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Beam & Shell Models for Composite Straight or Curved Bridge Decks with Intermediate Diaphragms & Assessment of Design Specifications

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Abstract. In this research effort, the generalized warping and distortional problem of straight or horizontally curved composite beams of arbitrary cross section, loading and boundary conditions is presented. An inclined plane of curvature is considered. Additionally, the stiffness of diaphragmatic plates has been introduced in the formulation in order to compare with the case where rigid diaphragms are assumed. Isogeometric tools (NURBS) are employed in order to obtain the results for the 1D formulation and 3D shell models are developed in FEM commercial software for composite cross sections with diaphragms. The number of intermediate diaphragms according to bridges design specifications is compared to the analyzed diaphragmatic arrangements in order to assess the overall structural behavior of bridges decks. For this purpose, examples of curved beam models with open or closed cross sections and various arrangements of diaphragms have been studied.

Keywords: Higher-Order-Beam-Theories; Finite element method-FEM; Distortion; Warping; Guidelines; Diaphragms.

1. Introduction

Straight or curved composite beams are largely used today in motorway and railway bridge superstructures mostly due to their high loading capacity combined to excellent mechanical behavior and ease of construction. They are usually composed of thin-walled open or closed cross sections with an upper concrete slab. Open I-girders or box cross-sections are the most commonly employed for bridge decks. Box-shaped composite decks have always been a popular type for curved plans due to their high ratio of torsional rigidity to self-weight, though susceptible to cross-sectional distortion under an eccentric load. In open sections, warping phenomena due to torsion and bending can be significant for curved arrangements as well as straight ones under eccentrically applied loading. In any case, curved beams are subjected to warping and distortion with various combinations of their magnitudes depending on many factors, even for dead loads [1].

Therefore, the investigation of torsion and distortion in the analysis of composite bridge decks is important with respect to practical aspects and has drawn the attention of researchers as well as designers for many years. However, comparing to straight beam formulations, the evaluation of these effects is far more complex in curved geometries. Shell or solid finite element (FE) models, which do not permit isolation of structural phenomena and results direct interpretation (warping parameters, distortional stress resultants etc. are also evaluated in contrast to solid models which yield only translations and stress components) [2], are usually employed. Additionally, consecutive straight-line segments are generally employed in practice in order to approximate the curved geometry ignoring warping and distortion transmissions between these segments. Vlasov [3] and Dabrowski [4] were the first that presented solutions for curved beams with open arbitrary or closed box-



shaped cross sections. Extending his previous formulation, Dabrowski [5] investigated the phenomenon of distortion in box members with a symmetric cross section. Later, the curved beam models that have been formulated are restricted to the analysis of the beam in the plane of curvature [6], do not take into account secondary shear deformation effect (SSDE) caused by nonuniform warping [7-8] or consider only doubly symmetric cross sections [9]. During the last decades, although the planar problem has been extensively studied, comparatively little work has been done towards developing a generalized three-dimensional (3D) curved beam model to study non-planar or coupled lateral-torsional responses [9-12]. In order to propose a more general formulation taking into account distortion, Arici and Granata [13] recently employed the Hamiltonian Structural Analysis Method (HSAM) for the analysis of straight or curved thin-walled structures on elastic foundation with open or closed cross section extending the so-called generalized beam theory (GBT). However, assumptions have been adopted in the evaluation of distortion in order to simplify equations and reduce couplings. To the authors' knowledge, there are no research efforts that introduce the full coupling of bending, torsion and distortion to the longitudinal analysis of curved beams with arbitrarily shaped composite cross sections ([except for the previous work of author in [1]). Usually, open or closed sections have been treated separately employing different governing differential equations.

Traditionally in practice, lateral (bracings at the upper flanges of open sections) and/or vertical stiffening systems (internal cross frames or rigid plates) are employed in order to reduce the twisting deformations (due to both torsion and distortion) of straight or curved beams during either the construction or service phase. Regarding a generalized analysis with intermediate diaphragms integrated, the number of research efforts is quite limited. In most research efforts, the distortional static analysis of structures was employed in order to give design guidelines on the placement of intermediate diaphragms of infinite stiffness (with respect to distortion). Design procedures of arranging intermediate diaphragms in curved girders have been presented and their importance in moderating distortional warping of girders has been noted [14-15]. Additionally, employing beam on elastic foundation (BEF) analogy for distortional analysis, spacing provisions of steel-plated intermediate diaphragms have been proposed [16]. More recently, straight [17] and curved [18] beam finite elements with box-shaped cross sections having nine degrees of freedom (DOF) per node have been developed in order to plot design charts that indicate an adequate maximum spacing of intermediate diaphragms. The displacement field is formulated according to a previous one proposed for doubly symmetric cross sections [19] with distortional functions derived for a mono-symmetric one. This study does not account for elastic constraints and lacks of generality due to other assumptions. However, exhibits high practical interest. Other recent research efforts, even though considering diaphragms of finite stiffness, mainly also provide design guidelines with new formulae for specific practical cases rather than a generalized theory. Particularly, a straight girder through the M/r method in [20] approximates a curved one and calculation formulae for determining the required diaphragm spacing under construction are obtained by regression analyses. Despite other simplifications and assumptions regarding the diaphragmatic action and its distribution along the beam, that study considers the influence of thickness in the additional torsional stiffness due to the diaphragms. Applying the concept of the BEF analogy for the analysis of distortional stresses of horizontally curved box-girders, a procedure capable of handling simple or continuous single cell box girders (or separated multi-cell box girders) with rigid or deformable interior diaphragms or cross-frames has been proposed [21]. Despite the design example from practice studied, only box-shaped sections have been considered and assumptions have been adopted regarding the cross sectional analysis, thus, limiting the application of the proposed method. In research efforts [22] and [23], the initial parameter method (IPM) has been employed in order to initially obtain distortional deformations and stresses for cantilever and simply supported girders, respectively. Afterwards, through parametric studies, the influence of number and thickness of diaphragms on the overall performance against distortion has been investigated. As in previous research efforts, despite the practical importance and the consideration of in-plane shear deformation of diaphragms in the evaluation of distortion (only due to torsion), the formulation is developed only for box-shaped straight beams.

Towards establishing more general/elaborate approaches for the study of diaphragmatic arrangements, a 3D solid-shell FE model has been developed in order to study the performance of straight Steel - Concrete Composite Box (SCCB) girders between 30 to 60 m lengths and a parametric study is undertaken to investigate the required number of intermediate diaphragms [24]. In this study, nonlinear inelastic analysis has been performed and the models have been verified by comparing with the experimental results previously obtained for a simply supported SCCB girder. Even though the suggested 3D FE model is highly accurate, analysis results have only been obtained for straight box-shaped girders. Additionally, a one-dimensional (1D) higher-order thin-walled beam model is employed to optimize the positions of diaphragms with specified thickness for straight or curved configurations [25]. Even though the generalized warping and distortion have been considered (meaning that bending higher-order phenomena have also been considered in addition to torsional ones), the formulation is developed for box-shaped quadrilateral homogenous straight or curved beams. However, the treatment of diaphragms' stiffness as finite and its integration to the 1D higher-order beam model constitutes a novel formulation.

In this research effort, the generalized warping and distortional problem of straight or horizontally curved composite beams of arbitrary cross section, loading and boundary conditions is presented. The 1D element developed is employed in the analysis of straight or curved bridge decks of composite thin-walled open (II-shaped) or closed (box-shaped) cross sections, taking into account shear lag (warping) and distortion due to both flexure and torsion. The coupling between tension, bending and torsion due to the curvature effect that is considered in curved geometries leads to the development of both angle of twist and displacement in the radial direction [26]. In addition to the author's most recent formulation [1], an inclined plane of curvature is considered (with respect to the horizontal plane) leading to a curvature equal to $\cos \phi / R$ aiming to establishing a more real approach. Additionally, the stiffness of plate diaphragms has been introduced in the formulation for the purpose of comparing with the case where rigid diaphragms are considered, as usually is the case for the models employed in practice. Isogeometric tools (B-Splines or NURBS) are employed in order to obtain the numerical results for the 1D formulation [1], both in the Finite

Element Method (FEM) or in a Boundary Element based Method (BEM) called Analog Equation Method (AEM) [27]. Moreover, bridges design specifications [28-30] are employed in order to arrange diaphragms and study the behavior of the created beam models. Particularly, AASHTO [28-29] specify the ratio of distortional to bending normal stresses, a limit value for transverse bending normal stress as well as the maximum spacing of the intermediate diaphragms through an approximate formula. The HEPC guidelines [30] provide also similar specifications but with different limit values and formulae. However, these design specifications do not provide the designer with any methodology and do not consider the boundary conditions, the cross section shape or any dynamic factors. Through the application of the aforementioned guidelines, for spacing of diaphragms, to the proposed 1D beam model the evaluation of these guidelines can now be performed with higher accuracy and additional factors can be considered in the analysis (beam geometry, boundary conditions etc.). This is a main scope of this research effort. However, even though the influence of a transient load on the number of diaphragms is indicated for validation reasons, the dynamic problem will be the subject of an authors' subsequent study, where other dynamic characteristics will be also considered. Up to 30 years ago, design codes for bridges usually provided guidance for the dynamic analysis of straight geometries (dynamic amplification factor, natural frequencies, modelling of vehicles, placement of diaphragms etc.). The same design recommendations used by some designers for curved bridges employing dynamic amplification factors based on the first natural frequency. However, through research efforts, the need to review previous guidelines was revealed and lead to new ones [31-34]. Much later, an extensive effort made by Hamed and Frosting [35] in order to introduce the effects of warping and distortion of bridge cross-sections. Some years ago, an analytical model is developed where the bridge is idealized as being made of panels which behave as plates in the transversal direction and as Euler-Bernoulli beams in the longitudinal direction [36]. Even though these efforts propose easy-to-apply methodologies, still rely on assumptions, thus, finally demanding more refined analysis methods. Additionally to the 1D proposed formulation, 3D FEM shell models for composite open and box-shaped cross sections are developed in this effort and analyzed for the static problem considering geometry and material nonlinearities. The commercial software ADINA [37] was employed for this purpose and parametric studies have been performed to investigate the effect of intermediate diaphragms. The goals of developing the 3D FEM models are: 1) validation as well as comparison with the proposed beam formulation, 2) suggestion of models that could be used for the simulation of real bridge decks and 3) study of various diaphragmatic arrangements as well as evaluation of the aforementioned guidelines.

The essential features and novel aspects of the present formulation compared with previous ones are summarized as follows.

I. The developed beam formulation is capable of the analysis of spatial curved beams of arbitrary composite thin-walled cross section with one, inclined or not, plane of constant curvature considering distortional effects and intermediate diaphragms to prevent them.

II. The numerical solution of advanced composite curved beam theories with intermediate diaphragms is based on NURBS (Isogeometric Analysis) offering the advantage of integrated computer aided design (CAD) in the analysis and design. Additionally, 3D FEM models with composite cross sections and diaphragms are developed employing the commercial software ADINA.

III. Both open and closed shaped cross section are studied and compared with respect to the influence of the number of diaphragms on their load carrying capacity. The magnitude of yield and ultimate displacements in the cross section's plane are compared, too.

IV. The influence of the diaphragms' plate thickness and their optimum positions are investigated through the proposed beam formulation.

V. The assessment of the design guidelines which specify the maximum spacing of intermediate diaphragms is achieved through the developed models, either beam or shell, and parametric studies in order to suggest further provisions and limitations on the application of the considered specifications.

In the following, primarily, the equations that have been altered, due to the new formulation, comparing to [1] are reported considering that the reader will refer mainly to [1], but also to [26] and [38], for the main theory behind the current formulation, which is actually the continuation of the aforementioned works of the authors.

2. Statement of the Problem (Composite Beam Formulation)

2.1 General theory

Let us consider a curved prismatic element (Fig. 1) of length L with a thick- or thin-walled arbitrarily shaped composite cross section of m homogenous, isotropic and linearly elastic materials with modulus of elasticity E_m , shear modulus G_m and Poisson ratio ν_m , occupying the region Ω_m of the yz plane (Fig. 2). Let also the boundaries of the regions Ω_m be denoted by Γ_m . This boundary curve is piecewise smooth, i.e. it may have a finite number of corners. $CXYZ$ is the principal bending coordinate system through the cross section's centroid C , while y_C, z_C are its coordinates with respect to $Sxyz$ reference coordinate system through the cross section's shear center S . It holds that $y_C = y - Y$ and $z_C = z - Z$. Some of the generalized loadings are also displayed in Fig. 1 in order to show the generalized formulation. Moment loads are indicated by m and forces by p . The initial radius of curvature, denoted by R is considered constant and it is parallel to Y axis. Additionally, even though not obvious in Fig. 1, a slight inclination of the XY plane has been considered, as previously mentioned. The displacement vector $\bar{u}(x, y, z)$ of an arbitrary point in the cross section's plane is obtained as the sum of Saint-Venant (SV) solution vector corresponding to the rigid body motion combined with a residual (index R) displacement vector due to end-effects which are responsible for the generation of self-equilibrating stress distributions (see [1] for details).

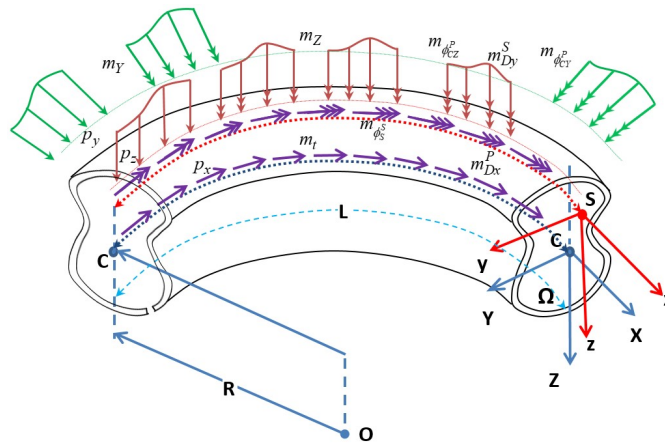


Fig. 1. Prismatic open or closed composite curved beam under axial (p_x) -flexural ($p_y, m_y, p_z, m_z, m_{\phi_{CZ}}^P, m_{\phi_{CY}}^P$ etc.) -torsional ($m_t, m_{\phi_S}^S$ etc.) -distortional loading (m_{Dx}^P, m_{Dy}^S etc.).

