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Research Paper

Perturbation-Iteration Algorithm for Solving Heat and Mass Transfer in the Unsteady Squeezing Flow between Parallel Plates

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Abstract. In this paper, heat and mass transfer in the unsteady squeezing flow between parallel plates is analyzed using a perturbation-iteration algorithm. The similarity transformation is used to transform the governing partial differential equations into ordinary differential equations, before being solved. The solutions of the velocity, temperature and concentration are derived and sketched to explain the influence of various physical parameters. The convergence of these solutions is also discussed. The numerical results of skin friction coefficient, Nusselt number and Sherwood number are compared with previous works. The results show that the method which has been used, in this paper, gives convergent solutions with good accuracy.

Keywords: Heat transfer, Mass transfer, Unsteady squeezing flow, Perturbation-iteration algorithm.

1. Introduction

Combined heat and mass transfer problems with two-dimensional unsteady squeezing viscous flow has attracted much interest from researchers in the past decades due to its wide range of scientific and engineering applications, such as drying, evaporation at the surface, polymer processing, hydrodynamic machines, chemical processing equipment, food processing and cooling towers etc.

There are many studies which have been successfully applied to solve the squeezing flow problems. A review of these studies reveals that most of the methods used to solve this problem can be divided into semi-analytical methods and other numerical methods. The following is a review of some of these studies: in [1], flow and heat transfer over a permeable sensor surface placed in a squeezing channel has been solved by the local non-similarity method, perturbation method and asymptotic method. This study is explained that perturbation solutions for small and large values of the local transpiration parameter are agree with local non-similarity solutions. The governing equations for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generation fluid on a continuously vertical porous surface in the existence of a radiation with the assumption that the plate moves at a constant velocity in the direction flow of fluid are solved analytically using two-terms harmonic and non-harmonic functions in [2]. The governing equations which are represented the fluid flow have been transformed into a nonlinear ordinary differential equation by using the similarity transformation in [3] and the homotopy analysis method (HAM) have been applied to solve the two-dimensional axisymmetric unsteady flow of nonconducting, incompressible second grade fluid between two circular plates. It has been given convergent series of solutions which depend on auxiliary parameter. Moreover, the similarity transformation and HAM have been utilized to solve the two-dimensional unsteady flow with heat and mass transfer of viscous fluid between the infinite parallel plates in [4]. It has been shown that only tenth-order of approximation gives the convergent solutions. The Adomian decomposition method (ADM) has also applied to solve the problem of unsteady squeezing nanofluid flow and heat transfer in [5]. The similarity transformations and least square method (LSM) are used to solve nanofluid flow and heat transfer problem between two parallel disks when the magnetic field is applied to the lower stationary disk and the upper disk can move towards or away from the lower stationary disk, and then the accuracy of the results is examined by comparing with the fourth order Runge-Kutta method in [6]. The model of two phase nanofluid flow and heat transfer between parallel plates which includes the effects of Brownian motion and thermophoresis is solved by the homotopy perturbation method (HPM) in [7].

In addition, the collocation method (CM), LSM and fourth-order Runge-Kutta method are used to solve the unsteady flow problem of a nanofluid squeezing between two parallel plates in [8]. It was observed that the results of LSM are more accurate than CM and they were approval of numerical results. The Nachtsheim and Swigert shooting iteration technique together with the sixth-order Runge-Kutta method are applied to study the effect of thermal radiation on the heat and mass transfer characteristics of an incompressible electrically conducting fluid squeezed between two parallel plates in the presence of magnetic field in [10]. In [11], the heat and mass transfer behavior of unsteady flow of squeezing nanofluid between two parallel plates in the sight of uniform magnetic field with slip velocity effect have been studied and solved numerically by using Runge-Kutta-Fehlberg method with shooting technique. The effect of Brownian motion and thermophoresis has been included in heat and mass transfer of unsteady nanofluid flow between parallel plates in [12], and solved by the differential transform method (DTM). the results were explained that the accuracy of DTM is higher than HPM. The double diffusive squeezing unsteady flow of electrically conducting nanofluid between two parallel disks under slip and temperature jump condition is solved by applying HPM in [13]. The Chebyshev wavelet method (CWM) are used in [9] to solve two-dimensional unsteady flow of a nanofluid squeezing between two parallel plates, and in [14] for the study of the effects of heat and mass transfer of three-dimensional, steady, electrically conducting, and viscous incompressible fluid which is bounded between two parallel plates. Heat and mass transfer in unsteady two-dimensional Magentohydrodynamics squeezing flow of Eyring-Powell fluid between two parallel infinite plates is solved by DTM in [15]. The comparisons between the results of DTM and numerical Runge-Kutta Fehlbrg method proved high precision of DTM. Wavelet Galerkin method (WGM) was proposed to discuss the heat and mass transfer analysis for an unsteady nanofluid flow, which is squeezed between two parallel plates in [16]. The comparison showed that the results of WGM were in excellent agreement with the results of the fourth-order Runge-Kutta method, HPM, DTM and CWM. In [17], the entropy generation on two-dimensional unsteady Casson fluid flow squeezing between two parallel plates is adopted where Cattaneo-Christov heat and mass flux is applied instead of Fourier's and Fick's laws, the suitable similarity transformation and the shooting method have been utilized.

The main aim of this study is to find approximate analytical solutions to heat and mass transfer in the unsteady squeezing flow between parallel plates by using the perturbation-iteration algorithm (PIA). This method was chosen for several reasons. Firstly, it can be applied to the problem directly by utilizing appropriate initial and boundary conditions, and without using linearization, discretization or any transformation. Secondly, it does not need to install a small auxiliary parameter in the equations, to reverse other perturbation methods which their solutions are restricted by the validity range of physical parameters. Moreover, this method was not previously used to solve this problem. It is worth to mention that PIA was proposed by Pakdemirli and Boyac in [18] to generate root finding algorithms. It is then applied by many authors to get the approximate analytical solution of nonlinear differential equations. In [19-21, 24, 25], PIA was applied to obtain the solution of nonlinear ordinary differential equations. Moreover, PIA was generalized to an arbitrary number of first-order coupled equations in [22]. This method was utilized to solve Fredholm and Volterra integral equations in [23]. Also, it has been developed in [26] to obtain the solutions of Lotka-Volterra differential equations. It is applied in [27] to solve a nonlinear system of differential equations which consist of five variables giving up smoking fractional mathematical model. In [28], some types of fractional differential equation systems were solved by using this method. PIA with Laplace transform method were combined in [29] to solve Newell-Whitehead-Segel equations. In [30], PIA has been used for solving the fractional Zakharov-Kuznetsov equation. PIA, in [31], is used to find the exact solution sets of nonlinear conformable time-fractional coupled Drinfeld-Sokolov-Wilson equation which arise in shallow water flow models.

The remaining parts of this study are as follows: In Section 2, the problem is formulated and a system of nonlinear ordinary differential equations for heat and mass transfer in the unsteady squeezing flow is derived. The mathematical properties of PIA are presented in Section 3. The convergence analysis of PIA is introduced in Section 4. Moreover, PIA is applied to solve our problem and then we discuss the convergence of the resulting solutions, in Section 5. The numerical results are presented and discussed in Section 6. Finally, in Section 7, some conclusions were presented.

