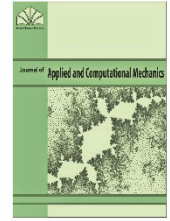


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Research Paper

Impact of Blood Vessel Wall Flexibility on the Temperature and Concentration Dispersion

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Abstract. The analysis of solute and thermal dispersion in pulsatile flow through the stenotic tapered blood vessel is presented. The present problem is an extension of the work done by Ramana *et al.* who considered the time-invariant arterial wall. In the present model, the flexible nature of the arterial wall through the obstruction (called stenosis) is considered and it is achieved with the help of period trigonometric function. In the present study, the impact of the time-dependent arterial wall on the blood flow dynamics is discussed in details. The rheology of the blood is modeled as a couple stress fluid. The proposed fluid model is the isothermal inclusion of temperature-sensitive drug coated Titanium dioxide Nano-particles in the couple stress fluid for examining the concentration and temperature dispersion. The effects of the catheter and permeability of the stenosis are considered in the model. Care has been taken to model the thermo-physical properties of the fluid with the immersed nanoparticle, e.g., TiO_2 , Ag and Cu . The modeled non-linear and coupled equations are solved by using the Homotopy Perturbation Method. The temperature and concentration dispersion effects are in the flexible stenotic arterial vessel under the pulsatile physiological pressure gradient are studied and reported in details. The alterations in the axial velocity, resistance to the flow, and wall shear stress are studied and found out that the high intense vortex regions are identified in the stenotic region. The model has direct applications in the pharmaceutical industry in design and developing the drug to treat stenotic conditions.

Keywords: Flexible arterial wall, Couple stress nano fluid, Homotopy perturbation method, Temperature dispersion, Drug delivery, Impedance, Wall shear stress.

1. Introduction

In physiology, the blood flow in the circulatory system is composed of the heart and blood vessels. The foremost cause for illness and death is the failure in the circulatory system. In the present world, 30% of the global deaths are associated with cardiovascular diseases, as reported in [1]. In India, it is observed [2, 3] that it is difficult to control non-communicable diseases when compared to communicable diseases. The large and middle-sized arteries are very elastic, exposing these to high pressure makes it firm and therefore resembles like a pipe. This change in nature from being elastic to firm is what is called as arteriosclerosis. Similar thing happens with the small arteries and arterioles wherein the arteries lose elasticity over a period of time. This is also termed as arteriosclerosis. This high blood pressure and several other physiological factors compromise the organ function. In particular, brain and heart get seriously affected, resulting in stroke and myocardial infarction, respectively. Therefore, incorporation of the pulsatile pressure gradient is effective in



understanding the characteristics of blood flow through unhealthy blood vessels. Also, one of the primary procedures to treat the stenotic conditions is to use a catheter. The insertion of an external object like catheter into the narrow artery results in substantial changes in blood flow dynamics. Therefore, it is important to understand the impact of the catheter on the hemodynamics in the treatment of unhealthy artery. Dejam [4] studied the advection-dispersive model to control the solute transport by the porous walls to the surrounding porous media. Here, the author also claimed that the investigation plays a vital role in assimilating the mechanism of drug delivery to the blood vessels.

The nanotechnology has been significantly contributing to pharmaceutical materials research. As of now, to the best of the authors' awareness, nanoparticles can be used to deliver the drug at the location of interest. The side effects of the medication can be reduced by adopting this procedure. The nanoparticles, in general, are extensively used for the treatment of brain tumors, cleaning water, etc. The challenges associated with the nanoparticle applications is to increase the amount of drug carried by decreasing the size, however, these particles already have an advantage of having more surface area per weight as compared to the large particles. The drug supply to the targeted organs can be achieved without the need of incomplete healing through the Drug-Coated Balloon therapy. The drug reaches the targeted organ through the diffusion procedure and will affect the targeted region only. In this way, the side effects of the drug can be minimized effectively. Modeling and simulation of such procedure lays the foundation to assimilate the drug delivery mechanism to the organ of interest by the drug coated with nanoparticles as demonstrated by Ramana et.al [5]. These new methods which are dependent on particles are beginning to influence the way drugs can be administered resulting in better health conditions.

The computational study of blood flow in stenosed arteries has usually been approached by the assumption that the considered arterial wall moments are significantly smaller. Accordingly, it was assumed that artery walls are rigid [6, 7]. However, it is reported that they exhibit anisotropic non-linear elastic response [8]. Further, it is well known that the blood vessels have delicate walls up to a certain extent. Few studies have developed the one-sided sinusoidal time-dependent function to represent the unsteady blood flow model [9]. In the present model, it is assumed that blood flow is driven by the pressure difference across the axial boundary. The physiological pressure and deformable wall due to change in pressure are overlapped by considering that they are in phase.

The concentration dispersion in a non-Newtonian fluid is considered by Jyotirmoy & Murthy [10]. Here authors considered the models like Carreau-Yasuda, Carreau and Casson fluid. The effective transport coefficients, i.e. convection, exchange and dispersion coefficients are evaluated. Elnaqeeb et al. [11] studied the flow through asymmetric stenosis and reported the influence of nanoparticle volume fraction on the blood flow dynamics. Here, the researcher considered classical viscous fluid to represent the blood, which is the base fluid. Researchers like Goldsmith & Skalak [12] and Bugliarello & Sevilla [13] studied blood flow experimentally and concluded that blood dynamics are unlike classical viscous fluid. The strong deviation is due to the presence of neutrally afloat blood components. Therefore, blood is having a non-Newtonian structure. Stress tensor expresses the related constitutive equation [5].

The hemodynamic properties are mainly affected by blood components. The change in the radius of gyration is used to incorporate the deformation of the particles in couple stress fluid model. It is identified that the couple stress fluid model reasonably resembles the blood flow as reported by Valanis & Sun [14] and Popel et al. [15]. Therefore, modeling the blood with the couple stress fluid model is useful in interpreting the abstract differences associated. Srinivasacharya & Srikanth [16, 17] and Ramana et al., [18, 6] highlighted various effects, for mild stenosis for different modeled parameters on fluid flow through a constriction. Here, the authors represented blood as a couple stress fluid and considered a rigid arterial wall. However, it is to be noted that the concentration and temperature dispersion in the considered domain in the presence of catheter in case of couple stress fluid has not been given much attention.

The ultimate goal is to maximize the therapeutic activity and minimize the side effects. An ideal drug distribution system should deliver the drug when it is required. In recent times, targeted and controlled drug delivery systems have gained cumulative attention. The rapid progress has been observed due to the advanced materials and technology. Over the last decade, the role of colloids has substantially increased. These engineered colloids have numerous applications such as in Nano Electro Mechanical Systems (NEMS), Micro Electromechanical Systems (MEMS), nuclear reactors, biomedical engineering (imaging and ablation), extraction of geothermal power, etc. [19]. Among the colloids, the Nano fluids own some exceptional features unlike other colloids, such as stability. To add to it, nano fluids possess improved thermal and physical features such as an increase in viscosity, thermal conductivity and diffusivity as compared to those of base fluids like oil or water [20].

As it is not possible to find out the closed-form solutions to the highly non-linear and coupled equations, the researchers came with the highly interesting way to find out the asymptotic series solutions based on the perturbation methods. Wang et al. [21] studied the concentration transport in the straight uniformly packed porous media, in order to understand the Taylor dispersion. Here, authors used series solutions. Nevertheless, the perturbation methods have their own limitations, like considering the small parameter from the modeled equations which is may not be always possible [22-24]. Therefore, recently several new methods are proposed without the necessity of the small parameter [25-27]. Among these, the Homotopy Perturbation Method (HPM) is prime wherein the intent is to capture the asymptotic form of infinite series solution which is convergent. The HPM method is proposed by He [28], in this method a homotopy is found out based on the externally imposed parameter q . Several researchers [29, 11, 5] studied the nanoparticle suspended blood flow and showed that this technique will enable us to get appropriate results when these kind of problems are modeled.

