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Micropolar Fluid Flow Induced due to a Stretching Sheet with Heat Source/Sink and Surface Heat Flux Boundary Condition Effects

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Abstract. Computational and mathematical models provide an important compliment to experimental studies in the development of solar energy engineering in case of electro-conductive magnetic micropolar polymers. Inspired by further understanding the complex fluid dynamics of these processes, we examine herein the non-linear steady, hydromagnetic micropolar flow with radiation and heat source/sink effects included. The transformed non-dimensional governing partial differential equations are solved with the R-K fourth order with shooting technique subjected to appropriate boundary conditions. The characteristics of the embedded parameters are obtained and presented through graphs. Velocity and microrotation of the fluid decreased with enhancing values of material parameter and suction/injection parameter. Electric field parameter has ability to enhance velocity, but temperature shows opposite behaviour. Microrotation increases for both magnetic field and surface temperature parameters.

Keywords: Micropolar fluid, Heat source/sink, Stretching sheet, Partial slip, Surface heat flux boundary conditions.

1. Introduction

The study of rheological fluid flows induced due to a permeable sheet stretching plays a significant role in science and industrial applications such as thermal insulation, fluid flowing in brain, exotic lubricants, chemical catalytic reactors and oil exploration etc. These fluids have complex nature in their structure and hence no single constitute equation will explain all the non-Newtonian characteristics. Micropolar fluid is a non-Newtonian fluid (lubricant fluids, colloids and polymer solutions, liquid crystal) in which the particles are suspended in a viscous medium with asymmetrical stress tensor. The micropolar theories was first developed by Eringen [1-2]. Mirzaaghaian and Ganji [3] considering the differential transformation method (DTM) to study the influence of a micropolar fluid through a channel with permeable walls. Siddiqua et al. [4] explored the convective magnetohydrodynamic micropolar fluid flow past a vertical surface with thermal radiation. Sui et al. [5] analysed the multi-dependent thermo diffusion influence on a viscoelasticity based micropolar fluid flow generated due to a sheet stretching with slip condition. The magnetohydrodynamic flow of a thermally radiative micropolar nanofluid in a porous channel was analysed by Alizadeh et al. [6]. Shamshuddin and Satya Narayana [7] numerically investigated the primary and secondary flows of a micropolar fluid past a plate. Miroshnichenko et al. [8] explored the influence of heat source on the natural convection flow of a micropolar fluid in a trapezoidal cavity. Abbas et al. [9] discussed the two-dimensional flow of a



micropolar fluid on a flat plate with the help of analytical methods. Venkateswarlu and Satya Narayana [10] developed a mathematical model to study the influence of heat source on unsteady free convection flow on a micropolar fluid along a plate.

The influence of a variable heat flux on a micropolar fluid has fascinated the attention of many recent researchers because of their numerous applications in many industrial (polymer sheet extrusion from a die), geophysical (production of oil and gas) medical problems (blood flow). Chen [11] considered the flexible heat flux as $\partial T / \partial y = -q_w / k$, where q_w is the surface heat flux. Kumar [12] proposed that the heat flux varies as the square of the distance from the origin in their study along a stretching wall. Elbashbeshy and Aldawody [13] analysed the unsteady fluid flow induced due to sheet stretching in the presence of heat flux $q_w(x, t) = bx / (1 - \gamma t)$. Turkyilmazoglu [14] presented the mixed convective magnetohydrodynamic flow of a non-Newtonian fluid past a permeable stretching surface in the presence of heat source. Khan et al. [15] provided impact of heat source/sink effects to Maxwell nanofluid flow. The magnetohydrodynamic flow of a micropolar liquid over a nonlinear stretching surface was analysed by Waqas et al. [16]. Baag et al. [17] presented the influence of a stagnation nature of a micropolar fluid over a vertical flat plate. Elbashbeshy et al. [18] examined numerically, the analysis of an unsteady Maxwell fluid flow caused due to a sheet stretching. Shaheen et al. [19] discussed the magnetohydrodynamic flow of a micropolar fluid flow induced by a stretchable porous disk. Mahmoud and Waheed [20] analysed the result of velocity slip on an electrically conducting and heat transfer flow of a rheological fluid over a permeable stretching surface. Doh and Muthamilselvan [21] analysed the unsteady MHD heat and mass transfer flow of a micropolar fluid due to a rotating disk. Ramzan et al. [22] explained the magnetohydrodynamic micropolar fluid flow generated by a permeable sheet stretching. Abbas et al. [23] discussed the magnetohydrodynamic stagnation point flow of a micropolar nanofluid over a circular cylinder.