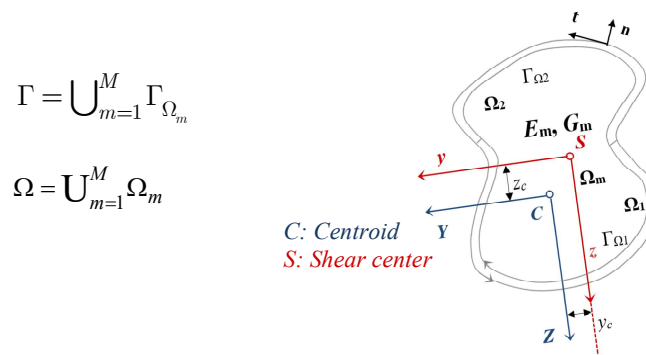


Fig. 2. Arbitrary thick- or thin-walled (with constant thickness and small inclination) composite cross section of m homogenous materials occupying the two dimensional region Ω .

After establishing the displacement field, the strain components for m th material due to end-effects can be computed as follows. It is considered that end-effects decay exponentially away from the support.

$$(\varepsilon_{xx})_m = (\bar{u}^R(x, y, z))_{,x} = \alpha_{,x}(W)_m \quad (1a)$$

$$(\varepsilon_{yy})_m = (\bar{v}^R(x, y, z))_{,y} = \alpha_{,x}(DY)_{,y} \quad (1b)$$

$$(\varepsilon_{zz})_m = (\bar{w}^R(x, y, z))_{,z} = \alpha_{,x}(DZ)_{,z} \quad (1c)$$

$$(\gamma_{xy})_m = (\gamma_{yx})_m = (\bar{v}^R(x, y, z))_{,x} + (\bar{u}^R(x, y, z))_{,y} = \alpha_{,xx}(DY)_m + \alpha(W)_{,y} \quad (1d)$$

$$(\gamma_{xz})_m = (\gamma_{zx})_m = (\bar{w}^R(x, y, z))_{,x} + (\bar{u}^R(x, y, z))_{,z} = \alpha_{,xx}(DZ)_m + \alpha(W)_{,z} \quad (1e)$$

$$(\gamma_{yz})_m = (\gamma_{zy})_m = (\bar{v}^R(x, y, z))_{,y} + (\bar{w}^R(x, y, z))_{,z} = \alpha_{,x}[(DZ)_{,y} + (DY)_{,z}] \quad (1f)$$

where $(\)_{,j}$ is for differentiation with respect to j , $\bar{u}$, $\bar{v}$, $\bar{w}$ are the axial, transverse and radial beam displacement components with respect to the $Sxyz$ system of axes, respectively, $W(y, z)$ is the warping function, $DY(y, z)$ and $DZ(y, z)$ are the distortional functions of the in-plane deformation mode $D(y, z)$ while $\alpha(x)$ is a function describing the decay of deformation along beam length, which equals to e^{-cx} (where c is a constant to be specified).

Employing the well-known stress-strain constitutive relationship for elastic media and isotropic solid, the stress components of the m th material are derived [1]. Employing local equilibrium equations of three-dimensional elasticity considering body

forces to be absent, substituting stress components and the exponential function $\alpha(x)$, the following system of partial differential equations can be derived for the m th material as

$$\left[\mu_m (\nabla^2 W)_m + c^2 (\mu_m + \lambda_m) (\nabla D)_m + c^2 (2\mu_m + \lambda_m) (W)_m \right] e^{-cx} = 0 \quad (2a)$$

$$\left[c\mu_m (\nabla^2 DY)_m + c(\mu_m + \lambda_m) (\nabla D)_{m,y} - c(\mu_m + \lambda_m) (W_{,y})_m + c^3 \mu_m (DY)_m \right] e^{-cx} = 0 \quad (2b)$$

$$\left[c\mu_m (\nabla^2 DZ)_m + c(\mu_m + \lambda_m) (\nabla D)_{m,z} - c(\mu_m + \lambda_m) (W_{,z})_m + c^3 \mu_m (DZ)_m \right] e^{-cx} = 0 \quad (2c)$$

where λ_m, μ_m are the two Lame parameters of the m th material. In case poisson ratio $\nu_m = 0$, $\lambda = 0$ and $E_m = 2\mu$. Thus, the following equations need to be satisfied

$$(\nabla^2 W)_m = c^2 \left[-\frac{2\mu_m + \lambda_m}{\mu_m} (W)_m - \frac{\mu_m + \lambda_m}{\mu_m} (\nabla D)_m \right] \quad (3a)$$

$$(\nabla^2 DY)_m + \frac{\mu_m + \lambda_m}{\mu_m} [(\nabla D)_{m,y} + (W_{,y})_m] = c^2 [-(DY)_m] \quad (3b)$$

$$(\nabla^2 DZ)_m + \frac{\mu_m + \lambda_m}{\mu_m} [(\nabla D)_{m,z} + (W_{,z})_m] = c^2 [-(DZ)_m] \quad (3c)$$

Together with the following boundary conditions

$$\begin{cases} (\sigma_{xn})_m = (\sigma_{xy})_m n_y + (\sigma_{xz})_m n_z = 0 & \text{on } (\Gamma_\Omega)_m \\ (\sigma_{xn})_m = -(\sigma_{xn})_n & \text{on } (\Gamma_\Omega)_m \cup (\Gamma_\Omega)_n \quad m \neq n \end{cases} \quad (3d)$$

$$\begin{cases} (\sigma_{yn})_m = (\sigma_{yy})_m n_y + (\sigma_{yz})_m n_z = 0 & \text{on } (\Gamma_\Omega)_m \\ (\sigma_{yn})_m = -(\sigma_{yn})_n & \text{on } (\Gamma_\Omega)_m \cup (\Gamma_\Omega)_n \quad m \neq n \end{cases} \quad (3e)$$

$$\begin{cases} (\sigma_{zn})_m = (\sigma_{zy})_m n_y + (\sigma_{zz})_m n_z = 0 & \text{on } (\Gamma_\Omega)_m \\ (\sigma_{zn})_m = -(\sigma_{zn})_n & \text{on } (\Gamma_\Omega)_m \cup (\Gamma_\Omega)_n \quad m \neq n \end{cases} \quad (3f)$$

Employing the relation $\nu_m = \lambda_m / [2(\lambda_m + \mu_m)]$ and expanding the stresses in the previous boundary conditions, the following boundary value problem is formulated (as in [1] and [38])

$$(\nabla^2 W)_m = c^2 \left[-\frac{2}{1-\nu_m^e} (W)_m - \frac{1+\nu_m^e}{1-\nu_m^e} (\nabla D)_m \right] \quad (4a)$$

$$(\nabla^2 DY)_m + \frac{1+\nu_m^e}{1-\nu_m^e} [(\nabla D)_{m,y} + (W_{,y})_m] = c^2 [-(DY)_m] \quad (4b)$$

$$(\nabla^2 DZ)_m + \frac{1+\nu_m^e}{1-\nu_m^e} [(\nabla D)_{m,z} + (W_{,z})_m] = c^2 [-(DZ)_m] \quad (4c)$$

$$\begin{cases} g_m \left\{ (DY)_{,n} \right\}_m + \left[(DY)_{,y} \right]_m n_y + (DZ)_{,y} \right]_m n_z \Big\} + g_m^* \left[(DY)_{,y} \right]_m + (DZ)_{,z} \right]_m n_y \\ = -g_m^* (W)_m n_y & \text{on free surface} \\ g_m \left\{ (DY)_{,n} \right\}_m + \left[(DY)_{,y} \right]_m n_y + (DZ)_{,y} \right]_m n_z \Big\} + g_m^* \left[(DY)_{,y} \right]_m + (DZ)_{,z} \right]_m n_y + \\ g_n \left\{ (DY)_{,n} \right\}_n + \left[(DY)_{,y} \right]_n n_y + (DZ)_{,y} \right]_n n_z \Big\} + g_n^* \left[(DY)_{,y} \right]_n + (DZ)_{,z} \right]_n n_y \\ = -(g_m^* - g_n^*) (W)_m n_y & \text{on Interfaces} \end{cases} \quad (4d)$$

$$\begin{cases} (W_{,n})_m = c^2 [-(DY)_m n_y - (DZ)_m n_z] & \text{on free surface} \\ g_m (W_{,n})_m + g_n (W_{,n})_n = c^2 (g_m - g_n) [-(DY)_m n_y - (DZ)_m n_z] & \text{on Interfaces} \end{cases} \quad (4e)$$

$$\begin{cases} g_m \left\{ (DZ_{,n})_m + [(DY_{,z})_m n_y + (DZ_{,z})_m n_z] \right\} + g_m^* [(DY_{,y})_m + (DZ_{,z})_m] n_z \\ = -g_m^* (W)_m n_z & \text{on free surface} \\ g_m \left\{ (DZ_{,n})_m + [(DY_{,z})_m n_y + (DZ_{,z})_m n_z] \right\} + g_m^* [(DY_{,y})_m + (DZ_{,z})_m] n_z + \\ g_n \left\{ (DZ_{,n})_n + [(DY_{,z})_n n_y + (DZ_{,z})_n n_z] \right\} + g_n^* [(DY_{,y})_n + (DZ_{,z})_n] n_z \\ = -(g_m^* - g_n^*) (W)_m n_z & \text{on Interfaces} \end{cases} \quad (4f)$$

where $\nu_m^e = \nu_m / (1 - \nu_m)$ is the effective Poisson ratio while $g_m = \mu_m / \mu_{ref}$, $g_m^* = \lambda_m / \mu_{ref}$ are weighted elastic constants with respect to μ_{ref} which is the shear modulus of reference material. If a plane stress assumption is employed, ν_m^e is substituted by ν_m . When $\nu_m = 0$, it holds that $g_m = E_m / E_{ref}$, $g_m^* = 0$, with E_{ref} being the elastic modulus of the reference material. Thus, the aforementioned boundary value problem is simplified. Afterwards, with a proper discretization of the cross section, the above coupled boundary value problem described by eqs. (4) will lead to the formulation of a generalized eigenvalue problem similar to $AF = c^2 BF$ where A, B are known coefficient matrices, c is the eigenvalue and $F = [W \quad DY \quad DZ]^T$ is the eigenvector of the problem. The solution of eigenvalue problem is a set of eigenvalues with the corresponding eigenvectors. The later constitute the cross sectional deformation modes employed for the distortional analysis of the composite beams.

The rigid body motions of the cross section (SV flexural and torsional warping modes) are employed for the initialization of the above stated boundary value problem. Afterwards, the number of modes (distortional and warping) that will be taken into account is specified (including also Poisson modes) and an iterative procedure is formulated in order to evaluate the secondary warping and distortional ones functions. It should be noted here that, before solving the system of equations, the right hand side (RHS) vector of the problem is re-arranged in order to bring interface quantities (due to composite cross section) to the bottom of the vector. Each functional vector F_{i+1} has to fulfil the orthogonality condition (after removing the rigid motions from the functions) with respect to the functions F_i corresponding to the previous set of modes and the functions are normalized [1].

In order to establish the displacement field of the problem, up to secondary warping as well as distortional displacements, which are actually the independent parameters, have been considered. The enriched kinematics of an arbitrary point of the beam for m th material at any time instant can, thus, be obtained. 16 DOFs have been employed in 3D space for each node of the beam, namely 6 for rigid body motions, 1 for primary warping due to torsion, 2 for primary warping due to bending, 1 for secondary warping due to torsion, 3 for primary distortion and 3 for secondary distortion (both due to torsion and bending). These activate 12 cross sectional deformation modes, namely rigid (4), primary (4) and secondary motions (4), including extension. If tertiary displacements have to be employed for accuracy reasons, the beam kinematics is enriched further and 22 DOFs have to be employed in order to describe the behavior of the beam. The additional 6 degrees account for 3 tertiary warping and 3 tertiary distortional effects, respectively. These activate 4 additional cross sectional deformation modes including extension. The enrichment of the beam kinematics can be done automatically by increasing the number of modes, which are an input value for the boundary value problem to be solved, as mentioned above. This results in the evaluation of additional cross sectional operators which will be employed in the analysis of the beam model, after establishing the strain components as it will be described in the following.