2. Mathematical Formulation

The governing equations for the heat and mass transfer in the unsteady two-dimensional squeezing flow of an incompressible viscous fluid between the infinite parallel plates consist of:

$$\left.\begin{aligned} u_x + v_y &= 0, \\ u_t + u\,u_x + v\,u_y + \frac{1}{\rho}\,p_x &= \nu\left(u_{xx} + u_{yy}\right), \\ v_t + u\,v_x + v\,v_y + \frac{1}{\rho}\,p_y &= \nu\left(v_{xx} + v_{yy}\right), \\ T_t + u\,T_x + v\,T_y &= \frac{k}{\rho\,C_p}\left(T_{xx} + T_{yy}\right) + \frac{1}{C_p}\left(4\,(u_x)^2 + (u_y + v_x)^2\right), \\ C_t + u\,C_x + v\,C_y &= D\left(C_{xx} + C_{yy}\right) - K_1(t)C, \end{aligned}\right\} \tag{1a}$$

with the following boundary conditions

$$u = 0, \quad v = \frac{dh}{dt}, \quad T = T_H, \quad C = C_H \quad at \quad y = h(t),$$
$$v = u_y = T_y = C_y = 0 \qquad\qquad at \quad y = 0, \tag{1b}$$

where u and v are the velocity components in the x and y directions, respectively, T is the temperature, C is the concentration, p is the pressure, ν is the kinematic viscosity, ρ is the fluid density, k is the thermal conductivity, D is the diffusion coefficient of the diffusing species, C_p is the specific heat, $K_1(t) = \frac{k}{1-\alpha\, t}$ is the reaction rate, α is rate of squeezing and $h(t) = l\sqrt{1-\alpha\, t}$ is the distance between two plates, so that the two plates are squeezed until they touch when $\alpha > 0$ and $t = \frac{1}{\alpha}$ and the two plates are separated when $\alpha < 0$. Flow geometry and the coordinate system are shown in the Fig. 1.

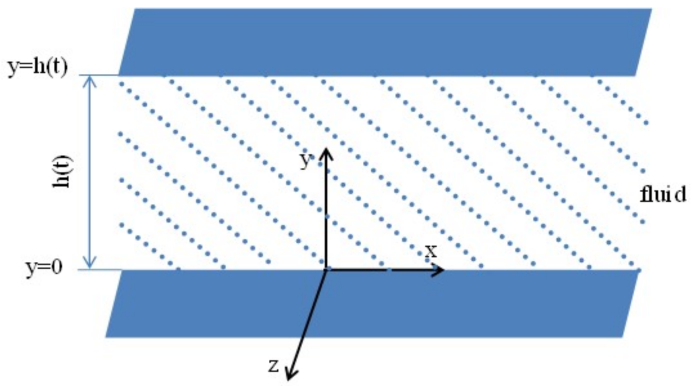


Fig. 1. Schematic diagram of the problem

By using the similarity transformation [4, 7, 8, 11, 13]:

$$\left.\begin{aligned} &\eta = \frac{y}{l\sqrt{1-\alpha\, t}}, \\ &u(x,y,t) = \frac{\alpha\, x}{2\,(1-\alpha\, t)}\, f'(\eta), \\ &v(x,y,t) = -\frac{\alpha\, l}{2\sqrt{1-\alpha\, t}}\, f(\eta), \\ &\theta = \frac{T}{T_H}, \qquad\qquad \phi = \frac{C}{C_H}, \end{aligned}\right\} \tag{2}$$

we have the following system of ordinary differential equations

$$\left.\begin{aligned} &\frac{1}{S}\, f^{(4)} + f\, f^{(3)} - 3\, f'' - \eta\, f^{(3)} - f'\, f'' = 0, \\ &\frac{1}{Pr}\, \theta'' + S\,(f-\eta)\,\theta' + Ec\,((f'')^2 + 4\,\delta^2\,(f')^2) = 0, \\ &\frac{1}{Sc}\, \phi'' + S(f-\eta)\,\phi' - \gamma\,\phi = 0, \end{aligned}\right\} \tag{3}$$

with the boundary conditions

$$f(0) = 0, \qquad f''(0) = 0, \qquad f(1) = 1, \qquad f'(1) = 0,$$
$$\theta'(0) = 0, \qquad \phi'(0) = 0, \qquad \theta(1) = 1, \qquad \phi(1) = 1, \tag{4}$$

where S is the squeeze number, Pr is the Prandtl number, Ec is the Eckert number, Sc is the Schmidt number and γ is the chemical reaction parameter, which are defined as

$$S = \frac{\alpha\, l^2}{2\,\nu}, \qquad Pr = \frac{\mu\, C_p}{k}, \qquad Ec = \frac{1}{C_p\, T_H}\left(\frac{\alpha\, x}{2\,(1-\alpha\, t)}\right)^2, \qquad Sc = \frac{\nu}{D}, \qquad \gamma = \frac{k_1\, l^2}{\nu}, \qquad \delta = \frac{l}{x} \tag{5}$$

It is worth mentioning here that the squeeze number S describes the movement of the plates such that $S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together, and it is noted $\gamma > 0$ represents the destructive chemical reaction and $\gamma < 0$ characterizes the generative chemical reaction.
The physical quantities discussed in this study are the skin friction coefficient, Nusselt number and Sherwood number which are defined [4, 5] as

$$C_f = \frac{\mu\,(\frac{\partial u}{\partial y})_{y=h(t)}}{\rho\, v_w^2}, \qquad \Rightarrow \qquad \frac{l^2}{x^2}\,(1-\alpha\, t)Re\, C_f = -f''(1),$$

$$Nu = \frac{-l\, k\,(\frac{\partial T}{\partial y})_{y=h(t)}}{k\, T_H}, \qquad \Rightarrow \qquad \sqrt{1-\alpha\, t}\, Nu = -\theta'(1), \tag{6}$$

$$Sh = \frac{-l\, D\,(\frac{\partial C}{\partial y})_{y=h(t)}}{D\, C_H}, \qquad \Rightarrow \qquad \sqrt{1-\alpha\, t}\, Sh = -\phi'(1).$$

3. Perturbation-Iteration Algorithm (PIA)

PIA is a combination of perturbation expansions and Taylor series expansions to construct an iteration scheme. It is obtained by taking different numbers of terms in the perturbation expansions and different order of correction terms in the Taylor series expansions. Therefore, the perturbation iteration algorithm is called PIA(m,n) where the m is the number of the correction terms in the perturbation expansion, and n is the highest order derivative term in the Taylor series such that m should always be less than or equal to n.

In order to obtain approximate analytical solutions for two-dimensional heat and mass transfer in the unsteady squeezing flow of an incompressible viscous fluid between the infinite parallel plates, PIA will be applied to the system Eqs.(3), Thus, we consider

$$\left.\begin{aligned} &G_1(f, f', f'', f^{(3)}, f^{(4)}, \varepsilon) = 0,\\ &G_2(f, f', f'', \theta', \theta'', \varepsilon) = 0,\\ &G_3(f, \phi, \phi', \phi'', \varepsilon) = 0, \end{aligned}\right\} \tag{7}$$

where ε is a small perturbation parameter. We define the following perturbation expansions with only one correction term:

$$\left.\begin{aligned} f_{n+1} &= f_n + \varepsilon (f_c)_n,\\ \theta_{n+1} &= \theta_n + \varepsilon (\theta_c)_n,\\ \phi_{n+1} &= \phi_n + \varepsilon (\phi_c)_n, \end{aligned}\right\} \tag{8}$$

where n represents the n-th iteration and f_c, θ_c and ϕ_c are the correction terms in the perturbation expansion. By substituting Eqs. (8) into Eqs.(7) and expanding the resulting equations in a Taylor series with first order derivative terms for applying PIA(1,1) about $\varepsilon = 0$, yields

$$\left.\begin{aligned} &G_1(f_n, f_n', f_n'', f_n^{(3)}, f_n^{(4)}, 0) + \varepsilon\,[G_{1f_{n+1}}\,(f_c)_n + G_{1f'_{n+1}}(f_c)'_n + G_{1f''_{n+1}}\,(f_c)''_n\\ &+G_{1f^{(3)}_{n+1}}\,((f_c)_n)^{(3)} + G_{1f^{(4)}_{n+1}}\,((f_c)_n)^{(4)} + G_{1\varepsilon}] = 0,\\ &G_2(f_n, f_n', f_n'', \theta_n', \theta_n'', 0) + \varepsilon\,[G_{2f_{n+1}}\,(f_c)_n + G_{2f'_{n+1}}\,(f_c)'_n + G_{2f''_{n+1}}\,(f_c)''_n\\ &+G_{2\theta'_{n+1}}\,(\theta_c)'_n + G_{2\theta''_{n+1}}(\theta_c)''_n + G_{2\varepsilon}] = 0,\\ &G_3(f_n, \phi_n, \phi_n', \phi_n'', 0)\\ &+\varepsilon\,[G_{3f_{n+1}}\,(f_c)_n + G_{3\phi_{n+1}}\,(\phi_c)_n + G_{3\phi'_{n+1}}\,(\phi_c)'_n + G_{3\phi''_{n+1}}(\phi_c)''_n + G_{3\varepsilon}] = 0. \end{aligned}\right\} \tag{9}$$