The energy and species transfer flow of an electrically conducting rheological fluid past a moving plate was introduced by Pal and Mondal [24]. The magnetohydrodynamic flow through a permeable enclosure was discussed by Sheikholeslami et al. [25]. Shah et al. [26] explored the influence of magnetic and electric fields between two parallel plates in a rotating system. Jusoh et al. [27] studied three-dimensional rotating flow of a Ferro fluid over an exponentially permeable stretching/shrinking sheet. Gupta et al. [28] presented the effect of Laurent's forces on mixed convective flow of an electrically conducting micropolar fluid over a porous shrinking sheet. Narla et al. [29] established the peristaltic flow of the magnetized viscoelastic fluid through a deformable curved channel. Din et al. [30] conducted a study on an electrically conducting and radiative micropolar nanofluid inside a porous channel. Soomro et al. [31] and Prabhakar et al. [32] reported that several factors influence the stagnation point flow of nanofluids. Qasim et al. [33] conducted extensive analysis of entropy generation in methanol-based nanofluid flow. Quite recently Ganesh et al. [34, 35] identified the major mechanism of Newtonian and non-Newtonian fluid flow past stretching sheet. Some of the recent scientists considered the different fluid models with different geometries [36-41].

The problem of micropolar fluid flow induced due to a sheet stretching in the presence of slip and convective boundary conditions has remained unexplored. So, the aim of this work is to study the influence of variable heat flux on a micropolar fluid flow generated due to a sheet stretching. The governing equations of the fluid flow are solved computationally, and the influences of variable flow physical parameters are discussed graphically. The results are validated through the comparison study made with the available results in the literature

2. Mathematical Formulation

Formulation of the problem is under the assumption that the fluid is incompressible, non-Newtonian (micropolar fluid), electrically conducting and magnetically susceptible, the permeable stretching surface which coincide with the sheet $y = 0$, the flow being in the region $y > 0$. The physical variables in this model in the Cartesian coordinate system are functions of x and y respectively. The physical model for the current flow to be studied is illustrated with the coordinate system in Fig. 1. It is assumed the sheet wall temperature T_w is sufficiently high to affect radiative heat transfer. So, if the axial velocity u , the velocity of the fluid v and it is the velocity at which the fluid is sucked by the wall, also in-comparison with applied magnetic field induced magnetic field is neglected so that $B = (0, B_0, 0)$ parallel to y -axis and electric field $E = (0, 0, -E_0)$ parallel to z -axis with slip velocity, a simple model can demonstrate the complex fluid dynamical transitions in material processing [42-44]. Suppose the fluid is electrically conducting, and a uniform transverse magnetic field of strength B_0 is applied, then the interaction between the motion and the magnetic field can be described by Maxwell equations. As in most problems involving conductors Maxwell's displacements currents are ignored. So that electric currents are regarded as flowing in closed circuits. Assuming that the velocity of flow is too small compared to the velocity of light, i.e., the relativistic effects are ignored the system of Maxwell's equation can be written in the following form:

$$\begin{aligned} \nabla \times \bar{B} &= \mu_e \bar{J}, \quad \nabla \cdot \bar{J} = 0 \\ \nabla \times \bar{E} &= -\frac{\partial \bar{B}}{\partial t}, \quad \nabla \cdot \bar{B} = 0 \end{aligned} \quad (1)$$

Ohm's law can be written in the form

$$\bar{J} = \sigma (\bar{E} + \bar{q} \times \bar{B}) \quad (2)$$



where $\vec{B}$ is the magnetic induction intensity, $\vec{E}$ is the electric field intensity, $\vec{J}$ is the electric current density, μ_e is the magnetic permeability and σ is the electrical conductivity. If we add the body force $\vec{J} \times \vec{B}$ per unit volume, the equation of the motion, then the body force represents the coupling between the magnetic field and the fluid motion which is called Lorentz force.

The induced magnetic field can be neglected under the assumption that the magnetic Reynolds number is small. This is rather important case for some practical engineering problems where the conductivity is not large in the absence of an externally applied field and with negligible effects of polarization of the ionized gas. Taking $\vec{E} = 0$, i.e. in the absence of convection outside the boundary layer, $\vec{B} = B_0$ and $\nabla \times \vec{B} = \mu_e \vec{J} = 0$. Then the equation (2) leads to $\vec{J} = \sigma(\vec{q} \times \vec{B})$. Thus, the Lorentz force becomes $\vec{J} \times \vec{B} = \sigma(\vec{q} \times \vec{B}) \times \vec{B}$. In the absence of the induced magnetic field, to get a better degree of approximation, the Lorentz force can be replaced by $\sigma(\vec{q} \times B_0) \times B_0 = -\sigma B_0^2 \vec{q}$.