After establishing the displacement field, the linear strain-displacement relations in the system (x, y, z) can be written as follows (and these differ from those in [1])

$$\varepsilon_{xx} = \left(\bar{u}_{,x} - \frac{\bar{v}}{R} \cos \phi \right) \cdot \left(\frac{R}{R-Y} \right), \quad \varepsilon_{yy} = \bar{v}_{,y}, \quad \varepsilon_{zz} = \bar{w}_{,z} \quad (5a)$$

$$\gamma_{xy} = \left(\bar{v}_{,x} + \frac{\bar{u}}{R} \cos \phi \right) \cdot \left(\frac{R}{R-Y} \right) + \bar{u}_{,y}, \quad \gamma_{xz} = \bar{w}_{,x} \cdot \left(\frac{R}{R-Y} \right) + \bar{u}_{,z}, \quad \gamma_{yz} = \bar{w}_{,y} + \bar{v}_{,z} \quad (5b)$$

2.2 Matrix formulation for the composite beam model

Employing the expressions of the displacement components [1], the strains and stresses can be computed. Applying the principle of virtual work or any other variational principle following standard arguments in the calculus of variations, the governing differential equations for the beam in terms of the kinematical components can be derived. Thus, the local stiffness



matrix $[k_r]$ of the spatial curved beam can be evaluated after solving a system of linear equations. Finally, the matrix form of stiffness matrix is derived as follows with the aid of eq. (5)

$$[Aux1] = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y & \phi_s^P & \phi_{CY}^P & \phi_{CZ}^P & \phi_s^S & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -z & 0 & 0 & 0 & 0 & 0 & 0 & v_s^P & v_{CY}^P & v_{CZ}^P & v_s^S & v_{CY}^S & v_{CZ}^S \\ 0 & 0 & 1 & y & 0 & 0 & 0 & 0 & 0 & 0 & w_s^P & w_{CY}^P & w_{CZ}^P & w_s^S & w_{CY}^S & w_{CZ}^S \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} e(R_0) \quad (6a)$$

$$[Aux2] = \begin{bmatrix} 0 & -\frac{1}{R}e(R) & 0 & \frac{z}{R}e(R) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{R}e(R) & 0 & 0 & 0 & \frac{Z}{R}e(R) & -1 - \frac{Y}{R}e(R) & \frac{\phi_s^P}{R}e(R) & \frac{\phi_{CY}^P}{R}e(R) & \frac{\phi_{CZ}^P}{R}e(R) & \frac{\phi_s^S}{R}e(R) & \dots \\ 0 & 0 & 0 & 0 & 1 & 0 & \phi_{s,z}^P & \phi_{CY,z}^P & \phi_{CZ,z}^P & \phi_{s,z}^S & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ \dots & \frac{v_s^P}{R}e(R) & \frac{v_{CY}^P}{R}e(R) & \frac{v_{CZ}^P}{R}e(R) & \frac{v_s^S}{R}e(R) & \frac{v_{CY}^S}{R}e(R) & \frac{v_{CZ}^S}{R}e(R) & \\ v_{s,y}^P & v_{CY,y}^P & v_{CZ,y}^P & v_{s,y}^S & v_{CY,y}^S & v_{CZ,y}^S & \\ w_{s,z}^P & w_{CY,z}^P & w_{CZ,z}^P & w_{s,z}^S & w_{CY,z}^S & w_{CZ,z}^S & \\ \dots & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ w_{s,y}^P & w_{CY,y}^P & w_{CZ,y}^P & w_{s,y}^S & w_{CY,y}^S & w_{CZ,y}^S & \\ +v_{s,z}^P & +v_{CY,z}^P & +v_{CZ,z}^P & +v_{s,z}^S & +v_{CY,z}^S & +v_{CZ,z}^S & \end{bmatrix} \quad (6b)$$

$$\varepsilon = [Aux1]u_{,x} + [Aux2]u \quad (6c)$$

$$\sigma = [C][Aux1]u_{,x} + [C][Aux2]u \quad (6d)$$

$$\delta U = \int_0^L \int_{\Omega} \left(\delta u_{,x}^T [Aux1]^T + \delta u^T [Aux2]^T \right) \left([C][Aux1]u_{,x} + [C][Aux2]u \right) \frac{1}{e(R)} d\Omega dx \Rightarrow \quad (6e)$$

$$\rightarrow \xrightarrow{\text{set } k_{22}, k_{12}, k_{21}, k_{11} \text{ as in (7a) \& integrate by parts}}$$

$$\Rightarrow \delta U = \int_0^L \left(\delta u^T \{ -k_{22}u_{,xx} + [k_{12} - k_{21}]u_{,x} + k_{11}u \} \right) dx + \left[\delta u^T \{ k_{22}u_{,x} + k_{21}u \} \right]_0^L \quad (6f)$$

where $[Aux1]$, $[Aux2]$ are auxiliary matrices to express strains ε in matrix form, $e(R_0) = R / (R - Y)$, $e(R) = e(R_0)\cos\phi$, $[1/e(R)] \times d\Omega dx = dV$ is the differential volume of the curved beam for constant radius of curvature, $[C]$ is the elasticity matrix employed to derive stresses σ , δU is the virtual strain energy and k_{11} , k_{12} , k_{21} and k_{22} are 16×16 coefficient matrices containing the geometric properties for each material m of the cross section. These are calculated as in eq. (7a) and the local stiffness matrix is obtained when all k_{ij} ($i, j = 1, 2$) are compiled in one matrix (eq. (7b)).

$$\begin{aligned} k_{11} &= \int_{\Omega_m} [Aux1]^T [C][Aux1] \frac{1}{e(R)} d\Omega, & k_{12} &= \int_{\Omega_m} [Aux1]^T [C][Aux2] \frac{1}{e(R)} d\Omega \\ k_{21} &= \int_{\Omega_m} [Aux2]^T [C][Aux1] \frac{1}{e(R)} d\Omega, & k_{22} &= \int_{\Omega_m} [Aux2]^T [C][Aux2] \frac{1}{e(R)} d\Omega \end{aligned} \quad (7a)$$

$$[k_l] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \quad (7b)$$

From eq. (6e) after integrating by parts, it holds that

$$NQM = k_{22}u_{,x} + k_{21}u \quad (8)$$

where NQM is the vector of the stress resultants at the beam ends. Moreover, the external work can be derived as follows. Employing the auxiliary matrix related to coefficients of the displacement field

$$[Aux] = \begin{bmatrix} 1 & 0 & 0 & 0 & Z & -Y & \phi_s^p & \phi_{CY}^p & \phi_{CZ}^p & \phi_s^s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -z & 0 & 0 & 0 & 0 & 0 & 0 & v_s^p & v_{CY}^p & v_{CZ}^p & v_s^s & v_{CY}^s & v_{CZ}^s \\ 0 & 0 & 1 & y & 0 & 0 & 0 & 0 & 0 & 0 & w_s^p & w_{CY}^p & w_{CZ}^p & w_s^s & w_{CY}^s & w_{CZ}^s \end{bmatrix} \quad (9)$$

and, then, the expression of virtual work can be obtained. Combining eqs. (6e) with virtual work, the expression of the variational total potential energy can be evaluated and, thus, the governing differential equations of the problem can be obtained together with the boundary conditions as

$$-k_{22}u_{,xx} + [k_{12} - k_{21}]u_{,x} + k_{11}u = p \quad (10a)$$

$$\alpha_1 u + \alpha_2 NQM = \alpha_3 \quad (10b)$$

where α_i are diagonal matrices and vector containing known coefficients according to the boundary conditions of the beam (i.e. for clamped end $\alpha_1 = 1$ and $\alpha_2 = \alpha_3 = 0$). Employing the expressions of the displacement components considering more independent parameters, the cross sectional operators (eqs. (7a)) and the governing differential equations of the curved beam can be obtained in a similar way when tertiary or higher warping and distortional effects are considered.

In order to derive the differential equations of motion with respect to the kinematical components, the terms of inertia contributions $\delta W_{\text{mass}} = \int_V \rho_m (\bar{u}_{,tt} \delta \bar{u} + \bar{v}_{,tt} \delta \bar{v} + \bar{w}_{,tt} \delta \bar{w}) dV$ (with $dV = [1/e(R_0)] \times d\Omega dx$) have to be added in the previous (eqs. 6e) and constitutive equations should be employed. ρ_m is the density of the m th material and $\bar{u}, \bar{v}, \bar{w}$ are the generalized displacements as previously described. Thus, the local spatial mass coefficient matrix $[m_l]$ can finally be derived. This can be extracted in matrix form from the following expression

$$\delta W_{\text{mass}} = \int_0^L \int_{\Omega} \rho \left(\delta u^T [Aux]^T [Aux] u_{,tt} \right) \frac{1}{e(R_0)} d\Omega_m dx \Rightarrow \delta W_{\text{mass}} = \int_0^L \left(\delta u^T [m_l] u_{,tt} \right) dx \quad (11)$$

with $[Aux]$ given in eq. (9), u representing the total displacement and $[m_l]$ being a 16×16 coefficient matrix when displacements with up to secondary higher order phenomena are employed.

Except for the boundary conditions there are also the initial conditions at $x \in (0, L)$. After establishing the stiffness and mass matrices of the spatial curved beam element, the equation of motion in matrix form can be derived as in [1].

The natural frequencies and modes in which the beam vibrates for the different motions (including also distortional ones) can be obtained by separation of variables ($u_i(x, t) = u_i(x) e^{i\omega t}$, with $i = \sqrt{-1}$ and ω = natural frequency). Finally, the typical generalized eigenvalue problem described by $([k_l] - \omega^2 [m_l])u_i = \{0\}$ is formulated and can be solved.

2.3 Diaphragmatic action in the composite beam model

The deformation of a diaphragm is generally considered as insignificant comparing to that of the thin-walled part of the composite cross section and its stiffness is taken as infinite in practice. To account for an infinite value, the boundary conditions related to distortional DOFs of the beam elements are treated accordingly (zero values are assigned). When high values of diaphragmatic stiffness, but not infinite, are considered, the calculated stiffness values are added to the corresponding components (warping and distortional terms of the cross sectional operators derived in eq. (7a), particularly from 7th to 10th column and row for warping terms and 11th to 16th for distortional terms) of the composite thin-walled cross section stiffness matrix at the node where a diaphragm is placed. In the calculation of composite thin-walled beam stiffness with diaphragm, the sequence of degrees of freedom follows the previous conventions (first in sequence are the rigid body movements, warping deformations follow and finally the distortional ones). Additionally, the case of a rectangular diaphragm with finite stiffness is considered. This is thin enough that its deformations can be described using the corresponding shape functions of the cross section. The Kirchhoff-Love theory assumptions are employed to describe the displacements of a diaphragm and, thus, strains,



considering the corresponding distortional and warping independent parameters previously calculated. The stresses are then obtained by using the known strain-stress relation for the plane stress case and the diaphragmatic stiffness for warping and distortion is derived by minimizing the total potential energy. Particularly, the warping and distortional stiffness due to torsion for a rectangular diaphragm are $k_w = (Gt_D^3 db)/3$ and $k_D = (Gt_D db(24d^2 + 40db + 24b^2))/(5(d+b)^2)$, respectively (where t_D, d and b are the thickness, width and height of the diaphragm, respectively). The assignments of values to the corresponding stiffness elements of the thin-walled beam at the positions of the diaphragms are performed as previously in the case of large stiffness values.

An optimization procedure and/or trial arrangements can find the optimum positions of diaphragms. However, in this case of curved beams, user's inspection is also important. A mean compliance minimization problem is formulated considering an objective function related to displacement and force vectors that is subjected to a constraint related to the number of nodes along the length and diaphragms. According to material parameterization of topology optimization [39], the stiffness of a possible diaphragm is penalized according to a parameter which is higher or equal to unity. The problem can be solved numerically using the method of moving asymptotes.

2.4 Design guidelines

The placement of diaphragms along the length of the beam, which consists of adjacent aligned beam elements, has been done according to the following guidelines. It is assumed that there are full-depth diaphragms provided at support lines (AASHTO [28], article 6.7.4.3). Based on the study in [40], the Guide Specifications for Horizontally Curved Highway Bridges by the American Association of State Highway and Transportation Officials – AASHTO [29] specify the maximum spacing of the intermediate diaphragms L_D as

$$L_D \leq L \left(\frac{R}{200L - 7500} \right)^{1/2} \leq 25 \text{ ft} \quad (12)$$

where L and R denote the span length and radius of curvature in feet, respectively. This provision (as well as AASHTO [28], Article 6.7.4.3). meets the requirement that the distortional normal stress (it is meant due to torsion) is limited within 10% of the bending normal stress and the transverse bending normal stress is limited to 137.3 MPa or lower. Moreover, regarding transverse bending, the distortional angle can be considered as insignificant in the design of modern box-girders (AASHTO [28], Article 6.7.4.3).

In addition to this, the Hanshin Expressway Public Corporation of Japan provides the Guidelines for the Design of Horizontally Curved Girder Bridges – HEPCJ [28] specifying the maximum spacing of the intermediate diaphragms in curved box girder with respect to that in straight box girders multiplied by a reduction factor, which is equal to unity for a span length less than 60 m. In these guidelines, design criteria for the spacing of the intermediate diaphragms are specified so that the distortional normal stress (due to torsion) is limited within 5% of the bending normal stress. In addition to this, the transverse bending normal stress due to distortion should be limited to about 4.90 MPa or lower. To account for these restrictions, the guidelines specify the maximum spacing of the intermediate diaphragms in curved box girder bridges as

$$L_D = L_{DS} \kappa(\phi, L) \quad (13)$$

with

$$L_{DS} = \begin{cases} 6m & \text{for } L < 60m \\ (0.14L - 2.4)m & \text{for } 60m \leq L \leq 160m \\ 20m & \text{for } L > 160m \end{cases} \quad (14)$$

$$\kappa(\phi, L) = \begin{cases} 1 & \text{for } L < 60m \\ 1 - \frac{\sqrt{\phi}(L - 60)}{100\sqrt{2}} & \text{for } 60m \leq L \leq 200m \end{cases} \quad (15)$$

with L_{DS} , $\kappa(\phi, L)$ and ϕ being the spacing of the intermediate diaphragms in straight box girder bridges (in m), the reduction factor and the angle formed by the arc at the center of the circle that it is a part of (in rad), respectively.

Regarding the provisions of Eurocode 3 [41], the designer should consider that the effects of warping and distortion are negligibly small in all box-girder analyses. Additionally, reduction factors or special arrangements in order to prevent distortion are suggested in different parts of Eurocodes [1].

3. Numerical Solution

3.1 Composite cross sectional problem with or without diaphragms

The evaluation of the warping and distortional functions is accomplished by solving the problems described by eqs. (4). The warping functions and their derivatives are at first computed by solving eqs. (4a, e). Afterwards, these values are inserted as generalized body forces in eqs. (4b, c, d, f) which are solved as a 2D elasticity problem in order to obtain the distortional functions. The solution of the problem is accomplished employing BEM within the context of the method of subdomains and BEM for the Navier operator [1]. Afterwards, the values are normalized [1] and the procedure is repeated for the desired number of modes. Finally, the functions calculated are employed in order to obtain the cross sectional operation factors given in eqs. (7) as well as mass operation factors derived by eq. (11). These are used as input values together with the elasticity and function matrices to solve the curved beam longitudinal problem with the method described below. When intermediate diaphragms are employed, their stiffness values are added during the aforementioned procedure to the corresponding elements of the cross sectional operation factors' matrices.

3.2 Composite beam with Isogeometric tools

A brief evaluation of a current technology, application or product. Publication of a submitted review paper is determined by the editor. After the precedent cross section analysis, the static and vibration analysis of the longitudinal problem of straight or curved beams with composite arbitrary cross section including generalized warping and distortional effects reduces in establishing the components the kinematical components u_i . Those u_i have continuous derivatives up to the second order with respect to x at the interval $(0, L)$, up to the first order at $x=0, L$, and for the dynamic problem up to the second order with respect to time t , satisfying the initial-boundary value problem described by the coupled governing differential equations of equilibrium in eqs. (10a) and (11) along the beam and the boundary conditions in eqs. (10b) at the beam ends, at $x=0, L$ as well as the initial conditions. The problem is solved using the AEM as described in [1].

