



Numerical Analysis of Transient Heat Transfer in Radial Porous Moving Fin with Temperature Dependent Thermal Properties

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Abstract. In this article, a time dependent partial differential equation is used to model the nonlinear boundary value problem describing heat transfer through a radial porous moving fin with rectangular profile. The study is performed by applying a numerical solver in MATLAB (*pdepe*), which is a centered finite difference scheme. The thermal conductivity and fin surface emissivity are linearly dependent on temperature while the heat transfer coefficient is given by power law function of temperature. The effects of thermo-physical parameters, such as the Peclet number, surface emissivity coefficient, power index of heat transfer coefficient, convective-conductive parameter, radiative-conductive parameter and non-dimensional ambient temperature on temperature are studied.

Keywords: Numerical analysis, Heat transfer, Thermal conductivity, Moving fin, Fin tip temperature.

1. Introduction

The study of heat transfer through fins (extended surfaces) is of immense interest in various industrial applications. An increasing number of engineering applications are concerned with energy transportation by requiring a rapid movement of heat away from a heat source. Fins are used to dissipate heat from a heated body and are seen in various applications such as heat exchangers, oil carrying pipelines and chemical processing equipment [1, 2]. Various studies have been devoted to finding efficient and effective ways of heat transportation through finned surfaces. Recent studies on porous stationary fins [3] and moving porous fins [4] have shown great improvements in the rate of heat transfer through fins when compared to solid stationary fins. Furthermore, the fin profile has been shown to have an impact on the rate of heat transfer. Ndlovu and Moitsheki [5, 6] studied the steady-state and transient behavior of fins with various profiles using analytical solutions. Mosayebidorcheh et al. [7] studied the transient heat transfer through radial fins of triangular, hyperbolic and rectangular profiles by applying hybrid analytical-numerical techniques.

In recent years, the study of heat transfer through moving fins has attracted the interests of many scholars. Roy et al. [8] studied the steady heat transfer through a triangular moving fin using a decomposition analytical solution. Turkyilmazoglu [9] further studied the heat transfer through exponential moving fins with internal heat generation. The temperature distribution profile was compared to that of a rectangular profile. Singla and Das [10] studied an inverse problem to estimate the speed of a moving fin using the binary-coded generic algorithm. Dogonchi and Ganji [11] applied differential transformation to study the simultaneous convection-radiation heat transfer through a moving fin with heat generation. Their results showed that the temperature inside the fin decreases with an increase on the speed of the moving fin. To the best knowledge of the author, the transient behavior of moving fins has been only studied by Ndlovu and Moitsheki [12]. Numerous studies in literature focused on the steady-state fin problems.



In this work, we study the heat transfer through a radial porous moving fin with temperature dependent thermal properties. The mathematical model is a highly nonlinear second order partial differential equation which is solved numerically using a built-in numerical solver in MATLAB. Numerical schemes have been used in various researches to benchmark the analytical results employed in the study of heat transfer [13, 14]. Moitsheki and Harley [15] applied classical Lie point symmetries to study the heat transfer through fins of various profiles. However, the resulting boundary value problem was not invariant under any Lie point symmetry and the built-in numerical solver in MATLAB was applied to study the problem. This paper is a great improvement of many studies which consider steady-state fin problems with the assumption that the transient thermal behavior dies out quickly [16, 17]. The current research paper considers a highly nonlinear boundary value problem including the porosity parameter, moving fin parameter, temperature dependent thermal conductivity, temperature dependent heat transfer coefficient, temperature dependent surface emissivity, radiation and convection parameters, all in a single model. The problems considering such nonlinearities are of fundamental importance in the thermal processing of materials [18, 19].

This article is organized as follows; the mathematical background of the problem under consideration is described in Section 2. A brief discussion on the fundamentals of the numerical solver is introduced in Section 3. The author provides numerical results and discussions in Section 4. Lastly, concluding remarks are presented in Section 5.