To start with basic governing equations Boussinesq approximation for this investigation is based on the balances of mass, linear momentum, and energy (see Ramzan et al., [22]) are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\frac{\mu + k}{\rho} \right) \frac{\partial^2 u}{\partial y^2} + \frac{\sigma}{\rho} (E_0 B_0 - B_0^2 u) + \frac{k}{\rho} \frac{\partial N}{\partial y} \quad (4)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \left(\frac{\gamma^*}{\rho j} \right) \frac{\partial^2 N}{\partial y^2} - \frac{k}{\rho j} \left(\frac{\partial u}{\partial y} + 2N \right) \quad (5)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho C_p} \right) \frac{\partial^2 T}{\partial y^2} - \frac{\sigma}{\rho C_p} (u B_0 - E)^2 + \frac{Q^*}{\rho C_p} (T - T_\infty) \quad (6)$$

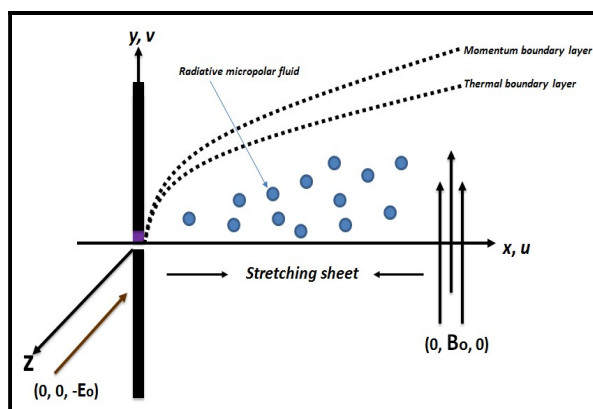


Fig. 1. Flow geometry and coordinate system

The following initial and boundary conditions

$$\left. \begin{aligned} u &= ax + \alpha^* \left[\left(\mu + k \right) \frac{\partial u}{\partial y} + kN \right], \quad v = v_w, \quad N = -n \frac{\partial u}{\partial y}, \quad \frac{\partial T}{\partial y} = -\frac{b}{k} x^m \quad \text{as } y = 0 \\ u &\rightarrow 0, \quad N \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \right\} \quad (7)$$

Aforementioned components of material parameter with velocity and microrotation components in x and y directions are represented as u , v and N ; B_0 , E_0 are the applied magnetic and electric field, μ , k are the dynamic and Eringen vortex viscosities, ρ , σ are the density, electrical conductivity of micropolar fluid, specific heat at constant pressure is denoted as C_p , α^* is slip coefficient, v_w is suction/injection velocity, a , b are constants. When $n = 0$, microelements close to the wall are not able to rotate [45]. When $n = 0.5$, this indicates weak concentration of micro-elements as elaborated by Ahmadi [46]. When $n = 1.0$, specifies turbulent boundary layer flows [47-48].

Using Rosseland's approximation as described by [49], the net radiative heat flux considered $\partial q_r / \partial y' = 16 \bar{\sigma} T_\infty^3 / 3 \bar{k}$

$\times(\partial^2 T' / \partial y'^2)$. It is pertinent to note that if this assumption is neglected, the radiative heat flux in Eqn. (6) results in a highly non-linear expression [50-52].

Introducing the stream function $\psi(x, y)$ which satisfies the continuity equation (3) as $u = \partial \psi / \partial y$, $v = -\partial \psi / \partial x$ with the consideration of following no-dimensional variables and parameters.

$$\eta = \sqrt{\frac{a}{v}} y, \quad \psi = \sqrt{a v} x f(\eta), \quad N = a x \sqrt{\frac{a}{v}} g(\eta), \quad T = T_\infty + \frac{q_w(x)}{\kappa} \sqrt{\frac{v}{a}} \theta(\eta) \quad (8)$$

We followed the recent studies of authors [53] by assuming the following gyroscopic viscosity $\gamma^* = (\mu + (k/2))j = \mu(1 + (K/2))j$ where $K = k/\mu (\geq 0)$ is material parameter and $j = v/a$ is micro inertia per unit mass.

Substituting Eqn. (8) into Eqns. (4) to (6) and using net radiative heat flux yields the following transformed, dimensionless nonlinear system of ordinary differential equations:

$$(K+1)f''' + f f'' - (f')^2 + K g' - M^2 f' + M^2 E = 0 \quad (9)$$

$$\left(\frac{K}{2} + 1\right) g'' - K(2g + f'') + f g' - f' g = 0 \quad (10)$$

$$(F+1)\theta'' + \text{Pr} f \theta' + M^2 \text{Ec} [f'^2 + E^2 - 2E f'] + \text{Pr} Q \theta = 0 \quad (11)$$

Initial and boundary conditions (7) are converted to:

$$\left. \begin{aligned} f &= S, \quad f' = 1 + \alpha(K+1)f'', \quad g = -n f'', \quad \theta'(0) = -1 \quad \text{as } \eta = 0 \\ f' &\rightarrow 0, \quad g \rightarrow 0, \quad \theta \rightarrow 0, \quad \text{as } \eta \rightarrow \infty \end{aligned} \right\} \quad (12)$$

