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Vibration Analysis of Different Types of Porous FG Conical Sandwich Shells in Various Thermal Surroundings

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Abstract. Vibration behavior of different types of porous functionally graded (FG) conical sandwich shells are investigated based on a modified high order sandwich shells theory for the first time. Sandwich shell includes FG face sheets covering a homogeneous core and the second one includes homogeneous face sheets and a FG core. Power law rule modified by considering two types of porosity distributions is used to model the functionally graded materials. All materials are temperature dependent and uniform, linear and nonlinear temperature distributions are used to model the effect of the temperature variation in the sandwiches. Governing equations are obtained by the Hamilton's energy principle and solved with Galerkin method. To verify the results, they are compared with ones achieved by finite element method obtained by Abaqus software for special cases with the results in literatures.

Keywords: Conical sandwich shell, Porosity, FG core, Temperature Dependent, Vibration.

1. Introduction

After the primary consideration sandwich constructions in research paper in 1944, these modern structures with high flexural stiffness to weight ratio have become favorite structures among the researchers. Sandwich constructions include two faces to resist the in-plane and bending loads and a core to resist the transverse shear loads and maintain the faces distance [1].

Application of classical composite material in high temperature environments at points resulted in the failure, delamination and thermal stress concentration. Researchers proposed functionally graded materials (FGMs) to overcome these problem. FGMs are inhomogeneous microscopic materials which are graded from a metal surface to a ceramic one gradually [2]. Chen et al. by applying the FGM in the faces of the sandwich plates studied the vibration and buckling behavior in the thermal condition. A power law rule was considered to model the material properties [3]. Benlahcen et al. by using FGM in the plates studied the buckling behavior of these structures in a simply supported condition [4]. Khayat et al. investigated the free vibration of FG cylindrical shells. The material properties of FGM varied gradually in thickness direction in accordant to power law rule [5].

During the production process of FGMs, some micro voids appear which affect the material properties.

Consideration of these porosities in the modeling of these materials is a development in the researches. Arefi et al. investigated the free vibration of sandwich nanoplates with porous FG core. Even and uneven porosity distributions were considered [6]. Akbas investigated the post-buckling behavior of porous FG beams. The power law rule was modified by considering different types of porosity [7]. Benferhat et al. studied the static behavior of porous FG plates. Since micro voids appear in FG material, they modified the power law rule by considering porosity [8].

There are several approaches to investigate the mechanical behavior of sandwich structures such as finite element model, shear deformation theories, 3D elastic theory and energy methods [9]. The core is a flexible layer in the thickness direction, but the height of the core is assumed constant in these theories. In classical theories, the localized effects in the core cannot be calculated, so to consider these effects, Frostig et al. presented a high order theory [10]. Salami [11] studied [12] the free vibration of sandwich beams by using a high order sandwich panel theory. Frostig and Kardomateas studied the thermomechanical responses of sandwich plates by applying a high order sandwich panel theory. Mohammadi et al. investigated the responses of low velocity impact in the sandwich plates with FG face sheets by using a high order sandwich plate theory [13]. Mohammadi and Khalili studied the behavior of sandwich beams with FG face sheets under indentation loading by a high order sandwich beams theory [14].

Since material properties are temperature dependent, distribution of the temperature in the thickness direction of the structure is important to model the mechanical behavior of the sandwich panels. A review in literatures shows there are limited researches which have taken temperature dependent behavior of materials in both faces and core concurrently into account. Van Tung studied the thermal buckling and bending behavior of FG sandwich plates with temperature dependent material properties in faces and core [15]. Khalili and Mohammadi studied the vibration of FG sandwich plates with temperature dependent material in faces and core in a uniform temperature distribution [16]. Duc et al. [17] studied the nonlinear dynamic and vibration of FG cylindrical sandwich panels based on a third order shear deformation shell theory. Material properties were temperature dependent. Fazzolari studied the free vibration and thermal stability of FG sandwich plates by using Ritz method. They considered the uniform, linear and nonlinear temperature distribution in the thickness direction [18]. Talebitooti investigated the effects of thermal load on the vibration of a rotating FG conical shells. Temperature distributed nonlinearly within the thickness direction [19].

