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Nonlinear Bending Analysis of Functionally Graded Plates Using SQ4T Elements based on Twice Interpolation Strategy

Hoang Lan Ton-That^{1,2}, Hieu Nguyen-Van², Thanh Chau-Dinh¹

¹ Faculty of Civil Engineering, Ho Chi Minh City University of Technology and Education
01 Vo Van Ngan Street, Thu Duc District, Ho Chi Minh City, Vietnam

² Faculty of Civil Engineering, Ho Chi Minh City University of Architecture
196 Pasteur Street, District 3, Ho Chi Minh City, Vietnam

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Corresponding author: Hoang Lan Ton-That (lan.tonthathoang@uah.edu.vn)

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Abstract. This paper develops a computational model for nonlinear bending analysis of functionally graded (FG) plates using a four-node quadrilateral element SQ4T within the context of the first order shear deformation theory (FSDT). In particular, the construction of the nonlinear geometric equations are based on Total Lagrangian approach in which the motion at the present state compared with the initial state is considered to be large. Small strain-large displacement theory of von Kármán is used in nonlinear formulations of the quadrilateral element SQ4T with twice interpolation strategy (TIS). The solution of the nonlinear equilibrium equations is obtained by the iterative method of Newton-Raphson with the appropriate convergence criteria. The present numerical results are compared with the other numerical results available in the literature in order to demonstrate the effectiveness of the developed element. These results also contribute a better knowledge and understanding of nonlinear bending behaviors of these structures.

Keywords: Functionally graded material, Nonlinear bending, First-order shear deformation theory (FSDT), Twice interpolation strategy (TIS), Von Kármán theory.

1. Introduction

Nowadays, composite materials play an important role in many manufacturing industries such as space shuttles, airplanes, ships, cars, etc. because of their high strength-to-weight, stiffness-to-weight ratios, excellent resistance to corrosive substances and potentially high overall durability. Laminated composite materials, however, often suffer from the delaminated phenomenon that occurs due to the weak bonds and stress concentration between laminas and causes to reduce lifetime of the materials. To overcome the delamination, the concept of functionally graded materials (FGMs) was firstly proposed by Japanese scholars in 1984 [1]. The FGMs are constituted by two different materials, which are usually ceramic and metal, with the material properties continuously changed in one direction. This makes the FGMs free-stress concentration, toughness and high-temperature resistance, and be widely applied in many structures. The rapid growth of FGM structures has required deeply research on behavior of these structures, especially FGM plates. Many analytical and numerical methods have been developed to simulate and analyze the behavior of FGM plates under various loading conditions. The analytical methods [2-6] limit on the FGM plates with simple geometry and boundary conditions. In contrast, the numerical methods such as finite element methods, smoothed finite element methods, mesh-free methods and isogeometric methods [7-29] can effectively compute the behavior of the FGM plates having arbitrary shape, loadings and boundaries. Research on the linear or nonlinear analysis of FGM plates can be found in the review of Jha et al. [17], the use of cell-based smoothed 4-noded quadrilateral element of Nguyen-Van et al. [18], the use of



higher order models of Moita et al. [20, 21], based on isogeometric analysis of Valizadeh et al. [22-25], the special cases of Karsh et al. [26, 27] or Zhang et al. [28], the improved moving Kriging mesh-free method based on a refined plate theory of Nguyen et al. [29] and so on. Within the above numerical methods, the finite element methods (FEM) employed for FGM plates using the first-order shear deformation theory (FSDT) instead of the higher order shear deformation theories (HSDTs) give reasonable results and easy implementation in the standard FEM codes. Beside the triangular finite elements which are most efficient for discretizing arbitrary geometries, the quadrilateral elements are also usually utilized. The difficulty in the development of the four-node element is that such elements are too stiff when simulating thin plates. This phenomenon is called shear locking and can be treated by using shear correction factors. It should be noted that the SQ4T element is firstly introduced in [19] for analysis of composite plate/shell structures, respectively. Therefore, the main objective of this work is to develop the flat four-node SQ4T element [19] for geometric nonlinear bending analysis of functionally graded plates. The von-Karman's large deflection theory and the total Lagrangian (TL) approach are utilized in the small strain-large deformation formulation and then the solution of nonlinear equilibrium equations is obtained by Newton-Raphson method with the automatic incremental algorithm. The evaluations of the membrane, bending and geometric stiffness matrices are built by twice interpolation strategy (TIS) [19, 30-32]. The first stage of TIS is the same as that of the standard FEM for quadrilateral element, but the averaged nodal gradients must be computed for the second stage of this interpolation. Then, the shape functions constructed by the TIS, exhibit more continuous nodal gradients and higher-order polynomial contrast compared to the standard FEM when analyzing the same mesh. Some admirable characteristics can be touched such as high accuracy in the field variables, high convergence rate, and the total number of the degree of freedom of the system does not increase. In the next section, a brief review of the FSDT finite element formulations for geometrically nonlinear analysis of FGM plates is presented. Then, the description of the TIS for the generalized strain and the tangent stiffness matrix of the SQ4T element is developed. Numerical examples of FGM plates are analyzed to verify the robustness and accuracy of the SQ4T elements. Finally, some concluding remarks are withdrawn.

2. Nonlinear Bending Formulation of the SQ4T Element

2.1 The first-order shear deformation theory (FSDT) for geometric nonlinear bending analysis

Consider a FGM plate made of metal and ceramic constituents as shown in Fig. 1. The FGM material continuously changes through the plate thickness h from the metal at the bottom to the ceramic at the top of the plate. Assume that through the FGM plate thickness, the Poisson's ratio $\nu(z)$ and the Young's modulus $E(z)$ is characterized by the following power law:

$$E(z) = (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n + E_m \quad (1)$$

$$\nu(z) = (\nu_c - \nu_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n + \nu_m \quad (2)$$

in which E_c, ν_c and E_m, ν_m are Young's moduli and Poisson's ratios of the ceramic and metal constituents, respectively; $n \geq 0$ is the power law index; and z is the coordinates in the direction normal to the middle plane Oxy of the plate

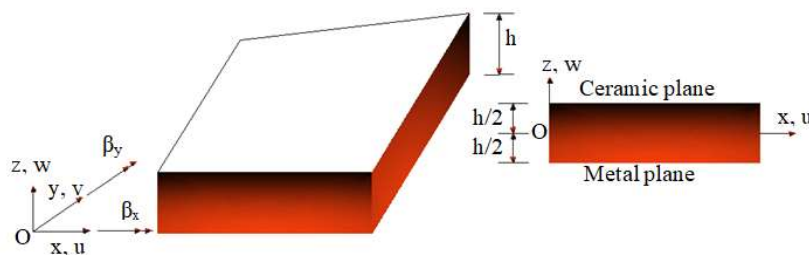


Fig. 1. Functionally graded plate and positive definition of displacements, rotations.