Here $M = \sigma B_o^2 / \rho a$ is Magnetic field parameter, $E = E_o^2 / u_w^2 B_o^2$ is Electric current, $F = (16\sigma^* / 3k^* \kappa) T_\infty^3$ is Radiation-conduction parameter, $\text{Pr} = \mu C_p / \kappa$ is Prandtl number, $\text{Ec} = u_w^2 / C_p (T_w - T_\infty)$ is the Eckert number, $Q = Q^* / a \rho C_p$ is heat source ($Q > 0$)/heat sink ($Q < 0$) parameter $\alpha = \alpha^* \mu \sqrt{a/v}$ is Slip parameter and $S = -(av)^{-0.5} v_w$ is the wall lateral mass flux (suction/injection) parameter in which for suction $S > 0$, for injection (blowing) $S < 0$ and for a solid (impermeable) plate surface, $S = 0$. It is noted that the case of uniform surface heat flux corresponds to $m = 0$ and also noted that all parameters are free from x which confirms the true similarity solution of Eqns. (9) to (11) subjected to boundary conditions (12).

Gradients of the key variables at the plate surface are important for engineering design considerations, in particular in materials processing. Here $\tau_w = ((\mu + k) \times (\partial u / \partial y) + kN)|_{y=0}$ is dimensional wall shear stress, $m_w = \gamma^* (\partial N / \partial y)_{y=0}$ is wall couple stress coefficient and $q_w = -\kappa (\partial T / \partial y)_{y=0}$ is wall heat transfer coefficient. The non-dimensional skin friction $C_f = 2\tau_w / \rho(ax)^2$, takes the form:

$$C_{f_x} = 2 \frac{(1 + (1-n)K)}{\sqrt{\text{Re}_x}} f''(0) \quad (13)$$

The non-dimensional wall couple stress coefficient of $Cw_x = m_w / \rho a v x^3$ becomes:

$$Cw_x = \frac{a}{v \text{Re}_x} \left(1 + \frac{K}{2}\right) g'(0) \quad (14)$$

The non-dimensional local Nusselt number of $Nu_x = x q_w / \kappa (T_w - T_\infty)$ emerges as:

$$Nu_x = -(1+F) \sqrt{\text{Re}_x} / \theta'(0) \quad (15)$$

3. Numerical Solution

The system of nonlinear coupled and inhomogeneous ordinary differential equations Eq. (9) to Eq. (11) subject to the boundary conditions in Eq. (12) are solved numerically using the Runge-Kutta method along with shooting technique [53]. Furthermore, this method is found to be suitable in dealing with nonlinear parabolic partial differential equations. The first step involves converting the Eqs. (4) to (6) into a system of first order ordinary differential equations. Thus, the coupled differential



equations of fourth order in $f(\eta)$ and second order in $g(\eta)$ and $\theta(\eta)$ has been reduced to a system of 9 simultaneous equations of first order for nine unknowns. To solve these system of equations, we adopted an iterative scheme with convergence criterion and is designated as: when the difference between two successive approximations is sufficiently small ($\leq 10^{-8}$), the solutions are taken to have converged to the requisite accuracy.

4. Results and Discussion

Figs. 2 and 3 demonstrate the influence of material parameter K on velocity and microrotation distributions. It is noticed that the velocity and microrotation of the fluid reduced with rising of K . Physically, higher K values corresponds to the lower viscosity nature and weaker rotation of the particle moment in the fluid. The fluid has maximum velocities at $\eta=0$, for different values of K . Also, as $\eta \rightarrow \infty$, the velocity and microrotation of the fluid converges to the boundary. The comparison of rate of heat transfer with that of [22] and present numerical method in the absence of heat source/sink is presented in Table 1 and found to be in excellent agreement. Further, Table 2 presents the numerical results of various parameters on Skin friction, wall couple stress and Nusselt number coefficients. These out comes are coincide with the results available in the literature (see Ref. [54]).

Table 1. Comparison of Local Nusselt number for E when $n = 0.5$ fixed

E	Results of [22] $-\theta'(0)$	Present results $-\theta'(0)$
0.2	0.08580	0.085781
0.6	0.08605	0.086011
0.9	0.08599	0.085975

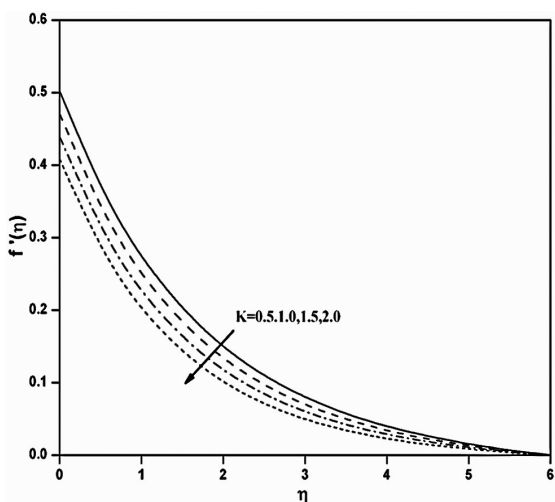


Fig. 2. Effect of material parameter (K) on velocity profiles.