Calculating all derivatives at $\varepsilon = 0$ and substituting the results in Eqs.(9) yields system of differential equations. The resulting system is solved by using the boundary conditions and an initial guesses to find $(f_c)_n$, $(\theta_c)_n$ and $(\phi_c)_n$. By substitution the values of $(f_c)_n$, $(\theta_c)_n$ and $(\phi_c)_n$ in (8) at $\varepsilon = 1$ we get f_{n+1}, θ_{n+1} and ϕ_{n+1}. The series of iterative solutions is obtained by repeating the same steps.

4. Analysis of Convergence

We now study the convergence analysis of the approximate analytical solutions which are computed from the application PIA on the system ordinary differential equations (3). Let us consider the Hilbert space $H = L^2(a,b)$ as defined by

$$u: H \to \mathbb{R}\ with \int_{(a,b)} u^2(x)\ dx < \infty, \tag{10}$$

and the norm

$$\| u \|^2 = \int_{(a,b)} u^2(x)\ dx. \tag{11}$$

We then define

$$\mathbf{u} = (f,\theta,\phi): H^3 \to \mathbb{R}^3 \; with \int_{(a,b)} (f^2(\eta) + \theta^2(\eta) + \phi^2(\eta))\, d\eta < \infty, \tag{12}$$

such that $\| \mathbf{u} \|^2 = \| f \|^2 + \| \theta \|^2 + \| \phi \|^2$.
To study the convergence of PIA, we write the approximate solutions in different form. To do this, we define

$$\begin{aligned}
\mathcal{B}_0 &= (\mathcal{B}_{10},\mathcal{B}_{20},\mathcal{B}_{30}) = (f_0(\eta),\theta_0(\eta),\phi_0(\eta)) = u_0(\eta),\\
\mathcal{B}_1 &= (\mathcal{B}_{11},\mathcal{B}_{21},\mathcal{B}_{31}) = ((f_c)_0(\eta),(\theta_c)_0(\eta),(\phi_c)_0(\eta)) = (u_c)_0(\eta),\\
&\vdots\\
\mathcal{B}_n &= (\mathcal{B}_{1n},\mathcal{B}_{2n},\mathcal{B}_{3n}) = ((f_c)_{n-1}(\eta),(\theta_c)_{n-1}(\eta),(\phi_c)_{n-1}(\eta)) = (u_c)_{n-1}(\eta).
\end{aligned} \tag{13}$$

Approximate solutions become formulas

$$\begin{aligned}
u_0 &= \mathcal{B}_0 = S_0,\\
u_1 &= u_0 + \varepsilon\,(u_c)_0 = \mathcal{B}_0 + \mathcal{B}_1 = S_1,\\
u_2 &= u_1 + \varepsilon\,(u_c)_1 = \mathcal{B}_0 + \mathcal{B}_1 + \mathcal{B}_2 = S_2,\\
u_3 &= u_2 + \varepsilon\,(u_c)_2 = \mathcal{B}_0 + \mathcal{B}_1 + \mathcal{B}_2 + \mathcal{B}_3 = S_3,\\
&\vdots\\
u_n &= u_{n-1} + \varepsilon\,(u_c)_{n-1} = \mathcal{B}_0 + \mathcal{B}_1 + \mathcal{B}_2 + \mathcal{B}_3 + \ldots + \mathcal{B}_n = \sum_{k=0}^{n} \mathcal{B}_k = S_n.
\end{aligned} \tag{14}$$

So the solutions, which are resulted from PIA, have the form

$$\mathbf{u}(\eta) = \sum_{k=0}^{\infty} \mathcal{B}_k \tag{15}$$

such that $\mathbf{S}_{n+1} = \mathcal{F}(\mathbf{S}_n)$. The sufficient condition for the convergence of the series solution $\{S_n\}_0^\infty$ is given in the following theorems:

Theorem 4.1. The series solution

$$\{S_n\}_0^\infty = \{(R_n,S_n,T_n)\}_0^\infty = \{\left(\sum_{k=0}^{n} \mathcal{B}_{1k}, \quad \sum_{k=0}^{n} \mathcal{B}_{2k}, \quad \sum_{k=0}^{n} \mathcal{B}_{3k}\right)\}_0^\infty, \tag{16}$$

converges whenever there is λ such that $0 < \lambda < 1$,

$$\lambda = \lambda_1 + \lambda_2 + \lambda_3, \quad and \quad \| \mathcal{B}_{i(k+1)} \| \; \lambda_i \; \| \mathcal{B}_{ik} \| \tag{17}$$

Proof : Firstly, we show that $\{\mathbf{S}_n = (R_n,S_n,T_n)\}_0^\infty$ is a Cauchy sequence in the Hilbert space H^3. For this reason, we suppose that

$$\begin{aligned}
\| R_{n+1} - R_n \| &= \| \mathcal{B}_{1(n+1)} \| \; \lambda_1 \;\; \| \mathcal{B}_{1n} \| \; \lambda_1^2 \;\; \| \mathcal{B}_{1(n-1)} \| \ldots \lambda_1^{n+1} \;\; \| \mathcal{B}_{10} \|,\\
\| S_{n+1} - S_n \| &= \| \mathcal{B}_{2(n+1)} \| \; \lambda_2 \;\; \| \mathcal{B}_{2n} \| \; \lambda_2^2 \;\; \| \mathcal{B}_{2(n-1)} \| \ldots \lambda_2^{n+1} \;\; \| \mathcal{B}_{20} \|,\\
\| T_{n+1} - T_n \| &= \| \mathcal{B}_{3(n+1)} \| \; \lambda_3 \;\; \| \mathcal{B}_{3n} \| \; \lambda_3^2 \;\; \| \mathcal{B}_{3(n-1)} \| \ldots \lambda_3^{n+1} \;\; \| \mathcal{B}_{30} \|.
\end{aligned} \tag{18}$$