When the FGM plate is stimulated by loads p normal to the top, displacements in the plate can be expressed by the first-order shear deformation theory as follows

$$\begin{aligned} u(x, y, z) &= u_0 + z\beta_x \\ v(x, y, z) &= v_0 + z\beta_y \\ w(x, y, z) &= w_0 \end{aligned} \quad (3)$$

Here u , v and w are the translational displacements in the x -, y -, and z - directions, respectively; u_0, v_0 and w_0 correspond to the displacements of the middle plane; β_x and β_y are respectively the rotation of the mid-plane about x - and y - axis with positive directions defined in Fig. 1.

For large deformation analysis, the in-plane vector of Green-Lagrangian strain in a plate element is:

$$\boldsymbol{\varepsilon} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}(u_{,x}^2 + v_{,x}^2 + w_{,x}^2) \\ v_{,y} + \frac{1}{2}(u_{,y}^2 + v_{,y}^2 + w_{,y}^2) \\ u_{,y} + v_{,x} + (u_{,x}u_{,y} + v_{,x}v_{,y} + w_{,x}w_{,y}) \end{Bmatrix} \quad (4)$$

Substituting Eq. (3) into Eq. (4) and considering the von Karman's large deflection assumption, the in-plane strain vector can be rewritten as:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_m + z\boldsymbol{\varepsilon}_b \quad (5)$$

in which

$$\boldsymbol{\varepsilon}_m = \begin{Bmatrix} u_{0,x} + \frac{1}{2}w_{,x}^2 \\ v_{0,y} + \frac{1}{2}w_{,y}^2 \\ u_{0,y} + v_{0,x} + w_{,x}w_{,y} \end{Bmatrix} = \underbrace{\begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}}_{\text{linear part}} + \underbrace{\begin{Bmatrix} \frac{1}{2}w_{,x}^2 \\ \frac{1}{2}w_{,y}^2 \\ w_{,x}w_{,y} \end{Bmatrix}}_{\text{nonlinear part}} = \boldsymbol{\varepsilon}_m^L + \boldsymbol{\varepsilon}_m^{NL} \quad (6)$$

$$\boldsymbol{\varepsilon}_b = \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix} \quad (7)$$

The transverse shear strain vector is given as:

$$\boldsymbol{\gamma} = \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \beta_x - w_{,x} \\ \beta_y - w_{,y} \end{Bmatrix} \quad (8)$$

The constitutive relationship of the FGM plate can be expressed as:

$$\boldsymbol{\sigma}^* = \mathbf{D}^* \boldsymbol{\varepsilon}^* \quad (9)$$

where

$$\boldsymbol{\sigma}^* = \begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{T} \end{Bmatrix}, \quad \boldsymbol{\varepsilon}^* = \begin{Bmatrix} \boldsymbol{\varepsilon}_m \\ \boldsymbol{\gamma} \end{Bmatrix}, \quad \mathbf{D}^* = \begin{Bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{B} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C} \end{Bmatrix} \quad (10)$$

and $\mathbf{N} = [N_x \ N_y \ N_{xy}]$ is the in-plane traction resultant, $\mathbf{T} = [T_x \ T_y]$ is the out-of-plane traction resultant and $\mathbf{M} = [M_x \ M_y \ M_{xy}]$ is the out-of-plane moment resultant. $\mathbf{A}, \mathbf{B}, \mathbf{D}, \mathbf{C}$ are the material constant matrices which are given as follow:

$$\begin{aligned} (\mathbf{A}, \mathbf{B}, \mathbf{D}) &= \int_{-h/2}^{h/2} (1, z, z^2) \mathbf{Q}(z) dz \\ \mathbf{C} &= \int_{-h/2}^{h/2} \mathbf{S}(z) dz \end{aligned} \quad (11)$$

with

$$\mathbf{Q}(z) = \frac{E(z)}{1-\nu(z)^2} \begin{bmatrix} 1 & \nu(z) & 0 \\ \nu(z) & 1 & 0 \\ 0 & 0 & (1-\nu(z))/2 \end{bmatrix}; \quad \mathbf{S}(z) = \frac{E(z)}{2(1+\nu(z))} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (12)$$

2.2 The TIS of SQ4T element for geometric nonlinear analysis

Let $\mathbf{x}_c$ be a point in a four-node quadrilateral element with nodes i, j, k, m as shown in Fig. 2. We denote by S_i, S_j, S_k and S_m for the elements that share nodes i, j, k and m . The supporting nodes for the point $\mathbf{x}_c$ in this quadrilateral element involve all nodes of elements S_i, S_j, S_k and S_m . The support domain of point $\mathbf{x}_c$ is much larger than the standard FEM support domain, and the trial solution at point $\mathbf{x}_c$ can be written as:

$$\tilde{u}(\mathbf{x}) = \sum_{l=1}^{n_{sp}} \tilde{N}_l(\mathbf{x}) d_l = \tilde{\mathbf{N}}(\mathbf{x}) \mathbf{d} \quad (13)$$