In the present study, blood flow is modeled with the couple stress nanofluid flow. The integrated fluid is assumed to

flow through the irregular annular region formed by the ω – shaped stenotic tapered arterial wall and the catheter wall. The HPM method has been used extensively to solve the modeled system. The reported study is expected to have applications in the drug delivery problems. The influence of the wall flexibility on the drug dispersion and the interrelation between the temperature and concentration dispersion has been investigated. Further, flow resistance and Shear-stress at the wall are analyzed. In this manuscript, the physical and mathematical models are explained in detail in section 2. The implementation of the HPM to get the perturbation solution to the modeled system of equations is clearly demonstrated in section 3. Thereafter, section 4 is for the discussions on the obtained results. Finally, few salient qualitative results are presented in the form of summary in section 5.

2. Mathematical Formulation

2.1 Stratified Viscosity

The blood components, Erythrocytes and Leukocytes are the deciding factors of the blood mechanics as they are present in majority. In the blood flow, the Erythrocytes deform and align with the flow in the blood vessels with less than $300\mu m$ in diameter. In this process, a thin layer will be formed on the blood vessel wall, and it leads to reduction of friction among the RBC and the blood vessel wall. This also results in lessening the red blood cell volume percentage in the blood, which decreases the effective blood viscosity [30].

Viscosity is one of the physical properties of the fluid. The immersed nanoparticles are aligned with the main flow region, which results in a denser main flow region as compared to the boundary. It is assumed and accordingly modeled that the particles are distributed in parabolic nature. High viscosity is observed in the core region, due to the alignment of particles. This is the cause of the apparent viscosity variation in the domain of interest. Einstein [31] developed the empirical formulation for the nanofluid viscosity based on the assumptions that the volume fraction is very low i.e., $\phi < 0.02$ and the immersed particles are spherical in shape. The suggested formula is

$$\mu_{nf} = \mu_f (1 + 2.5 \phi).$$

Here, the viscosity is in linear relationship with the particle volume concentration. Batchelor [32] improved the viscosity equation by considering isotropic suspension of spherical and rigid nanoparticles. The empirical formulation is given by

$$\mu_{nf} = \mu_f (1 + 2.5 \phi + 6.5 \phi^2).$$

The above variable viscosity models are the known standard nanofluids viscosity models. The classical models do not hold good for all types of nanofluids; therefore, new models are established by modifying these classical models. Some of the new models as reported in Table 1 for Cu , TiO_2 , and Ag nanoparticles submersion are experimentally verified.

Table 1. Thermal and physical properties of the base and nanofluids.

	Al_2O_3	Copper	TiO_2	Silver	Water	Hatami et. al. [31]
ρ	3970.0	8933.0	4250.0	10500.0	997.1	1050.0
C_p	765.0	385.0	686.2	235.0	4179.0	3617.0
κ	40.0	401.0	8.9538	429.0	0.613	0.52
$\gamma \times 10^{-5}$	0.85	1.67	1.9	1.89	21.0	-
μ_{nf} / μ_f	$1 + 39.11\phi + 533.9\phi^2$ as given by Bock Choon & Young Cho [34]	$\frac{1}{(1-\phi)^{2.5}}$ as given by Brinkman [35]	$1 + 5.45\phi + 108.2\phi^2$ as given by Bock Choon & Young Cho [34]	$1.005 + 0.497\phi - 0.1149\phi^2$ as found in Godson et. al. [36]	1.0	-
μ_{nf} / μ_f	$1 - 39.11\phi - 533.9\phi^2$	$\frac{1}{(1+\phi)^{2.5}}$	$1 - 5.45\phi - 108.2\phi^2$	$1.005 - 0.497\phi + 0.1149\phi^2$	1	-
κ_{nf} / κ_f	$1 + 7.47\phi$ as given by Bock Choon & Young Cho [34]	$\frac{\kappa_p + (n-1)\kappa_f + (n-1)\phi(\kappa_p - \kappa_f)}{\kappa_p + (n-1)\kappa_f + \phi(\kappa_p - \kappa_s)}$ as given by Brinkman [35]	$1 + 2.92\phi - 11.99\phi^2$ as given by Bock Choon & Young Cho [34]	$0.9508 + 0.9692\phi$ as given by Godson et. al. [36]	1	-

2.2 Geometry

The modeled asymmetric blood, flows through an stenotic tapered artery of ω -shape in the presence of the catheter. The schematic diagram is as shown in figure 1.

The incompressible couple stress nanofluid model has been used to model the blood with additive drug-coated nanoparticles. It is deliberated that the modeled fluid is flowing through the annular region, formed by the catheter and arterial wall. Also, it is considered the time-varying flexible arterial wall and rigid catheter wall due to which the change in annular region (i.e., $h(z, t)$) is observed. The mathematical model turns out to be two-dimensional with domain

$\Omega(=[r_c, h(z, t)] \times [0, L])$. The mathematical formula of the variable annular radius is described as [7, 5].

$$h(z, t) = \begin{cases} r_0 + \zeta z - \frac{3\varepsilon}{2L_1^4} (11(z-L_0)L_1^4 - 11(z-L_0)L_1^4 + 11(z-L_0)L_1^4 - 11(z-L_0)L_1^4) \Big] f(t), & \text{if } L_0 \leq z \leq L_0 + L_1 \\ (r_0 + \zeta z) f(t), & \text{otherwise} \end{cases} \quad (1)$$

where $f(t)$ is a special function which is responsible for the time dependent nature of the arterial wall. Mathematically, it is modeled with the one-sided sinusoidal function and is expressed as $[1 - b(\cos(2\pi f_p t) - 1)\exp(-2\pi f_p t b)]$. Here, f_p corresponds to the pulsatile frequency and b is an arbitrary constant.

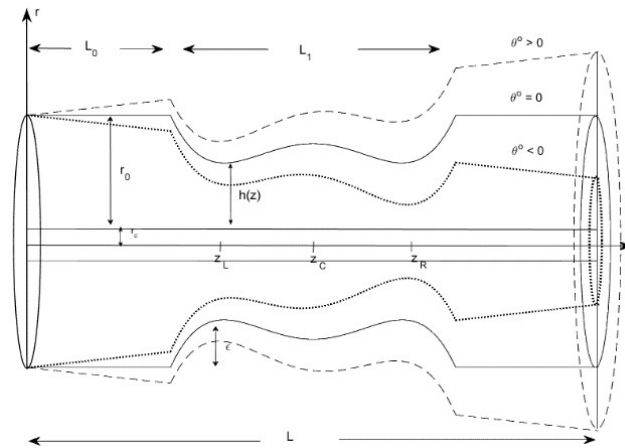


Fig. 1. Diagrammatic representation of the permeable omega-shaped stenotic tapered artery at $t=0$.

2.3 The equations governing the flow

The governing equations for unsteady incompressible couple stress axisymmetric fluid flow through the non-uniform region $[r_c, h(z, t)]$ are modeled with two-component mixture model. In this modelling, it is believed that, nanoparticles and base fluid are in thermal equilibrium and no chemical transformation takes place between these. Therefore, modeled coupled stress nanofluid flow equations are

$$\nabla \cdot \vec{V} = 0 \quad (2)$$

$$\rho_{nf} (\partial_t \vec{V} + \vec{V} \cdot \nabla \vec{V}) = -\nabla p + \mu_{nf} \nabla^2 \vec{V} - \eta_{nf} \nabla^4 \vec{V} + F \quad (3)$$

$$\partial_t T + (\vec{V} \cdot \nabla) T = \frac{\kappa_{nf}}{(\rho c_p)_{nf}} \nabla^2 T + \frac{(\rho c_f)_{nf}}{(\rho c_p)_{nf}} \left[D_B \nabla C \cdot \nabla T + \frac{D_T}{T_0} \nabla T \cdot \nabla T \right] \quad (4)$$

$$\partial_t C + (\vec{V} \cdot \nabla) C = D_B \nabla^2 C + \frac{D_T}{T_0} \nabla^2 T \quad (5)$$

where $\vec{V} = (u, 0, v)$ is the assumed two-dimensional velocity vector. C and T represent the concentration and temperature of the nanofluid. In the modeled nanofluid model μ_{nf} is the dynamic viscosity, η_{nf} is the couple stress viscosity. κ_{nf} is thermal conductivity of nanofluid, while $(\rho c_p)_{nf}$ is the heat capacitance of nanoparticle material $[\kappa_{nf} / (\rho c_p)_{nf} (:= \alpha_{nf})]$. The possible effects of the diffusion coefficients related to mass and heat are noted as D_B and D_T respectively.