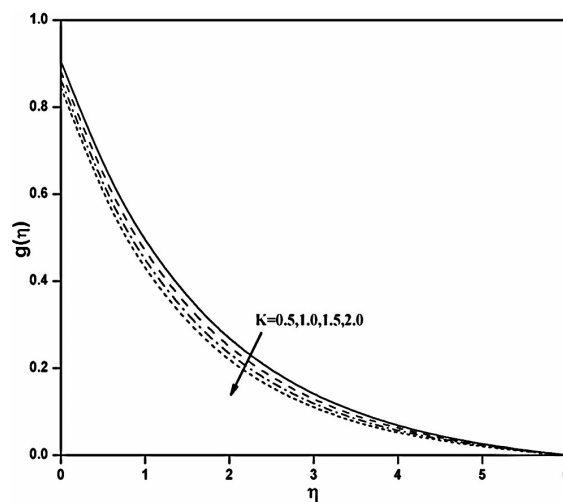


Fig. 3. Effect of material parameter (K) on microrotation profiles.

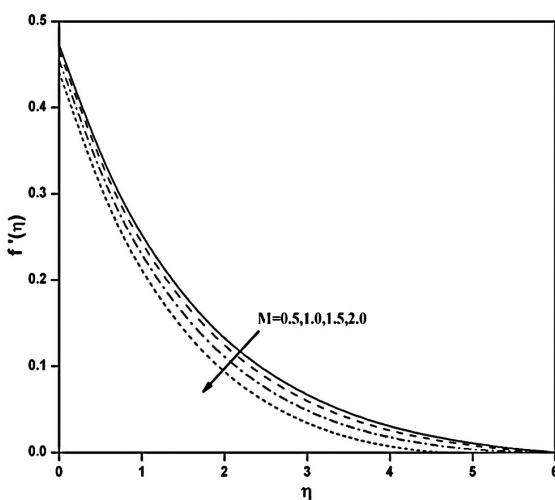


Fig. 4. Effect of Hartmann number (M) on velocity profiles.

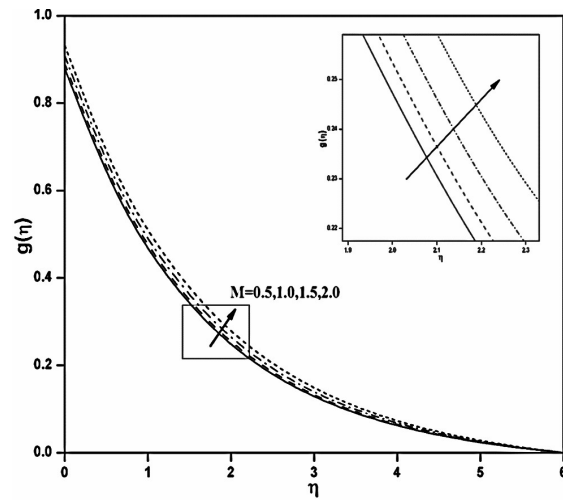


Fig. 5. Effect of Hartmann number (M) on microrotational profiles.

Table 2. Computation values of C_f , C_w and Nu

K	M	E	Pr	R	$Q > 0$	$Q < 0$	Ec	S	α	n	C_f	C_w	Nu
0.1											-0.271496	0.972268	-0.415711
0.2											-0.264776	0.992033	-0.500234
0.3											-0.258881	1.007200	-0.592763
0.4											-0.253688	1.019340	-0.695669
	0.1										-0.351605	0.905825	-0.504772
	0.2										-0.356126	0.911057	-0.544412
	0.3										-0.363328	0.91892	-0.611494
	0.4										-0.372739	0.92833	-0.707038
		0.1									-0.353035	0.909364	-0.500234
		0.2									-0.349924	0.906505	-0.466734
		0.3									-0.346863	0.903538	-0.435162
		0.4									-0.343849	0.900472	-0.405351
			0.1								-0.353035	0.909364	-0.078429
			0.2								-0.353035	0.909364	-0.145409
			0.3								-0.353035	0.909364	-0.219801
			0.4								-0.353035	0.909364	-0.302773
				0.5							-0.353035	0.909364	-1.30529
				0.6							-0.353035	0.909364	-1.1459
				0.7							-0.353035	0.909364	-1.01415
				0.8							-0.353035	0.909364	-0.903808
					0.5						-0.353035	0.909364	-0.903808
					0.6						-0.353035	0.909364	-1.15497
					0.7						-0.353035	0.909364	-1.48981
					0.8						-0.353035	0.909364	-1.96232
						-0.5					-0.353035	0.909364	1.06341
						-0.6					-0.353035	0.909364	1.12839
						-0.7					-0.353035	0.909364	1.18943
						-0.8					-0.353035	0.909364	1.24713
							1.0				-0.353035	0.909364	-0.899742
							10				-0.353035	0.909364	-0.861014
							20				-0.353035	0.909364	-0.821713
							30				-0.353035	0.909364	0.785844
								0.9			-0.461905	0.674534	0.541248
								1.1			-0.480778	0.767025	0.785681
								1.3			-0.498586	0.867175	1.02245
								1.5			-0.515182	0.974244	1.25236
									0.5		-0.512672	1.42086	-0.524513
									0.7		-0.433097	1.16028	-0.686987
									0.9		-0.376006	0.980124	-0.834441
									1.1		-0.332846	0.848037	-0.970733
										0.0	-0.577491	-0.072971	-0.200947
										0.3	-0.502876	0.109031	-0.363586
										0.6	-0.447001	0.241451	-0.527400
										0.9	-0.403387	0.342069	-0.696067