In Eq. (13), the twice-interpolation shape function $\tilde{N}_l$ is determined as follows:

$$\begin{aligned} \tilde{N}_l = & \underbrace{\phi_i N_l^{[i]} + \phi_{ix} \bar{N}_{l,x}^{[i]} + \phi_{iy} \bar{N}_{l,y}^{[i]}}_{\text{node } i} + \underbrace{\phi_j N_l^{[j]} + \phi_{jx} \bar{N}_{l,x}^{[j]} + \phi_{jy} \bar{N}_{l,y}^{[j]}}_{\text{node } j} \\ & + \underbrace{\phi_k N_l^{[k]} + \phi_{kx} \bar{N}_{l,x}^{[k]} + \phi_{ky} \bar{N}_{l,y}^{[k]}}_{\text{node } k} + \underbrace{\phi_m N_l^{[m]} + \phi_{mx} \bar{N}_{l,x}^{[m]} + \phi_{my} \bar{N}_{l,y}^{[m]}}_{\text{node } m} \end{aligned} \quad (14)$$

in which d_l denotes the nodal displacement vector, while $N_l^{[i]}$ is the shape function with respect to node i , and n_{sp} is the total number of the supporting nodes with regard to the point $\mathbf{x}_c$. The twice-interpolation shape function and the standard FEM shape function are plotted in Fig. 3a for one dimension, respectively.

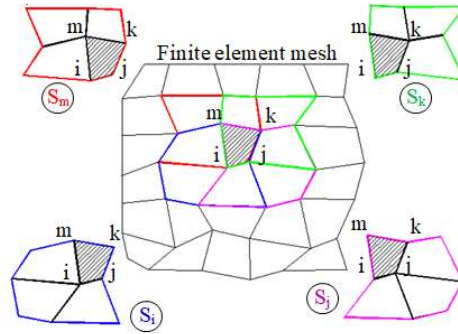


Fig. 2. Schematic sketch of the twice interpolation strategy for quadrilateral element in 2D.

The formulation of the average derivative of the shape functions at node i is given by (similar for other nodes):

$$\bar{N}_{l,x}^{[i]} = \sum_{e \in S_i} (\omega_e N_{l,x}^{[i][e]}) \quad \bar{N}_{l,y}^{[i]} = \sum_{e \in S_i} (\omega_e N_{l,y}^{[i][e]}) \quad (15)$$

In Eq. (15), the term $N_{l,x}^{[i][e]}$ is the derivative of $N_l^{[i]}$ computed in element e , and ω_e is the weight function of element $e \in S_i$, which is defined by:

$$\omega_e = \frac{\Delta_e}{\sum_{e \in S_i} \Delta_e} \quad \text{with } e \in S_i \quad (16)$$

and with Δ_e being the area of the element e . In Eq. (14), the functions ϕ_i, ϕ_{ix} and ϕ_{iy} forming the polynomial basis associated with node i must satisfy the following conditions:

$$\begin{aligned} \phi(\mathbf{x}_l) &= \delta_{il}, \quad \phi_{ix}(\mathbf{x}_l) = 0, \quad \phi_{iy}(\mathbf{x}_l) = 0, \quad \phi_{lx}(\mathbf{x}_l) = 0, \quad \phi_{lx,x}(\mathbf{x}_l) = \delta_{il} \\ \phi_{lx,y}(\mathbf{x}_l) &= 0, \quad \phi_{ly}(\mathbf{x}_l) = 0, \quad \phi_{ly,x}(\mathbf{x}_l) = 0, \quad \phi_{ly,y}(\mathbf{x}_l) = \delta_{il} \end{aligned} \quad (17)$$

with l is any one of the indices i, j, k and m ; we define

$$\delta_{il} = \begin{cases} 1 & \text{if } i = l \\ 0 & \text{if } i \neq l \end{cases} \quad (18)$$

The above conditions have to be applied in a similar manner to other functions, i.e., $\phi_j, \phi_{jx}, \phi_{jy}, \phi_k, \phi_{kx}, \phi_{ky}, \phi_m, \phi_{mx}$ and ϕ_{my} . These polynomial basis functions ϕ, ϕ_{ix} and ϕ_{iy} for the quadrilateral element are given by:

$$\begin{aligned}
\phi_i &= L_i + L_i^2 L_j + L_i^2 L_k + L_i^2 L_m - L_i L_j^2 - L_i L_k^2 - L_i L_m^2 \\
\phi_{ix} &= -(x_i - x_j)(L_i^2 L_j + 0.5 L_i L_j L_k + 0.5 L_i L_j L_m) - (x_i - x_k)(L_i^2 L_k + 0.5 L_i L_k L_j + 0.5 L_i L_k L_m) \\
&\quad - (x_i - x_m)(L_i^2 L_m + 0.5 L_i L_m L_j + 0.5 L_i L_m L_k) \\
\phi_{iy} &= -(y_i - y_j)(L_i^2 L_j + 0.5 L_i L_j L_k + 0.5 L_i L_j L_m) - (y_i - y_k)(L_i^2 L_k + 0.5 L_i L_k L_j + 0.5 L_i L_k L_m) \\
&\quad - (y_i - y_m)(L_i^2 L_m + 0.5 L_i L_m L_j + 0.5 L_i L_m L_k)
\end{aligned} \quad (19)$$

Other functions can be calculated in the same manner of Eq. (19) by a circulatory permutation of indices i, j, k and m . L_i, L_j, L_k and L_m are the area coordinates of the point $\mathbf{x}_c$ in the quadrilateral element with four nodes i, j, k and m see [19, 30-32] for more details. These shape functions are complete polynomials, satisfy properties of the partition of unity, and possess Kronecker's delta function property, respectively.