In the present analysis, some metallic and ceramic nanoparticles like Cu , Ag and TiO_2 are considered. It is assumed that in the fully-developed flow, the slip between the base fluid and immersed nanoparticles is neglected. The linear approximation is used to model the resultant density variation. The deviation in the thermo-physical features of the nanofluid from the base fluid is modeled as reported in Table 1. The component form of the above equations (2 - 5) can be written as

$$\partial_r u + u/r + \partial_z v = 0 \quad (6)$$

$$\rho_{nf} [\partial_t u + u \partial_r u + v \partial_z u] = -\partial_r p + \mu_{nf} [\partial_{rr} u + 1/r \partial_r u - u/r^2 + \partial_{zz} u] - \eta_{nf} [\partial_{rrr} u + 2\partial_{rrz} u + \partial_{zzz} u + 2/r \partial_{rr} u + 2/r \partial_{rz} u - 3/r^2 \partial_{rr} u - 2/r^2 \partial_{zz} u + 3/r^3 \partial_r u - 3u/r^4] \quad (7)$$

$$\rho_{nf} [\partial_t v + u \partial_r v + v \partial_z v] = -\partial_z p + \mu_{nf} [\partial_{rr} v + 1/r \partial_r v + \partial_{zz} v] - \eta_{nf} [\partial_{rrr} v + 2\partial_{rrz} v + \partial_{zzz} v + 2/r \partial_{rr} v + 2/r \partial_{rz} v - 1/r^2 \partial_{rr} v + 1/r^3 \partial_r v] + g(\rho\gamma)_{nf} (T - T_0) + g(\rho\gamma)_{nf} (C - C_0) \quad (8)$$

$$\partial_t T + u \partial_r T + v \partial_z T = \alpha_{nf} [\partial_{rr} T + 1/r \partial_r T + \partial_{zz} T] + \frac{(\rho c_f)_{nf}}{(\rho c_p)_{nf}} \left[D_B (\partial_r C \partial_r T + \partial_z C \partial_z T) + \frac{D_T}{T_0} \{ (\partial_r T)^2 + (\partial_z T)^2 \} \right] \quad (9)$$

$$\partial_t C + u \partial_r C + v \partial_z C = D_B [\partial_{rr} C + 1/r \partial_r C + \partial_{zz} C] + \frac{D_T}{T_0} [\partial_{rr} T + 1/r \partial_r T + \partial_{zz} T] \quad (10)$$

The resultant change in the thermo-physical properties are assumed to be linear in nature written as follows

$$\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s, \quad (\rho c_p)_{nf} = (1 - \varphi) (\rho c_p)_f + \varphi (\rho c_p)_s, \quad (\rho\gamma)_{nf} = (1 - \varphi) (\rho\gamma)_f + \varphi (\rho\gamma)_s. \quad (11)$$

2.3.1 The dimensionless governing equations

We scale the variables as follows:

$$r' = \frac{r}{r_0}, \quad z' = \frac{z}{L_1}, \quad p' = \frac{p r_0^2}{u_0 L_1 \mu_f}, \quad t' = \frac{t u_0}{L_1}, \quad v' = \frac{v}{u_0}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \quad \sigma = \frac{C - C_0}{C_1 - C_0}, \quad u' = \frac{u L_1}{u_0 \varepsilon} \quad (12)$$

where u_0 is characteristic velocity scale. Dimensionless variables are introduced into the governing equations (6)-(10) and the removal of dashes will result in

$$\xi (\partial_r u + u/r) + \partial_z v = 0 \quad (13)$$

$$\frac{\rho_{nf}}{\rho_f} \text{Re} \xi \delta^3 [\partial_t u + \xi u \partial_r u + v \partial_z u] = -\partial_r p + \mu_{nf} \xi \delta^2 [\partial_{rr} u + 1/r \partial_r u - u/r^2 + \delta^2 \partial_{zz} u] - \frac{\eta_{nf} \xi \delta^2}{\beta^2} [\partial_{rrr} u + 2/r \partial_{rr} u - 3/r^2 \partial_{rr} u + 3/r^3 \partial_r u - 3u/r^4 + \delta^2 (2\partial_{rrz} u + \delta^2 \partial_{zzz} u + 2/r \partial_{rz} u - 2/r^2 \partial_{zz} u)] \quad (14)$$

$$\frac{\rho_{nf}}{\rho_f} \text{Re} \delta [\partial_t v + \xi u \partial_r v + v \partial_z v] = -\partial_z p + \mu_{nf} [\partial_{rr} v + 1/r \partial_r v + \delta^2 \partial_{zz} v] - \frac{\eta_{nf}}{\beta^2} [\partial_{rrr} v + 2/r \partial_{rr} v - 1/r^2 \partial_{rr} v + 1/r^3 \partial_r v + \delta^2 (\partial_{rrz} v + 1/r \partial_{rz} v + \delta \partial_{rrz} v + \delta/r \partial_{zz} v + \delta^2 \partial_{zzz} v)] + \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} \theta \frac{r_0^2 (\rho\gamma)_f (T_1 - T_0)}{u_0 \mu_0} + \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} \sigma \frac{r_0^2 (\rho\gamma)_f (C_1 - C_0)}{u_0 \mu_0} \quad (15)$$

$$\text{Re Pr} \delta [\partial_t \theta + \xi u \partial_r \theta + v \partial_z \theta] = (\partial_{rr} \theta + 1/r \partial_r \theta + \delta^2 \partial_{zz} \theta) + N_b (\partial_r \sigma \partial_r \theta + \delta^2 \partial_z \sigma \partial_z \theta) + N_b \{ (\partial_r \theta)^2 + \delta^2 (\partial_z \theta)^2 \} \quad (16)$$

$$\text{Re Pr} \delta [\partial_t \sigma + \xi u \partial_r \sigma + v \partial_z \sigma] = (\partial_{rr} \sigma + 1/r \partial_r \sigma + \delta^2 \partial_{zz} \sigma) + N_t / N_b (\partial_{rr} \theta + 1/r \partial_r \theta + \delta^2 \partial_{zz} \theta) \quad (17)$$

where, $\text{Re} = \rho_f u_0 r_0 / \mu_f$ is the Reynolds number, $\delta = r_0 / L_1$, $\xi = \varepsilon / r_0$, $\beta^2 = \eta_f / \mu_f r_0^2$ is length dependent parameter, Grashof number is $Gr = r_0^2 g(\rho\gamma)_f (T_1 - T_0) / u_0 \mu_f$ and $Br = r_0^2 g(\rho\gamma)_f (C_1 - C_0) / u_0 \mu_f$ is corresponding to the solute Grashof number. While brownian motion parameter is $N_b = (\rho c_f)_f D_B (C_1 - C_0) / (\rho c_f)_{nf}$, the thermophoresis parameter is given by $N_t = (\rho c_f)_f D_B (T_1 - T_0) / (\rho c_f)_{nf}$.

The dimensionless form of the above governing equations (13) – (17) by considering above dimensionless parameters and the under the assumption of mild stenosis [5] get transformed to

$$\partial_z v = 0 \quad (18)$$

$$\partial_r p = 0 \quad (19)$$

$$\partial_z p = \mu_{nf} \left[\partial_r v + \frac{1}{r} \partial_r v \right] - \frac{\eta_{nf}}{\beta^2} \left[\partial_{rr} v + \frac{2}{r} \partial_{rr} v - \frac{1}{r^2} \partial_{rr} v + \frac{1}{r^3} \partial_r v \right] + \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} Gr \theta + \frac{(\rho\gamma)_{nf}}{(\rho\gamma)_f} Br \sigma \quad (20)$$

$$\partial_{rr} \theta + \frac{1}{r} \partial_r \theta + N_b (\partial_r \sigma \partial_r \theta) + N_b \{ (\partial_r \theta)^2 \} = 0 \quad (21)$$

$$\partial_{rr} \sigma + \frac{1}{r} \partial_r \sigma + \frac{N_t}{N_b} \left(\partial_{rr} \theta + \frac{1}{r} \partial_r \theta \right) = 0 \quad (22)$$

It is understood that the Pressure is independent of r as obtained from equations (19)-(20). It is assumed that the flow is driven by the pressure; therefore, the pressure gradient $\partial p / \partial z$ which appears due to the pumping action of the heart is taken as reported in [38]

$$-\frac{\partial p}{\partial z}(t) = A_0 + A_1 \cos(2\pi f_p t) \quad (23)$$

where, A_0 and A_1 are the mean and pulsatile components of the pressure gradient of the heart acting along the axis of the tube, f_p is the pulse frequency.