The analysis of translational velocity and micro-rotation profiles for diverse values of Hartmann number M is displayed respectively in Figs. 4 and 5. It is noticed that the fluid has less velocity due to the presence of Lorentz forces with rising values of M . Larger the magnetic values corresponds to smaller velocity and hence related to the thinner hydrodynamic boundary layer. On the other hand, microrotation profiles display the opposite nature with rising values of M . Also, the magnetic field shows higher impact on translational velocity than to that of microrotation.

Variation of electric field parameter E on translational velocity and micro-rotation profiles are visualized through Figs. 6 and 7. It can be noticed that the boost up velocity distributions with larger values of E . Physically, electric field behaves as an increasing force in diminishing the frictional resistance of the fluid and hence the fluid velocity increases. On the other hand, E shows opposite effect on microrotation. Moreover, electric field parameter shows significant effect on the translational velocity

profiles than to the microrotational distributions.

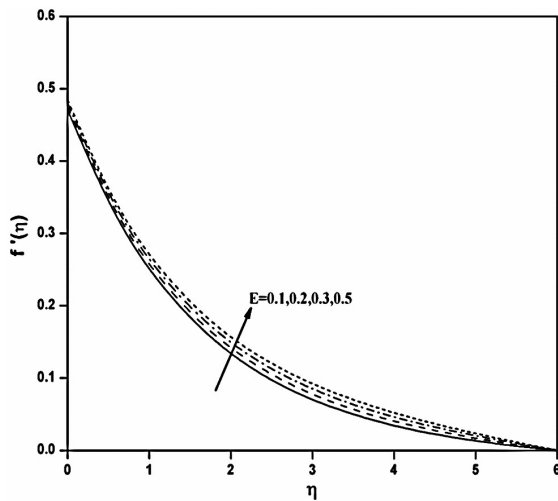


Fig. 6. Effect of Electric current (E) on velocity profiles.

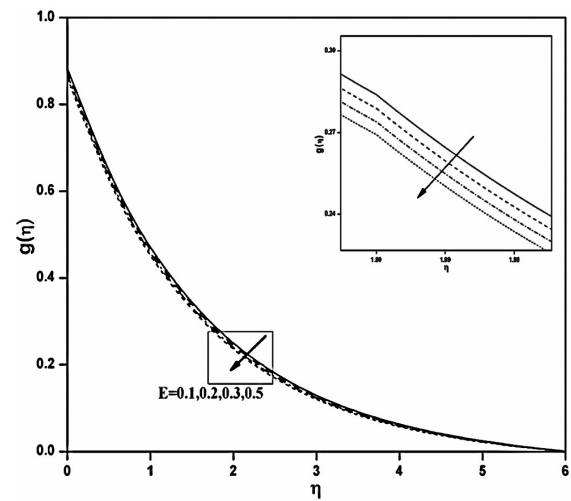


Fig. 7. Effect of Electric current (E) on microrotational profiles.

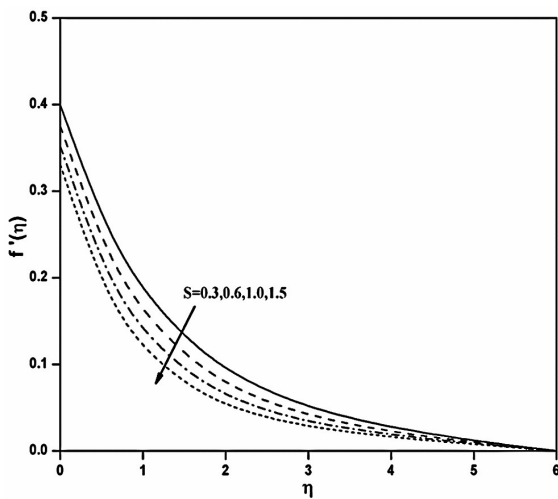


Fig. 8. Effect of Suction/Injection parameter (S) on velocity profiles.

