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Research Paper

Exploration of the Significance of Autocatalytic Chemical Reaction and Cattaneo-Christov Heat Flux on the Dynamics of a Micropolar Fluid

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Abstract. During the homogeneous-heterogeneous autocatalytic chemical reaction in the dynamics of micropolar fluid, relaxation of heat transfer is inevitable; hence Cattaneo-Christov heat flux model is investigated in this report. In this study, radiative heat flux through an optically thick medium is treated as nonlinear due to the fact that thermal radiation at low heat energy is distinctly different from that of high heat energy, hence classical approach of using Taylor series for simplification is ignored and implicit differentiation is used leading to temperature parameter. Uniqueness of the present analysis is the consideration of cubic autocatalytic chemical reaction between the homogeneous bulk fluid and two species of catalyst at the wall. Application of similarity analysis enabled us to recast the flow equations into a set of coupled nonlinear ODEs. The resulting equations along with the appropriate conditions are solved computationally. Graphical illustrations of the effect of pertinent parameters on momentum, heat and mass boundary layers are presented and discussed. The concentration of the homogeneous bulk fluid with microstructures and catalyst at the surface decreases and increases with diffusion ratio, respectively. Buoyancy has a decreasing effect on temperature distribution.

Keywords: Boundary layer flow, Non-linear thermal radiation, Auto catalysis, Cattaneo-Christov heat flux.

1. Introduction

Heat transfer is an extensive phenomenon that prevails in many industrial and engineering applications such as heat exchangers, refrigerators, cooling towers, and crops damage via freezing, etc. Pragmatic situations reveal that there are no materials that satisfy Fourier's law; see Fourier [1]. As a corrective measure of this unrealistic feature various modifications to the Fourier's law are suggested, which are all not found to be successful. It is noteworthy that the Maxwell-Cattaneo law is considered as the best. Cattaneo [2] explored that heat transport is related to the thermal wave dissemination with normal speed due to the presence of thermal relaxation time in Fourier's model. Christov [3] refined the Maxwell-Cattaneo model by incorporating the Oldroyd upper-convected derivative. Hayat et al. [4] studied the heat characteristics in a Maxwell fluid flow generated due to a sheet stretching by considering Cattaneo-Christov heat flux (CCH). In the report, it was suggested that temperature in the case of Fourier's law is seen to be higher than that of a CCH model. Li et al. [5] presented the influence of CCH on a non-Newtonian fluid with wall slip. It was concluded that the rate of heat transfer is enhanced for an increment in the mixed convection and relaxing time parameters.

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Muhammad et al. [6] developed a mathematical model to explore the heat and mass characteristics in a squeezing flow of a nanofluid using CCH. They found out that temperature and concentrations are respectively lowered by thermal and solutal relaxation parameters. Gnaneswara Reddy and Rama Subba Reddy [7] have reported the effects of a CCH and viscous dissipation on the flow of non-Newtonian fluids, viscoelastic fluid, and upper-convected Maxwell fluid. Recently, many scientists [8-12] performed the study on the boundary layer flow of a non-Newtonian fluid with CCH.

A chemical reaction is defined as a process in which one set of substances are transformed into one or more compounds (chemical elements). These reactions must be distinguished by a chemical change (8 Fe + $S_8 \rightarrow$ 8 FeS) and not by physical changes (ice melting to water and water evaporating to vapor). In a chemical reaction, no new elements (atoms) are created or lost (conservation mass), but chemical bonds may be cracked into different atoms. However, conservation of energy and mass are preserved in a chemical reaction. A catalyst (accelerator) in a reaction can be used in different phases (heterogeneous, examples: the reaction between a liquid and a gas) or in the same phase (homogeneous, examples: the reaction between two gases or between two liquids) as the reactants.

It is reported that homogeneous and heterogeneous chemical reactions (HHCR) [13-21] play a vital role in engineering and industrial applications such as manufacturing of polymer products, design of chemical processing equipment, and food processing, etc. Chaudhary and Merkin [22] presented a simple isothermal mathematical model to study the influences of homogeneous (bulk)-heterogeneous (surface) reactions on a boundary-layer flow with the kinetics of order one. Koriko et al. [23] and Koriko and Animasaun [24] not only appraised the HHCR but also explored and expanded it to the case where the species of the catalyst at the wall is three. In [24], the dynamics of a non-Newtonian fluid in which micro-rotation and deformation in the presence of HHCR are inevitable over a pointed surface of a rocket as in the case of space science is discussed. The quartic autocatalysis kind of HHCR has been discussed by Makinde et al. [25]. Heat and mass transfer of a nanofluid with motile gyrotactic microorganism and chemical reaction was presented by Ramzan et al. [26]. Recently, Hayat et al. [27-28] examined the chemical reaction effects on a nanofluid flow generated by a surface with nonlinear stretching. Satyanarayana et al. [29] formulated a mathematical model to study the effects of viscous dissipation and Joule heating on convectively magnetohydrodynamic flow due to the continuous stretching of a surface. Sarojamma et al. [30] made a comprehensive study to examine the influence of a non-linear radiative heat in a micropolar nanofluid flow through permeable collapsible walls. Vajravelu et al. [31] studied the effects of HHCR on MHD non-Newtonian boundary layer flow and concluded that the concentration of fluid decreases with the rise of chemical reaction parameter and opposite effect with Schmidt number. Ramzan et al. [32] addressed the influence of CCH on a non-Newtonian fluid flow and found that the concentration has a conflicting behavior for both HHCRs. Hashim and Khan [33] did a complete study on a Carreau fluid flow caused due to a slandering sheet and established that a higher value of HHCR relates to the decrease of concentration field. Sarkar et al. [34] explored that the CCH model has increased heat transfer compared to that of regular Fourier's model.