2.3.2 Boundary Conditions (Non-dimensional)

It is proposed to have a velocity discontinuity at the arterial wall due to the formation of the thin layer on it. And also, the special boundary condition (25) is considered to account for the slip and roughness at the permeable stenosis region as reported by Gordon & Daniel [39]. The slip is, therefore, due to the permeable nature of the stenosis. Hence

$$\begin{aligned} v &= u_s \text{ at } r = h(z, t); z \leq L_0, z \geq L_0 + L_1 \\ v &= 0 \text{ at } r = r_c \end{aligned} \quad (24)$$

$$\frac{\partial v}{\partial r} = \frac{\chi}{\sqrt{Da}} \left(u_s + \frac{Da}{\mu(r)|_{\text{arterial wall}}} \frac{\partial p}{\partial z} \right) \text{ at } r = h(z, t); L_0 \leq z \leq L_0 + L_1 \quad (25)$$

Here χ represents the surface roughness at the micro level, therefore, it is a material property. For this model, it is to be noted that the average roughness of the material is $O(\sqrt{Da})$. The correlation between χ and the average roughness (i.e., average pore diameter) of the material is reported at [39]. The modeled concentration and temperature boundary conditions are

$$\begin{aligned} \sigma &= 1, \quad \theta = 1, \text{ at } r = r_c \\ \sigma &= 0, \quad \theta = 0, \text{ at } r = h(z, t) \end{aligned} \quad (26)$$

These boundary conditions reveal that the drug-coated nanoparticles which are temperature-sensitive are placed on the catheter surface and inserted into the lumen of the artery.

3. Method of Solution

3.1 Introduction to Homotopy Perturbation Method

The series asymptotic solutions of the coupled non-linear modeled equations together with the reported boundary conditions have been worked out by the HPM method. The working procedure is as presented in [5], however, the outlines are presented here for the continuing purpose. Consider the system of differential equations with boundary condition as follows

$$S(V) = g(r), \quad B(V, \partial_n V) = 0; \quad r \in \Omega \quad (27)$$

where S is an operator which can be written as linear (L say) and non-linear (N say) operator respectively. The



$g(r)$ is a known analytic function. The boundary operator defined on the domain is B . Construct a function $U(r, q) : \Omega \times [0, 1] \rightarrow R^n$, such that

$$H(U, q) \equiv L(U) - L(V_0) + qL(V_0) + q[N(U) - g(r)] = 0. \quad (28)$$

where V_0 is an initial approximate of the equation (27) and also it is taken as a linear operator solution which satisfies the boundary condition. In this method, homotopy parameter $q \in [0, 1]$, the solution of the system deforms from $V_0(r)$ to $V(r)$ as the parameter q ranges from zero to one. Consider an infinite series solution in the powers of q for U i.e.,

$$U = U_0 + qU_1 + q^2U_2 + q^3U_3 + \dots \quad (29)$$

The approximate homotopy solution is obtained from the above equation by setting $q = 1$, i.e.,

$$V = \lim_{q \rightarrow 1} U = U_0 + U_1 + U_2 + U_3 + \dots \quad (30)$$

3.1.1 Convergence

Let us write an equation (28) in the following form

$$L(U) = L(V_0) - qL(V_0) - q[N(U) - g(r)]. \quad (31)$$

Applying the inverse operator to both sides of the equation (31), we obtain

$$U = V_0 + q[-V_0 - L^{-1}(N(U)) + L^{-1}(g(r))]. \quad (32)$$

Suppose that

$$U = \sum_{i=0}^{\infty} q^i u_i, \quad (33)$$

Substituting (33) into the right-hand side of the equation (32), we get

$$U = V_0 + q \left[-V_0 - L^{-1} \left(N \left(\sum_{i=0}^{\infty} u_i \right) \right) + L^{-1}(g(r)) \right]. \quad (34)$$

As discussed earlier the series asymptotic solution may be obtained as $q \rightarrow 1$, that is

$$V = \lim_{q \rightarrow 1} U = -L^{-1} \left(N \left(\sum_{i=0}^{\infty} u_i \right) \right) + L^{-1}(g(r)) = -\sum_{i=0}^{\infty} L^{-1}(N(u_i)) + L^{-1}(g(r)). \quad (35)$$

3.2 Implementation of HPM method to the modelled problem

This method is the union of the homotopy and the perturbation method. From the equations 21 and 22, the differential operator is split into linear and nonlinear parts as follows

$$\underbrace{\partial_{rr}\theta + \frac{1}{r}\partial_r\theta}_L + \underbrace{N_b(\partial_r\sigma\partial_r\theta) + N_b\{(\partial_r\theta)^2\}}_N = 0 \quad (36)$$

$$\underbrace{\partial_{rr}\sigma + \frac{1}{r}\partial_r\sigma}_L + \underbrace{\frac{N_t}{N_b} \left(\partial_{rr}\theta + \frac{1}{r}\partial_r\theta \right)}_N = 0 \quad (37)$$

with this identification of the operators, by using homotopy defined as the equation 28, the following are obtained.

$$H(q, \theta) = \partial_{rr}\theta + \frac{1}{r}\partial_r\theta - \left(\partial_{rr}\theta_0 + \frac{1}{r}\partial_r\theta_0 \right) + q \left(\partial_{rr}\theta_0 + \frac{1}{r}\partial_r\theta_0 \right) + q \left[N_b(\partial_r\sigma\partial_r\theta) + N_b\{(\partial_r\theta)^2\} \right] = 0 \quad (38)$$

$$H(q, \sigma) = \partial_{rr}\sigma + \frac{1}{r}\partial_r\sigma - \left(\partial_{rr}\sigma_0 + \frac{1}{r}\partial_r\sigma_0 \right) + q \left(\partial_{rr}\sigma_0 + \frac{1}{r}\partial_r\sigma_0 \right) + q \left[\frac{N_t}{N_b} \left(\partial_{rr}\theta + \frac{1}{r}\partial_r\theta \right) \right] = 0 \quad (39)$$

where θ_0 and σ_0 are the initial approximated solutions, currently, these expressions are the solutions of the appropriate linear operators with the appropriate conditions at the boundary. The infinite series expressions for the concentration and temperature in power of q are

$$\theta = \theta_0 + q\theta_1 + q^2\theta_2 + q^3\theta_3 + \dots \quad (40)$$

$$\sigma = \sigma_0 + q\sigma_1 + q^2\sigma_2 + q^3\sigma_3 + \dots \quad (41)$$

Putting these in equations (38) and (39) and equating the corresponding powers of q , the following equations will be realized

- Zeroth-order deformation equations

$$\partial_{rr}\theta_0 + \frac{1}{r}\partial_r\theta_0 = 0 \quad (42)$$

$$\partial_{rr}\sigma_0 + \frac{1}{r}\partial_r\sigma_0 = 0 \quad (43)$$

the corresponding boundary conditions are

$$\begin{aligned} \sigma_0 &= 1, \quad \theta_0 = 1, \quad \text{at } r = r_c \\ \sigma_0 &= 0, \quad \theta_0 = 0, \quad \text{at } r = h(z, t) \end{aligned} \quad (44)$$