Conical sandwich shells are important kinds of structural components. They have been applied in advanced industries such as aerospace, mechanical and nuclear engineering [20]. By using FGMs which have high thermal strength within the conical shells, application of these structures significantly have been increased. Despite the importance of these modern structures, due to the complex set of partial differential equations, there are little literature about conical sandwich shells comparing with cylindrical shells and circular plates [21]. Recently, FG truncated conical shells have been applied in military aircraft propulsion system, missile bodies, pressure vessels, oil tanks, nuclear reactors, rockets, water ducts, pipelines and casing pipes, process equipment, fuselage structures in the region of the exhaust ducts/propeller plane and rotary dryers [19, 22-26].

Studying the vibration behavior of conical shells has been of interest to researchers. Liu and Li used Galerkin method to solve the equations which were obtained by Hamilton's principle for nonlinear free vibration of conical sandwich shells [27]. By using love theory and generalized differential quadrature (GDQ), Shu studied the vibration of isotropic conical shells [28]. Tornabene et al. surveyed the dynamic behavior of FG conical shells and annular plates based on the first-order shear deformation theory (FSDT). The governing equations of motion are discretized by means of the GDQ method [29]. Sofiyev investigated the vibration behavior of FG conical shells based on large deformation theory and analyzed the frequency responses with the Superposition method, Galerkin method and Harmonic balance method [30]. Najafov et al. studied the vibration behavior of FG truncated conical shell based on von-Karman-Donnell type nonlinear kinematic [31]. Heydarpour et al. employed FSDT and DQM to analyze the free vibration of rotated truncated conical shells which made of carbon nanotube reinforced composite [32]. Sofiyev and Kuruoglu investigated the vibration of FG conical shell under mixed conditions by means of Airy stress function method [24]. Sofiyev studied the parametric vibration of FG truncated conical shells subjected to the different pressure loadings based on FSDT [20]. Sofiyev and Osmancebioglu by applying FSDT studied the vibration of sandwich truncated conical shells with a FG coatings [33]. By using Ansys software, Mouli et al. prepared a finite element model to investigate the free vibration of FG conical shells in a fully clamped condition [34]. Based on the FSDT and Donnell's theory, Kiani et al. studied the free vibration of composite conical panels which reinforced with FG carbon nanotube [35]. Sofiyev investigated the vibration behavior of laminated conical shells by employing FSDT and Galerkin method [36]. Shakouri studied the vibration behavior of temperature dependent FG rotating conical shells by using Donnell shell theory in thermal environments [37]. Sofiyev in a review paper gathered some researches on vibration and buckling of FG shells [38].

As a result of review in the accessible literatures, it have been found that there is no studying on the vibration of sandwich conical shells based on a modified high order sandwich shells theory in different thermal surroundings and considering the temperature dependent material for both faces and core, FG compressible core, FG face sheets and porosity, concurrently. In this study, for the first time, by applying a high order theory, which is modified by considering the flexibility of the core in the thickness direction, vibration behavior of two kinds of truncated conical sandwich shells are investigated in the uniform, linear and nonlinear temperature distributions. In first type, sandwiches consist of two FG faces which cover a homogeneous core, namely, type-I and in the second type, sandwiches with FG core which surrounded by two homogeneous face sheets, namely, type-II. Two types of porosity are considered in the FGMs properties. FG material properties are temperature and location dependent which graded in according to power law rules that include the volume fraction of the porosities. The homogeneous materials are temperature dependent, too. High

order stresses and thermal stress resultants, in plane stresses and thermal stresses of the core and face sheets are considered at the same time. Nonlinear strains are used for both mechanical and thermal stresses to obtain the more accurate equations that causes the problem be more complicated. Boundary condition is simply supported and equations are derived based on the Hamilton's energy principle. To obtain the frequencies, a Galerkin method is applied. In order to validate the present approach, the results of this analytical approach are compared with the numerical results which are obtained by Abaqus software and for a special case are compared with some literatures. Finally, the effects of the temperature variation, volume fraction distribution of FG face sheets and FG core, porosity and some geometrical effects on the vibration characteristics of defined sandwich shells are investigated.