To the best of our awareness, no other studies discussed previously the micropolar fluids with HHCR under the influence of magnetic field and CCH. Inspired by the foregoing studies, in this paper, a mathematical model is proposed to explore the influence of homogeneous-heterogeneous cubic auto catalysis on an electrically conducting convective micropolar fluid flow generated due to a stretched sheet. The heat transfer phenomenon in this work is coupled with a chemical reaction and nonlinear thermal radiation. The paper is organized as follows: In Section (2) mathematical formulation of the problem is presented. Method of the solution is discussed in Section (3). The results are discussed in Section (4) and the key features of this investigation are given in Section (5).

2. Mathematical Formulation

Two-dimensional steady free convective flow pertaining to an incompressible and electrically conducting non-Newtonian fluid induced by a sheet with continuous stretching (with velocity $u_w = cx$) is proposed. A constant Lorentz force of strength B_0 is considered in y-direction. The innovative CCH model is incorporated to explore the heat transfer phenomenon in the presence of nonlinear thermal radiation. Following the model of Chaudhary and Merkin [22], HHCR for the cubic autocatalysis is considered and is given by

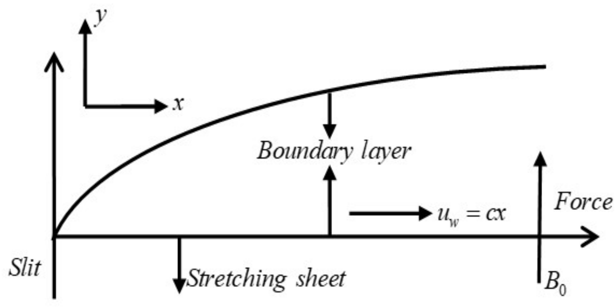

Fig. 1. Schematic diagram of the physical model

$$A+2B\rightarrow 3B, rate=k_c ab^2 \tag{1}$$

On catalyst surface, the chemical reaction is defined as

$$A\rightarrow B,\ rate=k_s a \tag{2}$$

The foregoing considerations and boundary layer approximation lead to the equations given here under:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{(\mu+k)}{\rho}(\frac{\partial^2 u}{\partial y^2})+\frac{k}{\rho}\frac{\partial N}{\partial y}-\frac{\sigma B_0^2}{\rho}u+g\beta_T(T-T_\infty) \tag{4}$$

$$u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}=\frac{\Omega}{\rho j}(\frac{\partial^2 N}{\partial y^2})-\frac{k}{\rho j}(2N+\frac{\partial u}{\partial y}) \tag{5}$$

$$\begin{aligned}&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_1(u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}+2uv\frac{\partial^2 T}{\partial y\partial x}+u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y})=\\&\frac{K}{\rho c_p}(\frac{\partial^2 T}{\partial y^2})+\frac{16\sigma^* T_\infty^3}{3k^*\rho c_p}\frac{\partial}{\partial y}(T^3\frac{\partial T}{\partial y})\end{aligned} \tag{6}$$

$$u\frac{\partial a}{\partial x}+v\frac{\partial a}{\partial y}=\mathrm{D}_A(\frac{\partial^2 a}{\partial y^2})-k_C ab^2 \tag{7}$$

$$u\frac{\partial b}{\partial x}+v\frac{\partial b}{\partial y}=\mathrm{D}_B(\frac{\partial^2 b}{\partial y^2})+k_C ab^2 \tag{8}$$

The boundary conditions are:

$$u=u_W, v=0, N=0, T=T_W, D_A\frac{\partial a}{\partial y}=k_s a, \mathrm{D}_B\frac{\partial b}{\partial y}=-k_s a\ \ at\ \ y=0 \tag{9}$$

$$u\rightarrow 0, N\rightarrow 0, T\rightarrow T_\infty, a\rightarrow a_0, b\rightarrow 0\ \ \text{as}\ \ y\rightarrow\infty \tag{10}$$

Considering the following similarity and dimensionless variables

$$\begin{aligned}&\eta=y\sqrt{\frac{cx}{\nu}},\ u=cxf'(\eta), \mathrm{v}=-\sqrt{c\nu}\,\mathrm{f}(\eta), N=cx\sqrt{\frac{c}{\nu}}h(\eta)\\&\theta(\eta)=\frac{T-T_\infty}{T_W-T_\infty},\ a=a_0\phi_1(\eta), \mathrm{b}=a_0\phi_2(\eta)\end{aligned} \tag{11}$$

The left-hand side of expression (3) trivially vanishes, while equations (4) – (10) yield

$$(1+\beta)f'''+ff''-(f')^2+\beta h'-Mf'+Gr\theta=0 \tag{12}$$

$$(1+\frac{\beta}{2})h''-\beta(2h+f'')+fh'-f'h=0 \tag{13}$$

$$\theta''+Nr[(1+(\theta_r-1)\theta)^3]\theta''+3Nr[(\theta_r-1)(1+(\theta_r-1)\theta)^2]\theta'^2+\Pr f\theta'-\Pr\gamma_1(ff'\theta'+f^2\phi'')=0 \tag{14}$$

$$\phi_1'+Sc(f\phi_1'-K_1\phi_1\phi_2^2)=0 \tag{15}$$

$$\Lambda\phi_2''+Sc(f\phi_2'+K_1\phi_1\phi_2^2)=0 \tag{16}$$

with the boundary conditions

$$f(\eta)=0, f^{'}(\eta)=1, h(\eta)=0, \theta(\eta)=1, \theta_1^{'}(\eta)=K_s\phi_1(\eta),\ \Lambda\theta_2^{'}(\eta)=-K_s\phi_1(\eta) \quad \text{at} \quad \eta=0 \tag{17}$$

$$f^{'}(\eta)\to 0, h(\eta)\to 0, \theta(\eta)\to 0=0, \phi_1(\eta)\to 0, \phi_2(\eta)\to 0 \ \text{ as } \eta\to\infty \tag{18}$$