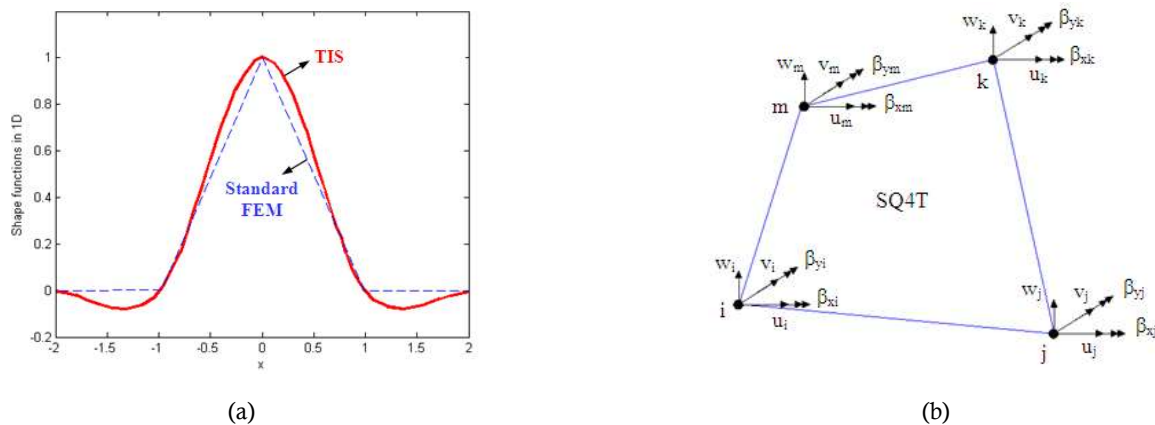


Fig. 3. (a) Comparison of the shape functions in one dimension, (b) the SQ4T element with degrees of freedom.

From the FSDT, the TIS and the SQ4T element as shown in Fig. 3b, the linear membrane strain at an arbitrary point $\mathbf{x}_c$ can be obtained as follows:

$$\boldsymbol{\varepsilon}_m^L(\mathbf{x}_c) = \mathbf{B}_m^L(\mathbf{x}_c) \mathbf{q} \quad (20)$$

where $\mathbf{B}_m^L$ is the linear gradient matrix, $\mathbf{q}$ is the vector of degrees of freedom associated with the supporting nodes as:

$$\mathbf{q}_i = [u_i \quad v_i \quad w_i \quad \beta_{xi} \quad \beta_{yi}]^T \quad (21)$$

$$\mathbf{B}_m^L(\mathbf{x}_c) = \begin{bmatrix} \tilde{N}_{1,x} & 0 & 0 & 0 & 0 & \dots & \tilde{N}_{n_{sp},x} & 0 & 0 & 0 & 0 \\ 0 & \tilde{N}_{1,y} & 0 & 0 & 0 & \dots & 0 & \tilde{N}_{n_{sp},y} & 0 & 0 & 0 \\ \tilde{N}_{1,y} & \tilde{N}_{1,x} & 0 & 0 & 0 & \dots & \tilde{N}_{n_{sp},y} & \tilde{N}_{n_{sp},x} & 0 & 0 & 0 \end{bmatrix}_{3 \times 5n_{sp}} \quad (22)$$

in which $\mathbf{q}_i$ is the vector of nodal degrees of freedom, n_{sp} is the total number of the supporting nodes in regard to the point $\mathbf{x}_c$. In a similar way, the nonlinear membrane strain can be written as:

$$\boldsymbol{\varepsilon}_m^{NL}(\mathbf{x}_c) = \mathbf{B}_m^{NL}(\mathbf{x}_c) \mathbf{q} \quad (23)$$

where $\mathbf{B}_m^{NL}$ is the nonlinear gradient matrix given as:

$$\mathbf{B}_m^{NL} = \mathbf{H}\mathbf{G} \quad (24)$$

in which

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & \tilde{N}_{1,x} & 0 & 0 & \dots & 0 & 0 & \tilde{N}_{n_{sp},x} & 0 & 0 \\ 0 & 0 & \tilde{N}_{1,y} & 0 & 0 & \dots & 0 & 0 & \tilde{N}_{n_{sp},y} & 0 & 0 \end{bmatrix}_{2 \times 5n_{sp}}, \quad (25)$$

$$\mathbf{H} = \begin{bmatrix} \tilde{N}_{i,x} w_i & 0 \\ 0 & \tilde{N}_{i,y} w_i \\ \tilde{N}_{i,y} w_i & \tilde{N}_{i,x} w_i \end{bmatrix}_{(i=1..n_{sp})} \quad (26)$$

and w_i is the deflection at the node i with i is one of the supporting nodes. The bending strain over the supporting domain is expressed as:

$$\boldsymbol{\varepsilon}_b(\mathbf{x}_c) = \mathbf{B}_b(\mathbf{x}_c) \mathbf{q} \quad (27)$$

where $\mathbf{B}_b$ is the bending gradient matrix given as:

$$\mathbf{B}_b(\mathbf{x}_c) = \begin{bmatrix} 0 & 0 & 0 & \tilde{N}_{1,x} & 0 & \dots & 0 & 0 & 0 & \tilde{N}_{n_{sp},x} & 0 \\ 0 & 0 & 0 & 0 & \tilde{N}_{1,y} & \dots & 0 & 0 & 0 & 0 & \tilde{N}_{n_{sp},y} \\ 0 & 0 & 0 & \tilde{N}_{1,y} & \tilde{N}_{1,x} & \dots & 0 & 0 & 0 & \tilde{N}_{n_{sp},y} & \tilde{N}_{n_{sp},x} \end{bmatrix}_{3 \times 5n_{sp}} \quad (28)$$

The shear strain is also expressed by:

$$\boldsymbol{\varepsilon}_s(\mathbf{x}_c) = \mathbf{B}_s(\mathbf{x}_c) \mathbf{q} \quad (29)$$

where $\mathbf{B}_s$ is the transverse shear gradient matrix given as:

$$\mathbf{B}_s(\mathbf{x}_c) = \begin{bmatrix} 0 & 0 & \tilde{N}_{1,x} & \tilde{N}_1 & 0 & \dots & 0 & 0 & \tilde{N}_{n_{sp},x} & \tilde{N}_{n_{sp}} & 0 \\ 0 & 0 & \tilde{N}_{1,y} & 0 & \tilde{N}_1 & \dots & 0 & 0 & \tilde{N}_{n_{sp},y} & 0 & \tilde{N}_{n_{sp}} \end{bmatrix}_{2 \times 5n_{sp}} \quad (30)$$