- Similarly, first-order deformation equations

$$\partial_{rr}\theta_1 + \frac{1}{r}\partial_r\theta_1 + \partial_{rr}\theta_0 + \frac{1}{r}\partial_r\theta_0 + N_b\partial_r\theta_0\partial_r\sigma_0 + N_t(\partial_r\theta_0)^2 = 0 \quad (45)$$

$$\partial_{rr}\sigma_1 + \frac{1}{r}\partial_r\sigma_1 + \partial_{rr}\sigma_0 + \frac{1}{r}\partial_r\sigma_0 + \frac{N_t}{N_b}\left(\partial_{rr}\theta_0 + \frac{1}{r}\partial_r\theta_0\right) = 0 \quad (46)$$

the corresponding boundary conditions are

$$\begin{aligned} \sigma_1 &= 0, \quad \theta_1 = 0, \quad \text{at } r = r_c \\ \sigma_1 &= 0, \quad \theta_1 = 0, \quad \text{at } r = h(z, t) \end{aligned} \quad (47)$$

- The deformation equations of order two are

$$\partial_{rr}\theta_2 + \frac{1}{r}\partial_r\theta_2 + N_b(\partial_r\theta_1\partial_r\sigma_0 + \partial_r\theta_0\partial_r\sigma_1) + N_t(2\partial_r\theta_0\partial_r\theta_1) = 0 \quad (48)$$

$$\partial_{rr}\sigma_2 + \frac{1}{r}\partial_r\sigma_2 + \frac{N_t}{N_b}\left(\partial_{rr}\theta_1 + \frac{1}{r}\partial_r\theta_1\right) = 0 \quad (49)$$

the corresponding boundary conditions are

$$\begin{aligned} \sigma_2 &= 0, \quad \theta_2 = 0, \quad \text{at } r = r_c \\ \sigma_2 &= 0, \quad \theta_2 = 0, \quad \text{at } r = h(z, t) \end{aligned} \quad (50)$$

and so on. The solutions of the above PDE's are

$$\theta_0 = \frac{\log(h(z, t)) - \log(r)}{\log(h(z, t)) - \log(r_c)} \quad (51)$$

$$\sigma_0 = \frac{\log(h(z, t)) - \log(r)}{\log(h(z, t)) - \log(r_c)} \quad (52)$$

$$\theta_1 = \frac{(N_b + N_t)(\log(h(z, t)) - \log(r))(\log(r) - \log(r_c))}{2(\log(h(z, t)) - \log(r_c))^2} \quad (53)$$

Similarly, the rest of the terms σ_1 , θ_i and σ_i are evaluated, where $i = 2, 3, 4, \dots$. The obtained solutions of concentration and temperature are used to calculate the axial velocity from the coupled equations. The corresponding axial velocity equation with the identification of linear and non-linear operator is given below

$$\underbrace{-\frac{\eta_{nf}}{\mu_{nf}\beta^2}\Psi_r(v) + \alpha_{nf}\theta Gr + \alpha_{nf}\sigma Br + \Gamma_r(v)}_N - \underbrace{\frac{1}{\mu_{nf}}\frac{\partial p}{\partial z}}_g = 0 \quad (54)$$

Here the operators Γ_r and Ψ_r are represented as $\partial_{rr} + (1/r)\partial_r$ and $\partial_{rrr} + (2/r)\partial_{rr} - (1/r^2)\partial_{rr} + (1/r^3)\partial_r$ respectively. The α_{nf} is denoted as $(1/\mu_{nf}) \times (\rho\gamma)_{nf} / (\rho\gamma)_f$. The HPM method is used to get the axial velocity profiles by using the boundary conditions Eqs. 24 and 25. The details are as follows

- Zeroth order deformation equation
 - In the non-stenotic region

$$\Gamma_r(v_0) = 0 \quad (55)$$

boundary conditions are

$$\begin{aligned} v_0 &= u_s \text{ at } r = h(z, t); z \leq L_0, z \geq L_0 + L_1 \\ v_0 &= 0 \text{ at } r = r_c \end{aligned} \quad (56)$$

- In the stenotic artery region

$$\Gamma_r(v_0) = 0 \quad (57)$$

the corresponding boundary conditions are

$$\begin{aligned} v_0 &= 0 \text{ at } r = r_c \\ \frac{\partial v_0}{\partial r} &= \frac{\chi}{\sqrt{Da}} \left(u_s + \frac{Da}{\mu(r)|_{arterial\ wall}} \frac{\partial p}{\partial z} \right) \text{ at } r = h(z, t); L_0 \leq z \leq L_0 + L_1 \end{aligned} \quad (58)$$

- The first order deformation equations
 - In the non-stenotic region

$$\Gamma_r(v_1) + \Gamma_r(v_0) - \frac{\eta_{nf}}{\beta^2\mu_{nf}}\Psi_r(v_0) - \frac{1}{\mu_{nf}}\partial_z p - \alpha_{nf}\theta_0 Gr - \alpha_{nf}\sigma_0 Br = 0 \quad (59)$$

with the following conditions at the boundary

$$\begin{aligned} v_1 &= 0 \text{ at } r = h(z, t); z \leq L_0, z \geq L_0 + L_1 \\ v_1 &= 0 \text{ at } r = r_c \end{aligned} \quad (60)$$

- In the stenotic artery region

$$\Gamma_r(v_1) + \Gamma_r(v_0) - \frac{\eta_{nf}}{\beta^2\mu_{nf}}\Psi_r(v_0) - \frac{1}{\mu_{nf}}\partial_z p - \alpha_{nf}\theta_0 Gr - \alpha_{nf}\sigma_0 Br = 0 \quad (61)$$

the corresponding boundary conditions are

$$\begin{aligned} v_1 &= 0 \text{ at } r = r_c \\ \frac{\partial v_1}{\partial r} &= 0 \text{ at } r = h(z, t); L_0 \leq z \leq L_0 + L_1 \end{aligned} \quad (62)$$

- The deformed second order equations
 - In the non-stenotic region

$$\Gamma_r(v_2) - \frac{\eta_{nf}}{\beta^2\mu_{nf}}\Psi_r(v_1) - \frac{1}{\mu_{nf}}\partial_z p - \alpha_{nf}\theta_1 Gr - \alpha_{nf}\sigma_1 Br = 0 \quad (63)$$

with the following conditions at the boundary

$$\begin{aligned} v_2 &= 0 \text{ at } r = h(z, t); z \leq L_0, z \geq L_0 + L_1 \\ v_2 &= 0 \text{ at } r = r_c \end{aligned} \quad (64)$$

➤ In the stenotic artery region

$$\Gamma_r(v_2) - \frac{\eta_{nf}}{\beta^2 \mu_{nf}} \Psi_r(v_1) - \frac{1}{\mu_{nf}} \partial_z p - \alpha_{nf} \theta_1 Gr - \alpha_{nf} \sigma_1 Br = 0 \quad (65)$$

with the following conditions at the boundary

$$\begin{aligned} v_2 &= 0 \text{ at } r = r_c \\ \frac{\partial v_2}{\partial r} &= 0 \text{ at } r = h(z, t); L_0 \leq z \leq L_0 + L_1 \end{aligned} \quad (66)$$

The velocity deformation zeroth order PDE solutions in the non-stenotic and stenotic regions respectively are

$$v_0 = \frac{u_s (\log(r) - \log(r_c))}{(\log(h(z, t)) - \log(r_c))} \quad (67)$$

$$v_0 = \frac{h(z, t) \chi (\log(r) - \log(r_c)) \left(Da \frac{\partial p}{\partial z} + u_s \mu(r)_{h(z, t)} \right)}{\sqrt{Da} \mu(r)_{h(z, t)}} \quad (68)$$

Similarly, $v_1, v_2, v_3, \dots$ are evaluated. The pressure gradient, volumetric flow rate is also derived from the computed primary physical quantities. Further, the resistance to the flow and shear stress is also computed.

3.2.1 Nusselt and Sherwood numbers

The convective temperature/concentration transfer between the fluid and the surrounding surface is useful to understand the convective boundary layer, which is formed due to the exchange of energy, emerging from the apparent temperature/concentration variation. To quantify this, the empirical relation for the physical quantities like Nusselt number and Sherwood number are formulated for temperature and concentration respectively.