The mass diffusion coefficients are equal i.e. $\Lambda=1$ this implies that

$$\phi_1(\eta)+\phi_2(\eta)=1 \tag{19}$$

Now, equation (15) becomes

$$\phi_1^{''}+Sc(f\phi_1^{'}-K_1\phi_1(1-\phi_1)^2)=0 \tag{20}$$

with boundary conditions

$$\theta_1^{'}(0)=K_s\phi_1(0), \theta_1(\infty)\to 1 \tag{21}$$

Coefficient of wall shear $\left(C_f\right)$, surface couple stress $\left(M_w\right)$ and rates of thermal energy $\left(Nu_x\right)$ can be given by

$$C_f=\frac{\tau_w}{\rho u_w^2}\ ,\ M_W=\frac{M_x}{\rho x u_w^2}\ ,\ Nu_x=\frac{xq_w}{k(\mathrm{T}_W-T_\infty)} \tag{22}$$

where

$$\tau_w=((\mu+k)\frac{\partial u}{\partial y}+kN)_{y=0}\ ,\quad m_x=\Omega(\frac{\partial N}{\partial y})_{y=0}\ , \mathrm{q}_w=-k(1+\frac{16\sigma^* T^B}{3Kk^*})(\frac{\partial T}{\partial y})_{y=0} \tag{23}$$

Using equation (23) in equation (22), we obtain

$$C_f\sqrt{\mathrm{Re}_x}=(1+\beta)f^{''}(0) \tag{24}$$

$$M_w\,\mathrm{Re}_x=(1+\frac{\beta}{2})h^{'}(0) \tag{25}$$

$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-(1+Nr[1+(\theta_r-1)\theta(0)]^3)\theta^{'}(0) \tag{26}$$

where $\mathrm{Re}_x=xu_w/v$ is the local Reynolds number.

3. Numerical Solution

Exact analytical solutions for the coupled ordinary differential equations (12) - (16) are not possible due to the nonlinearity of the equations. Hence, numerical solutions of the Equations (12) - (16) with the pertinent boundary conditions (17) and (18) are obtained with the aid of Runge-Kutta-Fehlberg (RKF-45) algorithm.

(I) The higher order coupled ODE's are first transformed into a system of first-order differential equations.
(II) Suitable guess values are chosen for unknown required boundary conditions.
(III) Runge-Kutta-Fehlberg method with shooting technique is utilized for step by step integration with the assistance of MATLAB software.
(IV) The calculated values of velocity, temperature and concentration are compared with the given boundary conditions for better accuracy.
(V) The above procedure is repeated until the results are obtained within a tolerance limit of 10^{-5}.

The accuracy of the numerical scheme is authenticated by comparing the results of the present analysis, i.e. the values of local temperature gradient as a particular case with those obtained by Grubka and Bobba [35], Ishak [36], Keimanesh and Aghanajafi [37] for different values of Prandtl number (Pr) in the absence of Lorentz force, thermal radiation, thermal buoyancy, for a viscous fluid flow, that is when $M=Gr=\gamma_1=Sc=Nr=0$ and are tabulated in Table 1 which reveals a very close agreement.

4. Results and Discussion

This investigation aims at analyzing the characteristics of the flow, energy and species transfer on the MHD flow of the micropolar fluid on a sheet of linear stretching under the influence of non-linear thermal radiation, CCH and HHCR. The flow variables are discussed for different variations of the governing parameters. The values of Pr and Sc are respectively fixed as 0.71 and 2.0. The ranges of M, Grashof number and β are between 0 and 3. The values of θ_r, Λ, and γ vary in the interval (1, 1.5). K_1, K_s, and Nr lie between 0 and 1.5. We consider two cases, which are:

Case (a): $\Lambda = 1$ in which case $\phi_1(\eta)+\phi_2(\eta)=1$ then the equations governing the concentrations reduces to a single equation (20) in terms of the bulk fluid namely micropolar fluid.

Case (b): $\Lambda \neq 1$ then we have two equations for the concentration of reactant $A(\phi_1(\eta))$ and concentration of reactant $B(\phi_2(\eta))$.

The plots of $\phi_1(\eta)$ in both cases indicate that the concentration values of $\phi_1(\eta)$ when $\Lambda = 1$ are meagerly higher than those corresponding to the case $\Lambda \neq 1$. In case (b) when $\Lambda = 1.2$ the concentrations $\phi_1(\eta)$ of reactant A and $\phi_2(\eta)$ of reactant B are discussed.