Then, by using the triangle inequality, we find that

$$\begin{aligned}
\| \mathbf{S}_n - \mathbf{S}_m \| &= \| (R_n,S_n,T_n) - (R_m,S_m,T_m) \|\\
&= \| (R_n - R_m, S_n - S_m, T_n - T_m) \|\\
&\le \| R_n - R_m \| + \| S_n - S_m \| + \| T_n - T_m \|\\
&\le \| R_n - R_{n-1} \| + \| R_{n-1} - R_{n-2} \| + \cdots + \| R_{m+1} - R_m \| + \| S_n - S_{n-1} \| +\\
&\qquad \| S_{n-1} - S_{n-2} \| + \cdots + \| S_{m+1} - S_m \| + \| T_n - T_{n-1} \| + \| T_{n-1} - T_{n-2}\\
&\qquad \| + \cdots + \| T_{m+1} - T_m \|\\
&\le (\lambda_1^n + \lambda_1^{n-1} + \cdots + \lambda_1^{m+1}) \| \mathcal{B}_{10} \| + (\lambda_2^n + \lambda_2^{n-1} + \cdots + \lambda_2^{m+1}) \| \mathcal{B}_{20}\\
&\qquad \| + (\lambda_3^n + \lambda_3^{n-1} + \cdots + \lambda_3^{m+1}) \| \mathcal{B}_{30} \|\\
&\le (\lambda^n + \lambda^{n-1} + \cdots + \lambda^{m+1})(\| \mathcal{B}_{10} \| + \| \mathcal{B}_{20} \| + \| \mathcal{B}_{30} \|)\\
&= \lambda^{m+1} (\lambda^{n-m-1} + \lambda^{n-m-2} + \cdots + 1)(\| \mathcal{B}_{10} \| + \| \mathcal{B}_{20} \| + \| \mathcal{B}_{30} \|)\\
&\le \frac{\lambda^{m+1}}{1-\lambda} \| \mathcal{B}_0 \|,
\end{aligned} \tag{19}$$

since $\| \mathcal{B}_0 \| < \infty$ and $0 < \lambda < 1$, we then have $\lim_{n,m\to\infty} \| \mathbf{S}_n - \mathbf{S}_m \| = 0$. Thus, we conclude that $\{\mathbf{S}_n\}_0^\infty$ is a Cauchy sequence in the Hilbert space H^3, thus, the series solution $\{S_n\}_0^\infty$ converges to some $\{\mathbf{S}\} \in H^3$.

Theorem 4.2 Let $\mathcal{F} = (\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3)$ be a nonlinear operator satisfies Lipschitz condition from a Hilbert space H^3 into H^3 and $\mathbf{u}(\eta)$ be the exact solution of INS equations. If the series solution $\{\mathrm{S}_n\}_0^\infty$ converges, then it is converged to $\mathbf{u}(\eta)$.

Proof: Let $\mathbf{u}_1(\eta), \mathbf{u}_2(\eta)$, then we have

$$\begin{aligned} \| \mathcal{F}(\mathbf{u}_1) - \mathcal{F}(\mathbf{u}_2) \| &= \| (\mathcal{F}_1(\mathbf{u}_1), \mathcal{F}_2(\mathbf{u}_1), \mathcal{F}_3(\mathbf{u}_1)) - (\mathcal{F}_1(\mathbf{u}_2), \mathcal{F}_2(\mathbf{u}_2), \mathcal{F}_3(\mathbf{u}_2)) \| \\ &= \| (\mathcal{F}_1(\mathbf{u}_1) - \mathcal{F}_1(\mathbf{u}_2), \mathcal{F}_2(\mathbf{u}_1) - \mathcal{F}_2(\mathbf{u}_2), \mathcal{F}_3(\mathbf{u}_1) - \mathcal{F}_3(\mathbf{u}_2)) \| \\ &\leq \| \mathcal{F}_1(\mathbf{u}_1) - \mathcal{F}_1(\mathbf{u}_2) \| + \| \mathcal{F}_2(\mathbf{u}_1) - \mathcal{F}_2(\mathbf{u}_2) \| + \| \mathcal{F}_3(\mathbf{u}_1) - \mathcal{F}_3(\mathbf{u}_2) \| \\ &\leq \sigma_1 \| \mathbf{u}_1 - \mathbf{u}_2 \| + \sigma_2 \| \mathbf{u}_1 - \mathbf{u}_2 \| + \sigma_3 \| \mathbf{u}_1 - \mathbf{u}_2 \| \\ &= (\sigma_1 + \sigma_2 + \sigma_3) \| \mathbf{u}_1 - \mathbf{u}_2 \| \\ &= \sigma \| \mathbf{u}_1 - \mathbf{u}_2 \| \end{aligned} \tag{20}$$

Therefore, from the Banach fixed-point theorem, there is a unique solution of the problem (3). Now we have to prove that $\{S_n\}_0^\infty$ converges to $\mathbf{u}(\eta)$

$$\mathbf{u}(\eta) = \mathcal{F}(\mathbf{u}(\eta)) = \mathcal{F}(\sum_{k=0}^{\infty} \mathcal{B}_k) = \mathcal{F}(\lim_{n\to\infty} \sum_{k=0}^{n} \mathcal{B}_k) = \lim_{n\to\infty} \mathcal{F}(\sum_{k=0}^{n} \mathcal{B}_k) = \lim_{n\to\infty} \mathcal{F}(\boldsymbol{S}_n) = \lim_{n\to\infty} \boldsymbol{S}_{n+1} = \boldsymbol{S}. \tag{21}$$

From the above theorems, we can deduce the following definition which we have adopted to discuss the convergence of solutions

Definition 4.1 For $\mathrm{i} = 1,2,3$ and $\mathrm{k} \in \mathbb{N} \cup \{0\}$, we define

$$\lambda_{ik} = \begin{cases} \dfrac{\| \mathcal{B}_{i(k+1)} \|}{\| \mathcal{B}_{ik} \|}, & \| \mathcal{B}_{ik} \| \neq 0, \\ 0, & \| \mathcal{B}_{ik} \| = 0. \end{cases} \tag{22}$$

then we can say that the series of approximate solutions $\{S_n\}_0^\infty$ converges to the exact solution $\mathbf{u}(\eta)$ when $\lambda_k = \lambda_{1k} + \lambda_{2k} + \lambda_{3k}$ and $0 < \lambda_k < 1$ for all $k \in \mathbb{N} \cup \{0\}$.

5. Application

In order to solve the system (3), we write this system as follows

$$G_1(f, f', f'', f^{(3)}, f^{(4)}, \varepsilon) = \frac{1}{S} f^{(4)} + \varepsilon \left(f\, f^{(3)} - 3\, f'' - \eta\, f^{(3)} - f' f''\right) = 0, \Big\} \tag{23}$$

$$\left.\begin{aligned} G_2(f, f', f'', \theta', \theta'', \varepsilon) &= \frac{1}{Pr} \theta'' + \varepsilon\, (S\, (f - \eta)\, \theta' + Ec\, ((f'')^2 + 4\, \delta^2\, (f')^2)) = 0, \\ G_3(f, \phi, \phi', \phi'', \varepsilon) &= \frac{1}{Sc} \phi'' + \varepsilon\, (S\, (f - \eta)\, \phi' - \gamma\, \phi) = 0, \end{aligned}\right\} \tag{23 cont.}$$

for apply PIA(1,1), substituting Eqs. (8) into Eqs. (23) yields

$$\left.\begin{aligned} G_1(f_{n+1}, f'_{n+1}, f''_{n+1}, f^{(3)}_{n+1}, f^{(4)}_{n+1}, \varepsilon) &= \frac{1}{S} f^{(4)}_{n+1} + \varepsilon\, (f_{n+1}\, f^{(3)}_{n+1} - 3\, f''_{n+1} - \eta\, f^{(3)}_{n+1} - f'_{n+1}\, f''_{n+1}) = 0, \\ G_2(f_{n+1}, f'_{n+1}, f''_{n+1}, \theta'_{n+1}, \theta''_{n+1}, \varepsilon) &= \frac{1}{Pr} \theta''_{n+1} + \varepsilon\, (S\, (f_{n+1} - \eta)\, \theta'_{n+1} + Ec\, ((f''_{n+1})^2 + 4\, \delta^2\, (f'_{n+1})^2)) = 0, \\ G_3(f_{n+1}, \phi_{n+1}, \phi'_{n+1}, \phi''_{n+1}, \varepsilon) &= \frac{1}{Sc} \phi''_{n+1} + \varepsilon\, (S\, (f_{n+1} - \eta) \phi'_{n+1} - \gamma\, \phi_{n+1}) = 0, \end{aligned}\right\} \tag{24}$$