To estimate and understand the heat transfer, the Nusselt number (Nu), which is the ratio of convective thermal to the conductive thermal transfer in the fluid across the boundary is calculated whose general expression is

$$Nu = \frac{\left. \frac{\partial (T_1 - T_0)}{\partial r} \right|_{r=0}}{(T_1 - T_0)} \quad (69)$$

The empirical relation in dimensionless form is

$$Nu = - \left(\frac{\partial \theta}{\partial r} \right) \bigg|_{r=0} \quad (70)$$

The ratio of the mass transfer due to convection to the diffusive mass rate (called Sherwood number) is calculated as follows

$$Sh = \frac{\left. \frac{\partial (C_1 - C_0)}{\partial r} \right|_{r=0}}{(C_1 - C_0)} \quad (71)$$

The dimensionless form is $Sh = -(\partial \sigma / \partial r) \big|_{r=0}$.

3.2.2 Estimation of impedance and Wall shear stress

The resistance to the flow directly impacts on the physiological dynamics of the flow and has a direct relation to the fluid rate of flow. The impedance is calculated as

$$\lambda = \frac{\Delta_z p}{Q} \quad (72)$$

where $\Delta_z p$ and Q are the derived quantities and are computed from the equations (23) and $Q = \int_{r_c}^{h(z,t)} 2rv dr$, respectively. It is important to understand physiological fluid dynamics and problems related to it. The shear-stress is one of the physical quantities which affects the fluid dynamics at the arterial wall in constricted arteries. The shear-stress expression in mathematical form is

$$\tau_{rz} = \underbrace{\mu_{nf} \frac{\partial v}{\partial r}}_{\tau_{rz}^S} + \underbrace{\frac{1}{4} \left(\frac{\eta_{nf}}{r^2} \frac{\partial v}{\partial r} - \frac{\eta_{nf}}{r} \frac{\partial^2 v}{\partial r^2} - \eta_{nf} \frac{\partial^3 v}{\partial r^3} \right)}_{\tau_{rz}^A} \quad (73)$$

where, τ_{rz}^A and τ_{rz}^S are corresponding to the separation of stress tensor as asymmetric and symmetric parts. The dimensionless equation corresponding to (73) after dropping dashes is given by

$$\tau_{rz} = \underbrace{\mu_{nf} \frac{\partial v}{\partial r}}_{\tau_{rz}^S} + \underbrace{\frac{1}{4\beta^2} \left(\frac{\eta_{nf}}{r^2} \frac{\partial v}{\partial r} - \frac{\eta_{nf}}{r} \frac{\partial^2 v}{\partial r^2} - \eta_{nf} \frac{\partial^3 v}{\partial r^3} \right)}_{\tau_{rz}^A} \quad (74)$$

4. Results

The article presents the mathematical model on drug delivery through the nanoparticles. The computations are carried out by the homotopy perturbation method and the obtained results are presented to understand the velocity in axial direction, impedance, shear stress at the wall, concentration dispersion and temperature enhancement. And also, the dispersion behavior in the presence of time dependent cardiac phases is presented. The physiological parameters along with the temperature and concentration profiles are presented in their graphical representation by fixing the parametric values as

$$r_0 = 0.152, L = 2, L_1 = 1, L_0 = 0.5, u_s = 0.01, r_c = 0.01, \beta = 0.5,$$

$$N_b = 2, N_t = 0.7, Gr = 2, Br = 1, Da = 0.01, \varepsilon = 0.01, \chi = 0.1, \zeta = 0.05.$$

To validate developed HPM code, the present simulation results are compared with the results obtained by Nadeem and Ijaz [40] using HPM method, the comparison can be observed from the figure 2. In this figure, it can be seen that both curves are in phase. The small difference at the extremes is because of the consideration of the pulsatile pressure gradient in the present problem and the same is not considered by Nadeem and Ijaz [40]. Here, it is noticed that the results obtained by employing the HPM method are in good agreement with the earlier results as far as trend is considered. Hence, it is clearly can be understood that the present methodology is suitable for these kinds of problems, thus validating the method and simulation code used in this analysis.

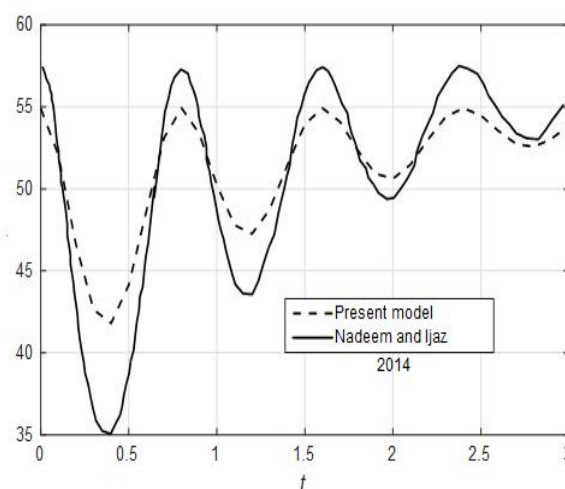


Fig. 2. Comparison of HPM method implementation by Nadeem and Ijaz [38] and present model in case of Impedance at $Br = 2$.

Further, from the resistance to the flow obtained by us for the present model and by Nadeem and Ijaz [40] with the HPM method, it is understood that the pulsatile pressure gradient decreases the resistance to the flow. This highlights the

valuableness of implementing the realistic systole and diastole nature of the heart in the present model.

The axial velocity variation of the blood in the constricted portion at $z = 0.6$ is discussed for the various nanoparticle from r_c to $h(z, t)$, and is as shown in figure 3. It is to be seen that the rate of change in axial velocity is more in case of couple stress fluid with silver nanoparticles than with TiO_2 nanoparticle. Higher the rate of change in velocity leads to more dispersion of the temperature and concentration. Nevertheless, Ag nanoparticles are metallic in nature and are costly as compared with the ceramic nanoparticles, e.g., TiO_2 . Further, the Titanium dioxide nanoparticles are having several biomedical applications as well. TiO_2 is a regular medication transporter in the case of chronic diseases, such as cancer. TiO_2 it is also used as a carrier material for various drugs [41–44]. The chemical equations of the titanium dioxide once activated under ultraviolet (UV) are as follows

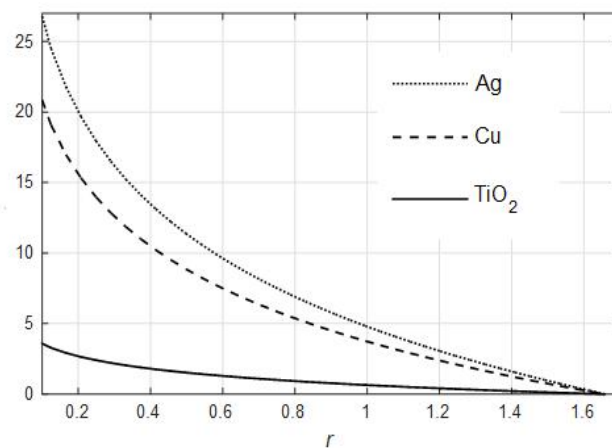
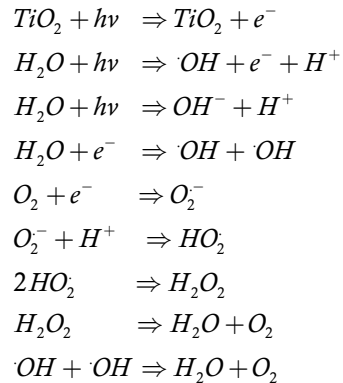


Fig. 3. Effect of nanofluid on axial velocity with respect to radial direction.

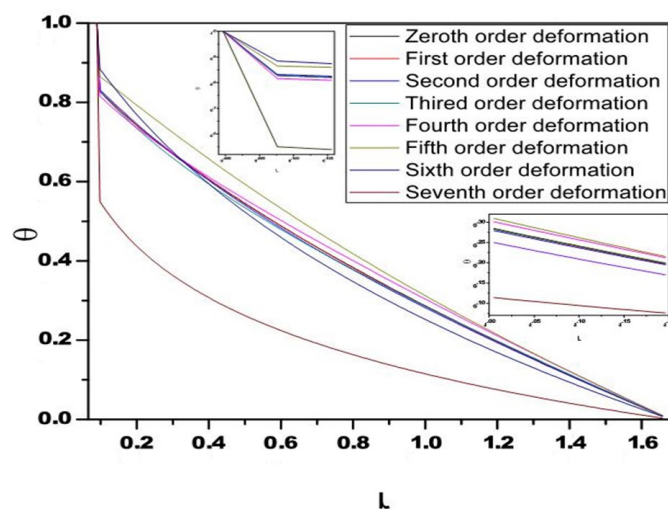


Fig. 4. Temperature change in radial direction for different deformation degrees.