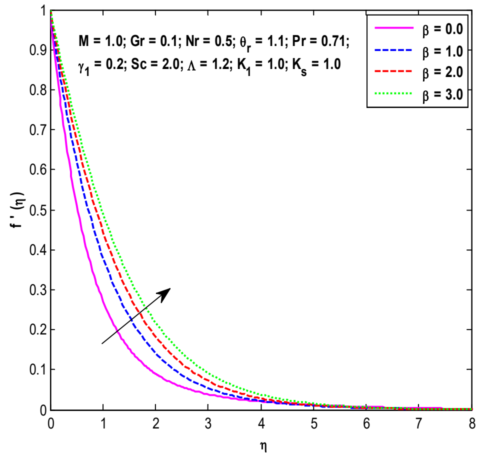

Fig. 2(a) Influence of β on Velocity

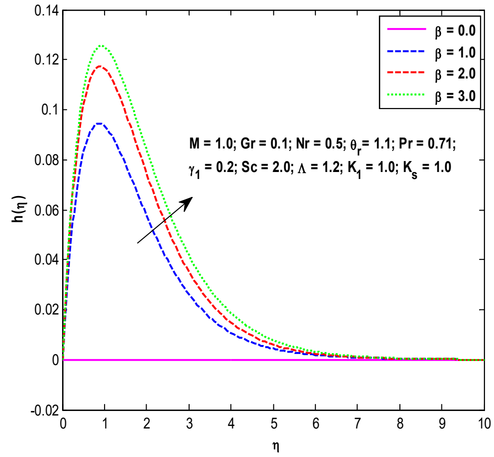

Fig. 2(b) Influence of β on Microrotation

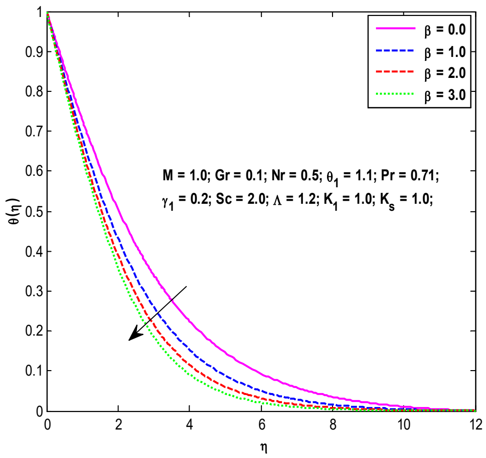

Fig. 2(c) Influence of β on Temperature

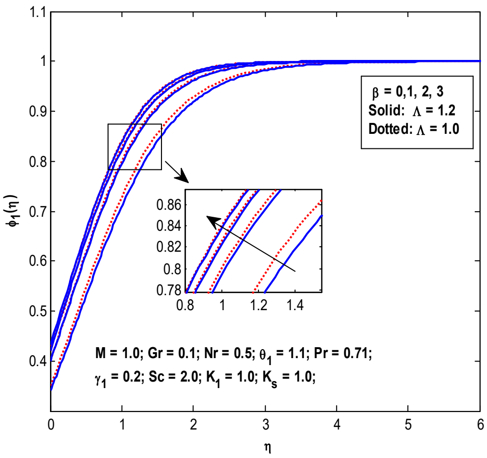

Fig. 2(d) Influence of β on Concentration

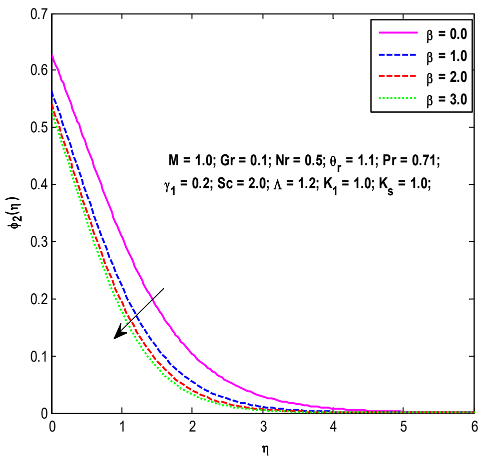

Fig. 2(e) Influence of β on Concentration

Table 1. Comparison between the results of present and previous study for $-\theta'(0)$ when $M = Gr = \gamma_1 = sc = Nr = \beta = \theta_r = \Lambda = K_1 = K_s = 0$

Pr	Grubka and Bobba [35]	Ishak [36]	Keimanesh and Aghanajafi [37]	Present
0.72	0.4631	0.4631	0.463592	0.463592
1.0	0.5820	0.5820	0.582011	0.582011
3.0	1.1652	1.1652	1.165240	1.165243
10.0	2.3080	2.3080	2.308000	2.308004
100.0	-	7.7657	-	7.765844