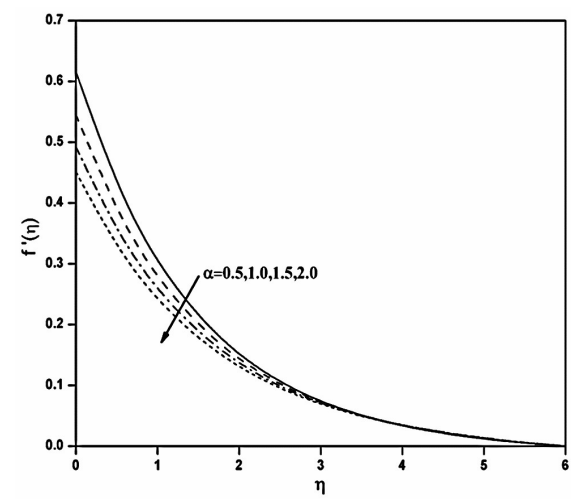


Fig. 9. Effect of Slip parameter (α) on velocity profiles.

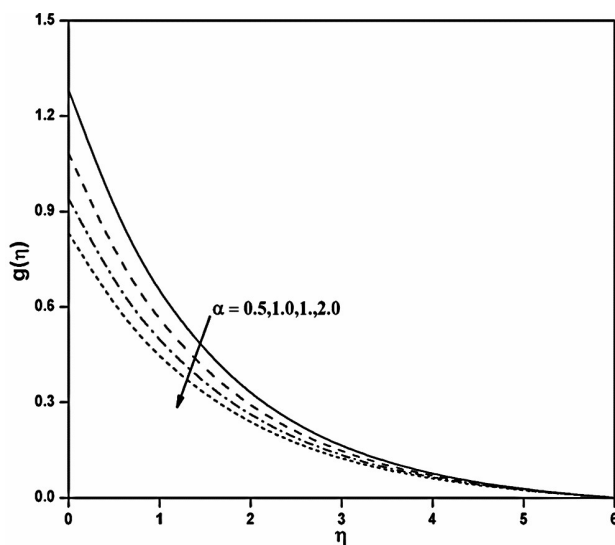


Fig. 10. Effect of Slip parameter (α) on microrotational profiles.

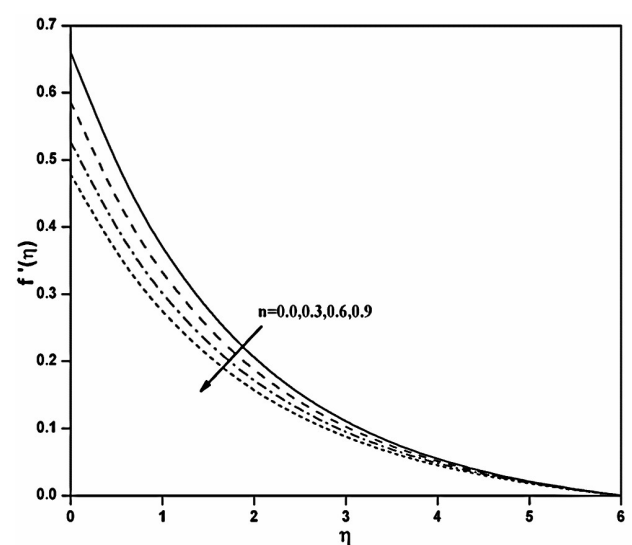


Fig. 11. Effect of Surface parameter (n) on velocity profiles.

The variations of $f'(\eta)$, $g(\eta)$ and $\theta(\eta)$, against η for different values of suction parameter S is respectively illustrated in Fig. 8. It can be inspected from Fig. 8 that the translational velocity reduces with an increasing values of S and hence decelerates the

boundary layer flow. Physically, increase of S indicates that $v_w < 0$ which gives the mass transfer at the sheet due to suction. Also, the velocity of the fluid is more at $\eta = 0$ for larger values of S . As $\eta \rightarrow \infty$ the velocity of the fluid converges to the boundary condition at $\eta = 6$.

Fig. 9 and 10 demonstrates the different values of α on $f'(\eta)$, $g(\eta)$ distributions. It is described from these figures that as α enhances, the translational liquid velocity and microrotation across the boundary layer declines. As, expected larger α signifies the maximum resistance for the fluid flow over the sheet due to the frictional forces between fluid particles and surface of the sheet. Fig. 11 are presented the velocity distribution for distinct values of n . It is noticeable from these figure that the increase in n , decline the translational fluid velocity. The influence of α and n on temperature profile $\theta(\eta)$ is elucidated in Fig.12. It is obvious that raising values of emerging parameters α and n , the magnitude of temperature distribution enhances. Physically, higher α corresponds to more opposition for the flow of stretching velocity of the fluid due to the adhesive forces between the sheet and particles. As a result, the temperature of the fluid increases. Also, it can be noticed that the temperature is higher in the case of α comparing to that of n .