2. Fundamental Equations

In this study, FGMs are used in the face sheets and the core in two types of sandwiches. First, a sandwich with FG face sheets and a homogeneous core, named, type-I and second, a sandwich with a FG core and two homogeneous face sheets, named, type-II. Since these sandwich structures are applied in high temperature conditions, all material properties should be assumed as temperature dependent. This dependency is expressed as a nonlinear function of temperature as follows [39]:

$$P = P_0 (P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (1)$$

where "P"s are unique coefficients of temperature for each material; and $T = T_0 + \Delta T$, which T_0 is the room temperature. A power law rule is applied to model the properties of the FGMs which usually include ceramic and metal and vary gradually in the thickness direction. By considering two kinds of porosity distribution which appear in the manufacturing process, the power law rule are modified to approach an accurate prediction of material properties. The first one is even porosity distribution that modifies the power law rule as follows [40]:

$$P_j(z_j, T) = g(z_j) P_{ce}^j(T) + [1 - g(z_j)] P_m^j(T) - (P_{ce}^j(T) + P_m^j(T)) \frac{\zeta}{2}, \quad j = (o, i, c) \quad (2)$$

$$g(z_o) = \left(\frac{\frac{h_o}{2} - z_o}{h_o}\right)^q; \quad g(z_i) = \left(\frac{\frac{h_i}{2} + z_i}{h_i}\right)^q; \quad g(z_c) = \left(\frac{\frac{h_c}{2} - z_c}{h_c}\right)^q \quad (3)$$

where "q" is the constant power law index; " ζ " is the porosity volume fraction; and subscripts "o", "i" and "c" refer to the outer and inner faces and the core, respectively. In the second approximation, it is considered that the porosities are distributed in the middle area of the FG layers and by approaching to the edges, they decrease and tend to the zero. Therefore, the equation of the material properties in the uneven case modified as follows [40]:

$$P_j(z_j, T) = g(z_j) P_{ce}^j(T) + [1 - g(z_j)] P_m^j(T) - (P_{ce}^j(T) + P_m^j(T)) \frac{\zeta}{2} \left(1 - \frac{2|z|}{h}\right) \quad (4)$$

To model the temperature variation in the thickness direction, uniform, linear and nonlinear temperature distributions are considered. In the linear case, the temperature distributions are assumed linearly through the thickness of each layer accordant to eqs. (5-7).

$$T_o(z_o) = r_1 z_o + r_2 \quad (5)$$

$$T_c(z_c) = r_3 z_c + r_4 \quad (6)$$

$$T_i(z_i) = r_5 z_i + r_6 \quad (7)$$

where "r"s are the unknown coefficients of the polynomials which obtained with six thermal boundary conditions in eqs. (8) and (9).

$$T_o(-h_o/2) = T_o; \quad T_o(h_o/2) = T_c(-h_c/2); \quad k_o(h_o/2, T_{io}) \frac{\partial T_o}{\partial z_o} = k_c(-h_c/2, T_{io}) \frac{\partial T_c}{\partial z_c} \quad (8)$$

$$T_i(h_i/2) = T_i; \quad T_c(h_c/2) = T_i(-h_i/2); \quad k_i(-h_i/2, T_{ii}) \frac{\partial T_i}{\partial z_i} = k_c(h_c/2, T_{ii}) \frac{\partial T_c}{\partial z_c} \quad (9)$$