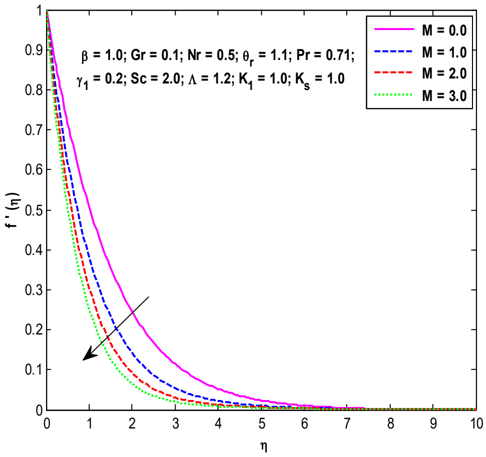

Fig. 3(a) Influence of M on Velocity

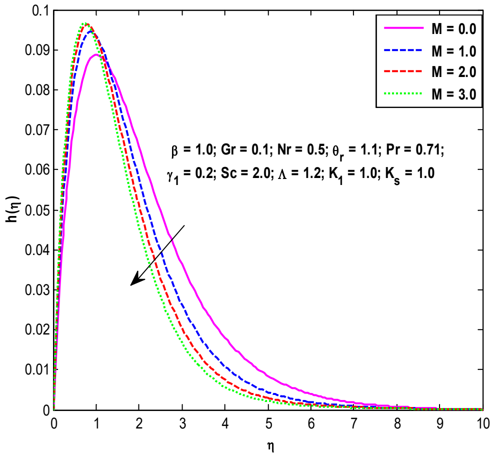

Fig. 3(b) Influence of M on Microrotation

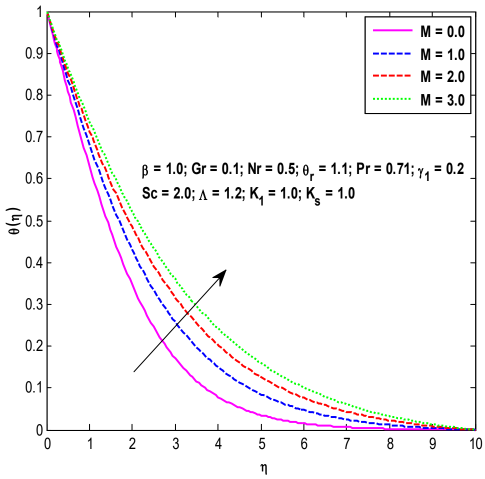

Fig. 3(c) Influence of M on Temperature

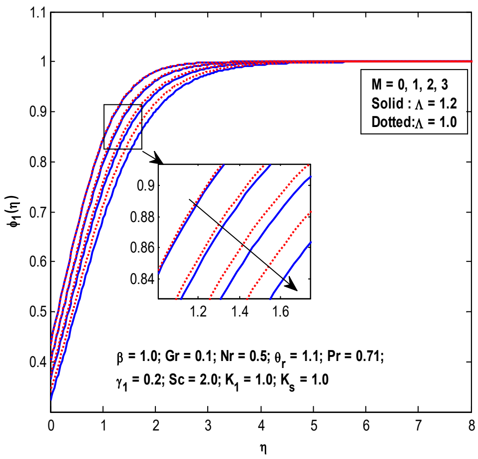

Fig. 3(d) Influence of M on Concentration

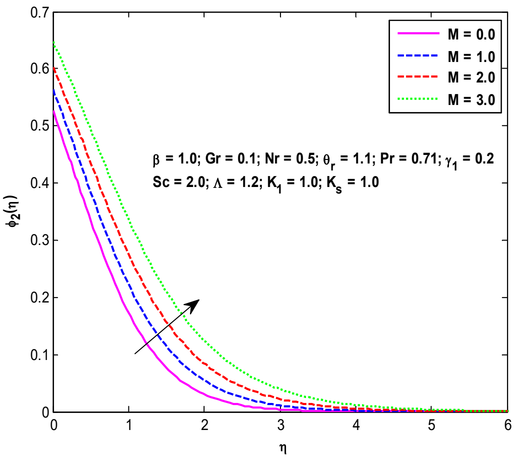

Fig. 3(e) Influence of M on Concentration

The influence of material parameter β on velocity, microrotation, temperature, and mass concentrations is illustrated in Figs. 2(a)- 2(e). As mentioned earlier, velocity is enhanced with increasing β and consequently the hydrodynamic boundary layers become thicker. When $\beta=1$, microrotation increases significantly in the vicinity of the boundary and attains its maximum then later descends reaching the upstream condition. As β enhances other profiles also follow the same pattern and it is observed that microrotation is an increasing function of β. It is pertinent to state that we have considered the strong concentration case (i.e., boundary parameter n = 0) which amounts to the zero microrotation at the boundary. The effect of β on temperature is observed to be contrast to that on velocity. The concentration of reactant A is seen to be higher than that corresponding to a Newtonian case. As β increases we notice an increasing tendency in the concentration. In the case of the reactant B, a totally opposite trend is observed. It is worth remarking that these results corroborate with that of Animasaun et al. [38].