2. Mathematical Model

The model formulation considers a one-dimensional radial porous moving fin of cross-sectional area A_c while it moves horizontally with a constant velocity U as depicted in Fig. 1. The fin is attached to a base surface of temperature T_b and extends into an ambient fluid of temperature T_a . The perimeter of the fin is denoted by P and the length of fin by $L = r_b - r_i$. We assume that at the tip of the fin, $r_i = 0$ and the fin thickness at the prime surface is given by δ_b . The fluid surrounding the moving fin is heated by the convective-radiative fin and induces a horizontal flow field (advection). The porous medium is assumed to be isotropic, homogeneous and saturated with a single-phase fluid. The fluid and solid matrix are not at local thermal equilibrium and the fluid-solid interaction in the porous medium is stimulated by Darcy's model. The energy balance equation for the moving porous fin losing heat by simultaneous convection and radiation can be expressed as, (see also, [1, 12]):

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left(RK(T) \frac{\partial T}{\partial R} \right) - \frac{P}{A_c} H(T)(T - T_a) - \frac{mc}{A_c} (T - T_a) - \frac{\varepsilon(T)\sigma P}{A_c} (T^4 - T_a^4) - \rho c U \frac{\partial T}{\partial R}, r_b \leq R \leq r_i. \quad (1)$$

The mass flow rate of the fluid passing through the porous media is given by [3]:

$$\dot{m} = \rho v_w W \Delta R. \quad (2)$$

The velocity of the fluid passing through the porous media is given by Darcy's model,

$$v_w = \frac{gKA}{g} (T - T_a). \quad (3)$$

For most materials, the thermal conductivity may be given by a linear function of temperature, that is,

$$K(T) = k_c [1 + \gamma (T - T_a)], \quad (4)$$

the surface emissivity is also assumed to vary linearly with temperature,

$$\varepsilon(T) = \varepsilon_a [1 + \mu (T - T_a)], \quad (5)$$

and the heat transfer coefficient may be given by a power law function of temperature,

$$H(T) = h_b \left(\frac{T - T_a}{T_b - T_a} \right)^n \quad (6)$$

The fin tip is assumed to be adiabatic (insulated) and the base temperature is kept constant. The boundary conditions are then given by [1],

$$T(t, R) = T_b \quad (7)$$

and

$$\frac{\partial T}{\partial R} \Big|_{R=0} = 0. \quad (8)$$

Initially, the fin is kept at the ambient temperature [6, 7],

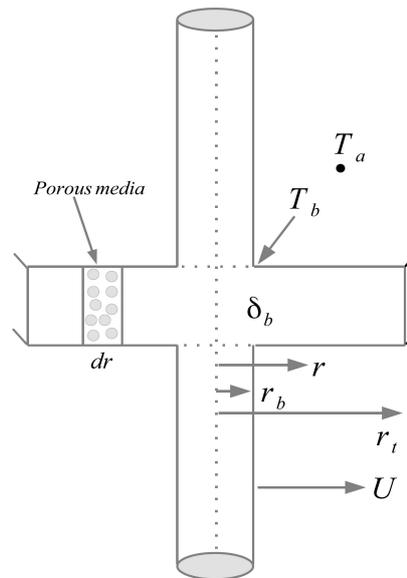


Fig. 1. Schematic representation of a radial fin.

$$T(0, R) = 0. \quad (9)$$

Here, r_b and r_t are the radii of the fin base and fin tip, respectively. The thermal conductivity of the fin at ambient temperature is k_c , γ is a measure of thermal conductivity variation with temperature, μ is a measure of surface emissivity variation with temperature, ρ is the density of fluid, c is the specific heat of fluid, ε and σ are the emissivity and Boltzman constant respectively, h_b is the heat transfer coefficient at the fin base and n is a constant. In most practical applications, the convection coefficient n lies between -3 and 3 [20]. The exponent n represents nucleate boiling when $n = 2$, radiation when $n = 3$ and $n = 0$ implies a constant heat transfer coefficient. For the porous media formulation, g is the gravitational acceleration, K is the permeability, ϑ and A are the kinematic viscosity and thermal expansion coefficient respectively. Introducing the following dimensionless variables,

$$\begin{aligned} r &= \frac{R}{L}, \tau = \frac{k_c t}{\rho c L^2}, \theta = \frac{T}{T_b}, \beta = \gamma T_b, \zeta = \mu T_b, k = \frac{K}{k_a}, h = \frac{H}{h_b}, \alpha = \frac{k_c}{\rho c}, \\ N_c &= \frac{Ph_b T_a^n L^2}{k_c A_c (T_b - T_a)^n}, N_r = \frac{\varepsilon_a \sigma PL^2 T_b^3}{k_c A_c}, N_p = \frac{\rho c g K A W L^2 T_b}{\vartheta k_c A_c}, Pe = \frac{UL}{\alpha}, \end{aligned} \quad (10)$$