where "k" is the thermal conductivity; " T_o " and " T_i " are the temperatures of outer and inner surfaces of the sandwiches; " T_{io} " and " T_{ii} " are the temperatures of the top and bottom interfaces of the core with outer and inner face sheets. To model the nonlinear temperature distribution, the steady state, one dimensional heat conduction equations are considered for two face sheets and the core, separately. The nonlinear temperature rises equations of the face sheets and

the core can be considered as [19]:

$$-\frac{d}{dz_o} \left[k_o \frac{dT_o}{dz_o} \right] = 0 \quad (10)$$

$$-\frac{d}{dz_c} \left[k_c \frac{dT_c}{dz_c} \right] = 0 \quad (11)$$

$$-\frac{d}{dz_i} \left[k_i \frac{dT_i}{dz_i} \right] = 0 \quad (12)$$

It should be noted that in sandwich type-I, the thermal conductivity in the FG faces are both temperature and location dependent and in the homogeneous core is just temperature dependent. Also, in sandwich type-II, the thermal conductivity in the FG core are both temperature and location dependent and in the homogeneous FG faces is just temperature dependent. In this study, all material properties that appear in the following, conform this rule. To solve the equations, the boundary conditions are as follows:

$$T_c(h_c/2) = T_{ii}; \quad T_c(-h_c/2) = T_{io} \quad (13)$$

$$T_o(h_o/2) = T_{io}; \quad T_o(-h_o/2) = T_o; \quad k_o \frac{\partial T_o}{\partial z_o} \Big|_{z_o=h_o/2} = k_c \frac{\partial T_c}{\partial z_c} \Big|_{z_c=-h_c/2} \quad (14)$$

$$T_i(-h_i/2) = T_{ii}; \quad T_i(h_i/2) = T_i; \quad k_i \frac{\partial T_i}{\partial z_i} \Big|_{z_i=-h_i/2} = k_c \frac{\partial T_c}{\partial z_c} \Big|_{z_c=h_c/2} \quad (15)$$

By using eqs. (10-12) and boundary conditions in eqs. (13-15), the nonlinear temperature distribution in the FG layers can be determined as eq. (24) of reference [19] and the temperature rise in the homogeneous can be obtained by eq. (16) as follows:

$$T_j(z_j) = \left(\frac{T_{ii} - T_{oi}}{h_j/2} \right) \int_{-h_j/2}^{z_j} \frac{dz_j}{k_j(T_{j1})} + T_{oi}; \quad (j = i, o, c) \quad (16)$$

To study the vibration behavior of two types of conical sandwich shells, and obtain the governing equations, Hamilton's energy principle is applied which include the variation of the kinetic energy, δK , and the strain energy variation, δU as follows [41]:

$$\int_{t_1}^{t_2} (-\delta K + \delta U) dt = 0 \quad (17)$$

The variation of the kinetic energy is calculated as following:

$$\begin{aligned} \int_{t_1}^{t_2} \delta K dt = & - \int_{t_1}^{t_2} \left\{ \int_0^L \int_0^{2\pi} \int_{-\frac{h_o}{2}}^{\frac{h_o}{2}} \rho_o (\ddot{u}_o \delta u_o + \ddot{v}_o \delta v_o + \ddot{w}_o \delta w_o) r d\chi d\psi dz_o + \int_0^L \int_0^{2\pi} \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}} \rho_i (\ddot{u}_i \delta u_i + \ddot{v}_i \delta v_i + \ddot{w}_i \delta w_i) r d\chi d\psi dz_i \right. \\ & \left. + \int_0^L \int_0^{2\pi} \int_{-\frac{h_c}{2}}^{\frac{h_c}{2}} \rho_c (\ddot{u}_c \delta u_c + \ddot{v}_c \delta v_c + \ddot{w}_c \delta w_c) r d\chi d\psi dz_c \right\} dt \end{aligned} \quad (18)$$