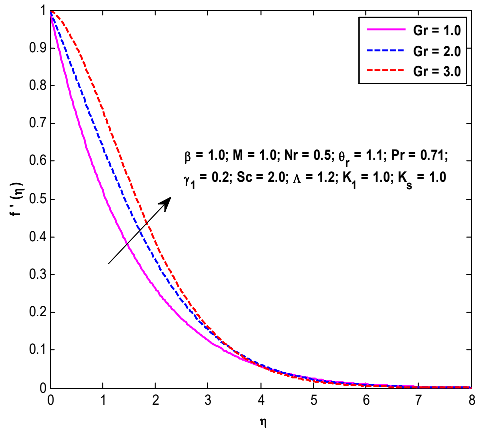

Fig. 4(a) Influence of *Gr* on Velocity

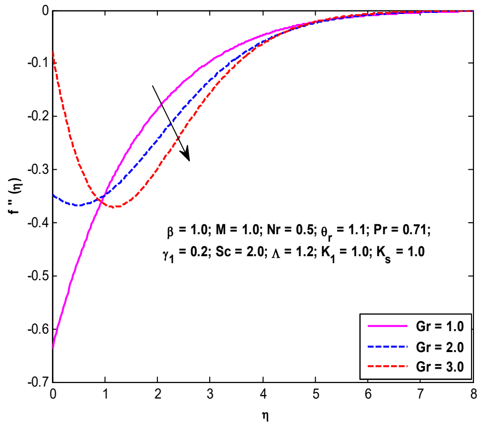

Fig. 4(b) Influence of *Gr* on Shear stress

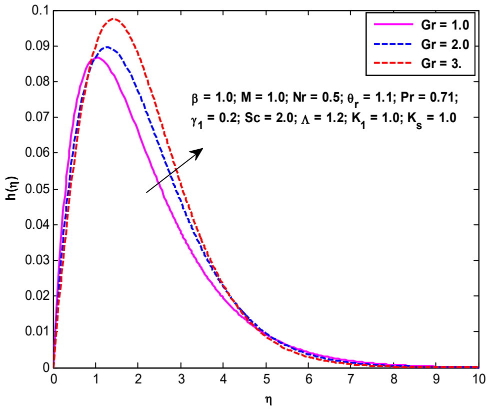

Fig. 4(c) Influence of *Gr* on Microrotation

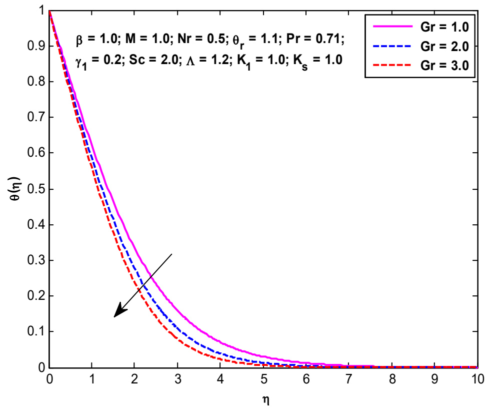

Fig. 4(d) Influence of *Gr* on Temperature

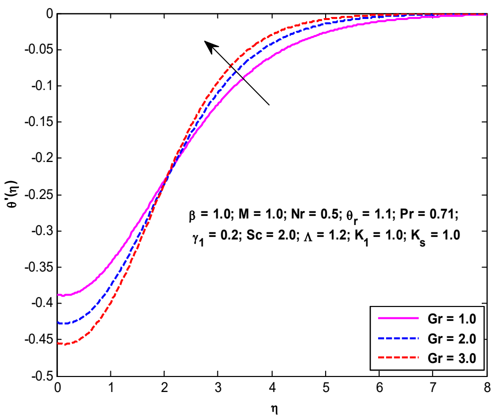

Fig. 4(e) Influence of *Gr* on Temperature

Figs. 3(a)-3(e) display the influence of the magnetic field on velocity, micro rotation, temperature and mass concentrations. The presence of magnetic field diminishes the velocity as a result of the stabilizing effect of the magnetic field owing to the Lorentz force. Stronger magnetic fields result in further reduction in velocity and hence the corresponding hydrodynamic boundary layers are thinner. In the absence of magnetic field, microrotation is seen to boost rapidly across the fluid layers near the boundary and attains its maximum followed by a descent fulfilling its free stream condition asymptotically. In the presence of magnetic field the microrotation increases. Increasing strength of magnetic field increases the peak value more and the point of maximum microrotation velocity is skewed towards the boundary, proceeding further along the curve, a reversal behavior is noticed. The temperature is boosted with a rise in Lorentz force strength and thus thermal boundary layers are enlarged. Magnetic field shows a reduction in the concentration of the species *A* while concentration of the species *B* experiences an enhancement for the same variation in *M*.

The effect of Grashof number (*Gr*) is portrayed in Figs. 4(a)-4(g). It is observed that the influence of *Gr* enhances the velocity in the boundary layer. This shows that the role of thermal buoyancy is to accelerate the flow. At this point, following the analysis of Shah et al. [39] the slope of linear regression through data points (slp) on the wall is estimated as 0.280419551616302 and far upstream, that is, at $\eta = 8$ it is 0.000509958731604 which accounts for the increase in the velocity. Shear stress at the wall $f''(0)$ increases for an increment in *Gr* in the region $0 < \eta < 1$ and diminishes in the rest of the region. The slope of linear regression at the wall estimated through data points is -0.045612643871131which accounts for the decrease in microrotation while at η =1.7 it is 0.005388466547168 that represents the increase. The microrotation velocity also improves with increasing values of Grashof number with its peak value at $\eta = 1$. The microrotation starts from its zero value at the surface and rises till it hits the peak value and then starts decreasing until the boundary condition satisfies as $\eta \to \infty$ for all values of *Gr*. The impact of *Gr* is observed to be more predominant at the point of peak value i.e., increment in Grashof number enhances significantly the peak value of the microrotation and the point of maximum microrotation drifts away from the boundary. The temperature is observed to fall with increasing values of *Gr*. This may be explained as follows. An increment in *Gr* corresponds to an increase in buoyancy force due to a large temperature difference between the boundary and the ambient fluid. This large temperature is initialized to heat up the fluid which leads to the reduction of the density. Temperature gradient is seen to decrease in the region $0 < \eta < 2.4$ and later it shows a reverse trend. *Gr* is seen to have an increasing influence on the concentration of the reactant *A* and an opposite influence on the concentration of the species *B*.