reduces Eq. (1) to,

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left[r \{1 + \beta(\theta - \theta_a)\} \frac{\partial \theta}{\partial r} \right] - N_c (\theta - \theta_a)^{n+1} - N_p (\theta - \theta_a)^2 - N_r [1 + \zeta(\theta - \theta_a)] (\theta^4 - \theta_a^4) - Pe \frac{\partial \theta}{\partial r}, 0 \leq r \leq 1, \quad (11)$$

and Eq. (11) admits the following boundary conditions,

$$\theta(\tau, 1) = 1, \quad (12)$$

$$\frac{\partial \theta}{\partial r} \Big|_{r=0} = 0, \quad (13)$$

and the initial condition becomes,

$$\theta(0, r) = 1. \quad (14)$$

The dimensionless variable θ represents the temperature, θ_a denotes the dimensionless ambient temperature, r is the dimensionless space variable, τ is the dimensionless time variable, β is the thermal conductivity gradient, ζ is the surface emissivity gradient, N_c is the convection-conduction parameter, N_r is the radiation-conduction parameter, N_p is the porosity parameter, Pe is the Peclet number which represents the dimensionless speed of the moving fin and α is the thermal diffusivity of the fin.

3. Fundamentals of the Numerical Solver

MATLAB (*pdepe*) specifies the partial differential Eq. (11) in the following form,

$$c(r, t, \theta, \theta_r) \theta_t = r^{-m} \frac{\partial}{\partial r} (r^m b(r, t, \theta, \theta_r)) + s(r, t, \theta, \theta_r) \quad (15)$$

with boundary conditions,

$$p(r_l, t, \theta) + q(r_l, t) \cdot b(r_l, t, \theta, \theta_r) = 0 \quad (16)$$

$$p(r_r, t, \theta) + q(r_r, t) \cdot b(r_r, t, \theta, \theta_r) = 0 \quad (17)$$

where r_l represents the left endpoint of the boundary and r_r represents the right endpoint of the boundary, and initial condition,

$$\theta(0, r) = f(r). \quad (18)$$

Based on the problem under consideration, (Eq. (11)), we have,

$$c(r, t, \theta, \theta_r) = 1 \quad (19)$$

$$f(r, t, \theta, \theta_r) = \{1 + \beta(\theta - \theta_a)\} \frac{\partial \theta}{\partial r} \quad (20)$$

$$s(r, t, \theta, \theta_r) = -N_c (\theta - \theta_a)^{n+1} - N_p (\theta - \theta_a)^2 - N_r [1 + \zeta(\theta - \theta_a)] (\theta^4 - \theta_a^4) - Pe \frac{\partial \theta}{\partial r}. \quad (21)$$

The boundary conditions are given as follows,

$$p(0, t, \theta) = 0 \quad (22)$$

$$p(1, t, \theta) = \theta - 1 \quad (23)$$

$$q(0, t, \theta) = 1 \quad (24)$$

$$q(1, t, \theta) = 0, \quad (25)$$

and the initial condition becomes,

$$f(r) = 0. \quad (26)$$

The code below executes the numerical solution for the problem under consideration,

```
function pdesolver()
m = 1;
x = linspace(0,1,20);
t = linspace(0,0.6,20);
% Define analytical parameter values here
solution = pdepe(m,@pdex1pde,@pdex1ic,@pdex1bc,x,t,[],b,A,S,n,G,R,Z,P);
% Model specification function
function [c,f,s] = pdex1pde(x,t,u,DuDx,b,A,S,n,G,R,Z,P)
c = 1;
f = (1+b*(u-A))*DuDx;
s = -S*(u-A)^(n+1)-G*(u-A)^2-R*(1+Z*(u-A))*(u^4-A^4)-P*DuDx;
% Defining the initial condition
function u0 = pdex1ic(x,b,A,S,n,G,R,Z,P)
u0 = 0;
% Defining the boundary conditions
function [pl,ql,pr,qr] = pdex1bc(xl,ul,xr,ur,t,b,A,S,n,G,R,Z,P)
pl = 0;
ql = 1;
pr = ur-1;
qr = 0;
```

The variables on the above code represent the model parameter values. Unless stated otherwise, the following values are used in generating the numerical results; $u = \theta$, $b = \beta = 1$, $A = \theta_a = 0.50$, $S = N_c = 0.75$, $n = 2$, $G = N_p = 0.50$, $R = N_r = 4$, $Z = \zeta = 0.75$, $P = Pe = 3$.