Finally, the element tangent stiffness matrix is modified as:

$$\mathbf{K}_T = \mathbf{K}_L + \mathbf{K}_{NL} + \mathbf{K}_g \quad (31)$$

where $\mathbf{K}_L$ and $\mathbf{K}_{NL}$ present the linear and nonlinear stiffness matrices, respectively, and $\mathbf{K}_g$ is the geometric stiffness matrix. They are obtained as follows:

$$\mathbf{K}_L = \mathbf{B}_L^T \mathbf{D}^* \mathbf{B}_L \quad (32)$$

$$\mathbf{K}_{NL} = \mathbf{B}_{NL}^T \mathbf{D}^* \mathbf{B}_{NL} \quad (33)$$

$$\mathbf{K}_g = \mathbf{G}^T \hat{\mathbf{N}} \mathbf{G} \quad (34)$$

with

$$\mathbf{B}_L = \begin{bmatrix} \mathbf{B}_m^L \\ \mathbf{B}_b \\ \mathbf{B}_s \end{bmatrix}, \quad \mathbf{B}_{NL} = \begin{bmatrix} \mathbf{B}_m^{NL} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad \hat{\mathbf{N}} = \begin{bmatrix} N_x & N_{xy} \\ N_{xy} & N_y \end{bmatrix} \quad (35)$$

Based on the Total Lagrangian approach, the internal forces at the loop rep computed from the stress state in the structures can be rewritten as:

$${}^{rep}\mathbf{F} = \int_{\Omega} (\mathbf{B}_L + \mathbf{B}_{NL}) {}^{rep}\boldsymbol{\sigma}^* d\Omega \quad (36)$$

in which the stress resultant after the i^{th} iteration is:

$${}^{rep}\boldsymbol{\sigma}_{i+1}^* = {}^{rep}\boldsymbol{\sigma}_i^* + {}^{rep}\Delta\boldsymbol{\sigma}^* \quad (37)$$

Finally, the nonlinear equations can be rewritten as:

$${}^{rep}\mathbf{K}_T \Delta\mathbf{q} = {}^{rep+\Delta rep}\mathbf{P} - {}^{rep}\mathbf{F} \quad (38)$$

3. Numerical Results and Discussion

In this section, we will test and assess the SQ4T element through numerical examples. In all examples, the Newton-Raphson method and automatic incremental algorithm are used to solve the nonlinear finite element equations. The convergence tolerance of displacement is taken to be 0.001. The shear correction factors are equal to $\pi^2/12$, and SI units are used. Material properties of the FGM plates used in the numerical simulations are provided in Table 1.

Table 1. Material properties of FGM plates.

Skew FGM plate & Arbitrarily straight-sided quadrilateral FGM plate		
Properties	Ti-6Al-4V	Aluminum oxide
E (GPa)	105.7	320.2
ν	0.298	0.26
Circular FGM plate		
Properties	Aluminum(Al)	Zirconia (ZrO_2-2)
E (GPa)	70	151
ν	0.3	0.3

3.1 A skew FGM plate

The bending behavior of a skew plate is often considered as a corner stress concentration problem due to a strong singularity in bending moments at the obtuse vertex. It is often avoided for nonlinear analyses of plate bending problems. Therefore, this section deals with the nonlinear bending analysis of clamped skew plates with three skew angles, namely $\alpha = 0^\circ, 30^\circ, 60^\circ$ under a uniform load q . The studied plate have a side length of $a = b = 0.2\text{m}$ and thickness $h = a/20$. The full plate is modelled using 8×8 elements as shown in Fig. 4. The normalized parameters of the present results are the central deflection $w^* = w/h$ and the load parameter $P = qa^4 / E_m h^4$. The clamped boundary conditions are expressed as $u_0 = v_0 = w_0 = \beta_x = \beta_y = 0$ on all edges.

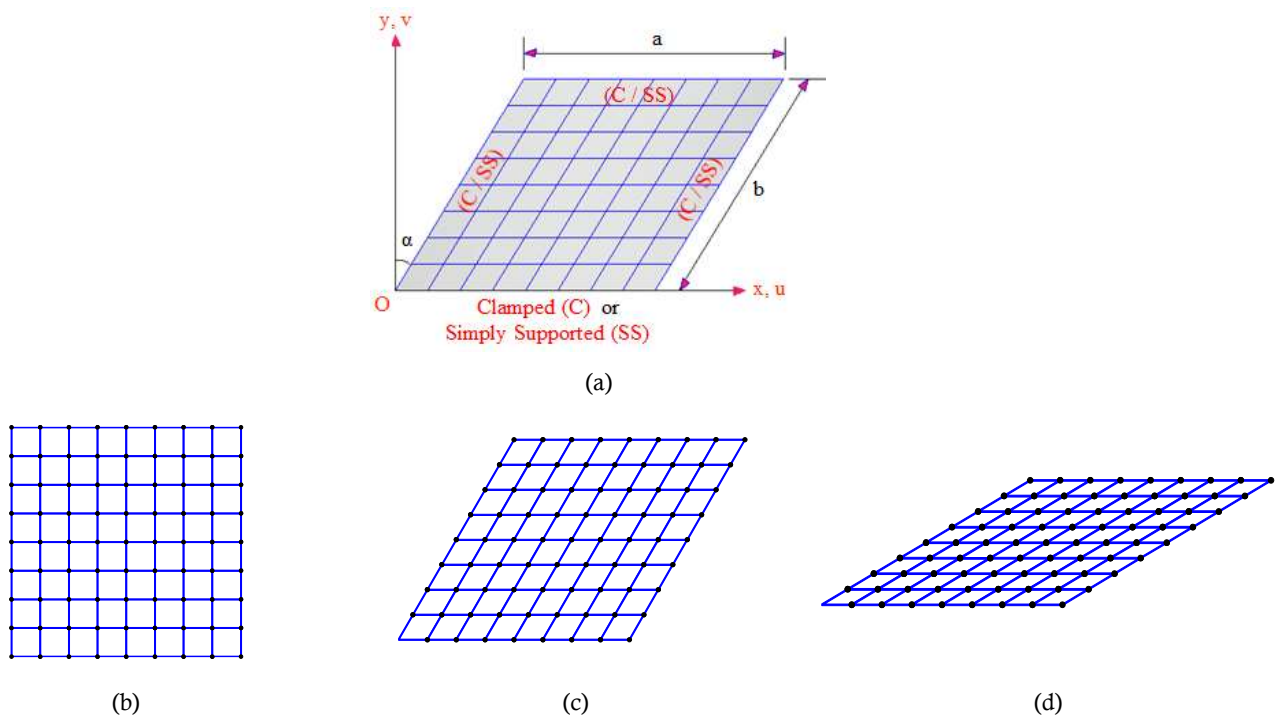


Fig. 4. A skew plate: (a) geometry, (b) a mesh with $\alpha = 0^\circ$, (c) a mesh with $\alpha = 30^\circ$, (d) a mesh with $\alpha = 60^\circ$.