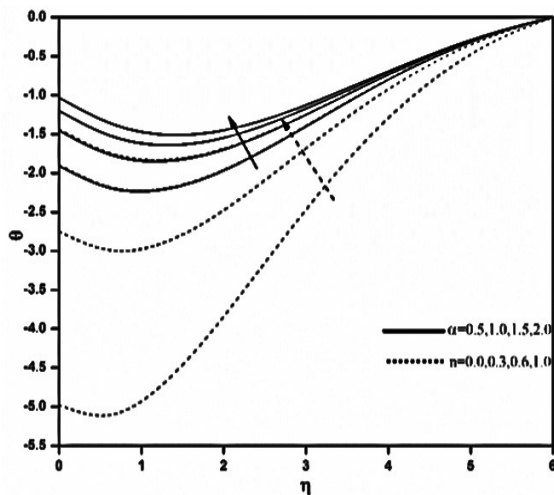


Fig. 12. Effect of Slip parameter (α) and Surface parameter (n) on temperature.

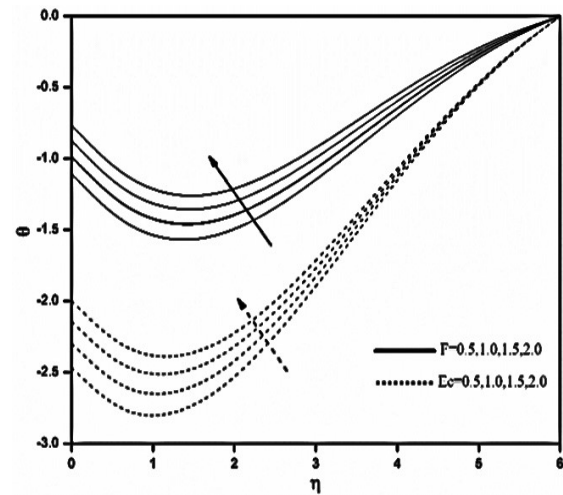


Fig. 13. Effect of Radiation parameter (F) and Eckert number (Ec) on temperature.

The impact of radiation (F) and Eckert number (Ec) on $\theta(\eta)$ is elucidated in Fig.13. It is observed that raising values of F and Ec enhances the magnitude of temperature distribution. As expected, increase of F is related to the decrease of average Rosseland absorption coefficient and hence causes to rise the temperature. Moreover, greater Ec values are corresponds to the more heat generation in the fluid is because of the frictional heating of the particles. Also, we can notice that the temperature is higher in the case of F than Ec .

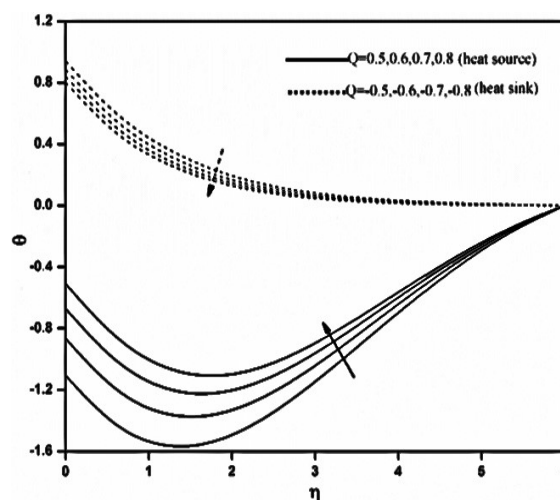


Fig. 14. Effect of Heat Source/sink (Q) on temperature

Fig. 14 exhibits the variation of $\theta(\eta)$ for various positive and negative values of Q . It is witnessed that the magnitude of temperature profile increase for positive values of Q (heat source), while contrary trend is seen for negative values of Q (heat sink). It is interesting to observe that for larger values of $Q > 0$, the temperature suddenly drops near the sheet and converges to boundary condition as $\eta \rightarrow \infty$. Also, we can perceive that temperature profile inferior in the case of $Q > 0$ than $Q < 0$.

5. Conclusion

A computational study on the MHD micropolar fluid flow induced due to a porous sheet stretching in the presence of variable heat flux is carried out. The flow generating equations are converted into a system of coupled nonlinear ODEs and are solved numerically. The interesting conclusions of the present work are presented as follows:

- Electric field parameter has an important enhancing influence on velocity and opposing nature on temperature.
- Material and heat source parameters have very less effect on the temperature.
- Microrotation is increased for magnetic field and surface temperature parameters
- Velocity and microrotation of the fluid decrease with enhancing values of K and α
- Temperature shows opposite effect for increasing values of M and E .

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Conflict of Interest

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