such that

$$\begin{aligned} G_1(f_n, f'_n, f''_n, f^{(3)}_n, f^{(4)}_n, 0) &= \frac{1}{S} f^{(4)}_n, \\ G_2(f_n, f'_n, f''_n, \theta'_n, \theta''_n, 0) &= \frac{1}{Pr} \theta''_n, \\ G_3(f_n, \theta_n, \phi'_n, \phi''_n, 0) &= \frac{1}{Sc} \phi''_n, \end{aligned} \tag{25}$$

$$G_{1f_{n+1}} = \varepsilon\, f_{n+1}^{(3)}, \quad G_{1f'_{n+1}} = -\varepsilon f''_{n+1}, \quad G_{1f''_{n+1}} = -\varepsilon\,(3 + f'_{n+1}), \quad G_{1f^{(3)}_{n+1}} = \varepsilon\,(f_{n+1} - \eta), \quad G_{1f^{(4)}_{n+1}} = \frac{1}{S},$$

$$G_{2f_{n+1}} = \varepsilon\, S\, \theta'_{n+1}, \quad G_{2f'_{n+1}} = 8\,\varepsilon\, Ec\, \delta^2\, f'_{n+1}, \quad G_{2f''_{n+1}} = 2\,\varepsilon\, Ec\, f''_{n+1}, \quad G_{2\theta'_{n+1}} = \varepsilon\, S\,(f_{n+1} - \eta), \quad G_{2\theta''_{n+1}} = \frac{1}{Pr},$$

$$G_{3f_{n+1}} = \varepsilon\, S\, \phi'_{n+1}, \quad G_{3\phi_{n+1}} = -\varepsilon\, \gamma,$$

$$G_{3\phi'_{n+1}} = \varepsilon\, S\,(f_{n+1} - \eta), \quad G_{3\phi''_{n+1}} = \frac{1}{Sc},$$

$$G_{1\varepsilon} = f_{n+1}\, f_{n+1}^{(3)} - 3\, f''_{n+1} - \eta\, f_{n+1}^{(3)} - f'_{n+1}\, f''_{n+1},$$

$$G_{2\varepsilon} = S\,(f_{n+1} - \eta)\theta'_{n+1} + Ec\,((f''_{n+1})^2 + 4\,\delta^2\,(f'_{n+1})^2)$$

$$G_{3\varepsilon} = S\,(f_{n+1} - \eta)\phi'_{n+1} - \gamma\, \phi_{n+1}.$$

By calculating all derivatives at $\varepsilon = 0$ and then substituting the results into Eqs. (9), yields the following system of linear ordinary differential equations

$$\left.\begin{aligned} ((f_c)_n)^{(4)} &= -\frac{1}{\varepsilon}\, f_n^{(4)} - S\,(f_n\, f_n^{(3)} - 3 f''_n - \eta\, f_n^{(3)} - f'_n\, f''_n), \\ (\theta_c)''_n &= -\frac{1}{\varepsilon}\, \theta''_n - Pr\,(S\,(f_n - \eta)\theta'_n + Ec\,((f''_n)^2 + 4\,\delta^2\,(f'_n)^2)), \\ (\phi_c)''_n &= -\frac{1}{\varepsilon}\phi''_n - Sc\,(S\,(f_n - \eta)\phi'_n - \gamma\, \phi_n). \end{aligned}\right\} \tag{26}$$

We start with the initial guesses $f_0(\eta) = \frac{1}{2}\,(3\,\eta - \eta^3)$, $\theta_0(\eta) = 1$, $\phi_0(\eta) = 1$, and the boundary conditions to find iterative solutions for this system at $\varepsilon = 1$. The iterative solutions by PIA(1,1) are

$$\begin{aligned} f_1(\eta) &= \frac{1}{2}\,(3\,\eta - \eta^3) + \frac{S}{280}\,(x^7 - 28\,x^5 + 53\,x^3 - 26\,x), \\ \theta_1(\eta) &= 1 - \frac{3\,Pr\,Ec}{40}\,\big(10\,(\eta^4 - 1) + 4\,\delta^2\,(\eta^6 - 5\,\eta^4 + 15\,\eta^2 - 11)\big), \\ \phi_1(\eta) &= 1 + \frac{Sc\,\gamma}{2}\,(\eta^2 - 1), \end{aligned} \tag{27a}$$

$$\begin{aligned} f_2(\eta) = f_1(\eta) &+ S^2\,\eta\,(\frac{1}{92400}\,\eta^{10} + \frac{1}{2016}\,\eta^8 - \frac{193}{19600}\,\eta^6 + \frac{53}{1400}\,\eta^4 + \frac{1}{6}\,(\frac{169391}{294294000}\,S - \frac{1871}{6468})\,\eta^2 - \frac{1009}{12841920}\,S \\ &+ \frac{25477}{1293600}) + \frac{S^3\,\eta^7}{100}\,(\frac{1}{305760}\,\eta^8 - \frac{1}{6006}\,\eta^6 + \frac{1801}{388080}\,\eta^4 - \frac{31}{2352}\,\eta^2 + \frac{1147}{164640}), \end{aligned}$$

$$\begin{aligned} \theta_2(\eta) = \theta_1(\eta) &+ \frac{Ec\,Pr\,S}{504504000}\,[990\,S\,\delta^2\,(18\,Pr - 7)(\eta^{14} - 1) - 1365\,S\,(564\,\mathrm{Pr}\,\delta^2 - 280\,\delta^2 - 30\,Pr + 63)(\eta^{12} - 1) \\ &+ (\mathrm{Pr}\,\delta^2\,(5453448\,S - 5045040) - \delta^2\,(6242236\,S - 1681680) - 1681680\,S\,Pr \\ &+ 3363360\,S)\eta^{10} \\ &- (\mathrm{Pr}\,\delta^2\,(19845540\,S - 35135100) - \mathrm{Pr}(5115825\,S - 13513500) \\ &- \delta^2\,(20630610\,S - 56756700) + 39105495\,S - 8108100)\eta^8 \\ &+ (\mathrm{Pr}\,\delta^2\,(38017980\,S - 126126000) - \mathrm{Pr}(4684680\,S - 25225200) \\ &- \delta^2\,(27937338\,S - 215495280) + 76396320\,S - 201801600)\eta^6 \\ &- (\mathrm{Pr}\,\delta^2\,(35135100\,S - 189189000) - \delta^2\,(17734860\,S - 333333000) + 54227745\,S \\ &- 286486200)\eta^4 + \mathrm{Pr}\,\delta^2\,(11509212\,S - 93153060) + \mathrm{Pr}(1250535\,S - 11711700) \\ &- \delta^2\,(4185896\,S - 172912740) + 13573560\,S - 92792700 - 669240\,\delta^2\,(13\,S - 420)(x^2 - 1)], \end{aligned} \tag{27b}$$

$$\begin{aligned} \phi_2(\eta) = \phi_1(\eta) &+ Sc^2\,[\frac{\gamma^2}{24}\,(\eta^4 - 6\,\eta^2 + 5) - S\,\gamma\,(\frac{S}{25200}\,\eta^{10} - \frac{S}{560}\,\eta^8 + \frac{1}{8400}\,(53\,S - 140)\,\eta^6 \\ &- \frac{1}{1680}\,(13\,S - 70)\,\eta^4 + \frac{S}{315} - \frac{1}{40})], \end{aligned}$$

$$\vdots$$

The condition of the convergence of these solutions is verified by applying **Definition** 4.1 in the domain $[0,1]$

$$\lambda_{10} = \frac{\| (f_c)_1(\eta) \|}{\| f_0(\eta) \|} = \frac{\sqrt{41973646}\,|S|}{255255},$$