First, the accuracy of the present solutions is investigated for a square plate (skew angle $= 0^\circ$). The load-deflection curves obtained by the present elements are plotted and compared with numerical solutions of Do and Lee [13] using a modified radial point interpolation mesh-free method as shown in Fig. 5a. It can be seen that the present results with mesh 8×8 elements are in good agreement with those from [13]. Next, the effect of skew angle on the normalized central deflection is studied with the load parameter increasing from 0 to 220. It is followed from Figs. 5b-d that the maximum central deflection decreases as the skew angle of the plate increases with the increase in load for any gradient index. The normalized central deflections are also increased with the increasing of the gradient index under the same load.

According to these results, it is inferred that the rigidity of skew plates can be improved by increasing the skew angles or decreasing the gradient index.

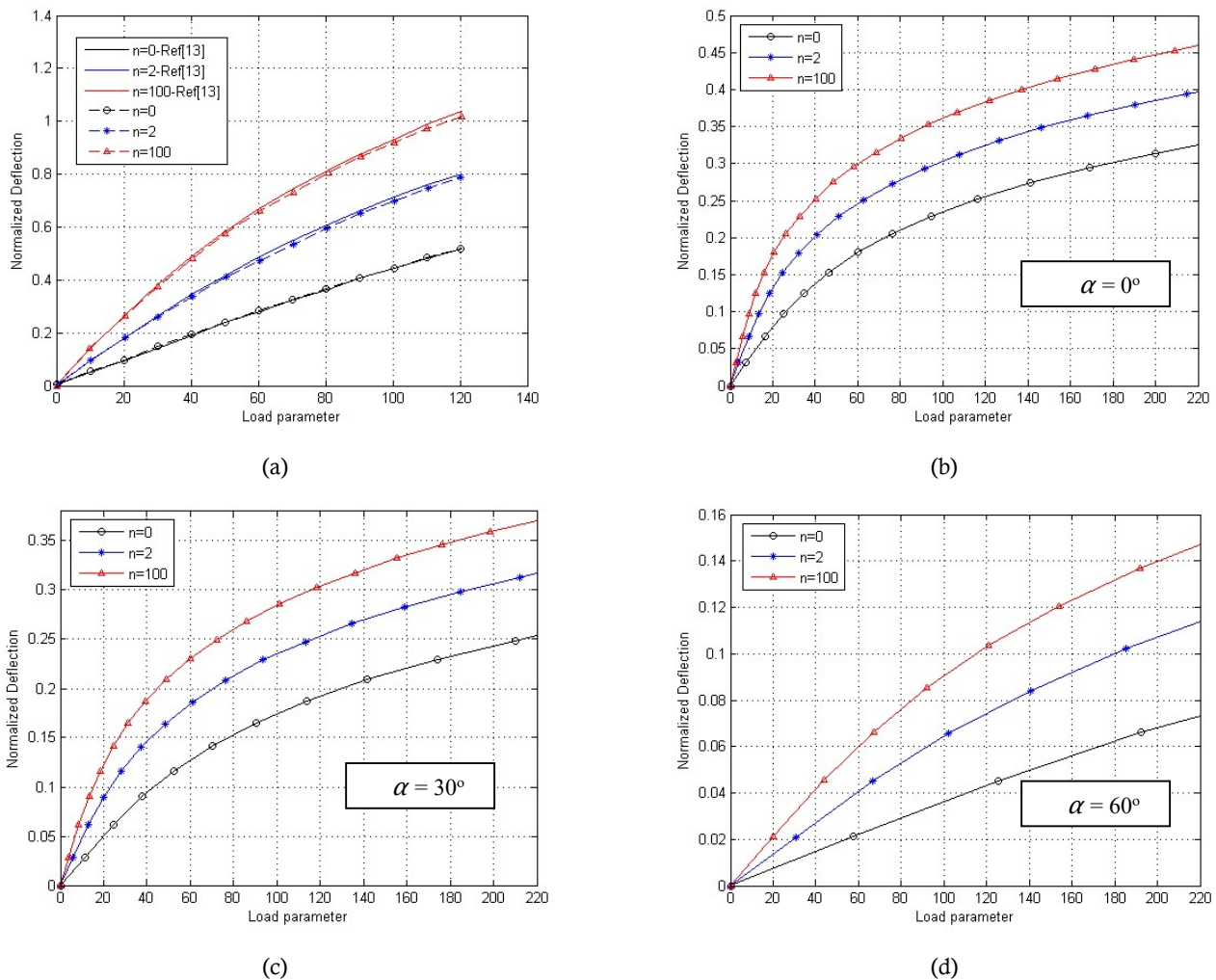


Fig. 5. (a): Load-deflection curves of the clamped skew plate, (b): nonlinear behavior with $\alpha = 0^\circ$, (c) nonlinear behavior with $\alpha = 30^\circ$, (d) nonlinear behavior with $\alpha = 60^\circ$.

Furthermore, by changing the boundary conditions from clamped to simply supported, the effects of skew angle and the gradient index on the normalized central deflection are also studied in Figs. 6a and 6b. The simply supported boundary conditions are expressed as $u_0 = w_0 = \beta_x = 0$ on edges in x- direction and $v_0 = w_0 = \beta_y = 0$ on edges in y-direction, respectively.