where $(\ddot{\cdot})$ denotes the second derivative with respect to time; The density is " ρ " which in the functionally graded layers is the function of the displacement and the temperature, and in the homogeneous layer is only dependent on temperature; the meridional, circumferential and normal directions of the sandwich shells are shown by χ , ψ and z , respectively. The variation of the total strain energy includes all mechanical and thermal stresses and linear and nonlinear strains of the layers of sandwich shell that make the mechanical and thermal energies. In addition, the compatibility conditions which assumed the core to be stucked to the faces completely at the interfaces of the core and the face-sheets, are attended in the Hamilton's principle as the constraints by using six Lagrange multipliers. Finally, by considering the in-plane stresses of the core in this study, δU is as follows:

$$\begin{aligned}
\delta U = & \int_{V_o} ((\sigma_{\chi\chi}^o + \sigma_{\chi\chi}^{oT}) \delta \varepsilon_{\chi\chi}^o + (\sigma_{\psi\psi}^o + \sigma_{\psi\psi}^{oT}) \delta \varepsilon_{\psi\psi}^o + \tau_{\chi\psi}^o \delta \gamma_{\chi\psi}^o + \tau_{\chi z}^o \delta \gamma_{\chi z}^o + \tau_{\psi z}^o \delta \gamma_{\psi z}^o) dV_o \\
& + \int_{V_i} ((\sigma_{\chi\chi}^i + \sigma_{\chi\chi}^{iT}) \delta \varepsilon_{\chi\chi}^i + (\sigma_{\psi\psi}^i + \sigma_{\psi\psi}^{iT}) \delta \varepsilon_{\psi\psi}^i + \tau_{\chi\psi}^i \delta \gamma_{\chi\psi}^i + \tau_{\chi z}^i \delta \gamma_{\chi z}^i + \tau_{\psi z}^i \delta \gamma_{\psi z}^i) dV_i \\
& + \int_{V_c} ((\sigma_{\chi\chi}^c + \sigma_{\chi\chi}^{cT}) \delta \varepsilon_{\chi\chi}^c + (\sigma_{\psi\psi}^c + \sigma_{\psi\psi}^{cT}) \delta \varepsilon_{\psi\psi}^c + (\sigma_{zz}^c + \sigma_{zz}^{cT}) \delta \varepsilon_{zz}^c + \tau_{\chi\psi}^c \delta \gamma_{\chi\psi}^c + \tau_{\chi z}^c \delta \gamma_{\chi z}^c + \tau_{\psi z}^c \delta \gamma_{\psi z}^c) dV_c \\
& + \delta \int_0^L \int_0^{2\pi} [\lambda_{\chi o} (u_o(z_o = h_o/2) - u_c(z_c = -h_c/2)) + \lambda_{\psi o} (v_o(z_o = h_o/2) - v_c(z_c = -h_c/2)) \\
& + \lambda_{zo} (w_o(z_o = h_o/2) - w_c(z_c = -h_c/2)) + \lambda_{\chi i} (u_c(z_c = h_c/2) - u_i(z_i = -h_i/2)) \\
& + \lambda_{\psi i} (v_c(z_c = h_c/2) - v_i(z_i = -h_i/2)) + \lambda_{zi} (w_c(z_c = h_c/2) - w_i(z_i = -h_i/2))] r dr d\psi
\end{aligned} \tag{19}$$

where $\sigma_{\chi\chi}$, $\sigma_{\psi\psi}$ and $\tau_{\chi\psi}$ display the in-plane normal and shear stresses; $\varepsilon_{\chi\chi}$, $\varepsilon_{\psi\psi}$ and $\gamma_{\chi\psi}$ are the in plane normal and shear strains of the layers; $\sigma_{\chi\chi}^T$ and $\sigma_{\psi\psi}^T$ express the thermal stresses; σ_{zz}^c and ε_{zz}^c present the lateral normal stress and strain in the core; $\tau_{\chi z}^c$, $\tau_{\psi z}^c$, $\gamma_{\chi z}^c$ and $\gamma_{\psi z}^c$ declare shear stresses and shear strains in the core; and λ_{χ} , λ_{ψ} and λ_z are the Lagrange multipliers at the face sheet-core interfaces.

