer. In this case, a thermal boundary layer will be formed. The influence of the viscous dissipation parameter on the isotherms is similar to the Rayleigh number. Further, by ascending the viscous dissipation parameter, the high-temperature isotherm moves upward along the vertical adiabatic wall. This fact is more notable for higher Rayleigh number values.

Figs. 4 and 5 represent the diversity of $Nu_{loc.}$ and $Nu_{ave.}$ for different values of the viscous dissipation parameter, Rayleigh number, and nanofluid's volume fraction, respectively. Based on Eq. (15), $Nu_{loc.}$ and the temperature gradient share a direct relationship, and since there is a high temperature gradient near the vertical adiabatic wall (See Fig. 3), it is seen that the optimal value of the local Nusselt number happens near the vertical adiabatic wall. It can be also deduced that by ascending Rayleigh number and viscous dissipation parameter, the local Nusselt number's maximum value increases.

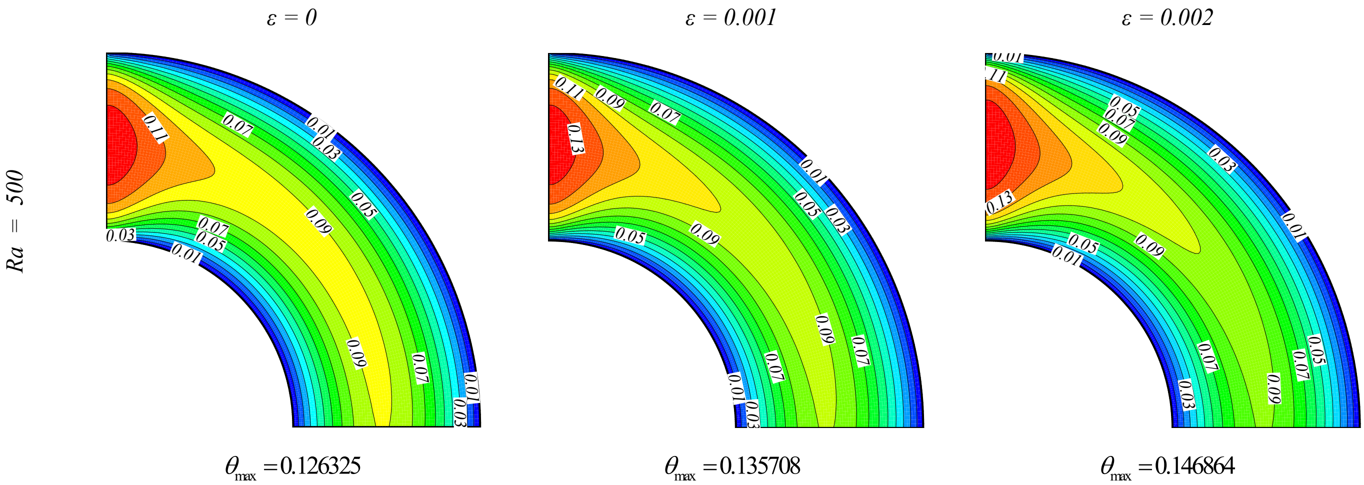

Fig. 3. Isotherms contours for different values of *Ra* and ε when $\phi = 2\%$

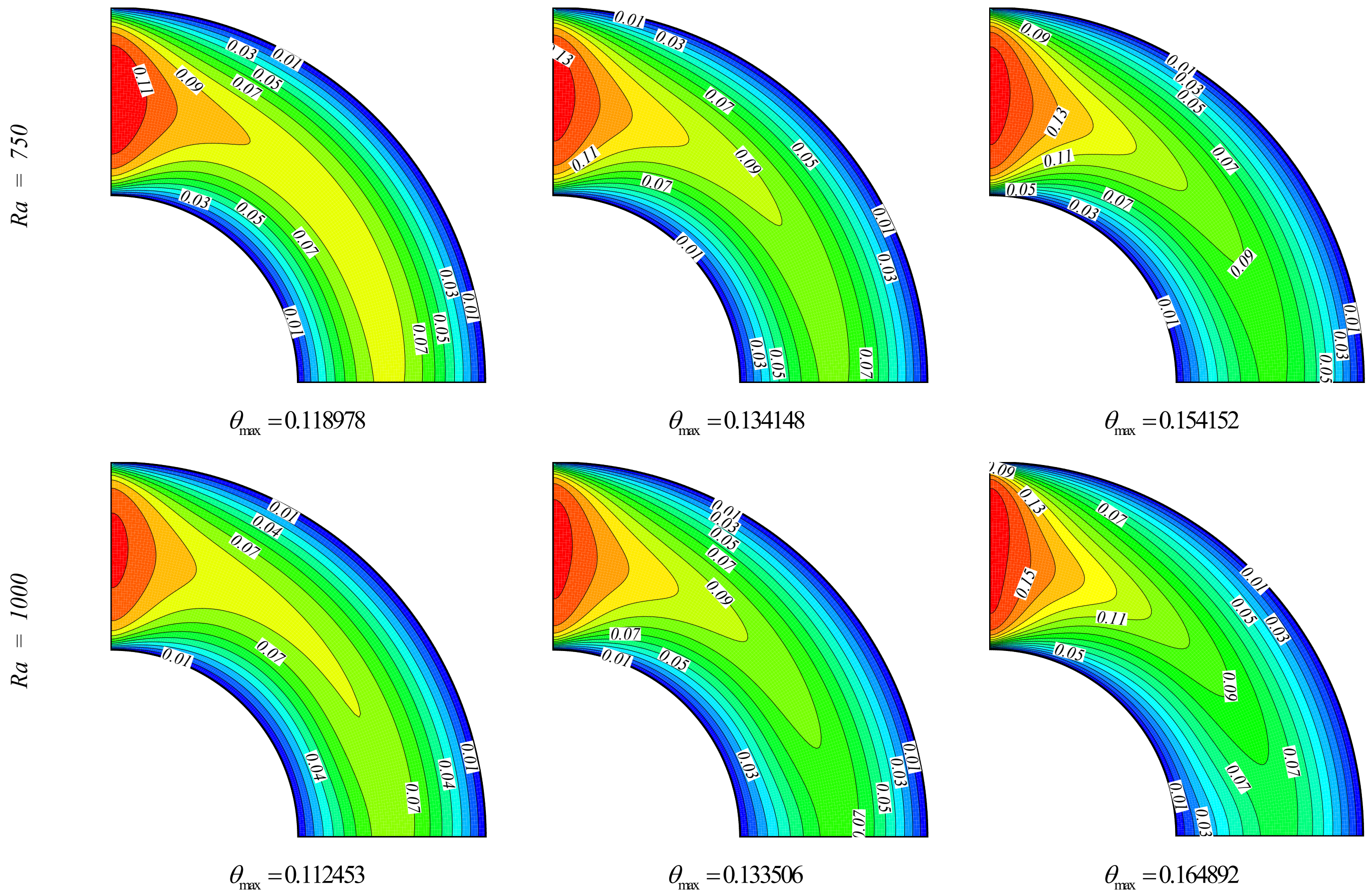

Fig. 3. Continued

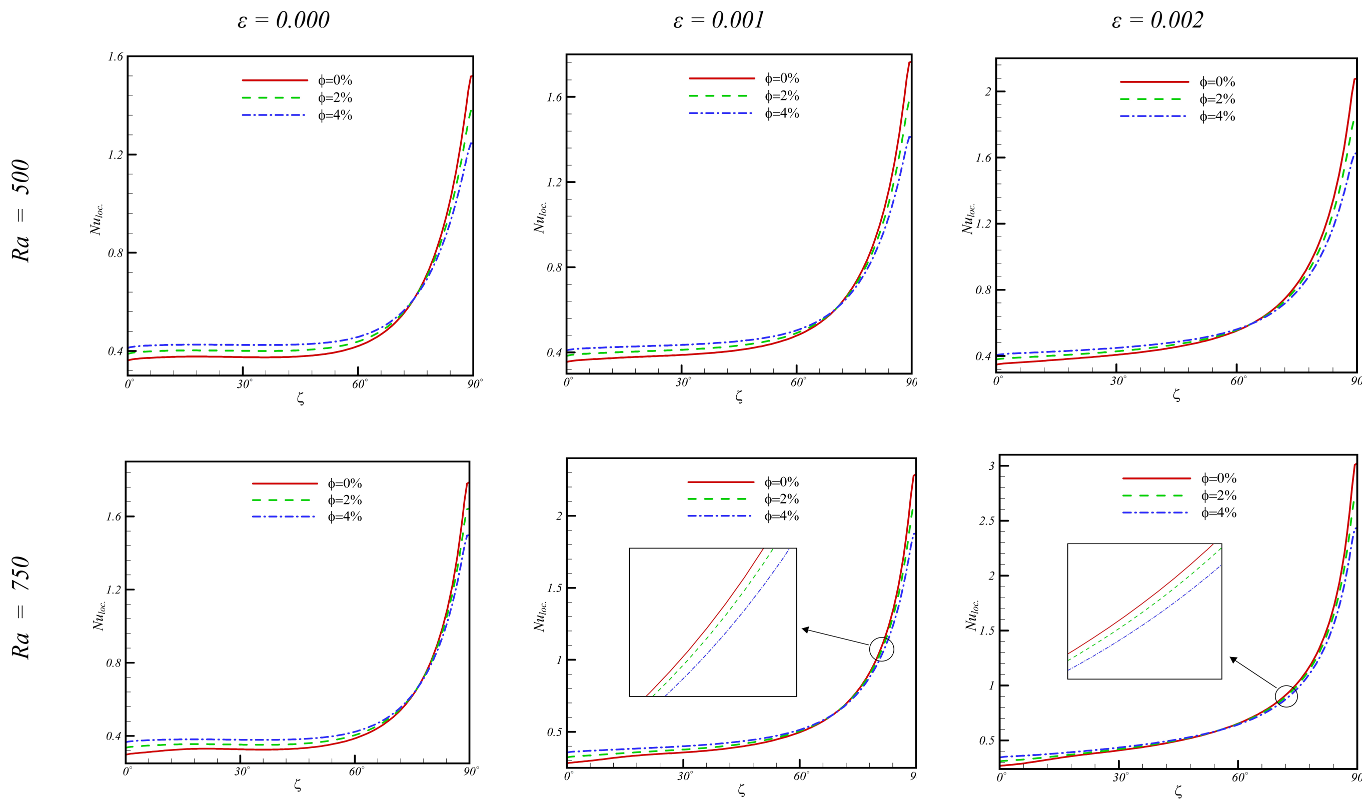

Fig. 4. Local *Nu* number ($Nu_{loc.}$) for different values of *Ra*, *ε* and ϕ

On the other hand, a rise in the nanofluid's volume fraction leads the maximum value of $Nu_{loc.}$ to decrease. In addition, $Nu_{ave.}$ is an ascending function of the volume fraction of a nanofluid and the viscous dissipation parameter. Finally, we attain the correlation for $Nu_{ave.}$ in terms of impressive parameters of the current study which is defined as follow:

$$Nu_{ave} = 0.59801 - 2.52375\times10^{-4}\times Ra - 0.69629\times\phi - 101.18379\times\varepsilon - 9.38661\times10^{-3}\times Ra\times\phi + 0.24447\times Ra\times\varepsilon \\ - 871.76736\times\phi\times\varepsilon + 1.24073\times10^{-7}\times Ra^2 + 5.88264\times\phi^2 + 20348.14048\times\varepsilon^2 \tag{16}$$

The R-Squared for this correlation is equal to 0.9930. The mentioned correlation is valid for Ra = 500-1000, ϕ = 0-0.04 and ε = *0-0.002*.