Having understood this, titanium dioxide nanoparticles were well thought out in the present modelling, simulation and

analysis. The article studied up to seven terms in the HPM power series expansion. The behavior of the sequence of partial sums up to the seventh term on temperature is depicted in figure 4. The results show that the curves are swinging above and below, and later from the fifth order deformation onwards the curves are settling to a mean position. It is also observed that the difference is decreasing and the curves are almost coinciding from sixth and seventh order deformation onwards. Comparable outcomes are found in case of axial velocity and concentration. Therefore, the reported problem is studied up to the seventh order deformation.

4.1 Temperature

Across the arterial segment, the temperature dispersion between the arterial wall and catheter surface at $t = 0$, is as shown in figure 5. The rate of dispersion in temperature is decreasing with maximum at catheter surface. High-temperature dispersion near the catheter surface is observed due to high-temperature on the catheter surface. The fact that the dispersion rate is more at high temperature region is also observed from figure 5. It is also observed that, temperature dispersion is less at the first extrema of the considered stenosis than at the second extrema in case of diverging tapered artery. Figure 7 is to understand the effect of the flexible arterial wall on the temperature dispersion. For diverging tapered artery, low-temperature dispersion is observed at the first extremum than at the other extreme of the stenosis. The temperature dispersion is in-phase with the time-dependent nature of the arterial wall. However, Isothermal regions are found at the peaks of the stenosis which is an interesting phenomenon to observe.

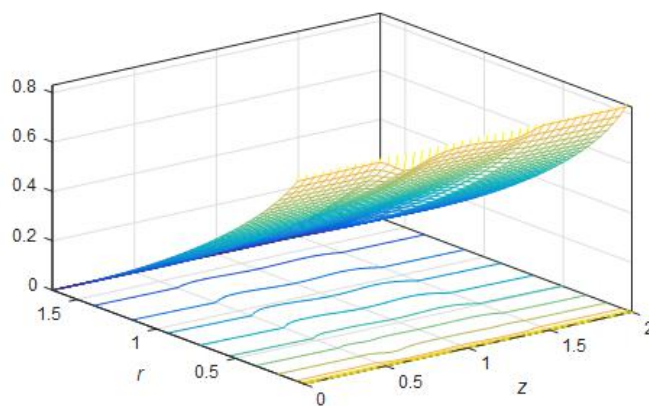


Fig. 5. Temperature change in the radial and axial direction at $\varepsilon = 0.2, \zeta = 0.05$, and $t = 0$.

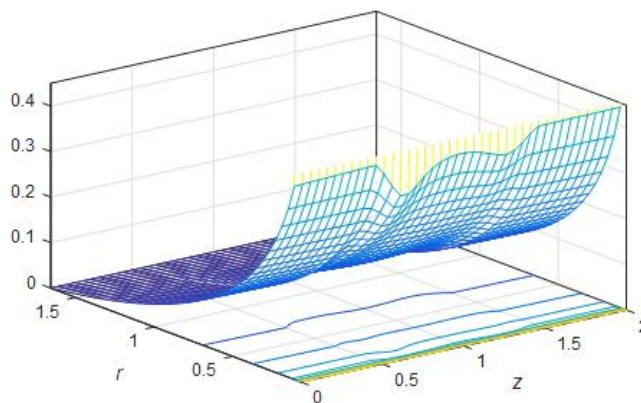


Fig. 6. Concentration change across radial and axial direction at $\varepsilon = 0.2, \zeta = 0.05$, and $t = 0$.

4.2 Concentration

The concentration dispersion up to the arterial wall from the catheter surface at $t = 0$ is shown in figure 5 across the arterial segment. The dispersion profiles here are in similar lines as that of temperature. In particular, the influence of the considered boundary condition, drug concentration is high at the wall of the catheter as compared to the arterial boundary surface. Further, the rate of concentration dispersion is more as compared with the dispersion of temperature at the catheter surface, whereas the dispersion rate is less near the arterial wall. Drug mixing is more at the second peak of the stenosis as compared with the other extremes in case of tapered artery which is diverging. The effect of the non-rigid nature of the arterial wall on the concentration dispersion is depicted in figure 8. The concentration dispersion regions are noticed at the peaks of the stenosis. And also, the concentration dispersion is in-phase with the non-rigid nature of the arterial wall. Further, from figures 5 and 6, it is understood that the temperature dispersion rate is less than

drug dispersion at the wall of the catheter and reversed behavior is recorded when it comes to the arterial wall. These investigations indicate that the amount of drug near the wall of the catheter is more in comparison to the drug near the wall of the artery, i.e., maximum drug is observed in the stenotic region. Thus, the technique of administering the drug with the help of a catheter is very effective and can be done with minimum/no side effects.

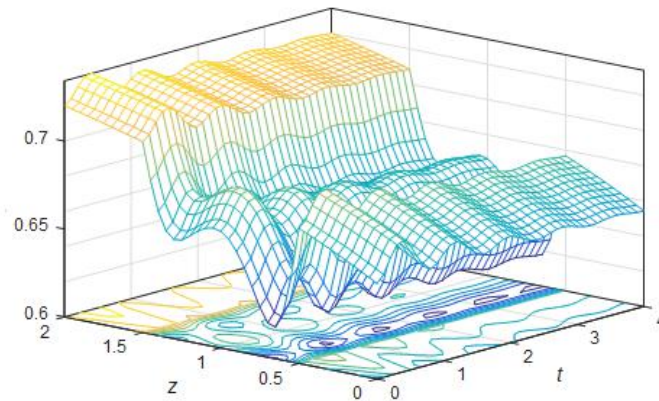


Fig. 7. Effect of wall flexibility on temperature dispersion across the axial direction at $r = 0.1282$, and $\zeta = 0.05$.

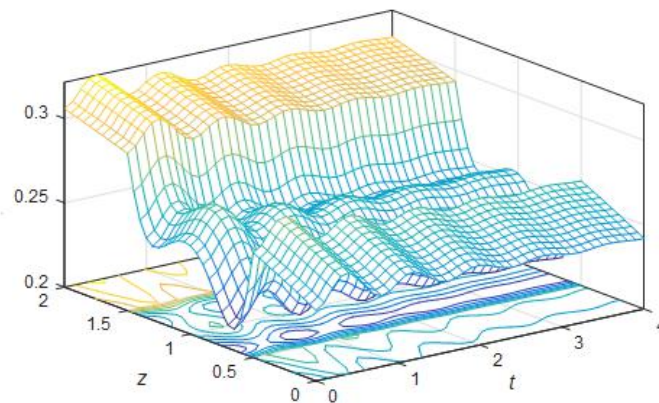


Fig. 8. Effect of wall flexibility on concentration dispersion across the axial direction at $r = 0.1282$, and $\zeta = 0.05$.

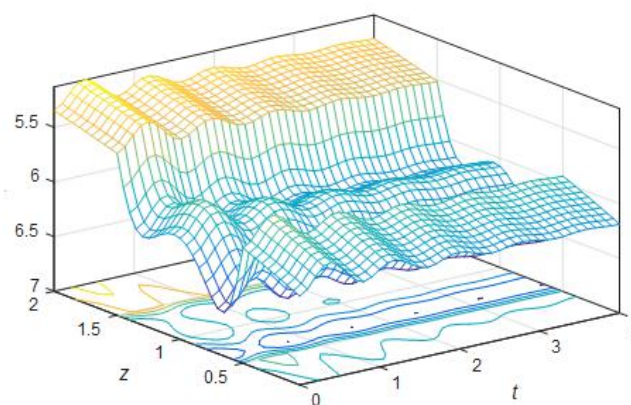


Fig. 9. Axial velocity variation with respect to t in the axial direction when $\varepsilon = 0.05$, and $\zeta = 0.05$.