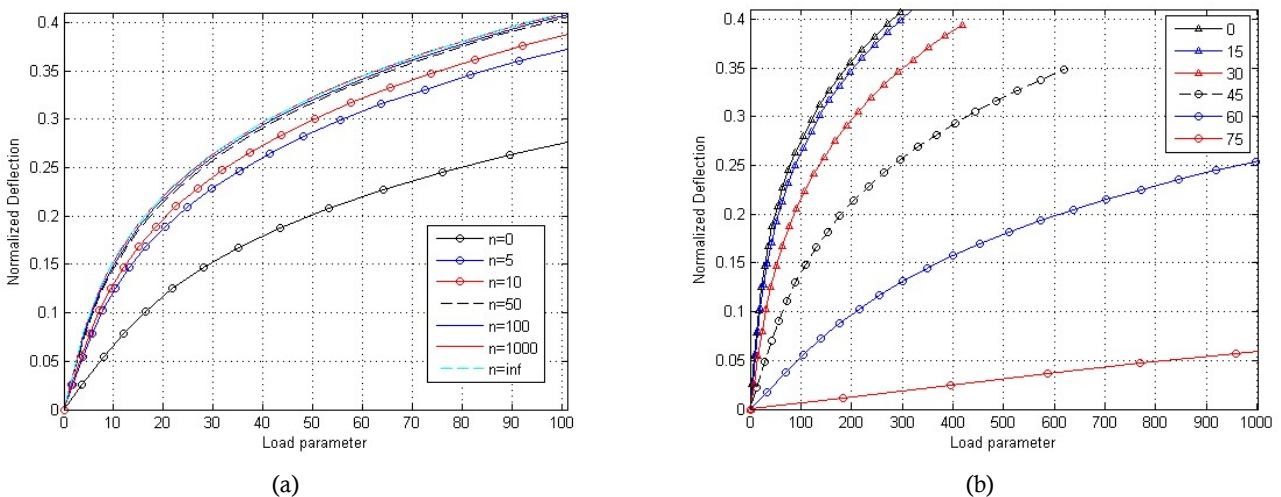


Fig. 6. Load-deflection curves of the simply supported skew plate, (a): nonlinear behavior with $\alpha = 0^\circ$ and $n=0, 5, 10, 50, 100, 1000$ & ∞ , (b) nonlinear behavior with $n=0$ and $\alpha = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ$ & 75° .

3.2 A circular FGM plate

The large deformation analysis of a clamped circular plate under uniform pressure q is considered in this section. The plate has the radius $R = 1\text{m}$ and the thickness $h = 0.1\text{m}$. Owing to symmetry, a quadrant of the plate is modeled with a 27-element mesh as shown in Fig. 7. The symmetrical boundary conditions are presented as $v_0 = \beta_x = 0$ for edge along the x-direction and $u_0 = \beta_y = 0$ for edge along the y-direction, respectively.

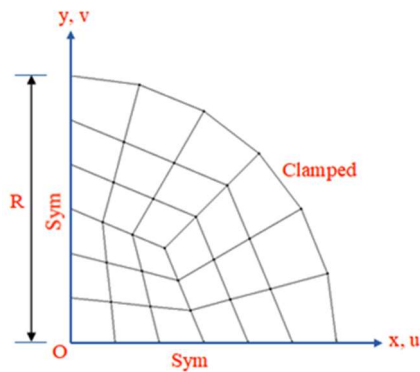


Fig. 7. A mesh of a quadrant of a clamped circular FGM plate.

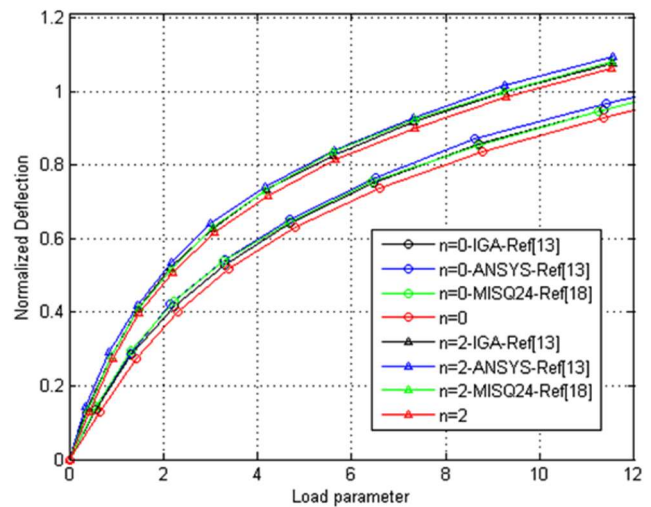


Fig. 8. Load-deflection curves of the circular plate under effect of volume fraction exponent with $R/t=10$.

The computed normalized central deflection $w^* = w/h$ versus the normalized load parameter $P = qR^4 / E_m h^4$ of the present analysis using SQ4T element together with the solutions by the FSDT-based isogeometric analysis [13], ANSYS results [13] and mixed interpolation smoothed quadrilateral MISQ24 element [18] are displayed in Fig. 8. The convergence of the present solutions is investigated through Table 2 that shows the normalized central deflection w^* of the circular FGM plate with the volume fraction exponents $n = 0$ and 2.

From Fig. 8 and Table 2, it is interesting to note that the obtained numerical results match very well with those from other numerical methods.

3.3 An arbitrarily straight-sided quadrilateral FGM plate

Finally, the nonlinear bending behavior of an arbitrarily straight-sided quadrilateral FGM plate under a uniform load q is also considered as shown in Fig. 9. The studied plate has a side length $a = 0.254\text{m}$ and thickness $h = a/100$. Similarly, the simply supported boundary conditions are expressed as $u_0 = w_0 = \beta_x = 0$ on edges a , d and $v_0 = w_0 = \beta_y = 0$ on edges b , c .

With the normalized parameters $w^* = w/h$ and $P = qa^4 / E_m h^4$, the effect of side angles on the deflection of this FGM plate is investigated and illustrated in Figs. 10a, b. Results obtained by the present method are in close agreement with those obtained by the element-free IMLS-Ritz method [28].