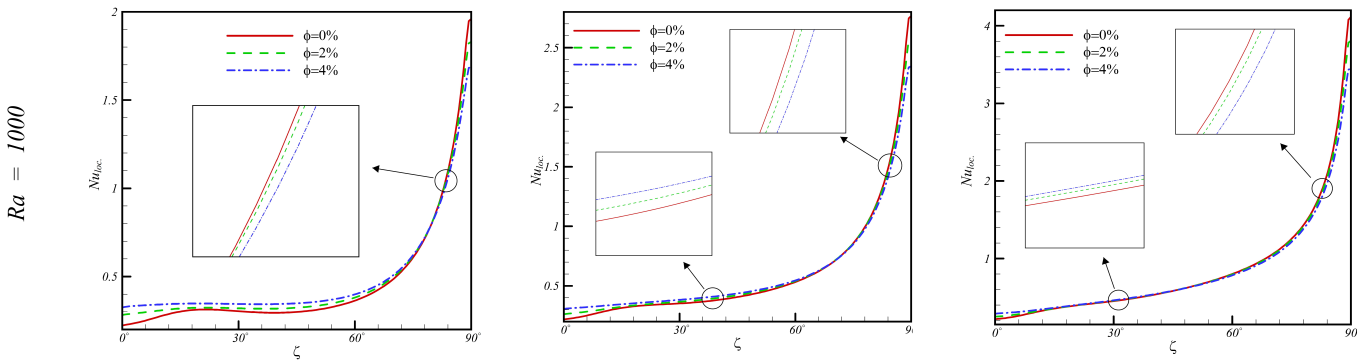

Fig. 4. Continued

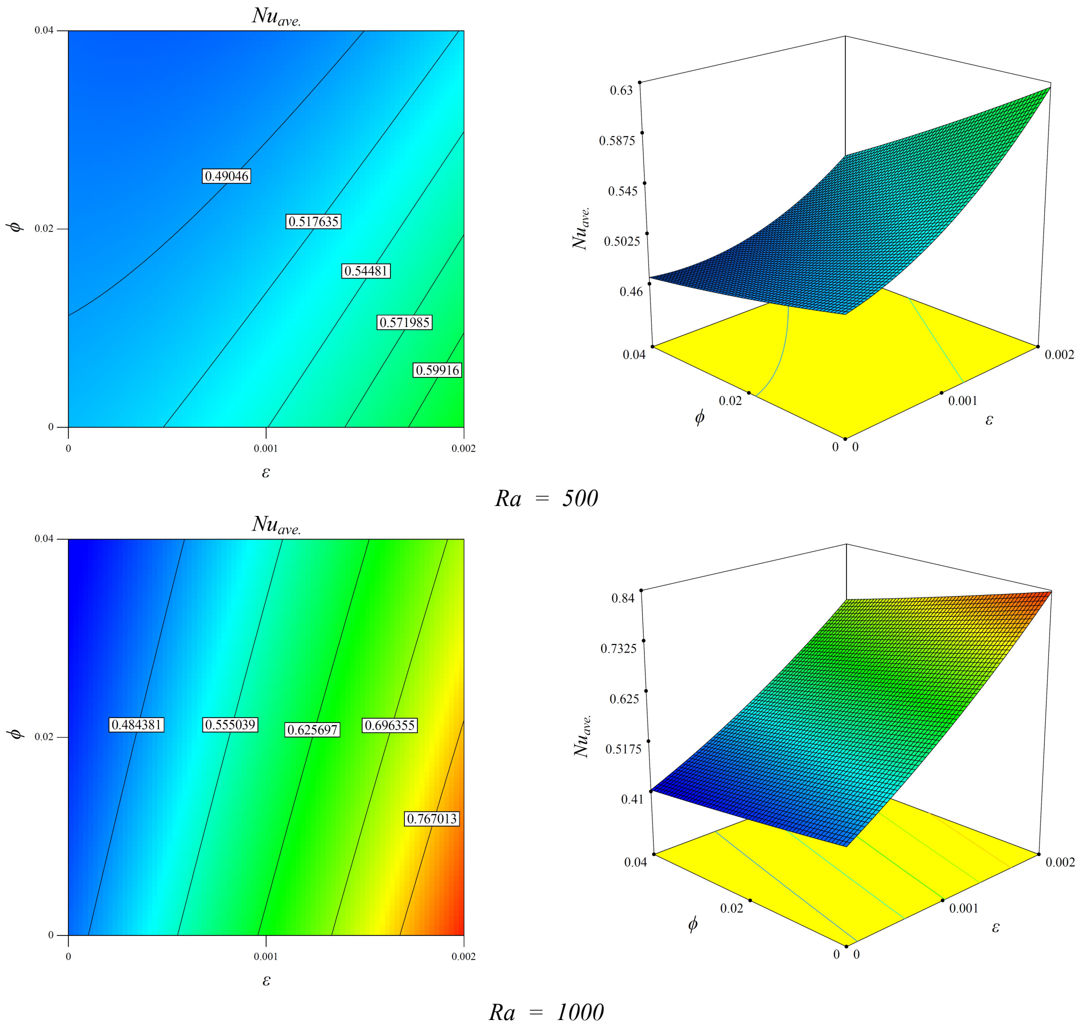

Fig. 5. Average Nu number ($Nu_{ave.}$) for different values of Ra, ε and ϕ

5. Conclusion

In this work, nanofluid free convection in a porous circular enclosure is studied by considering the internal heat generation and viscous dissipation effects using control volume finite element method. The comparison between the CVFEM and other works in the literature indicates the legitimacy of this course of research. The impacts of some parameters like Rayleigh number, viscous dissipation parameter and Nanofluid's volume fraction on flow and heat transfer features are examined. The results state that power of convective flow ascends with a growth in Rayleigh number and viscous dissipation parameter. In addition, the maximum value of temperature is proportional to viscous dissipation parameter. Furthermore, the maximum value of the local Nusselt number is calculated near the vertical adiabatic wall. Besides, in this work, a correlation for average Nusselt number with R-Squared of 0.9641 is obtained.

Conflict of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

K	Permeability of the porous medium	*Greek symbols*	
Pr	Prandtl number	β	Thermal expansion coefficient [K^{-1}]
u, v	Velocity components in x and y directions, respectively [$m\ s^{-1}$]	ρ	Density [$kg\ m^{-3}$]
C	Specific heat [$J\ kg^{-1}\ K^{-1}$]	μ	Dynamic viscosity [$kg\ m^{-1}\ s^{-1}$]
T	Temperature [K]	ν	Kinematic viscosity [$m^2\ s^{-1}$]
$Nu_{loc.}$	Local Nusselt number	ζ	Rotation angle
$Nu_{ave.}$	Average Nusselt number	ϕ	Nanoparticles volume fraction
k	Thermal conductivity [$W\ m^{-1}\ K^{-1}$]	*Subscripts*	
Ra	Rayleigh number	f	Base fluid
p	Pressure term	nf	Nanofluid
ε	Viscous dissipation parameter	s	Solid nanoparticles

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