The global equation system is formulated and the unknowns are evaluated. The result is not the displacement vector as in traditional FEM but it is either the fictitious load of the AEM, which represents the second derivative of the kinematical components, or the control points of NURBS (when NURBS are employed in AEM). The kinematical components and their first derivatives can eventually be obtained [1]. These are employed as input values together with the function matrices in a post-processing procedure in order to derive the total displacements, stresses and stress resultants along the straight or curved geometry.

Instead of B-splines, NURBS curves in terms of B-spline basis functions can be employed either in FEM or AEM. The description of the numerical procedures is given in [1]. The geometry of the beam is described by a NURBS structure with initial control points given by the following spatial coordinates $(x_i, y_i, z_i, w_i) : (-R, 0, 0, 1), (-R \cos(\phi/2), R \sin(\phi/2), 0, \cos(\phi/2)), (-R \cos(\phi), R \sin(\phi), 0, 1)$, and the arc which describes the longitudinal axis of the curved beam is obtained in this way. w_i ($\in \epsilon$) are weights of the control polygon which defines the arc and is ϕ the angle formed by the arc at the center of the circle that it is a part of. After establishing the kinematical components and their derivatives, total displacements can directly be plotted on the curved geometry. Finally, stresses and stress resultants can also be derived without the need for excessive post-processing.

Regarding the vibration analysis, the Newmark time integration scheme, which is widely used in structural dynamics, can be employed. The equation of motion can be expressed at time $t+Dt$ and the step-by-step solution can be computed. Aforementioned procedure to the corresponding elements of cross sectional operation factors.

3.3 FEM 3D composite models

In the present study, 3D finite element models of composite straight or curved bridge decks with open or close-shaped cross sections are developed and analyzed using the commercial software ADINA [37]. ADINA system has been employed in previous research efforts for the numerical analysis of structural models with challenging aspects in the simulations giving highly accurate results [42-44].

For the models studied in this paper, 4-node shell elements have been employed in order to discretize both the thin (dimension ratios are less than 0.1) concrete slab (thickness = 0.3 m and width = 12 m) and the thin-walled steel sections of the beams (thickness = 0.03 m). The meshes exhibit element sizes from 0.02 to 0.1 m depending on the demanded accuracy (e.g. for regions suffering from bucking a more dense mesh has been used). Additionally, the diaphragms along the length of the beam or at the supports have been simulated with the aid of rigid links, which connect all the discretization points of the cross section with its centroid. Rigid links have also been accomplished for the boundary condition connecting the nodes of the cross-section with a point outside the cross-section in the middle of its height. In the case of a hinge the three translational (x, y, z) and one rotational (y) degrees of freedom have been restrained, whereas in the case of a roll two translational (x and z) degrees of freedom have been restrained.

Both linear and nonlinear analyses have been performed. Regarding the nonlinear analyses, either only geometry/material nonlinear analysis (GNA/MNA) has been considered or both geometry and material nonlinearities (GMNA) have been considered. A linear elastic isotropic material with the following properties has been employed for the C30/37 concrete slab: Density = 2500 kg/m³, Modulus of Elasticity = 32 GPa and Poisson's ratio = 0.2. Moreover, a plastic bi-linear stress-strain model with the following properties has been used for the S355 steel of the thin-walled steel cross sections: Density = 7850 kg/m³, Modulus of Elasticity = 210 GPa, Yield strength (f_y) = 355 MPa and Poisson's ratio = 0.3. In all previous nonlinear

cases the arc-length method has been employed with a collapse analysis type. The number of iterations in most cases is 200 with a 0.005 step.

The ADINA 3D models are finally employed in order to compare with the equilibrium paths obtained by the solution of the proposed 1D beam formulation and validate it.

4. Examples

The computer programs formulated as described in previous sections are employed and/or compared to FEM plate/shell elements (FEMAP [45] for the homogenous beam, ADINA for the composite beams). Either the static or dynamic problem is examined. However, the dynamic problem, as also mentioned in the Introduction, is mainly presented in order to show the importance of geometry (straight or curved beam) with respect to the placement of diaphragms. Design guidelines for specifying the maximum spacing of intermediate diaphragms have been applied to the examined models and the obtained results have been compared to the proposed models in order to assess the number of diaphragms suggested by the codes. One homogenous beam of box-shaped thin-walled cross section is initially studied mostly for validation. Afterwards, two different examples of composite open or closed thin-walled cross sections, which are usually employed in practice of bridge design and mainly suffer from warping (open) or distortion (closed), with different geometric ratios have been examined. According to thin-walled theory, the upper bounds of these ratios are $t/d < 0.1$ and $d/L < 0.1$, where t , d and L are the thickness, width and length of the straight or curved beam, respectively.

4.1 Homogenous box-shaped thin-walled beam with intermediate diaphragms

The first beam model under consideration has a rectangular box-shaped homogenous cross section 5.0×3.5 m with plate thickness 0.30 m ($E = 3 \times 10^7$ kN/m², $G = 1.5 \times 10^7$ kN/m², $R = 25.465$ m, $\rho = 2.5$ kN sec²/m⁴, $\nu = 0$, $t/d = 0.085$, $d/L = 0.087$) and an arc length of 40 m as shown in Fig. 3. The material properties have been selected in a way that Poisson ratio will not have any influence on the obtained stress results. By taking $\nu = 0$, only distortion due to torsion is considered in the formulation (and not due to flexure), as it is usually the case in most studies (a material that could be related to these properties is concrete with cracks, but this is not within the scope of this work). The beam is considered as cantilevered with an inclination of curvature plane equal to 3° and is mainly studied for comparison as well as validation reasons.

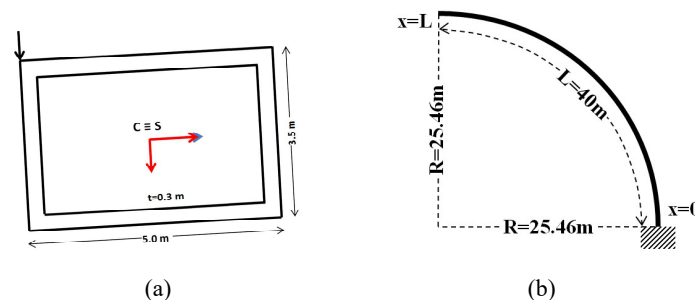


Fig. 3. Thin-walled rectangular cross section (a) and the plan view (b) of the curved beam formulation

Comparing to the results compiled in Table 2 of [1] for the static problem with a concentrated force of 5000 kN at the free end eccentrically applied (Fig. 3), it should be noted here that, regarding the value of the maximum vertical displacement at the tip, the difference with the present inclined formulation is 3% and, thus, insignificant. However, the difference for the rotation θ_x due to torsion is 7% and for the rotation θ_y due to bending is 5% .

The vibration problem is then considered to validate the proposed beam model described in section 2 (for the homogenous case only) and show the importance of the curved geometry in the arrangement of diaphragms. The beam is subjected to a transient concentrated force of 5000 kN at the free end eccentrically applied for 3 seconds. The response of the beam with respect to the vertical displacement at the free edge is shown in Fig. 4 for both straight and curved beam formulations of the same length.

Fig. 4 illustrates the difference in the dynamic responses of straight or curved beam formulations with intermediate diaphragms. It is obvious that the number of diaphragms, which are equally spaced in this case, alters only the behavior of the curved beam and, thus, should be considered in the analysis. Due to the large magnitude of cross section's dimensions, it is difficult in practice to employ a diaphragm with thickness (>1 m) that will ensure similar results to a diaphragm of infinite stiffness. Thus, a plate of thickness equal to 0.50 m is considered (even if a plate with smaller thickness leads to similar conclusions since this section is not very thin-walled). The results are compared to those obtained by a 3D shell model developed in FEMAP with diaphragms of infinite stiffness. Arising errors of the proposed 1D formulation are less than 5% for both the peak vertical displacement as well as the rotations due to torsion and the lateral displacement.

In Table 1, the maximum von mises stresses and displacements at the free edge of the curved formulation have been obtained for the static case and for three different cases of diaphragmatic stiffness, namely infinite stiffness, plate thickness equal to 0.50 m and plate thickness equal to 0.30 m. For all of the aforementioned cases the diaphragm is placed at the optimum position calculated at 34.8 m (considered at 35 m, which is at 0.875 of the length). The optimum position has been obtained through the comparison of the Von Mises stresses where their difference in values accounts for the different magnitudes of distortional stresses. It is obvious from the stress values that there is a considerable difference when infinite stiffness of the

diaphragm is considered for both maximum and minimum values. Additionally, when comparing the displacements, it can be deduced that both the warping and the distortional stiffness of the diaphragm influence the behavior of the beam since maximum displacements differ to each other in all directions including the longitudinal one, which is related to warping.

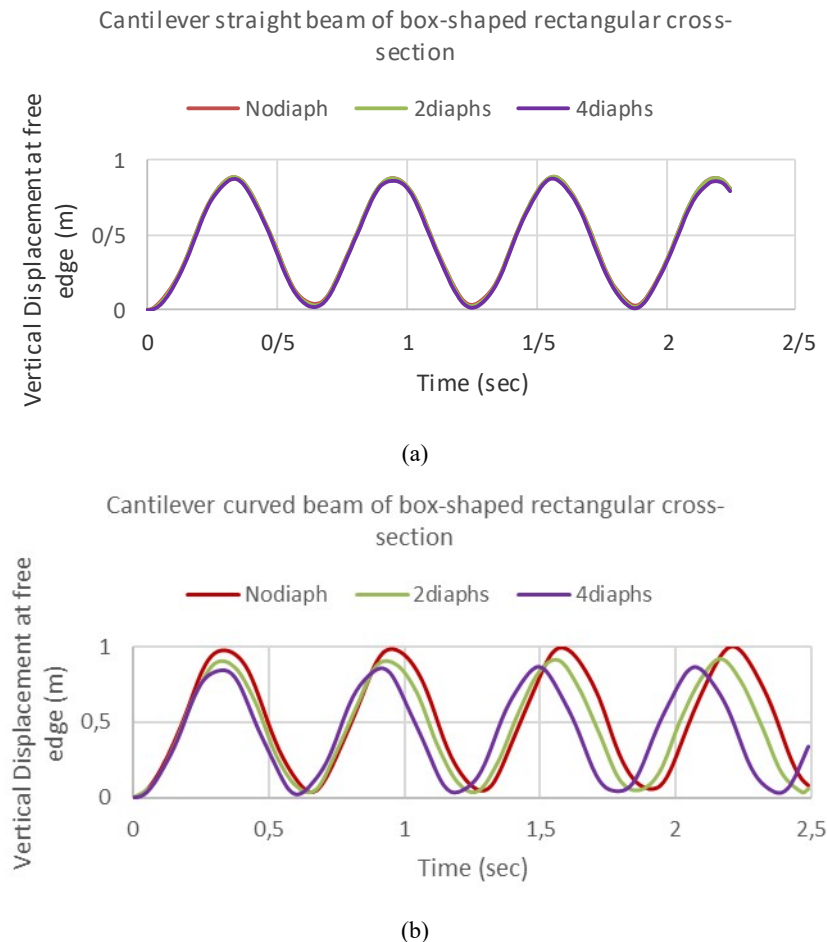


Fig. 4. Vertical displacement at free edge of a straight (a) and curved (b) beam for transient load.

According to the provisions of AASHTO [29], 6 intermediate diaphragms need to be placed along this curved beam length at equal distances in order to prevent excessive distortion. Additionally, 7 diaphragms have to be employed for the same purpose either in curved or straight beam formulation according to HEPCJ guidelines [30]. It should be noted here that according to the results significant normal σ_{zz} and σ_{yy} as well as shear stresses arise only when no diaphragms exist in the model. Only 1 diaphragm needs to be employed in order to prevent distortion instead of the 7 diaphragms required according to HEPCJ. In Fig. 4, in the case of curved configuration, kinematical components are significantly decreased when employing diaphragms. However, not that much with more than 4. In addition to this, in the case of the 2-diaphragmatic arrangement, σ_{yy} remains significant comparing to the model with 6 or 7 diaphragms (quadruple in magnitude). When employing 4 diaphragms, the values of stresses are closer to the arrangement with 7 diaphragms and much lower than the cases of 1 or 2 diaphragms. Therefore, it seems to be more cost-effective to use fewer diaphragms than those required by the provisions (while fulfilling the limits for stresses) with respect to safety.

Table 1. Von Mises Stress and displacements for different diaphragmatic stiffness for the curved beam of Fig. 3.

	$\sigma_{vm} - \sigma_{max}/\sigma_{min}$ (% difference A-B)	Max. displacements (z/y/x according to axes notation in Fig.1)
A. Infinite Diaph.	120576/1245 (5.79/10.28)	-0.6589/0.0159/0.0256
B. Thickness=0.50m	113587/1117	-0.7143/0.0189/0.0324
C. Thickness=0.30m	110591/1054	-0.7423/0.0205/0.0534

4.2 Open thin-walled composite beam with intermediate diaphragms

The second beam model under consideration has the shape shown in Fig. 5 (the steel part is the one initially considered before the final design) and is studied both as straight and curved. The employed materials are those mentioned in section 3.3 for the FEM models (Poisson ratio $\nu = 0.3$ for the steel part, $\nu = 0.2$ for the concrete part, $t/d = 0.014$, $d/L = 0.0438$, where t is the minimum thickness, d is the height and L is the length). The beam is simply supported with a length of 50 m and a radius of curvature $R = 100m$ in the case of a curved geometry without any inclination of the curvature plane ($\phi = 0^\circ$).