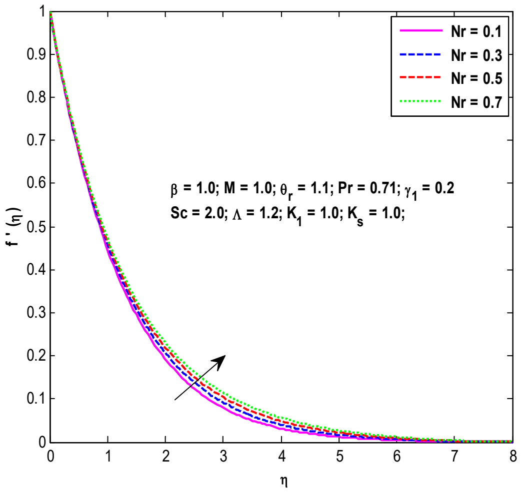

Fig. 5(a) Influence of *Nr* on Velocity

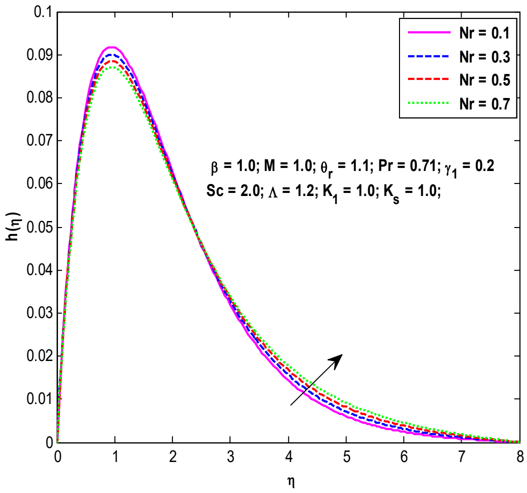

Fig. 5(b) Influence of *Nr* on Microrotation

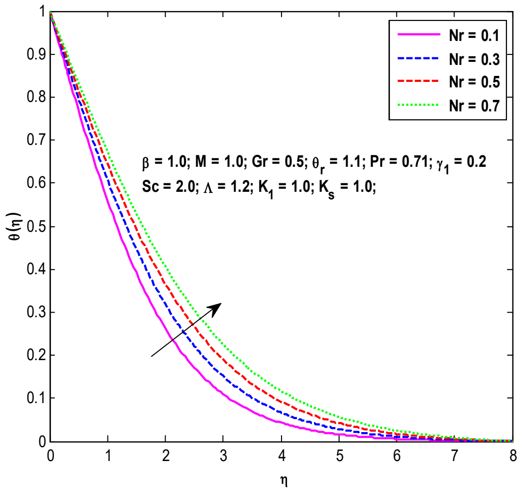

Fig. 5(c) Influence of *Nr* on Temperature

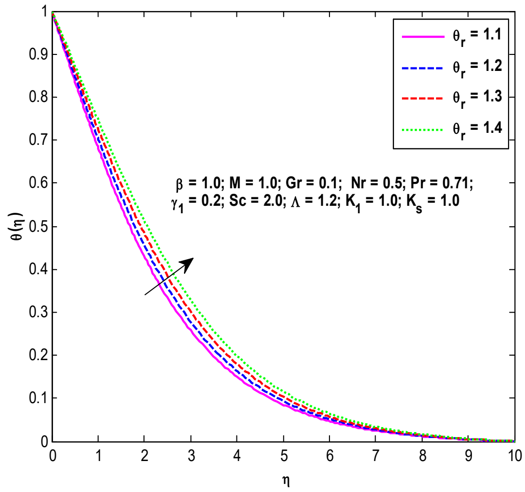

Fig. 6 Influence of θ_r on Temperature

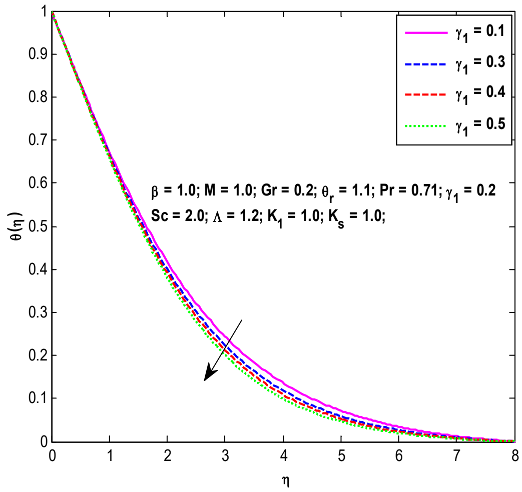

Fig. 7 Influence of γ_1 on Temperature

From Figs. 5(a) – 5(c), it is observed that the velocity is found to be enhanced with increasing values of thermal radiation parameter. Microrotation decreases in $0 \leq \eta \leq 2.424$ and later it shows a reversal trend. It is observed that increasing thermal radiation parameter enhances the thickness of the boundary layer with a rise in temperature. Fig. 6 is the plots of temperature for a variation in temperature ratio parameter θ_r, which emerges due to non-linear thermal radiation. Temperature is raised across the fluid region with increasing values of θ_r and thickness of the corresponding boundary layers is increased.