The strain components of the face sheets and the core are defined in accordance with von Karman nonlinear relations as follows:

$$\varepsilon_{\chi\chi}^j = u_{j,\chi} + \frac{1}{2} (w_{o,\chi})^2, \quad j = (c, i, o) \tag{20}$$

$$\varepsilon_{\psi\psi}^j = \frac{1}{r} [\nu_{j,\psi} + u_{j,\psi} \sin \gamma + w_j \cos \gamma] + \frac{1}{2r^2} (w_{o,\psi})^2 \tag{21}$$

$$\varepsilon_{zz}^j = w_{j,z} \tag{22}$$

$$\gamma_{\chi\psi}^j = \frac{1}{r} [u_{j,\psi} - v_j \sin \gamma] + \nu_{j,\chi} + \frac{1}{r} [w_{o,\chi} w_{o,\psi}] \tag{23}$$

$$\gamma_{\chi z}^j = u_{j,z} + w_{j,\chi} \tag{24}$$

$$\gamma_{\psi z}^j = \frac{1}{r} [w_{j,\psi} - v_j \cos \gamma] + \nu_{j,z} \tag{25}$$

Based on a high order sandwich shell theory, distinct displacement fields should be considered for each layer. So, the FSDT is employed to model the displacement fields of the face-sheets in two types of sandwiches:

$$u_j(\chi, \psi, z_j, t) = u_{0j}(\chi, \psi, t) + z_j \phi_{\chi}^j(\chi, \psi, t), \quad (j=o, i) \tag{26}$$

$$v_j(\chi, \psi, z_j, t) = v_{0j}(\chi, \psi, t) + z_j \phi_{\psi}^j(\chi, \psi, t) \tag{27}$$

$$w_j(\chi, \psi, z_j, t) = w_{0j}(\chi, \psi, t); \quad j = (o, i) \tag{28}$$

where subscript "0" denotes values with correspondence to the middle surface of the layers; and " ϕ "s are the rotation of the normal to the middle surface. Also, cubic patterns are applied to model the kinematic relations of the core with twelve unknown coefficients for the in-plane and vertical displacement components as follows:

$$u_c(\chi, \psi, z_j, t) = u_0(\chi, \psi, t) + z_c u_1(\chi, \psi, t) + z_c^2 u_2(\chi, \psi, t) + z_c^3 u_3(\chi, \psi, t) \tag{29}$$

$$v_c(\chi, \psi, z_j, t) = v_0(\chi, \psi, t) + z_c v_1(\chi, \psi, t) + z_c^2 v_2(\chi, \psi, t) + z_c^3 v_3(\chi, \psi, t) \tag{30}$$

$$w_c(\chi, \psi, z_j, t) = w_0(\chi, \psi, t) + z_c w_1(\chi, \psi, t) + z_c^2 w_2(\chi, \psi, t) + z_c^3 w_3(\chi, \psi, t) \tag{31}$$

For expanding the eqs. (18) and (19), some basic relations and expresses must be introduced to determine the stress and moment resultants for the face sheets as:

$$\begin{bmatrix} N_{\chi\chi}^j \\ N_{\psi\psi}^j \\ N_{\chi\psi}^j \\ M_{\chi\chi}^j \\ M_{\psi\psi}^j \\ M_{\chi\psi}^j \end{bmatrix} = \begin{bmatrix} A_{11}^j & A_{12}^j & 0 & B_{11}^j & B_{12}^j & 0 \\ A_{12}^j & A_{22}^j & 0 & B_{12}^j & B_{22}^j & 0 \\ 0 & 0 & A_{66}^j & 0 & 0 & B_{66}^j \\ B_{11}^j & B_{12}^j & 0 & D_{11}^j & D_{12}^j & 0 \\ B_{12}^j & B_{22}^j & 0 & D_{12}^j & D_{22}^j & 0 \\ 0 & 0 & B_{66}^j & 0 & 0 & D_{66}^j \end{bmatrix} \begin{bmatrix} \varepsilon_{\chi\chi}^0 \\ \varepsilon_{\psi\psi}^0 \\ \varepsilon_{\theta\theta}^0 \\ k_{\chi\chi} \\ k_{\psi\psi} \\ k_{\chi\psi} \end{bmatrix} - \begin{bmatrix} N_{\chi\chi}^{Tj} \\ N_{\psi\psi}^{Tj} \\ 0 \\ M_{\chi\chi}^{Tj} \\ M_{\psi\psi}^{Tj} \\ 0 \end{bmatrix}, j = (o, i) \quad (32)$$

$$\begin{bmatrix} Q_{sj}^0 \\ Q_{\theta j}^0 \end{bmatrix} = \frac{\pi^2}{12} \begin{bmatrix} A_{44}^j & 0 \\ 0 & A_{55}^j \end{bmatrix} \begin{bmatrix} \gamma_{szj}^0 \\ \gamma_{\theta sj}^0 \end{bmatrix} + \frac{\pi^2}{12} \begin{bmatrix} B_{44}^j & 0 \\ 0 & B_{55}^j \end{bmatrix} \begin{bmatrix} \gamma_{szj}^1 \\ \gamma_{\theta sj}^1 \end{bmatrix} \quad (33)$$

$$Q_{\theta j}^1 = \frac{\pi^2}{12} B_{55}^j \gamma_{\theta sj}^0 + \frac{\pi^2}{12} D_{55}^j \gamma_{\theta sj}^1$$

where the "N"s and "N^T"s depict the in-plane stress resultants and thermal stress resultants, respectively; "Q"s declare the out of plane shear stress resultants; "M"s and "M^T"s refer to the moment resultants and thermal moment resultants, respectively; and, the constant coefficients "A", "B" and "D" indicate the stretching stiffnesses, bending-stretching stiffnesses, and bending stiffnesses, respectively, which defined in reference [41]. By substituting the kinematic relations and compatibility conditions, after some algebraic operations, the twenty eight equations are obtained. These equations include twenty eight unknowns: ten displacement unknowns for face sheets, twelve displacement unknowns for the core, and six Lagrange multipliers as follows:

$$-N_{\chi\chi}^o \sin\beta - rN_{\chi\chi,\chi}^o - N_{\chi\chi}^{oT} \sin\beta - rN_{\chi\chi,\chi}^{oT} + N_{\psi\psi}^o \sin\beta + N_{\psi\psi}^{oT} \sin\beta - N_{\chi\psi,\psi}^o + r\lambda_{\chi o} = +I_{0o} \ddot{r}u_0^o + I_{1o} r \ddot{\phi}_\chi^o \quad (34)$$

$$-N_{\psi\psi,\psi}^o - N_{\psi\psi,\psi}^{oT} - 2N_{\chi\psi}^o \sin\beta - rN_{\chi\psi,\chi}^o - Q_{\psi 0}^o \cos\beta + r\lambda_{\psi o} = +I_{0o} r \ddot{v}_0^o + I_{1o} r \ddot{\phi}_\psi^o \quad (35)$$