$$\lambda_{20} = \frac{\| (\theta_c)_1(\eta) \|}{\| \theta_0(\eta) \|} = \frac{\sqrt{2002\,Pr^2\,Ec^2\,(66112\,\delta^4 + 35672\,\delta^2 + 5005)}}{5005},$$

$$\lambda_{30} = \frac{\| (\phi_c)_1(\eta) \|}{\| \phi_0(\eta) \|} = \sqrt{\frac{2\,\gamma^2\,Sc^2}{15}},$$

$$\lambda_{11} = \frac{\|(f_c)_2(\eta)\|}{\|(f_c)_1(\eta)\|} = [41224731221245\,(215336567632814\,S^2 - 786456997481759 94\,S + 7453096073864298585)^{0.5}\,|S|] \div 90900532342845225,$$

$$\lambda_{21} = \frac{\|(\theta_c)_2(\eta)\|}{\|(\theta_c)_1(\eta)\|} = \frac{\sqrt{462120945}}{5094883418625\sqrt{66112\,\delta^4+3567\quad\delta^2+50}}[((257481345358464192\,Pr^2 + 1497188849878404 48\,Pr + 22009438312443776)\,\delta^4 + (54165816926371080\,Pr^2 + 57728259397656840 0\,Pr + 163665362012901720)\,\delta^2 + 2868935117650875\,Pr^2 + 5896414208553277 5\,Pr + 306502870328383365)\,S^4 - ((400272198834152736 0\,Pr^2 + 4622860456511955840\,Pr + 1031584724473255680)\,\delta^4 + (957173622469155000\,Pr^2 + 8650638475408074600\,P + 4899551959843304400)\,\delta^2 + 56850065717445000\,(Pr^2 + \frac{12389037}{720824}\,Pr + \frac{130680439}{1802060}))\,S^3 + 15573557591832240000\,((Pr^2 + \frac{4410299}{2571135}\,Pr + \frac{7342357}{9256086})\,\delta^4 + (\frac{52521}{195896}\,Pr^2 + \frac{129925717}{61707240}\,Pr + \frac{5658227}{3428180})\,\delta^2 + \frac{170335}{9403008}\,Pr^2 + \frac{24985}{100032}\,Pr + \frac{700739}{783584})\,S^2]^{0.5}, \tag{28}$$

$$\lambda_{31} = \frac{\|(\phi_c)_2(\eta)\|}{\|(\phi_c)_1(\eta)\|} = \frac{1}{72747675}[230945\,Sc^2\,(1094567\,S^4 - 17667720\,S^3 - 127675440\,S^2\,\gamma + 71391075\,S^2 + 1027285350\,S\,\gamma + 3758629875\,\gamma^2)]^{0.5},$$

For example, if $Pr = Ec = Sc = 1$, $\gamma = 0.5$ and $\delta = 0.1$ for all η in the domain, then

$$\begin{aligned} &\lambda_0 = 0.8378375091, && \lambda_1 = 0.2074674406,\ldots && as \quad S = 0.01,\\ &\lambda_0 = 0.8502743526, && \lambda_1 = 0.4404368540,\ldots && as \quad S = 0.5,\\ &\lambda_0 = 0.8756556660, && \lambda_1 = 0.8516660647,\ldots && as \quad S = 1.5, \end{aligned} \tag{29}$$

if $S = Ec = Sc = 1$, $\gamma = 0.5$ and $\delta = 0.1$ then

$$\begin{aligned} &\lambda_0 = 0.5354602542, && \lambda_1 = 0.6134701939,\ldots && as \quad Pr = -0.5,\\ &\lambda_0 = 0.8629650093, && \lambda_1 = 0.6568294044,\ldots && as \quad Pr = 1,\\ &\lambda_0 = 0.6664621563, && \lambda_1 = 0.6481282532,\ldots && as \quad Pr = 0.7, \end{aligned} \tag{30}$$

and if $S = Pr = Ec = 1$, $\gamma = 0.5$ and $\delta = 0.1$ then

$$\begin{aligned} &\lambda_0 = 0.7716779164, && \lambda_1 = 0.5313042051,\ldots && as \quad Sc = 0.5,\\ &\lambda_0 = 0.8447075908, && \lambda_1 = 0.6317243645,\ldots && as \quad Sc = 0.9,\\ &\lambda_0 = 0.8994798465, && \lambda_1 = 0.7070394840,\ldots && as \quad Sc = 1.2. \end{aligned} \tag{31}$$

From the above, it can be concluded that the use of PIA gives solutions that satisfy the convergence condition. Furthermore, the values of the parameters in this problem affect the convergence of the resulting solutions.

6. Results and Discussion

In this section, we introduce the numerical computations of velocity, temperature, concentration, skin friction coefficient, Nusselt number and Sherwood number. Moreover, we show the influence of physical parameters on the computed velocity, temperature and concentration profiles.

In Table 1, the values of velocity $f(\eta)$, temperature $\theta(\eta)$ and concentration profiles $\phi(\eta)$ for some values of η in the domain $[0,1]$ are computed for different values of the active parameters when $Ec = 0.5$, $\delta = 0.1$ and $\gamma = 1$. The calculation time that is required to find these solutions was 0.047s. In Table 2, L_1, L_2 and L_∞ errors are calculated at 128 grid points of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ which are resulted from PIA with CPU time. Note that the value of errors in $f(\eta)$ is lower than in $\theta(\eta)$ and $\phi(\eta)$, and the largest computational time period is for $\theta(\eta)$.

The computed values of skin friction coefficient and Nusselt number are compared with the results which are calculated by HAM in [4], CWM in [9] and WEM in [16] for various values of S when $Pr = Ec = Sc = \gamma = 1$ and $\delta = 0.1$ and presented in Table 3. At $S = 0.5$ and $\delta = 0.1$, the comparison between Nusselt and Sherwood numbers which are calculated by PIA with the results of HAM in [4], ADM in [5] and CM and LSM in[8], are introduced in Tables 4 and 5, respectively.

Table 1. The computed values of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ when $Ec = 0.5$, $\delta = 0.1$ and $\gamma = 1$

	$S = 1$,	$Pr = 0.7$,	$Sc = 1$	$S = 0.5$,	$Pr = 2$,	$Sc = 0.5$
η	$f(\eta)$	$\theta(\eta)$	$\phi(\eta)$	$f(\eta)$	$\theta(\eta)$	$\phi(\eta)$
0	0	1.2236097254	0.6252752485	0	1.6853846217	0.7930840909
0.1	0.1419117706	1.2234540791	0.6289263532	0.1453798050	1.6849083511	0.7950970401
0.2	0.2817138715	1.2228264571	0.6398836783	0.2882508397	1.6828642524	0.8011410434
0.3	0.4172100402	1.2212227749	0.6581613693	0.4260542054	1.6773661830	0.8112320454
0.4	0.5460294073	1.2177260123	0.6837901525	0.5561306350	1.6651314686	0.8253982358
0.5	0.6655356316	1.2108783111	0.7168284992	0.6756700441	1.6412547142	0.8436825495
0.6	0.7727317561	1.1984745309	0.7573793436	0.7816608303	1.5988535623	0.8661463397
0.7	0.8641593624	1.1772442822	0.8056139999	0.8708389416	1.5285420825	0.8928744148
0.8	0.9357906565	1.1423709152	0.8618063847	0.9396368409	1.4176628081	0.9239817330
0.9	0.9829122828	1.0867687304	0.9263831913	0.9841326623	1.2491703840	0.9596221744
1	1	1	1	1	1	1

Table 2. The computed errors of $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ when $Ec = 0.5$, $\delta = 0.1$ and $\gamma = 1$