4. Results and Discussion

In this section, we provide the numerical analysis of the model covering all embedding parameters. Some interesting results are observed, and we explain the physics behind the effects of varying the thermo-physical parameters. Figure 2 shows the temperature distribution in a radial fin for varying time. We observe that the temperature inside the fins increases as the time increases and eventually reaches steady state. Figure 2 also shows a benchmark analysis of validating the numerical scheme against analytical solutions generated by two-dimensional differential transform method (DTM). We observe that the numerical and analytical solutions are in agreement. Figure 3 depicts the effect of the speed of moving fin on the temperature distribution. As Pe increases, (the speed of the moving fin increases), the temperature inside the fin decreases rapidly due to increased advective effect on the surface of the fin. We also observe that for $Pe = 0$, i.e. stationary fin, the cooling of the fin takes more time as shown by higher temperatures when compared to a moving fin. In Figure 4, we observe that for $n \geq 3$ the temperature inside the fin does not change significantly when other parameters are kept constant. For $n = 3$, the radiation becomes more prominent and any further increase in the convection exponent n has no influence on the fin temperature. This is a result of radiation being modeled by an additional term N_r .

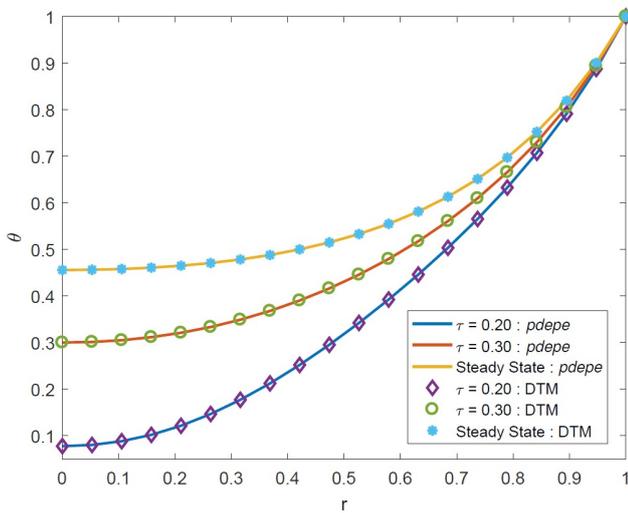


Fig. 2. Temperature distribution along a radial porous moving fin for varying time.

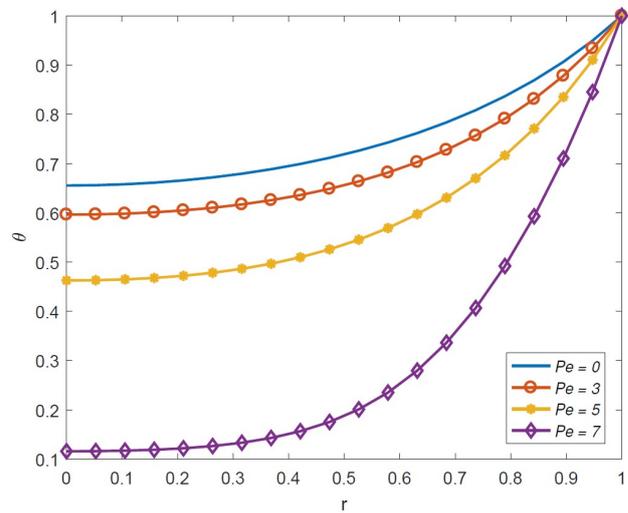


Fig. 3. Temperature distribution along a radial porous moving fin for varying fin velocity.