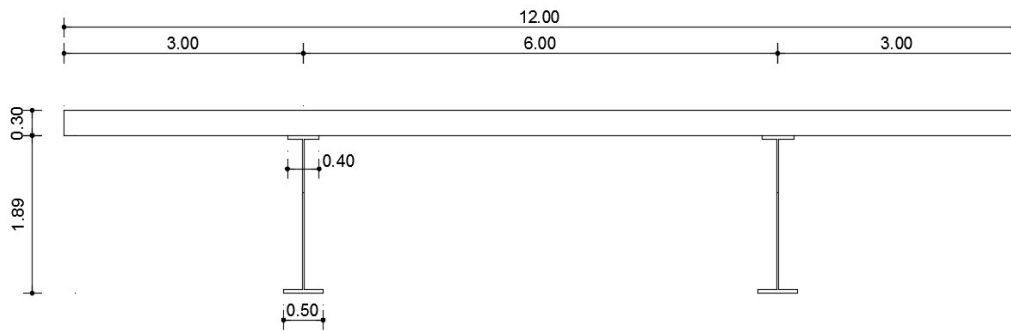


Fig. 5. Open composite thin-walled cross section for a straight or curved bridge deck.

The steel part of the composite section has been designed according to the required plastic moment of resistance and the final dimensions obtained are compiled in Table 2. This has been calculated for a linear distributed design load according to Eurocode 3: Design of steel structures- Part 2: Steel bridges ($q_{UDL} = 9 \text{ kN/m}^2$ distributed load per lane and $Q_{TS} = 300 \text{ kN}$ concentrated load per lane and per axis of a vehicle) $q_{lim} = 183.3 \text{ kN/m}$ taking into account a safety factor of 1.35. After designing the steel part, the composite cross section in Fig. 5 has been considered. The resistance load q_{rd} of the model has been defined against distortional buckling according to the relevant guidelines of Eurocode 3 and it is higher than the relevant design load (for dead loads and different positions of a moving vehicle).

Table 2. Steel cross-sectional dimensions of the open the section shown in Fig. 5.

	b (mm)	h (mm)	A (mm²)
<i>Flange 1</i>	700	60	42000
<i>Flange 2</i>	600	60	36000
<i>Web</i>	40	2500	100000
Total	-	2620	178000

Regarding the position of the vehicle load moving on the concrete slab, three cases have been studied, namely vehicle in the middle of the cross section's width (12 m), vehicle near the edge of the width and vehicle above the steel cross section. GMNA has been performed for each case and comparative equilibrium paths have been obtained. These graphs indicate that the position of load has influence only on the lateral displacement of the cross section and not the vertical one. Thus, the use of diaphragms is considered in order to decrease the magnitude of lateral displacements. In particular, cases with 1, 2, 3 or 4 equidistant intermediate diaphragms have been studied and the equilibrium paths for a straight beam are shown in Fig. 6 for both vertical and lateral displacements. In addition to the previous arrangements, cases with 2 diaphragms placed at the supports and 1, 3 or 7 equidistant intermediate diaphragms have been studied. The last 3 cases are indicated with letter A in Fig. 6. The positions of diaphragms have also been calculated using the proposed beam formulation in order to be defined as optimum. Due to the symmetry of the beam, these positions are almost equidistant. This can also be indicated by the derived contours of the von mises stresses (Fig. 7).

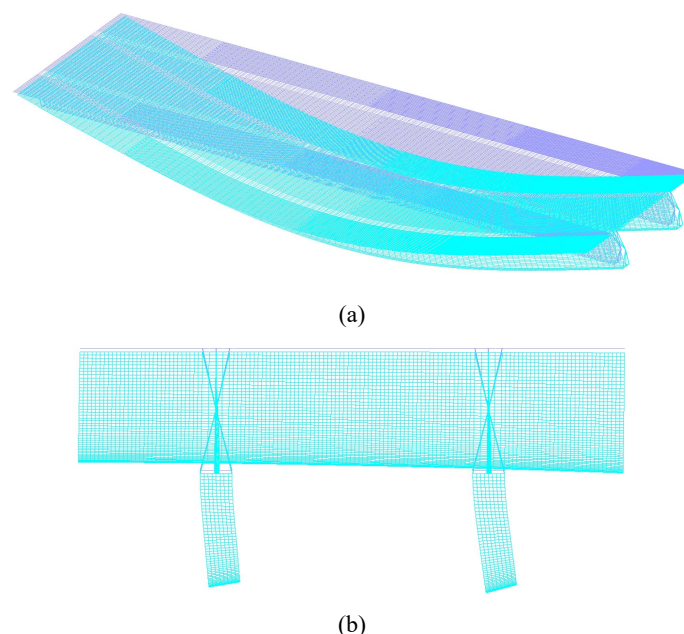


Fig. 6. Deformed shape of the straight deck for GMNA with no diaphragms, both along the length (a) and in the cross section's plane (b), as well as equilibrium paths for various diaphragmatic arrangements for the vertical (c) and lateral (d) displacements.

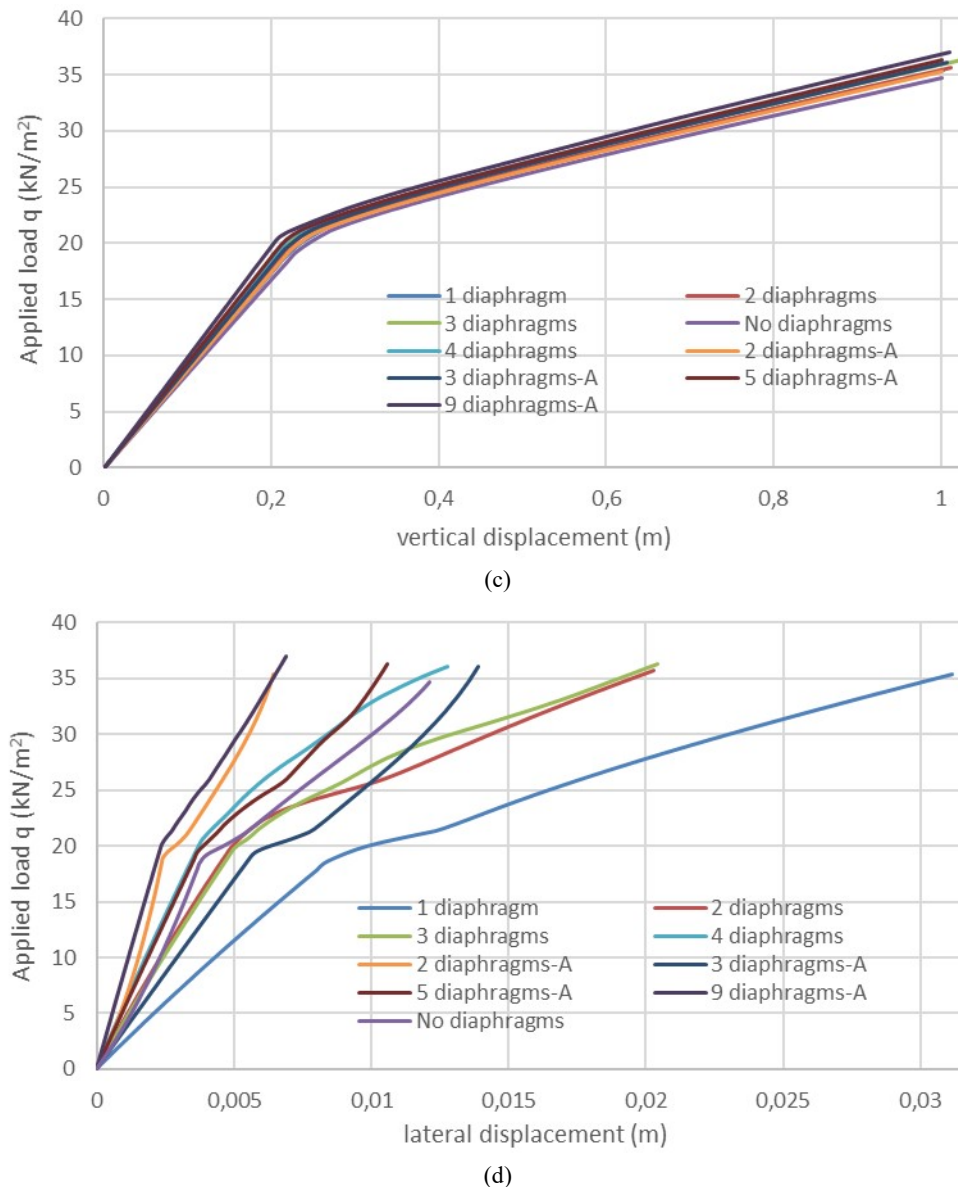


Fig. 6. Continued.

In Fig. 8, various diaphragmatic cases have been studied for the curved bridge deck for the two displacements in the cross section's plane. The cases studied are in analogy to the previous straight beam. The cases with 2 diaphragms placed at the supports are indicated with letter A, as previously.

In Fig. 9 various diaphragmatic cases have been studied for the same straight and curved bridge deck for the vertical displacement of the cross section employing the proposed beam formulation. Thus, the analyses are linear with respect to both material and geometry. As it is illustrated in Fig. 9, the placement of diaphragms is not significant for the straight beam (and lateral displacements are of very low magnitude comparing to vertical ones). Considering the curved case (where both displacements in cross section's plane are of the same magnitude), it is obvious that the placement of 3 diaphragms, two at the supports and one in the middle, reduces the vertical displacement comparing to the cases of 0 or 2 diaphragms. However, when placing 5 diaphragms, the difference becomes much more significant. On the contrary, the differences between the arrangements with 5, 7 and 9 diaphragms become much smaller indicating that 5 diaphragms are sufficient. It is worth noting here that the same conclusion can be drawn both for lateral and vertical displacements (for 5, 7 and 9 diaphragms) when observing Fig. 8 even though nonlinear analyses have been performed in order to obtain these results.

According to the provisions of AASHTO [29], 7 intermediate diaphragms need to be placed along this curved beam length at equal distances in order to prevent excessive distortion. Additionally, 9 diaphragms have to be employed for the same purpose either in curved or straight beam formulation according to HEPCJ guidelines [30]. Therefore, it seems to be more optimum to use fewer diaphragms than those required by the provisions with respect to safety and serviceability (e.g. limit of vertical displacement for a small highway bridge can be $L/200$).

In Fig. 10, the equilibrium paths are compared for both the proposed 1D beam formulation with warping/distortion effects and the FEM-shell models. The comparison graphs are only shown for the elastic region (since the proposed model is linear and elastic) with a distributed load 21kN/m². Results are compiled for the cases of 0, 5 and 9 diaphragms (similar results have been

obtain for the rest diaphragmatic cases) and exhibit small deviations, which validates the accuracy of the proposed beam formulation.

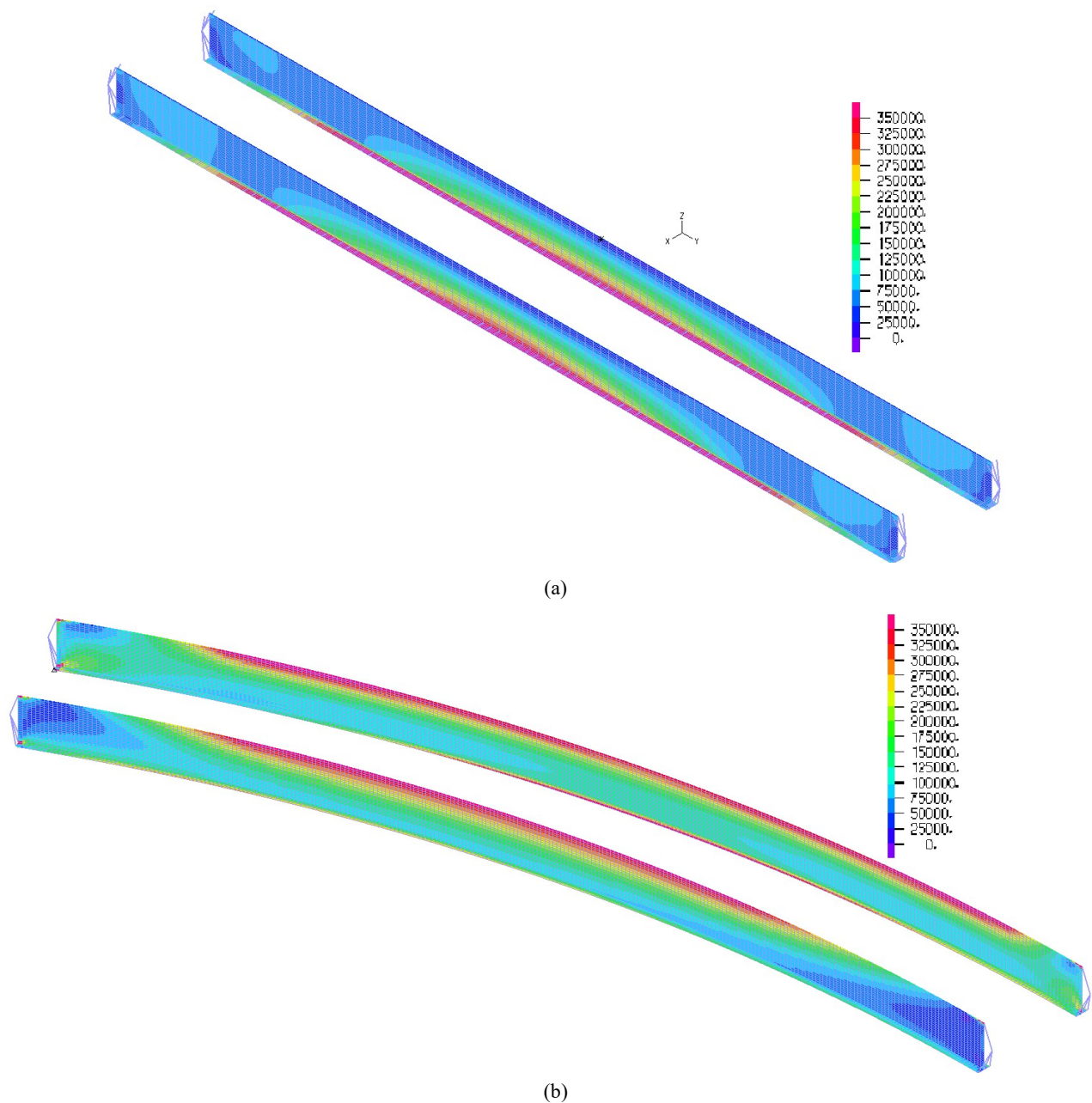


Fig. 7. Von Mises Stress (kPa) contours of the steel part of the composite cross section for the straight (a) and curved (b) beam formulation.