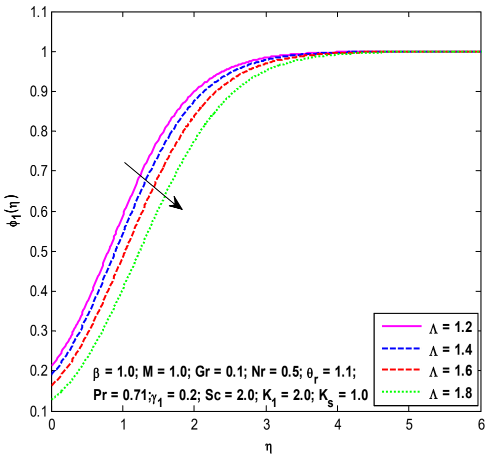

Fig. 8(a) Influence of Λ on Concentration

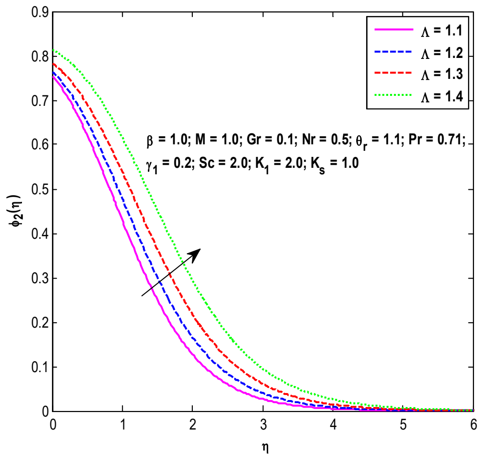

Fig. 8(b) Influence of Λ on Concentration

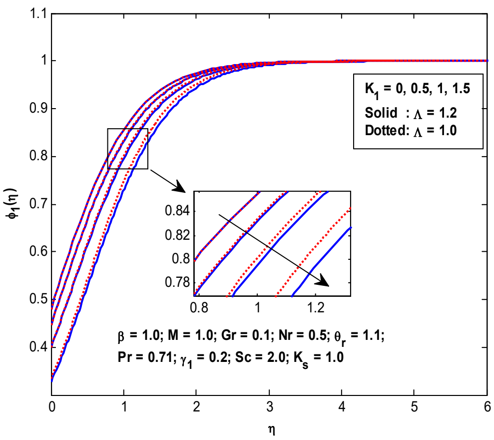

Fig. 9(a) Influence of K_1 on Concentration

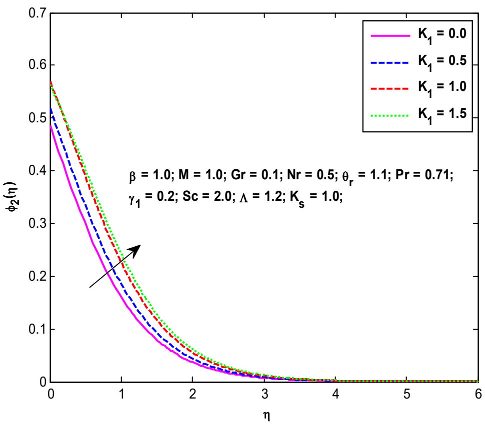

Fig. 9(b) Influence of K_1 on Concentration

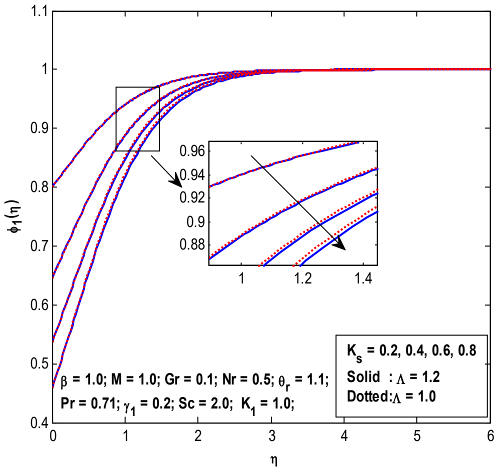

Fig. 10(a) Influence of K_s Concentration

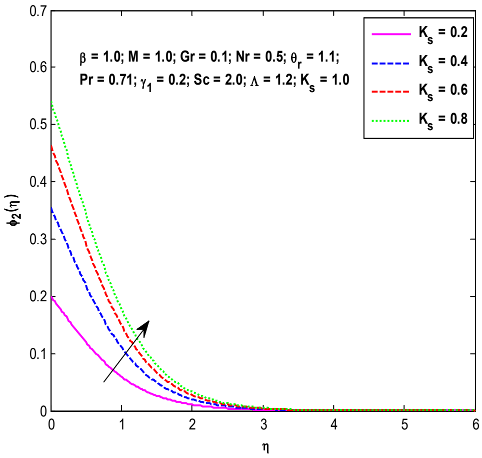

Fig. 10(b) Influence of K_s on Concentration

Fig. 7 is sketched to illustrate the influence of the thermal relaxation parameter (γ_1) on temperature. The temperature falls for stronger thermal relaxation parameter γ_1 with thinner thermal boundary layers. Physically for higher values of γ_1, material particles need more time to transport thermal energy to the neighbouring particles. In other words, it can be said that for higher values of thermal relaxation parameter, material shows a non-conducting behavior which is responsible in reduction of temperature distribution. When γ_1 =0, the CCH model reduces to the classical Fourier's law of conduction. It can be concluded that the temperature in CCH model is less than that in the Fourier's model due to the instant heat transfer in the entire medium. This result corroborates the observation by Li et al. [40] and Khan and Hammad [41]. Figs. 8(a) and 8(b) indicate that concentration of the chemical species *A* is diluted with increasing values of ratio of mass diffusion coefficient *A* while concentration of the species *B* shows an opposite trend. This is evident as the mass diffusivity of the species *B* is greater than that of species *A*.

Figs. 9(a) and 9(b) respectively are the plots of concentration of the reactant *A* and concentration of the reactant *B* for a variation in strength of homogeneous reaction parameter K_1. Physically the rate of diffusion reduces as the rate of reaction enhances. The effect of strength of heterogeneous reaction parameter K_s (catalyst at the surface) on ϕ_1 and ϕ_2 is illustrated in Figs. 10(a) and 10(b). It is observed that for small values of K_s i.e., K_s =0.2 the concentration of the species *A* gradually increases and attains saturation. As K_s takes higher values, concentration falls sharply across the fluid region in $0 < \eta < 2.054$ and gets saturated. Also, the concentration of reactant B, i.e. φ_2 decreases gradually in the region $0 < \eta <$ 2.138 and fulfills the freestream condition. With an increase in K_s, a rapid increase in ϕ_2 takes place. At η =0, ϕ_2 varies from 0.2 to 0. 542 when K_s changes from 0.2 to 0.8. Physically K_s refers to the strength of the heterogeneous (catalyst at the surface) reaction, which implies that an increment in K_s enhances the concentration of the catalyst at the surface.

5. Conclusion

In this analysis the effect of auto catalytic chemical reaction and CCH on the steady free convective flow of a micropolar fluid is studied by considering the non-linear thermal radiation. Using suitable scaling transformations followed by the numerical scheme the computational results of the governing equations of the problem are deduced and are illustrated through graphs. Some of the salient traits are:

- Material parameter and Grashof number accelerate the flow.
- Temperature experiences an enhancement with thermal radiation parameter and temperature ratio parameter while a reduction with thermal relaxation parameter.
- Higher values of parameters pertaining to HHCR correspond to a dilution in the concentration of bulk fluid and stronger concentration of the catalyst at the surface.
- The concentration of reactant *A* decreases while a contrast nature prevails in the case of *B*.

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Conflict of Interest

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Nomenclature

$a_{0,} c$	Dimensional constants
a, b	Chemical Species
u, v :	Velocity components Along X, Y-axis
c_f	Wall Drag Force
c_p	Specific heat at constant pressure
$(D_A\ D_B)$	The diffusion coefficients
f	Dimensionless Stream Function
f'	Dimensionless velocity
Gr	Thermal Grashof number
g	Acceleration due to gravity
Nr	Thermal Radiation Parameter
j	Micro-Inertia Density
k^*	Mean Absorption Coefficient
K_1	Strength Of Homogeneous Reaction Parameter
k	Vortex Viscosity
K	Thermal Conductivity Of The Medium
K_s	Strength of heterogeneous reaction parameter
Sc	Schmidt number
M	Magnetic field parameter
k_c, k_s	Reaction rate constants
m_x	Wall couple stress
Nr	Thermal radiation parameter
N	Microrotation
Nu_x	Measure of heat transfer
Pr	Prandtl Number

q_w	Surface Heat Flux
Re_x	Reynolds number
T_∞	Ambient fluid temperature
T	Fluid temperature
T_w	Temperature Near The Surface

Greek Symbols

θ_r	Temperature ratio parameter
β_T	Coefficient of thermal expansion
ϕ_1, ϕ_2	Dimensionless concentration
γ_1	Thermal Relaxation parameter
τ_w	Wall shear stress
θ	Dimensionless temperature
ν	Kinematic viscosity
Ω	Spin gradient viscosity
ρ	Fluid density
μ	Dynamic viscosity
λ_1	The relaxation time
β	Material parameter
σ^*	Stefan-Boltzmann constant
σ	Electrical conductivity
λ	Thermal buoyancy ratio parameter

Subscripts

∞	Condition at free stream

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