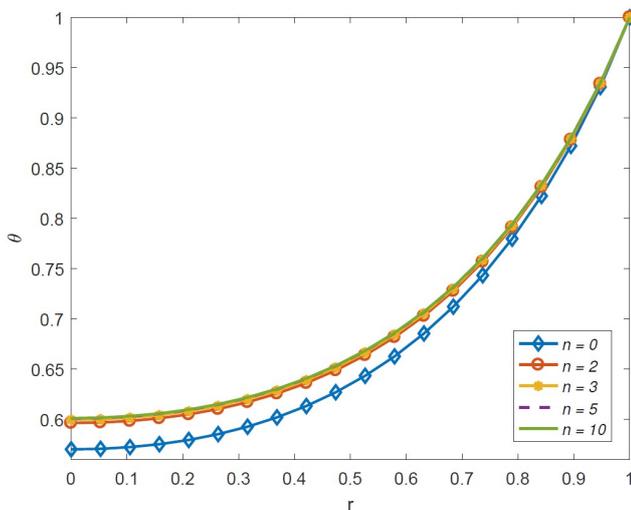


Fig. 4. Effect of varying the power of convection-conduction term on temperature distribution.

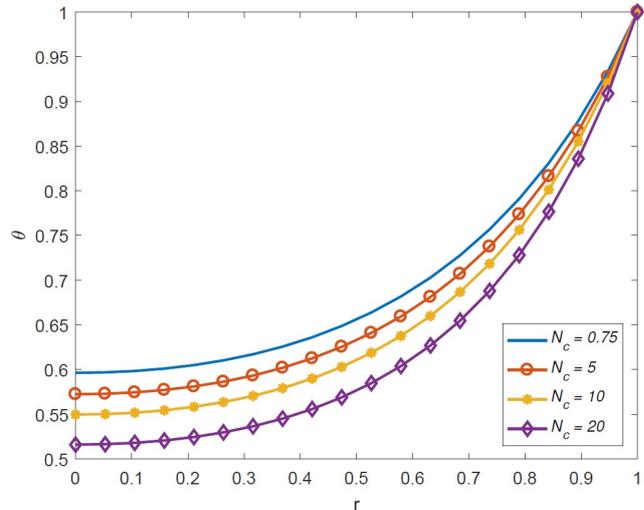


Fig. 5. Effect of varying the convection-conduction parameter on temperature distribution.



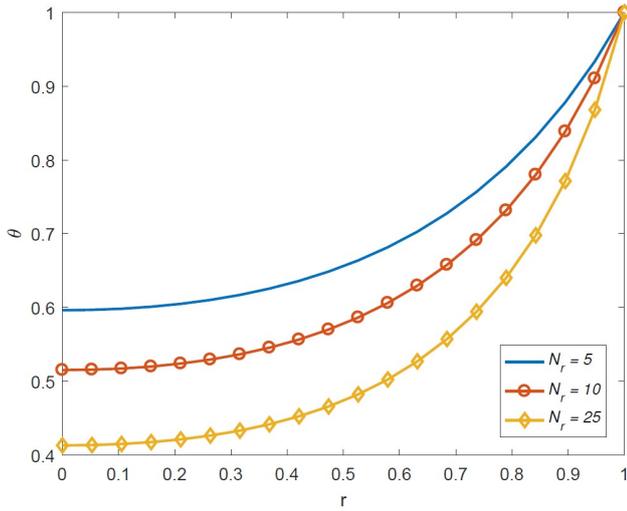


Fig. 6. Effect of varying the radiation-conduction parameter on temperature distribution.

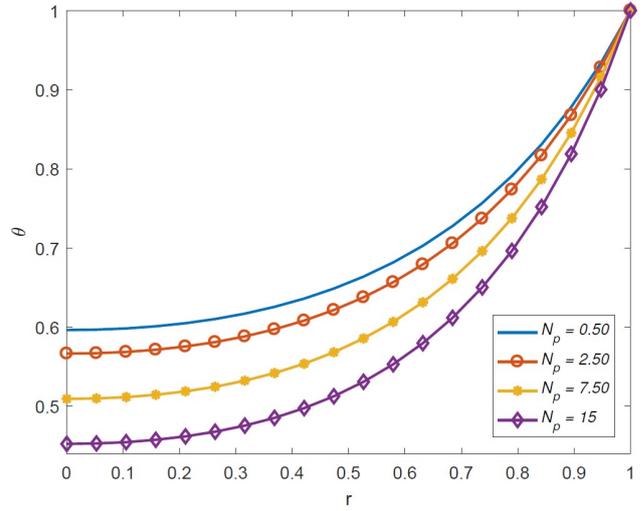


Fig. 7. Effect of varying the porous media parameter on temperature distribution.