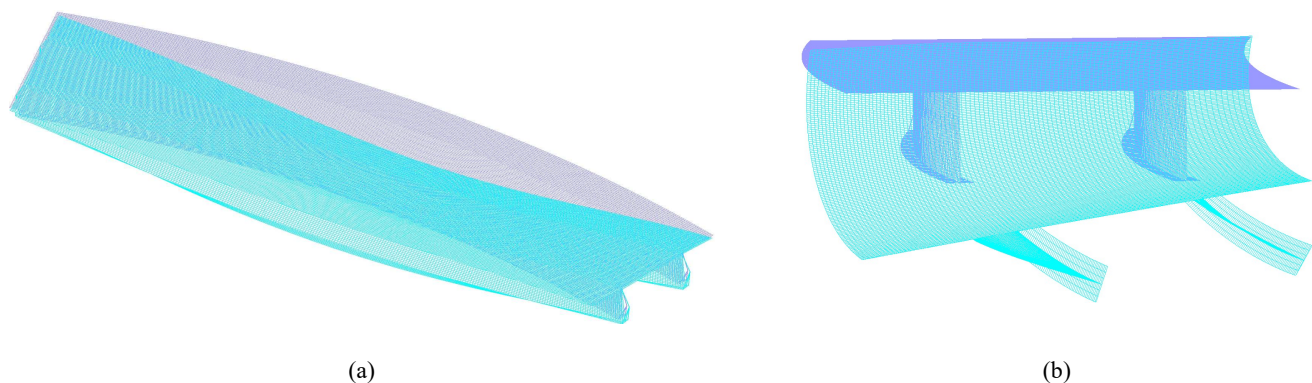
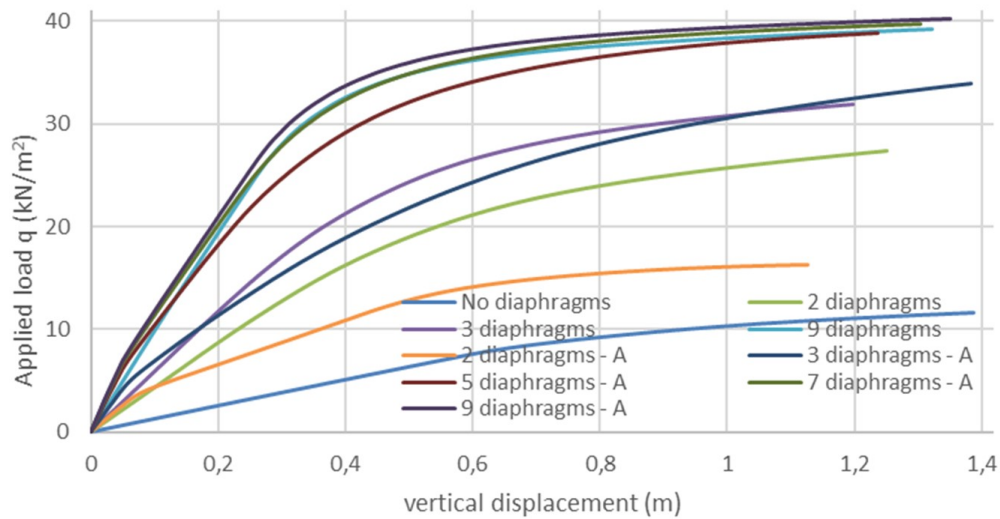
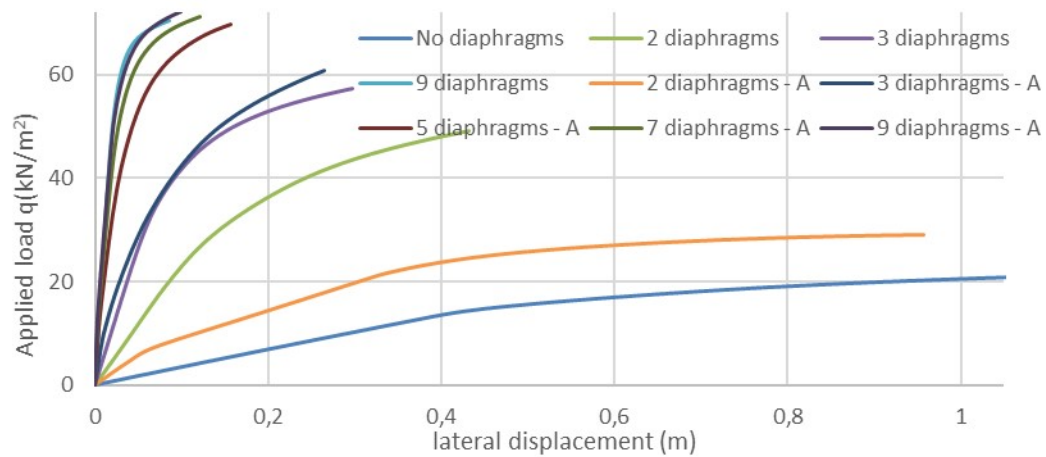


Fig. 8. Deformed shape of the curved deck for GMNA with no diaphragms, both along the length (a) and in the cross section's plane (b), as well as equilibrium paths for various diaphragmatic arrangements for the vertical (c) and lateral (d) displacements.

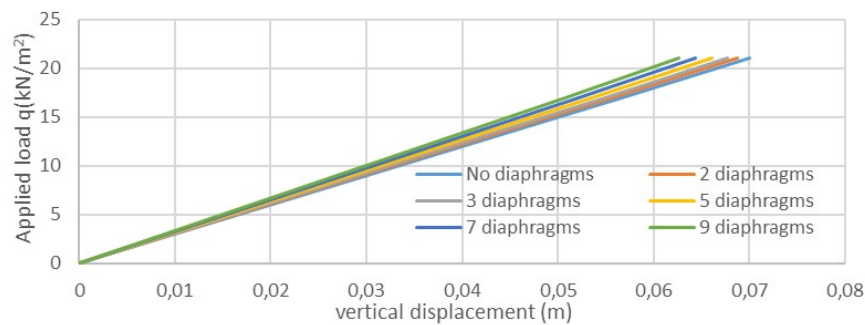


(c)

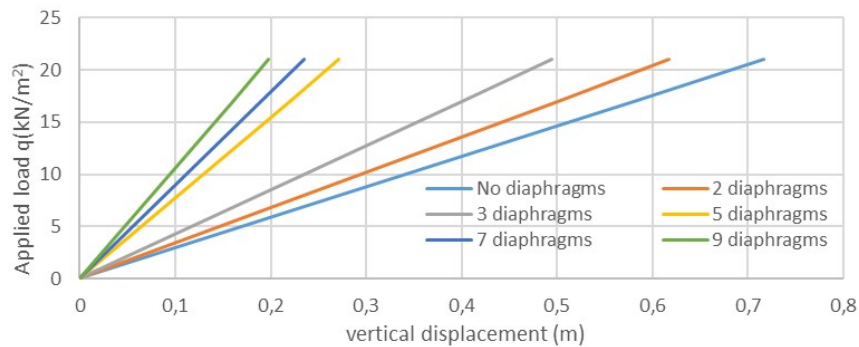


(d)

Fig. 8. Continued.



(a)



(b)

Fig. 9. Equilibrium paths for various diaphragmatic arrangements for the vertical displacement of a straight (a) and curved (b) open bridge deck formulation.

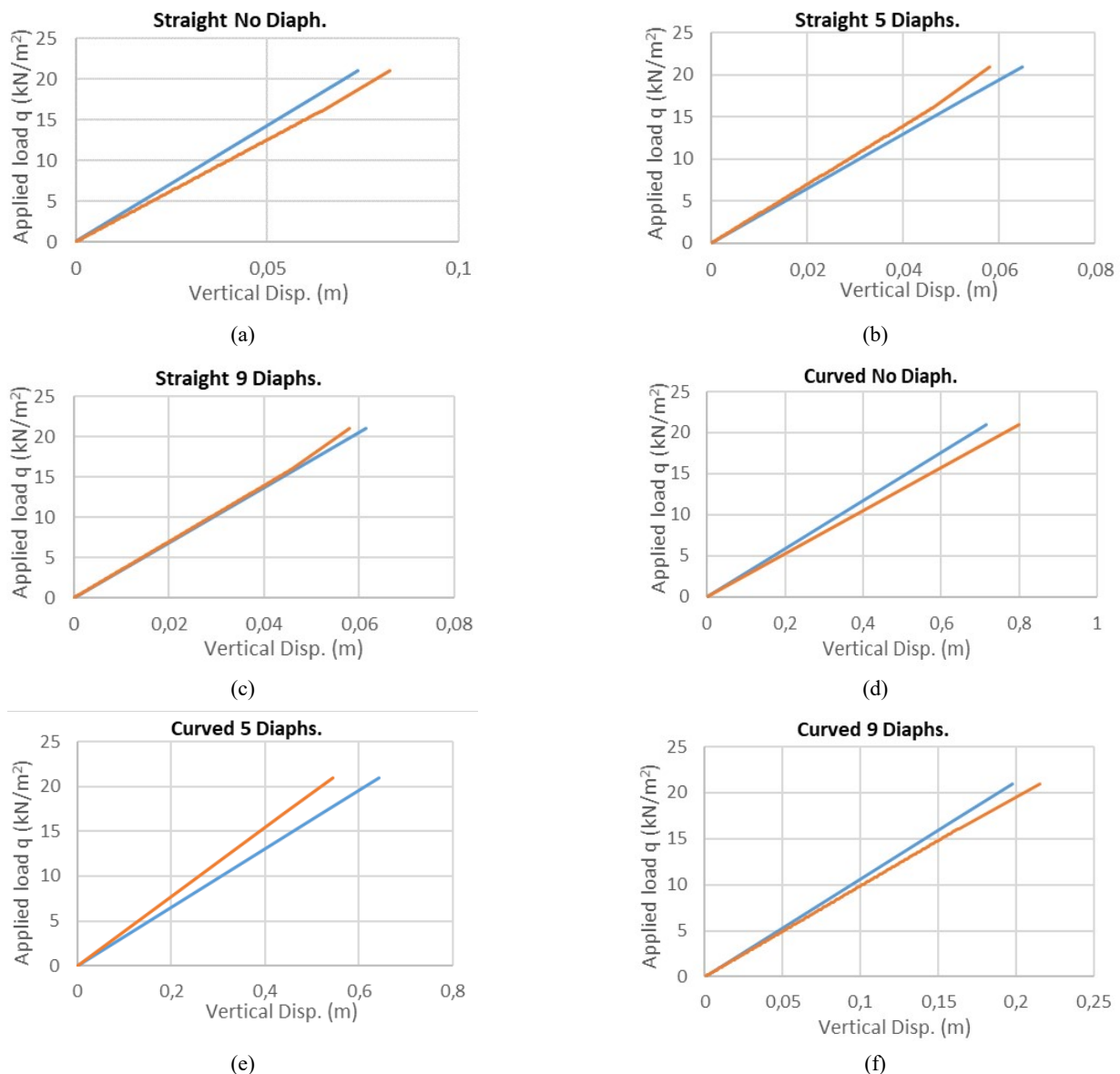


Fig. 10. Comparison of equilibrium paths for various arrangements for the vertical displacement of an open shaped bridge deck (blue line is for the proposed beam element and orange line is for the FEM-shell models): (a) straight beam without any diaphragms, (b) straight beam with 5 diaphragms, (c) straight beam with 9 diaphragms, (d) curved beam without any diaphragms, (e) curved beam with 5 diaphragms and (f) curved beam with 9 diaphragms.

In Fig. 11, the equilibrium paths have been illustrated with respect to the vertical displacement for different meshes of the FEM-shell beam model. Results are compiled for the cases of 0.1, 0.05, 0.03 and 0.02 element sizes and exhibit small deviations, especially after reducing the element size to 0.05 or smaller. It is obvious that convergence is achieved quickly by reducing the element size from 0.1 to the half of it.

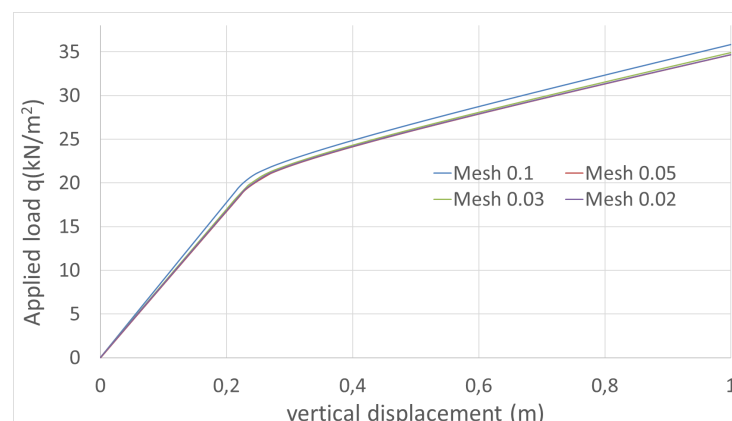


Fig. 11. Equilibrium paths for various meshes of the FEM-shell beam model with open cross section.

4.3 Closed trapezoidal thin-walled composite beam with intermediate diaphragms

The third beam model under consideration has the shape shown in Fig. 12 (the final dimensions of the steel part are compiled in Table 3 similarly to the previous example) and is studied both as straight and curved. The employed materials are those mentioned in section 3.3 for the FEM models and similar to the previous example ($t/d = 0.015$, $d/L = 0.0392$, where t is the minimum thickness, d is the height and L is the length). The beam is again simply supported with the same length, same radius of curvature in the case of a curved geometry and without any inclination of the curvature plane ($\phi = 0^\circ$).