	$S = 1$,	$Pr = 0.7$,	$Sc = 1$	$S = 0.5$,	$Pr = 2$,	$Sc = 0.5$
Error	$f(\eta)$	$\theta(\eta)$	$\phi(\eta)$	$f(\eta)$	$\theta(\eta)$	$\phi(\eta)$
L_1	5.9583020e-4	7.1631685e-3	6.8635271e-2	7.5094500e-5	8.8104100e-3	7.7333712e-3
L_2	6.9736960e-4	8.0913435e-3	7.5631410e-2	8.7872200e-5	9.6783185e-3	8.5425263e-3
L_∞	1.0629667e-3	1.0673028e-2	1.0488348e-1	1.3389270e-4	1.2372027e-2	1.1925829e-2
CPUs	0.031	0.291	0.063	0.063	0.296	0.063

Table 3. Comparison of the skin friction coefficient $-f''(1)$ and Nusselt number $-\theta'(1)$ with the results in [4, 9, 16] for different values of S when $Pr = Ec = 1$, $\delta = 0.1$

S	$-f''(1)$				$-\theta'(1)$			
	PIA	HAM [4]	CWM [9]	WEM [16]	PIA	HAM [4]	CWM [9]	WGM [16]
-1	2.17504630	2.170090	2.17009087	2.17009087	3.29678519	3.319899	3.31989925	3.31989931
-0.5	2.61766739	2.614038	2.61740384	2.61740384	3.12783101	3.129491	3.12949109	3.12949108
0.01	3.00713376	3.007134	3.00713375	3.00713375	3.04709194	3.047092	3.04709193	3.04809193
0.5	3.33665212	3.336449	3.33644946	3.33644946	3.02666155	3.026324	3.02632345	3.02632351
2	4.20599272	4.167389	4.16738921	4.16738919	3.09288959	3.118551	3.02632345	3.11854932

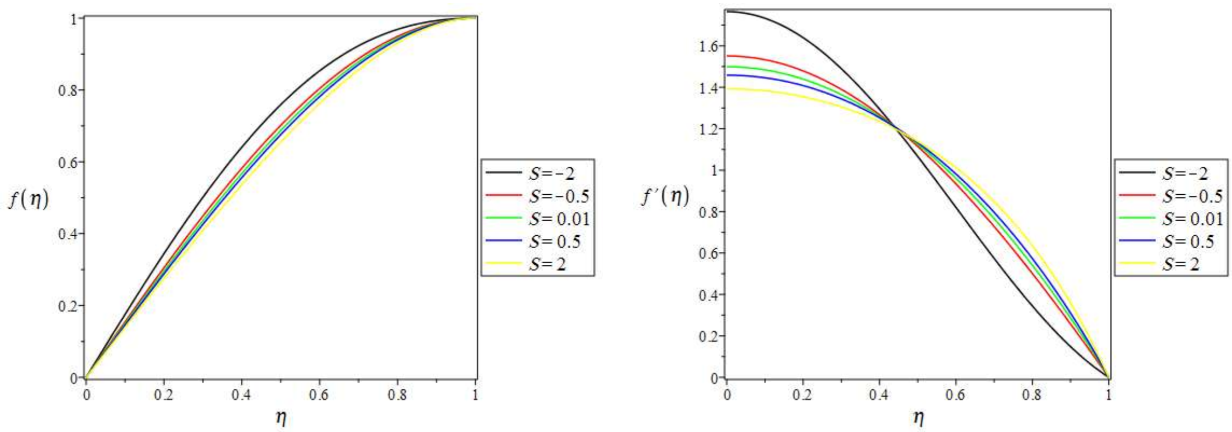
It is observed that, all these comparisons illustrate that PIA gives results close to other results for the present problem. Fig. 2 shows the combined effects of positive and negative the squeeze number on $f(\eta)$ and $f'(\eta)$. We note that, the velocity $f(\eta)$ increases while $f'(\eta)$ decreasing from $\eta = 0$ to $\eta = 1$, the velocity decreases slightly when the squeeze number increases and it takes the highest values at the lowest value of S, while $f'(\eta)$ is changing its impact by value of S as $\eta > 0.4$.

Table 4. Comparison of Nusselt number $-\theta'(1)$ with the results in [4, 5, 8] for different values of Pr and Ec when $S = 0.5$ and $\delta = 0.1$

Pr	Ec	PIA	HAM [4]	ADM [5]	LSM [8]	CM [8]
0.5	1	1.522364231	1.522368	1.522367495	1.520649143	1.526577091
1	1	3.026661548	3.026324	3.026323559	3.023438178	3.03732009
2	1	5.982991585	5.980530	5.980530397	5.976762396	6.012625387
5	1	14.46871534	14.43941	14.43941323	14.44158461	14.59172334
1	0.5	1.513330774	1.513162	1.513161806	1.511719088	1.518660044
1	0.2	3.631993859	3.631588	3.631588268	3.628125812	3.644784107
1	2	6.053323094	6.052647	6.052647107	6.046876352	6.074640177
1	5	15.13330774	15.13162	15.13161783	15.11719088	15.18660044

Table 5. Comparison of local Sherwood number $\phi'(1)$ with with the results in [4] for different values of Sc and γ when $S = 0.5$ and $\delta = 0.1$

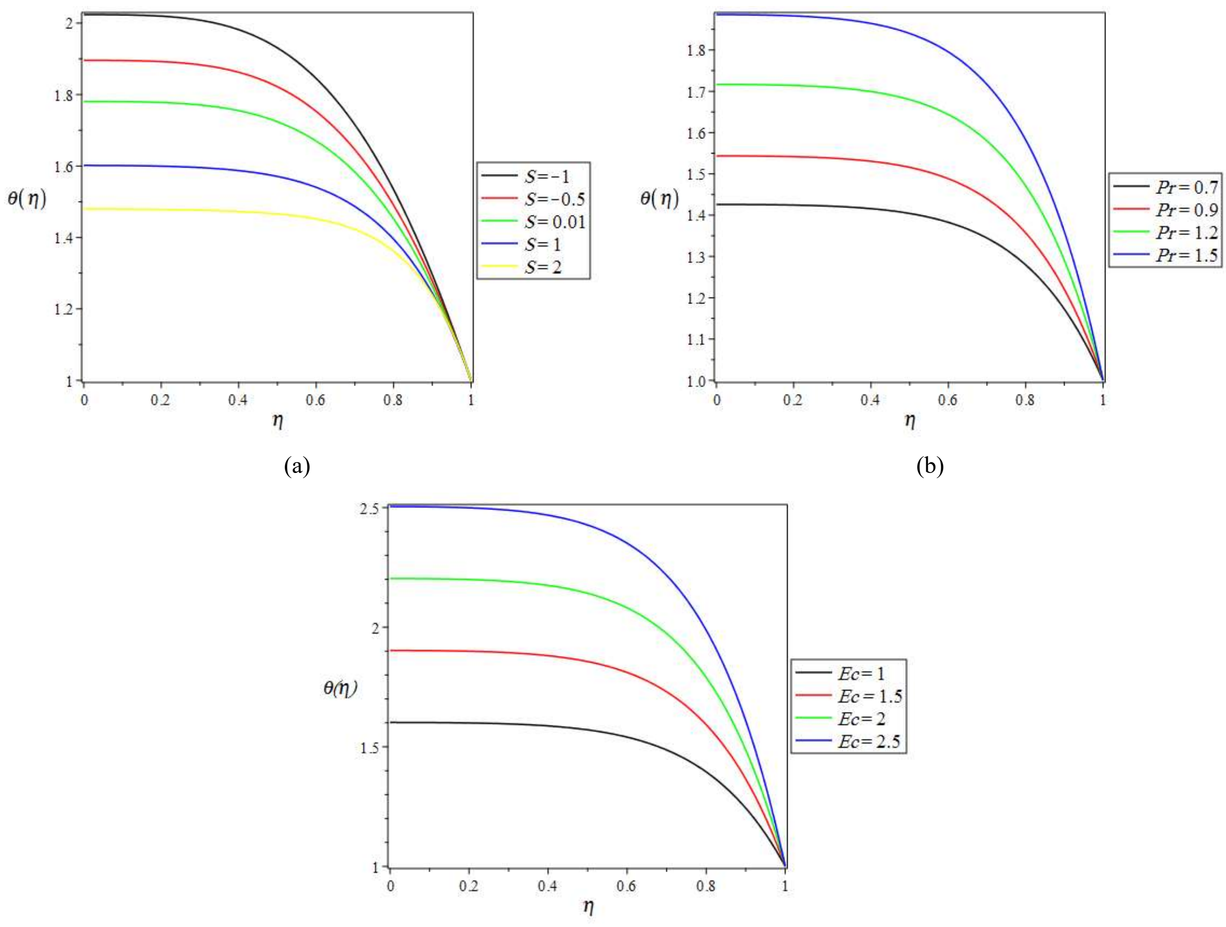
Sc	γ	PIA	HAM [4]
0.5	1	0.428301	0.424826
1	1	0.791485	0.744224
1.5	1	1.206977	0.990645
2	1	1.792201	1.211277
1	-0.5	-0.578851	-0.581206
1	-0.1	-0.100138	-0.100139
1	0.5	0.423445	0.419225
1	1	0.791485	0.744224