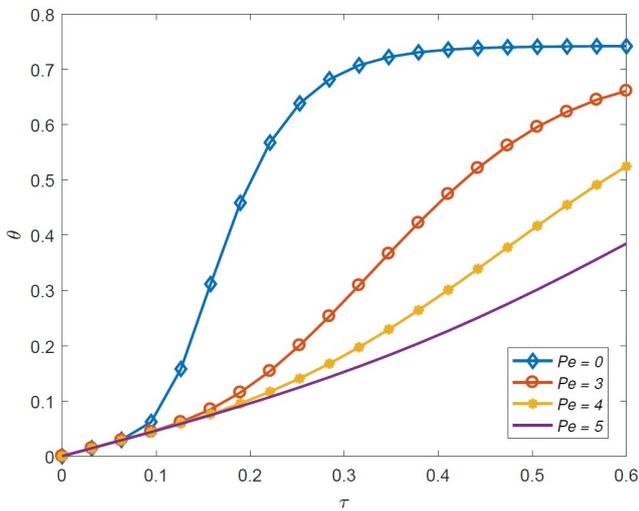


Fig. 8. Temperature distribution on a radial porous moving fin tip for varying fin velocity.

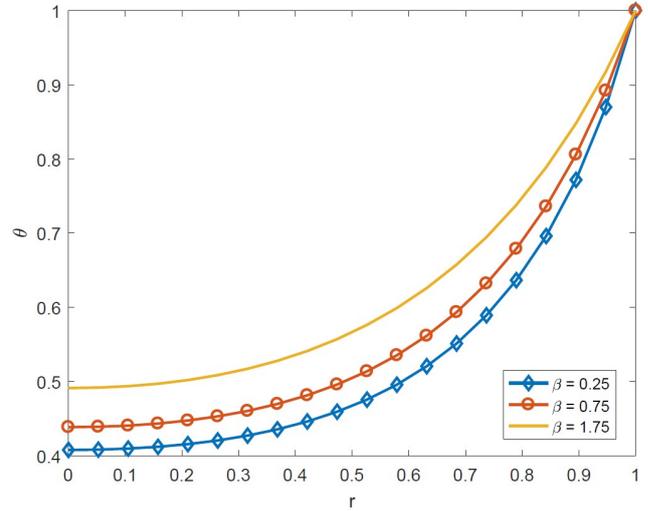


Fig. 9. Effect of varying the thermal conductivity gradient parameter on temperature distribution.

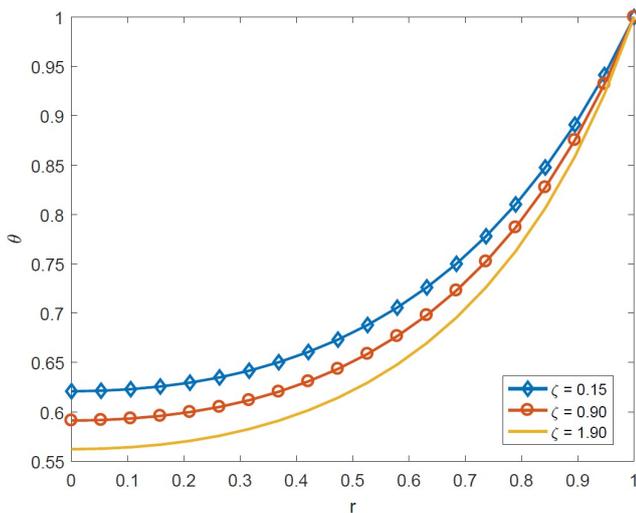


Fig. 10. Effect of varying the surface emissivity gradient parameter on temperature distribution.

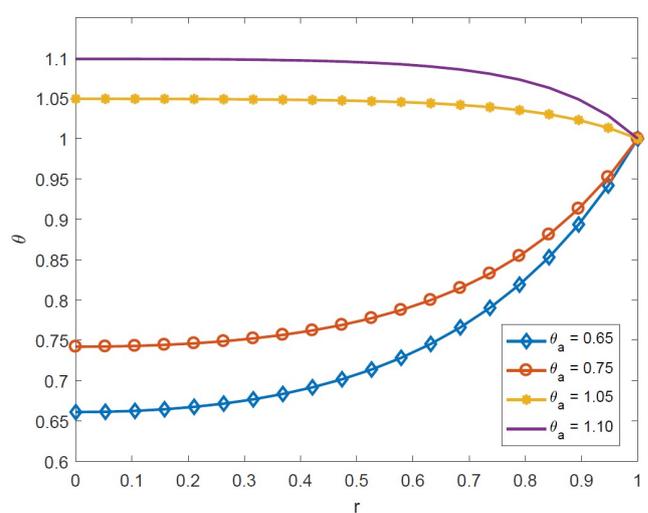


Fig. 11. Effect of varying the ambient temperature parameter on temperature distribution.