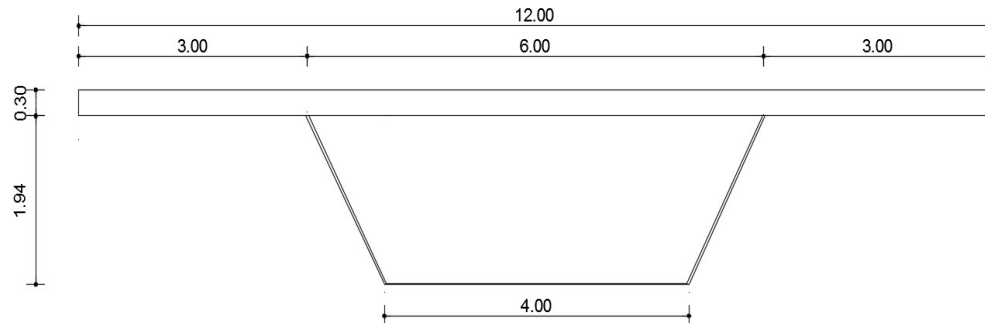
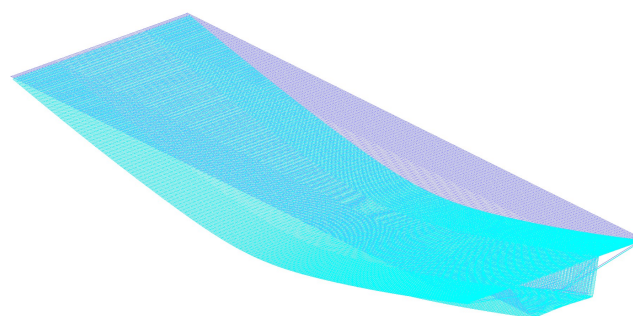


Fig. 12. Closed composite thin-walled cross section for a straight or curved bridge deck.

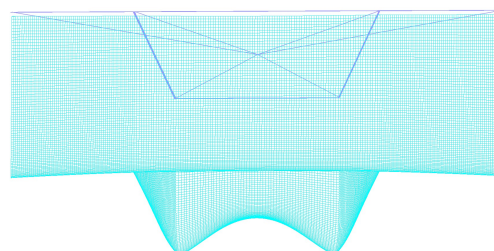
Table 3. Steel cross-sectional dimensions of the closed the section shown in Fig. 11.

	b (mm)	l (mm)	h (mm)	A (mm²)
<i>Left web</i>	30	2200	196	66000
<i>Right web</i>	30	2200	196	66000
<i>Bottom Flange</i>	4000	4000	40	120000
Total				252000

As in the previous example, regarding the position of the vehicle load moving on the concrete slab, three cases have been studied, namely vehicle in the middle of the cross section's width (12 m), vehicle near the edge of the width and vehicle above the steel cross section. GMNA has been performed for each case and comparative equilibrium paths have been obtained. The diaphragmatic cases studied, except for that of no diaphragms, for this cross section are: 2, 3, 5, 7 or 9 almost equidistant intermediate diaphragms with 2 of them being placed at the supports for each case. The equilibrium paths for a straight beam are shown in Fig. 13 for both vertical and lateral displacements. As it is expected the case of 2 diaphragms, which are actually placed at the supports, is almost identical to the case where no diaphragms have been employed. However, their placement is important in order to avoid local buckling phenomena for low values of the load factor, as it has been noticed through the results of the linearized buckling analysis, which is not within the scope of the current paper. The placement of more diaphragms is obviously decisive for the equilibrium paths under both displacements. The optimum positions of diaphragms have also been calculated using the proposed beam formulation and these are almost equidistant, as already mentioned. This can also be indicated by the derived contours of the von mises stresses (Fig. 14). It is worth noting here that the placement of more than 3 diaphragms is insignificant for the arising stresses of the primary beams of the straight bridge deck. In particular, the first yield is observed under almost the same load ($\approx 25 \text{ kN/m}^2$) for all cases.



(a)



(b)

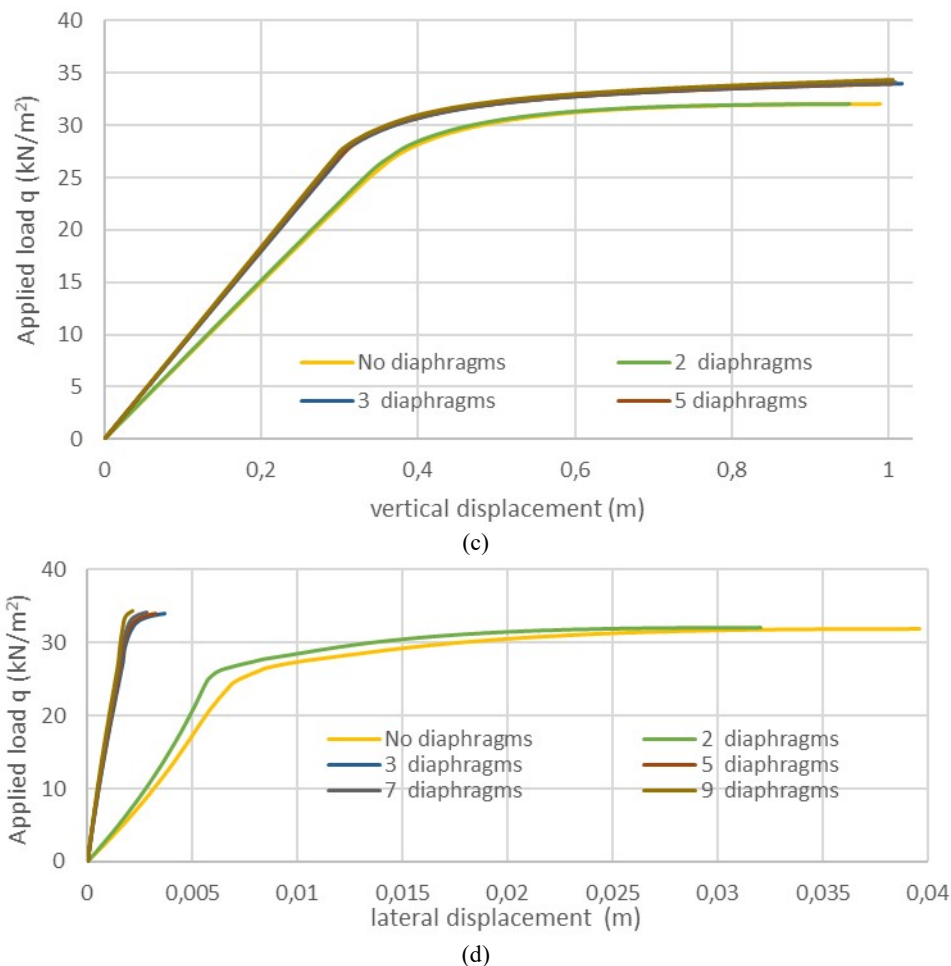
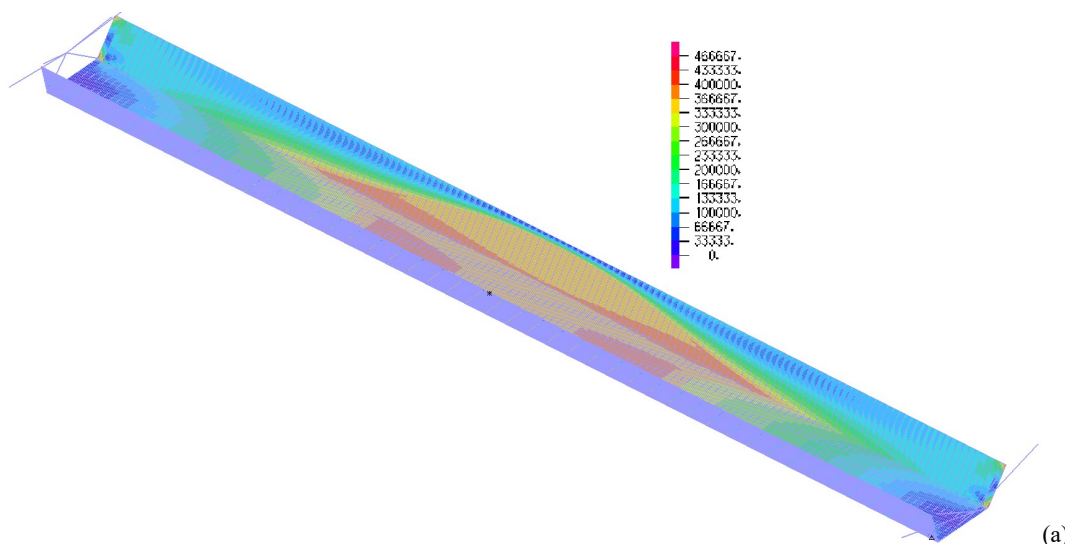


Fig. 13. Deformed shape of the curved deck for GMNA with no diaphragms, both along the length (a) and in the cross section's plane (b), as well as equilibrium paths for various diaphragmatic arrangements for the vertical (c) and lateral (d) displacements.

In Fig. 15, the same diaphragmatic cases have been studied for the curved bridge deck for both displacements in the cross section plane. The GMNA have been performed under the same conditions and loading as defined in previous cases. Comparing to the previous results for the straight bridge deck, the placement of the 2 diaphragms at the supports has a significant influence on the equilibrium paths of both displacements while the placement of intermediate diaphragms affects (insignificantly for more than 3 in total) only the lateral displacement. The curved geometry leads to a smaller yield load and to a slightly higher ultimate load comparing to the straight beam. However, the ultimate vertical displacements are obviously increased when placing more than 2 diaphragms but with much smaller deviations for more than 3 diaphragms. It should be noted here that, even though the magnitude of the lateral displacements is smaller than that of the open cross section, the closed cross section suffers mainly from distortional effects, which leads to a broader nonlinear part and a smaller linear one comparing to the open cross section, which suffers mainly from warping.



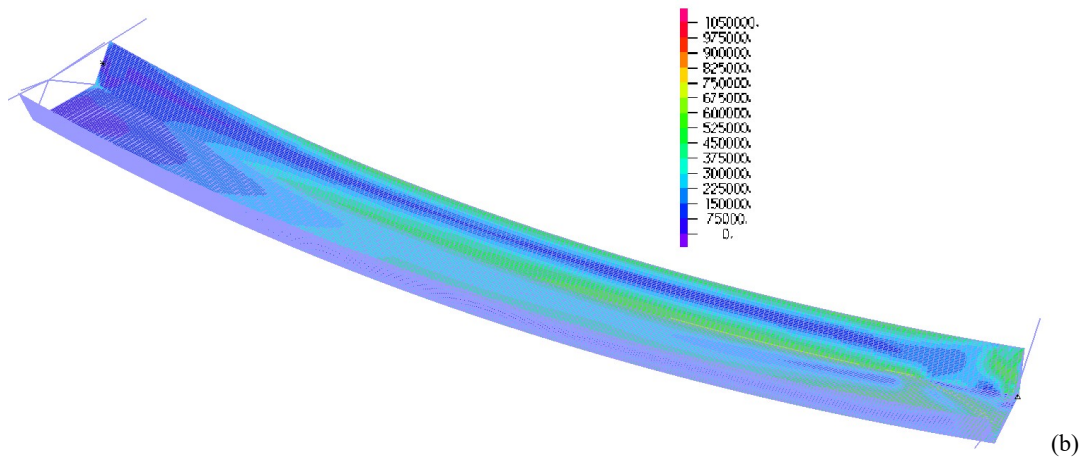


Fig. 14. Von Mises Stress (kPa) contours of the steel part of the composite cross section for the straight (a) and curved (b) beam formulation.

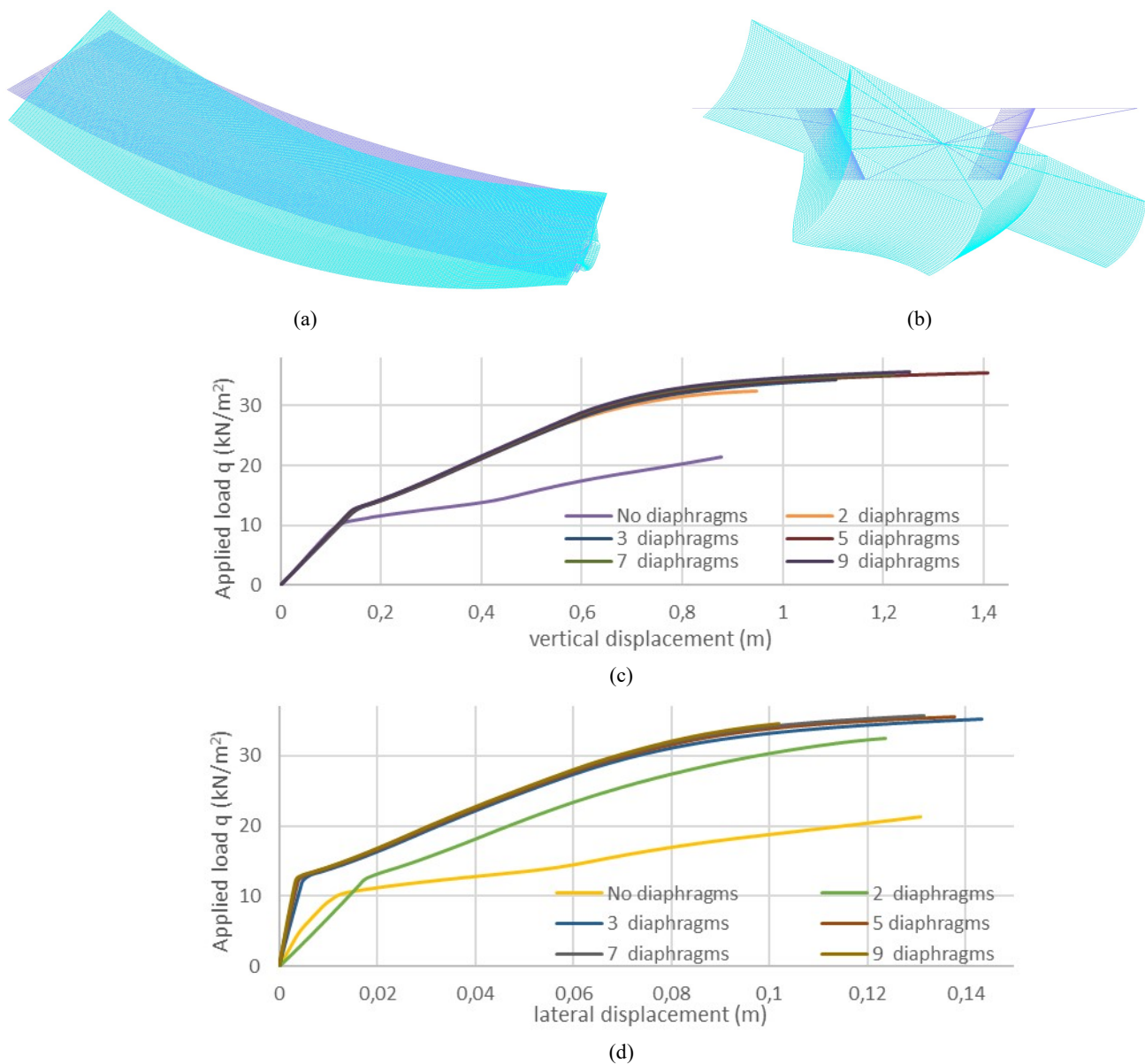


Fig. 15. Deformed shape of the curved deck for GMNA with no diaphragms, both along the length (a) and in the cross section's plane (b), as well as equilibrium paths for various diaphragmatic arrangements for the vertical (c) and lateral (d) displacements.