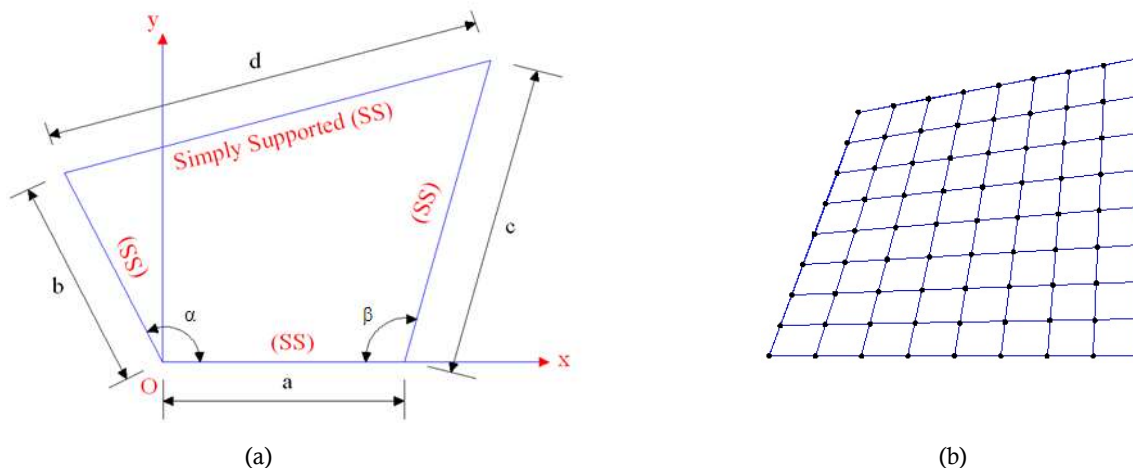
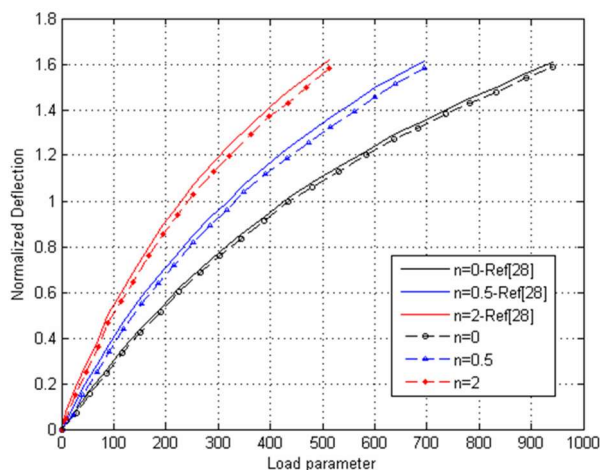


Fig. 9. (a) An arbitrarily straight-sided quadrilateral plate, (b) a mesh with $b/a = 0.7$, $c/a = 0.8$, $a/h = 100$, $\alpha = 70^\circ$ and $\beta = 90^\circ$

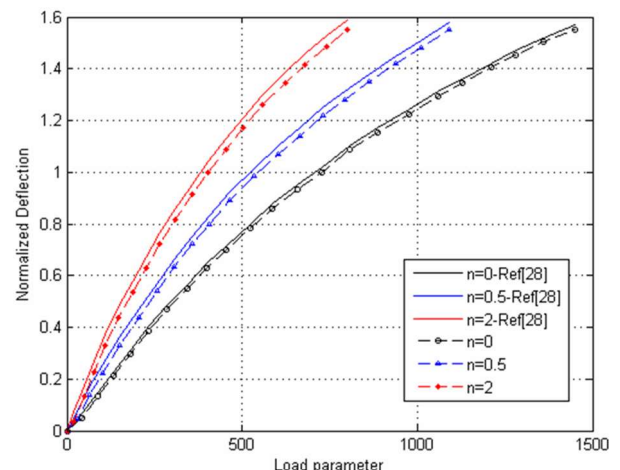
Table 2. Convergence study of FGM circular plate with the volume fraction exponent $n = 0$ and 2.

P	n=0			
	IGA	ANSYS	MISQ24	SQ4T
0	0	0	0	0
0.580645	0.137969	0.142023	0.142857	0.128407
1.327190	0.284083	0.288137	0.293878	0.275662
2.156680	0.414018	0.422107	0.428571	0.402536
3.317970	0.527774	0.543991	0.538776	0.517490
4.686640	0.641596	0.653758	0.644898	0.629909
6.511520	0.751457	0.767683	0.755102	0.738576
8.626730	0.857367	0.873565	0.853061	0.836053
11.40550	0.951208	0.967434	0.946939	0.930131

P	n=2			
	IGA	ANSYS	MISQ24	SQ4T
0	0	0	0	0
0.386207	0.134909	0.143119	0.134890	0.128342
0.910345	0.281991	0.290220	0.281991	0.274239
1.46207	0.408577	0.420864	0.412654	0.397947
2.15172	0.522782	0.535050	0.518668	0.508573
3.06207	0.628645	0.640970	0.632741	0.615643
4.19310	0.734358	0.742568	0.730281	0.717967
5.60000	0.827596	0.839863	0.835787	0.815179
7.31034	0.916531	0.928817	0.924703	0.900840
9.26897	1.001200	1.017580	1.001220	0.984266
11.5586	1.077450	1.093840	1.081570	1.062871



(a)



(b)

Fig. 10. Load-deflection curves of the arbitrarily straight-sided quadrilateral plate, (a): nonlinear behavior with $b/a = 0.7$, $c/a = 0.8$, $a/h = 100$, $\alpha = 70^\circ$ and $\beta = 90^\circ$, (b): nonlinear behavior with $b/a = 0.6$, $c/a = 0.7$, $a/h = 100$, $\alpha = 70^\circ$ and $\beta = 80^\circ$

4. Conclusion

In this paper, the SQ4T element was developed and successfully applied to geometrically nonlinear analysis of functionally graded plate structures in the framework of the FSDT. Numerical examples were carried out and the present element was found to achieve satisfactory results in comparison with the other available numerical results using finite element as well as isogeometric methods. It was observed that the present approach remains accurate for nonlinear analysis of both moderately thin and thick plates. In addition, the present element had the advantage of being simple in formulation and ready for use in analysis of both plate and shell structures. The success of the present flat element provided a further demonstration of efficient flat quadrilateral elements for nonlinear analysis.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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