4.3 Axial velocity

In order to analyze the axial velocity impact due to flexible arterial wall and the catheter surface, axial velocity within the flow domain is studied and the obtained axial velocity contour plots are presented in the form of figure 9. From this figure, it is realized that in the starting portion of the constricted region is possessing less axial velocity as compared with the later part of it when $\zeta = 0.05$. As described in the problem formulation, the stenosis permeability which has been considered allows the flow of modeled fluid through it. The secondary flow is observed prior to the second extrema of

the ω – shaped stenosis. The strong secondary flow is going on declining in the axial direction. However, more impact is observed in the initial part of the stenotic portion. The magnitude of the axial velocity is intensified in the latter portion of the stenosis segment. The demeanor of the velocity profiles described through figure 9 is drawn with respect to the variables t and z . This graphical representation will help us apprehend the velocity variation and impact of temporal dependency of the arterial wall on axial velocity near the surface of the catheter surface and at the wall of the artery in a conclusive manner.

4.4 Resistance to the flow & Wall shear stress

The impedance behavior is studied and graphically depicted in figure 10. Here, the consideration of the slip and no-slip at the arterial wall and the catheter wall respectively can be seen proficiently, because the high impedance at the surface of the catheter and low impedance at the wall of the artery is observed. The high impedance at the first extrema is observed than at the second extrema in the diverging tapered arterial segment. It is worth to mention the physical reason behind such observation. The obstructed fluid in the constriction region quickly moves towards the main flow region and therefore the fluid about the catheter enhances the resistance to the flow in the pre-stenotic region for a transitory period before attaining the minimum in the post stenotic region. Similar reason is attributed to the reduction in resistance in the post stenotic region.

Likewise, from the adjacent chassis, it is reported that in ω – shaped stenosis the shear stress at the wall is maximum at the second extreme than at the first extrema. The wall shear stress occurring at the first extrema tip of the stenosis is the onset of the secondary flow in the stenotic region. This leads the sudden rise in wall shear stress for a brief period near the surface of the artery as shown in figure 11. Understanding the wall shear stress is very critical to find out the things which are happening with the small arteries and arterioles. The pulsatile pressure gradient and the wall shear stress significantly impact on the arteries and these arteries become very firm over time, and they lose their elasticity. These effected arteries when exposed to high blood pressure leads to the rupture on the arterial wall. As a consequence, the saclike widening of an artery resulting from weakening of the artery wall called “aneurysm” occurs. It can be noticed from figures 9, 10 and 11 that the axial velocity is inversely proportional to the impedance and directly related to the wall shear stress, which can be noticed from the formulation thus which justifying the analysis and the results obtained.

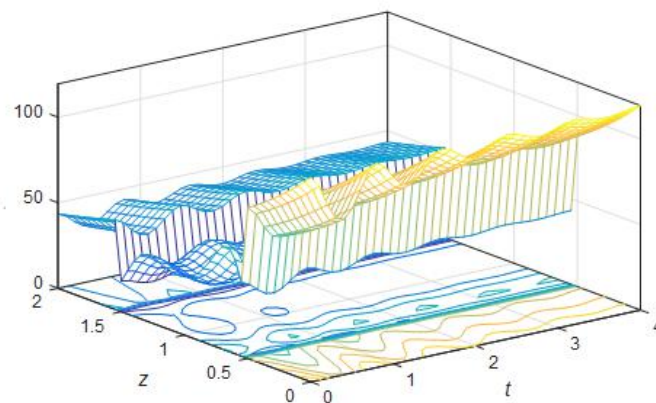


Fig. 10. The change in impedance across the axial direction.

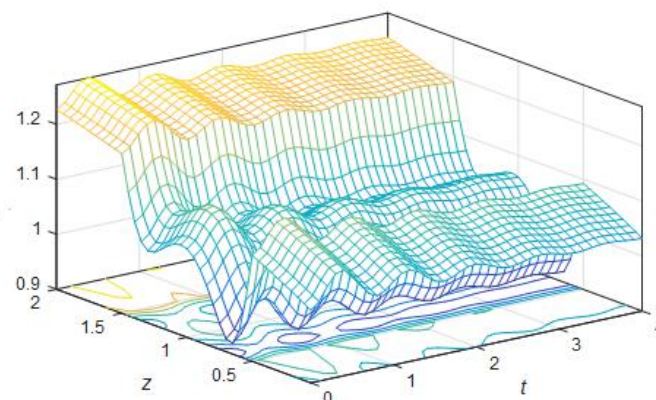


Fig. 11. Shear stress variation across the axial direction.

5. Summary and Conclusion

The article is to understand the drug delivery to constrict region. Therefore, the effect of the elastic nature of the blood vessel on the dispersion of temperature and concentration is considered. The schematic diagram has been represented to investigate the above computational model. The permeable stenosis has been considered, as the stenosis is formed due to the settlement of unabsorbed sodium, platelets, etc. Hemodynamically we modeled as the couple stress fluid together with embedded TiO_2 nanoparticles. TiO_2 has strong oxidizing power, long-term photo stability, low toxicity and ease of availability and therefore, it has been the most widely used ceramic in biological applications. Some of the highlights of the present investigation are listed as follows:

- The homotopy based perturbation method has been used to compute the expressions for the temperature, concentration and the axial velocity. It has been verified and reported that from the fifth-order deformation onwards the temperature solution curves are converging. Therefore, in the present analysis, we studied deformation solutions up to the order of seven. The same is observed in the case of dispersion of concentration and in the profiles of velocity.
- The method used and the developed code have been validated by comparing the results of our case with that of Nadeem and Ijaz [40]. Both of these results are in similar lines, however, the slight variation at the extremes is occurring due to the consideration of the pulsatile pressure gradient. The peristaltic nature of the pressure gradient is enhancing the physical property at the extremes of the stenosis.
- The impact of time-dependent nature of the arterial wall on concentration, temperature, velocity in axial direction, impedance and shear stress at the wall are investigated. The higher temperature dispersion and higher drug dispersion is observed in the post stenosis region in the diverging tapered artery.
- At $r = 0.1282$ level, the temperature dispersed from 60 to 70 percent, whereas the drug is delivered from 23 to 28 percent in the stenotic region. Here, it seems that the drug impacts the stenotic region less, however, its influence is appreciably high in the stenotic (targeted) region due to its extended presence caused by the developed secondary flow.
- Velocity in the axial direction is less in the pre-part of the considered stenosis for the tapered stenosis which is diverging. This shows the high impedance in the pre-stenosis as compared to the post-stenosis region, which is in line with the authors' investigation in [5].
- In the diverging tapered artery, a secondary flow is observed at the first extrema of the ω – shaped stenotic region due to which the drug finds difficulty in reaching the targeted stenotic region. However, the secondary flow will arrest the flow in it, therefore, the drug will stay for more time in the stenotic region once it is released from the nanoparticles.
- Impedance for the flow nearer to the wall of the catheter is observed to be high due to the no-slip boundary condition.

The aforementioned results could be used to realize the improvement of temperature and concentration dispersion, which could be used to deliver the drug in the treatment of abnormal narrowing of a blood vessel. Further, the related finding in the present physical model will act as a prototype for pharmaceutical and biomedical scientists who are involved in R&D. This mathematically oriented study could act as a prototype in biomedical engineering for the cure of vascular-related diseases using angioplasty through the Balloon coated with the drug.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

L	Length of the considered arterial segment	χ	Material property (surface roughness)
L_1	Length from the left to starting portion of stenosis	ϕ	Volume fraction
L_0	Length of the stenosis spread over the region	ε	Maximum height of the stenosis
r_c	Catheter radius	θ°	Tapering angle
$h(z, t)$	Radius of the fluid flow region	$\zeta (= \tan(\theta^\circ))$	Tapering parameter and
u, v	Radial and Axial component of velocity	θ, σ	Local mean temp. and concentration
p	Pressure across the region	μ	Dynamic viscosity
Re	Reynolds number	η	Couple stress viscosity



Da	Darcy number	β	Length dependent parameter
Gr	Thermal Grashof number	λ	Impedance
Br	Solutal Grashof number	τ	Shear stress
N_t	Thermophoresis parameter	Subscript	
N_b	Brownian motion parameter	nf	Property of the nanofluid
Nu	Nusselt's number	f	Property of the base fluid
Sh	Sherwood number	p	Property of the immersed particles

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