In Fig. 16 various diaphragmatic cases have been studied for the same straight and curved bridge deck for the vertical displacement of the cross section employing the proposed beam formulation. Thus, the analyses are linear with respect to both

material and geometry. As it is illustrated, the placement of diaphragms is only significant for the curved configuration (where vertical displacements in cross section's plane are of a higher magnitude). However, as it has also been deduced from the GMNA results, only the placement of only 2 or 3 diaphragms reduces the vertical displacement.

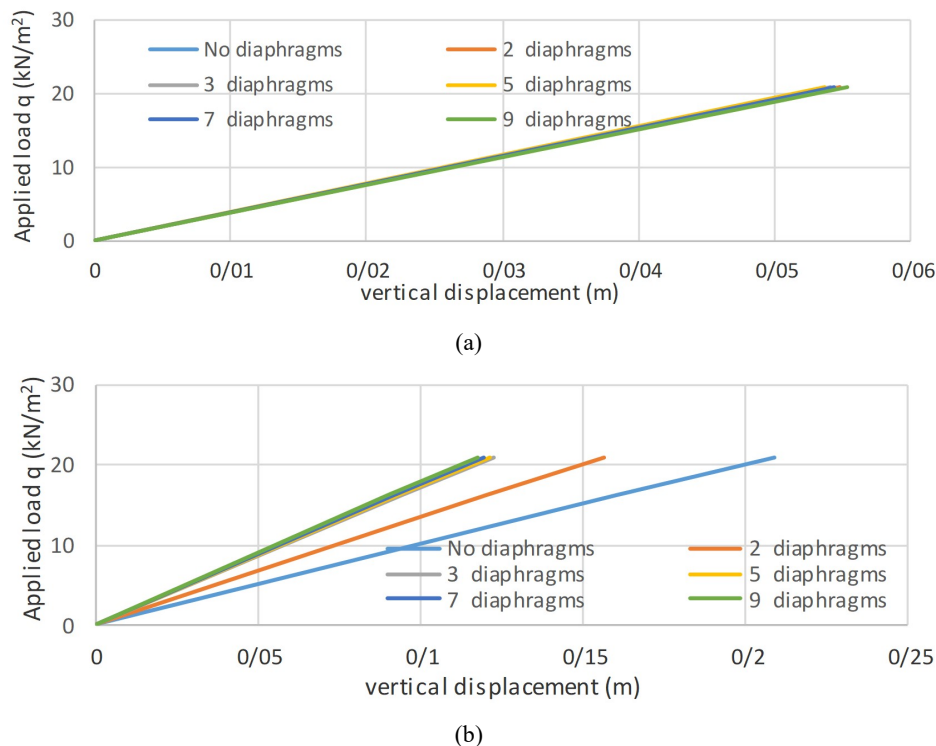


Fig. 16. Equilibrium paths for various diaphragmatic arrangements for the vertical displacement of a straight (a) and curved (b) closed bridge deck formulation.

According to the provisions of AASHTO [29] and HEPCJ guidelines [30], the same number of diaphragms is calculated as for the deck with the open cross section. When considering the ratio of distortional (due to torsion) to the bending stress, it is proved (employing the values of the proposed beam formulation) that the limitations are fulfilled for the whole bridge deck when considering 3 diaphragms along the straight deck and 5 along the curved one. These coincide with the diaphragmatic cases for GMNA that exhibit the maximum ultimate vertical displacements (more profound differences for the curved beam). Therefore, it seems to be again more optimum to use fewer diaphragms than those required by the provisions. It should be noted here that the number of diaphragms could not be accurately predicted by the equilibrium paths for this case. The reason for that is the excessive distortion (with its intrinsic nonlinear behavior) of the cross section, which changes the merits that the various structural phenomena (bending, distortion etc.) have in the magnitude of displacements and stresses. These merits can be easily assessed by the use of the proposed beam formulation, which permits the isolation of higher order phenomena in order to predict their magnitude. However, this is not an easy task in ADINA FEM software.

In Fig. 17, the equilibrium paths are compared for both the proposed beam formulation with warping/distortion effects and the FEM-shell models. The comparison graphs are only shown for the elastic region with a distributed load 21 kN/m^2 (as for the open cross section for comparison reasons). Results are compiled for the cases of 0, 5 and 9 diaphragms and exhibit small deviations, which validates the accuracy of the proposed beam formulation.

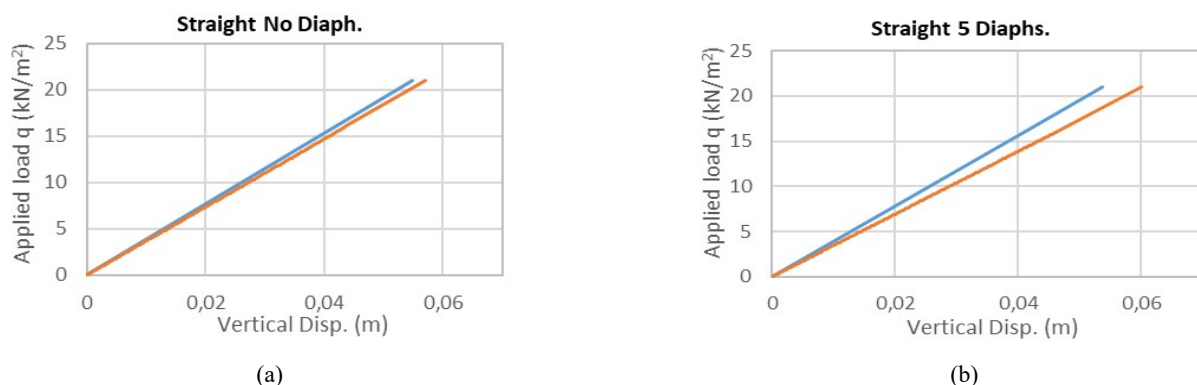


Fig. 17. Comparison of equilibrium paths for the vertical displacement of a closed shaped bridge deck (blue line is for the proposed beam element and orange line is for the FEM-shell models): (a) straight beam without any diaphragms, (b) straight beam with 5 and (c) 9 diaphragms, (d) curved beam without any diaphragms, (e) curved beam with 5 diaphragms, (f) curved beam with 9 diaphragms.

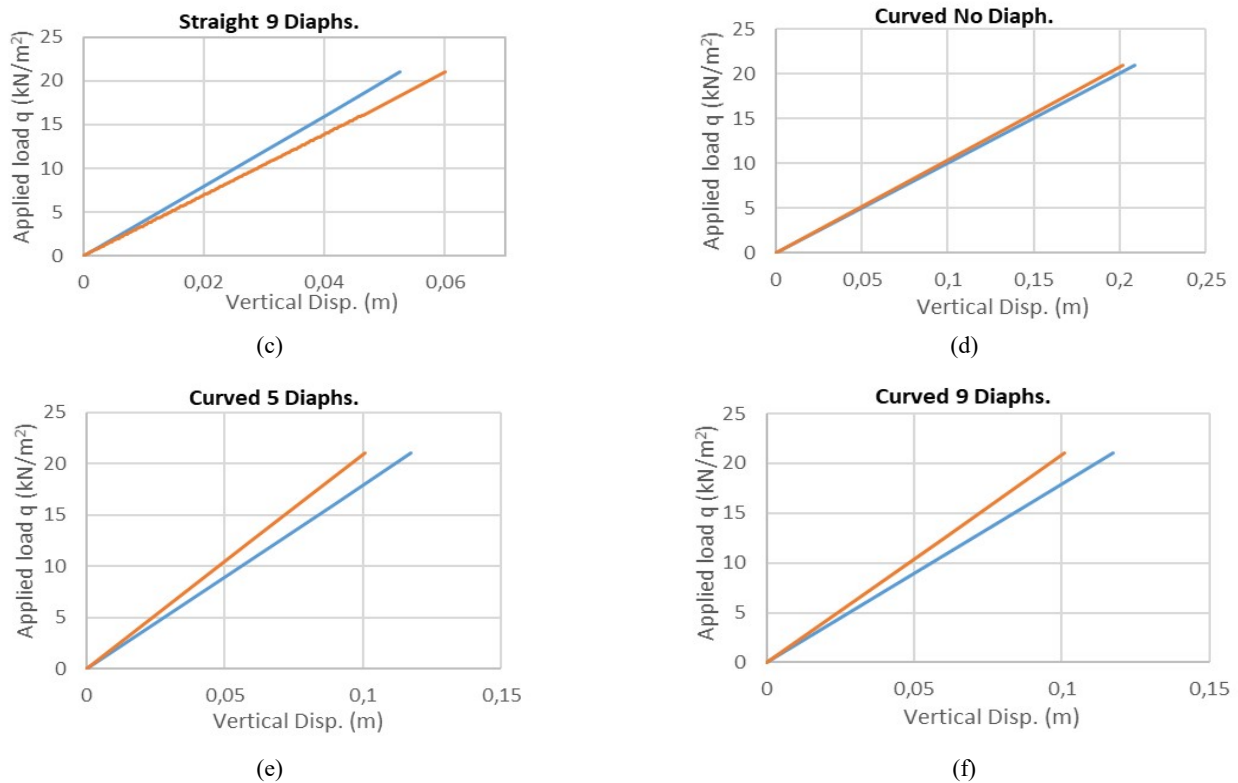


Fig. 17. Continued.

In Fig. 18, the equilibrium paths have been illustrated with respect to the vertical displacement for different meshes of the FEM-shell beam model, similarly to the previous example. Results are compiled for the cases of 0.1, 0.05, 0.03 and 0.02 element sizes and exhibit relatively small deviations, especially after reducing the element size to 0.05 or smaller. It is obvious that convergence is achieved quickly by just reducing the element size from 0.1 to the half of it. Errors follow almost the same pattern with similar magnitudes as in the previous example, for the open cross section, and convergence is achieved with the same rate.

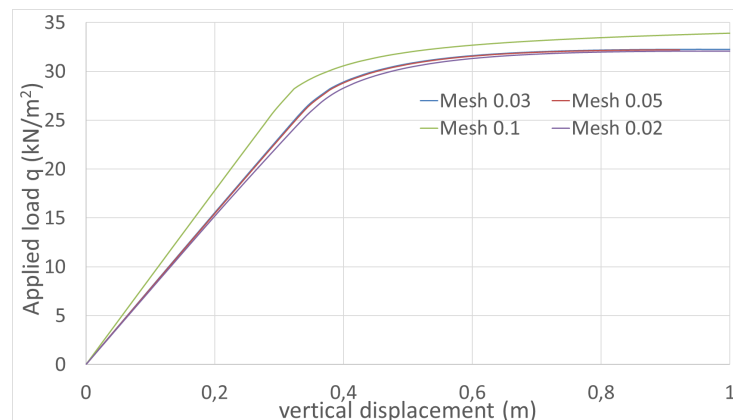


Fig. 18. Equilibrium paths for various meshes of the FEM-shell beam model with closed cross section.

5. Conclusions

The proposed formulation is employed for the static and dynamic analysis of composite thin-walled straight or curved beams together with FEM-shell models in order to highlight its importance when considering the advantages of beam models compared with 3D ones and assess the design guidelines regarding the placement of intermediate diaphragms to prevent distortion. Results have been obtained after placing intermediate diaphragms either in the beam formulation or in the shell models. The influence of diaphragms' plate thickness and optimum positions are also investigated through the proposed beam model. Comparisons have been performed to the solutions obtained according to guidelines of AASHTO and HEPCJ for both open and closed cross sections. As it has also been discussed in previous works [1, 46], warping and distortion influence beams of open or closed sections in a different way both for the static and dynamic cases. However, in this investigation more accurate numerical approaches have been employed. The main conclusions that can be drawn from this investigation are:

- i. In structures with higher thickness to width ratios, it seems that the guidelines applied in this study might give more conservative solutions in order to moderate distortional effects for both straight and curved geometries as well as both open and closed cross sections.
- ii. The specification of the maximum spacing of intermediate diaphragms should be encountered as a multi-parameter problem considering cross sectional geometry (open or closed) together with the plan view dimensions and boundary conditions.
- iii. Both the proposed beam formulation with higher order phenomena included and the nonlinear FEM models lead to the same optimum number of diaphragms both for closed and open cross sections.
- iv. The proposed beam formulation is capable of finding the optimum position of intermediate diaphragms and takes into account the thickness of a diaphragm in the warping and distortional stiffness of a cross section.
- v. In GMNA, due to the nonlinear part of the derived equilibrium paths, the ultimate displacement does not significantly change after placing more diaphragms than the optimum number obtained (the optimum number is calculated through the beam model).

The material of the beam is also important and its influence on the number of diaphragms needs to be further investigated as structures of different materials are treated differently in practice (i.e. steel or concrete bridge decks). Moreover, the dynamic case, as mentioned also earlier, will be studied in more detail as part of subsequent work of the authors. The proposed beam formulation can be suggested for a preliminary design in order to define the optimum number of diaphragms and take advantage of the proposed beam model in order to perform parametric studies and trials easily. Afterwards, more elaborate analyses can be employed (as GMNA) for the detailed design of the bridge deck. Finally, in order to improve the efficiency of the optimization involved in the presented beam formulation and create a unified framework of exact modeling, IGA and optimization, NURBS will be employed for the derivation of the sensitivity and this will advance the gradient-based method. Particularly, there will not be anymore gap among geometry, analysis and optimization as well as the convergence rate and computational cost of the gradient-based optimization will be improved, as in [47-49].

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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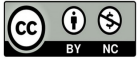
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