(a) (b)

Fig. 2. Influence of S on (a) $f(\eta)$ (b) and $f'(\eta)$

The effects of the squeeze, Prandtl and Eckert numbers on the temperature $\theta(\eta)$ are described in Fig. 3. We can note that the decrease in temperature values is observed with the increasing of S values, since any increase in S can be related with the increase in the distance between the plates, the increase in the plate movement and the decrease in the kinematic viscosity. The values of Pr are lesser than one characterize liquid materials with high thermal diffusion and low viscosity while the values of Pr are greater than one represent high viscosity liquids, so it is observed that the increase in the values of Pr lead to increase temperature value. Eckert number Ec expresses the relationship between a flow kinetic energy and the thermal boundary layer thickness, so we note that the increase in Ec leads to an increase in $\theta(\eta)$. In Fig. 4, the influence of squeeze, Schmidt and chemical reaction numbers on the concentration are explained. It is noticed that the increase in S leads to an increase in $\phi(\eta)$ while the molecular diffusivity decreases with increasing the values of Sc because it represents the ratio between kinematic viscosity and molecular diffusivity. We also observe that the chemical reaction has an opposite effect on the values of $\phi(\eta)$.



(a) (b)

(c)

Fig. 3. Influence of S, Pr and Ec on $\theta(\eta)$ at $\delta = 0.1$, $(a)\ Pr = Ec = 1$ $(b)\ S = Ec = 1$ $(c)\ S = Pr = 1$

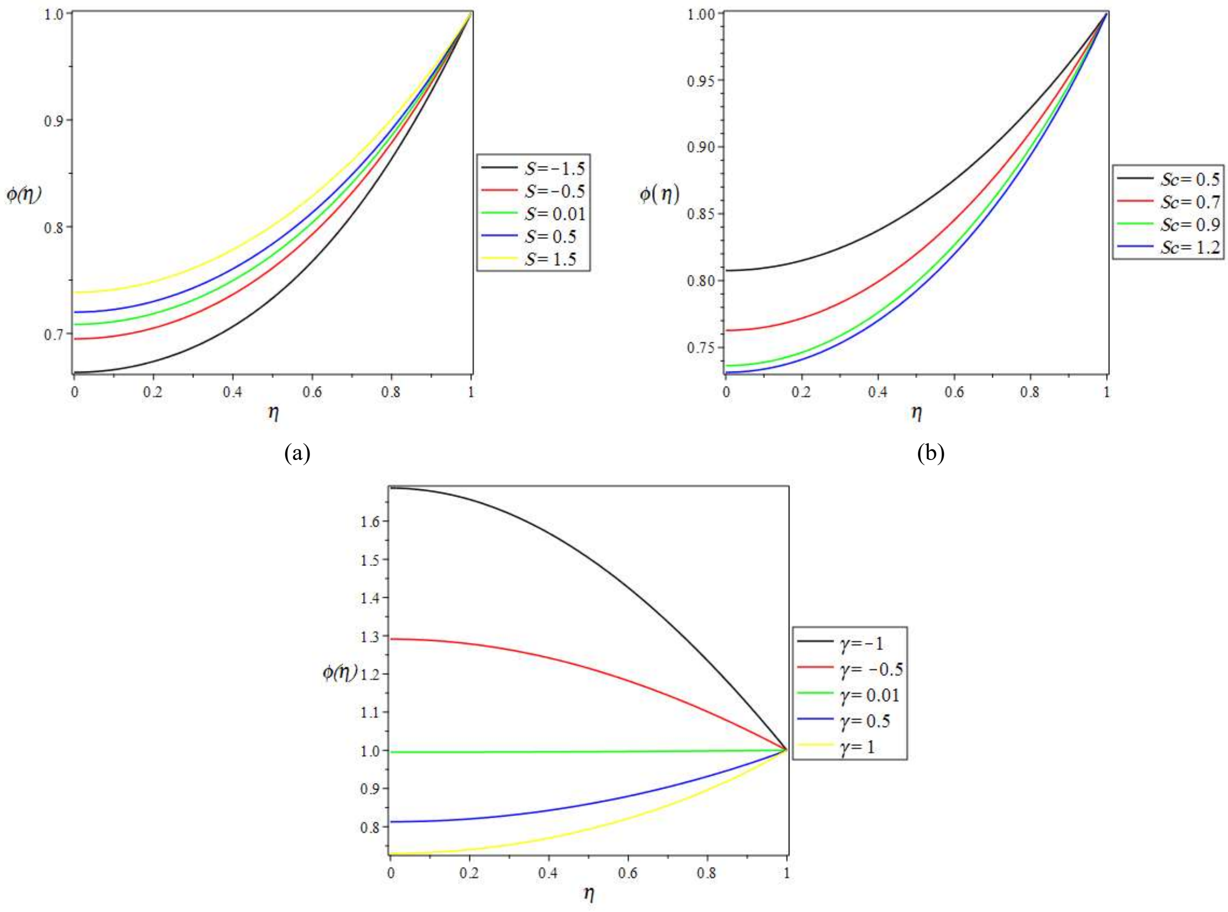


Fig. 4. Influence of S, Sc and γ on $\phi(\eta)$, $(a) Sc=\gamma=1$ $(b)\ S=\gamma=1$ $(c) S=Sc=1$

7. Conclusions

This paper presented the approximate analytical solutions for the heat and the mass transfer of the unsteady two-dimensional squeezing viscous flow between two parallel plates. The governing equations were converted into a set of ordinary differential equations and then solved by using the perturbation-iteration algorithm. The numerical results of the proposed method were reviewed and compared with the results of previous work. The effects of the squeeze number, Prandtl number, Eckert number, Schmidt number and the chemical reaction parameter on the velocity, temperature and concentration were investigated. The main conclusions are as follows:

1. PIA method is fast and easy to implement when compared with other methods.
2. PIA gives a series of convergent solutions at all physical parameter values which is used in this problem.
3. All results of PIA are close to other results of previous studies for this problem. Therefore, the application of PIA method can be extended to include various and multi-dimensions of flow problems.
4. The increase in value of the squeeze number S has an opposite effect on the velocity $f(\eta)$ and the temperature $\theta(\eta)$ in this flow problem and a positive effect on the concentration $\phi(\eta)$, while its effect on $f'(\eta)$ varies depending on the value of η.
5. The Prandtl and Eckert numbers are positively affected on the temperature $\theta(\eta)$ and oppositely effected on the concentration $\phi(\eta)$.

Conflict of Interest

The author(s) declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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