Figure 5 illustrates the temperature distribution for different values of the convective-conductive parameter N_c . For $N_c = 0$ this indicates the absence of convection while $N_c = \infty$ indicates that convection and conduction are at equal rates [8]. An increase in the values of the thermo-geometric fin parameter leads to a decrease in temperature in the fin. A similar result is observed in Fig. 6 which represents the impact of the radiation parameter $N_r = 0$ on the temperature distribution. Figure 7 shows the effect of varying the porous media parameter $N_p = 0$ on the temperature distribution. We observe that the temperature decreases with an increasing value of the porous parameter. The high rate of heat transfer is a result of higher porosity due to changes in permeability parameter K included in $N_p = 0$.

In Fig. 8, as expected, the fin temperature increases with increasing values of the Peclet number representing the speed of the moving fin and the dimensionless time. We also observe that the transient fin behavior dies out quickly on a stationary fin when compared to a moving fin. Figure 9 shows the effect of thermal conductivity gradient on the temperature distribution. We observe that as the values of β decreases, the temperature in the fin decreases signifying an increased heat loss to the ambient fluid. Increasing the thermal conductivity gradient enhances heat conduction from the fin base and leads to increased temperatures inside the fin. The effect of varying surface emissivity parameter ζ is depicted in Fig. 10. As expected, we observe that as the surface emissivity increases, the radiative heat loss decreases resulting in lower fin temperatures.

Figure 11 shows the variation in the temperature distribution for varying values of the ambient temperature parameter θ_a . The ambient temperature plays an important role in transient heat dissipation. Higher ambient temperature leads to a reduced heat transfer rate as indicated by high values of fin temperature for higher values of the ambient temperature parameter. Lastly, Fig. 12 depicts the temperature distribution along the three-dimensional spaces and this corresponds to the imposed boundary conditions and to all the results described above.

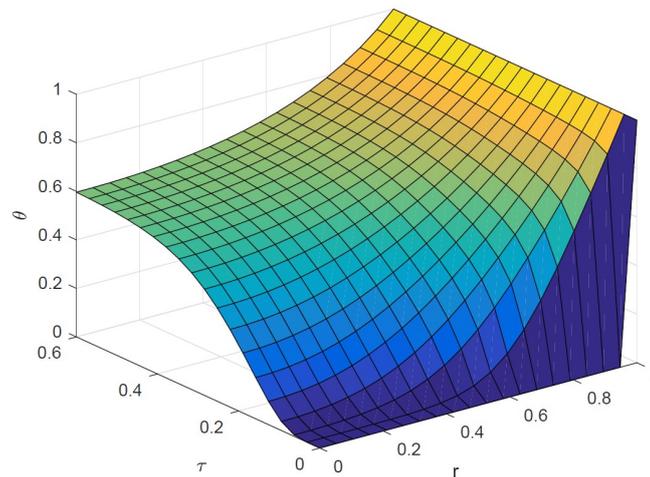


Fig. 12. Transient temperature distribution along a radial porous moving fin.

5. Concluding Remarks

In this study, we derived a mathematical model describing heat transfer through a radial porous fin of a rectangular profile as it moves with a constant velocity. The analysis was performed using built-in numerical scheme in MATLAB software package. MATLAB does not require a compiler to execute and this increases the productivity as well as coding efficiency. Furthermore, the software has inbuilt library of neural networks, fuzzy logic, Simulink and many more, making it easy to handle highly nonlinear problems. In this study, it was observed that the rate of heat transfer through extended surfaces can be influenced by the embedding parameters. An increase in the speed of the moving fin as well as an increase in fin permeability led to increased fin efficiency. The results were in line with some of the studies performed for similar models. The impact of various model parameters on temperatures distribution were illustrated graphically and discussed.

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Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship and publication of this article.



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