$$\begin{aligned} & -N_{\chi\chi}^o \sin\beta w_{0,\chi}^o - N_{\chi\chi,\chi}^o r w_{0,\chi}^o - N_{\chi\chi}^o r w_{0,\chi\chi}^o - N_{\chi\chi}^{oT} \sin\beta w_{0,\chi}^o - N_{\chi\chi,\chi}^{oT} r w_{0,\chi}^o - N_{\chi\chi}^{oT} r w_{0,\chi\chi}^o + N_{\psi\psi}^o \cos\beta \\ & -r^{-1} N_{\psi\psi,\psi}^o w_{0,\psi}^o - r^{-1} N_{\psi\psi}^o w_{0,\psi\psi}^o + N_{\psi\psi}^{oT} \cos\beta - r^{-1} N_{\psi\psi,\psi}^{oT} w_{0,\psi}^o - r^{-1} N_{\psi\psi}^{oT} w_{0,\psi\psi}^o - N_{\chi\psi,\chi}^o w_{0,\psi}^o - 2N_{\chi\psi}^o w_{0,\chi\psi}^o \\ & -N_{\chi\psi,\psi}^o w_{0,\chi}^o - Q_{\chi 0}^o \sin\beta - rQ_{\chi 0,\chi}^o - Q_{\psi 0,\psi}^o + r\lambda_{zo} = +I_{0o} r \ddot{w}_0^o \end{aligned} \quad (36)$$

$$-M_{\chi\chi}^o \sin\beta - rM_{\chi\chi,\chi}^o - M_{\chi\chi}^{oT} \sin\beta - rM_{\chi\chi,\chi}^{oT} + M_{\psi\psi}^o \sin\beta + M_{\psi\psi}^{oT} \sin\beta - M_{\chi\psi,\psi}^o + rQ_{\chi 0}^o + r \frac{h_o}{2} \lambda_{\chi o} = +I_{1o} \ddot{r}u_0^o + I_{2o} r \ddot{\phi}_\chi^o \quad (37)$$

$$-M_{\psi\psi,\psi}^o - M_{\psi\psi,\psi}^{oT} - 2M_{\chi\psi}^o \sin\beta - rM_{\chi\psi,\chi}^o - Q_{\psi 1}^o \cos\beta + rQ_{\psi 0}^o + r \frac{h_o}{2} \lambda_{\psi o} = +I_{1o} r \ddot{v}_0^o + I_{2o} r \ddot{\phi}_\psi^o \quad (38)$$

$$-N_{\chi\chi}^i \sin\beta - rN_{\chi\chi,\chi}^i - N_{\chi\chi}^{iT} \sin\beta - rN_{\chi\chi,\chi}^{iT} + N_{\psi\psi}^i \sin\beta + N_{\psi\psi}^{iT} \sin\beta - N_{\chi\psi,\psi}^i - r\lambda_{\chi i} = +I_{0i} \ddot{r}u_0^i + I_{1i} r \ddot{\phi}_\chi^i \quad (39)$$

$$-N_{\psi\psi,\psi}^i - N_{\psi\psi,\psi}^{iT} - 2N_{\chi\psi}^i \sin\beta - rN_{\chi\psi,\chi}^i - Q_{\psi 0}^i \cos\beta - r\lambda_{\psi i} = +I_{0i} r \ddot{v}_0^i + I_{1i} r \ddot{\phi}_\psi^i \quad (40)$$

$$\begin{aligned} & -N_{\chi\chi}^i \sin\beta w_{0,\chi}^i - N_{\chi\chi,\chi}^i r w_{0,\chi}^i - N_{\chi\chi}^i r w_{0,\chi\chi}^i - N_{\chi\chi}^{iT} \sin\beta w_{0,\chi}^i - N_{\chi\chi,\chi}^{iT} r w_{0,\chi}^i - N_{\chi\chi}^{iT} r w_{0,\chi\chi}^i + N_{\psi\psi}^i \cos\beta \\ & -r^{-1} N_{\psi\psi,\psi}^i w_{0,\psi}^i - r^{-1} N_{\psi\psi}^i w_{0,\psi\psi}^i + N_{\psi\psi}^{iT} \cos\beta - r^{-1} N_{\psi\psi,\psi}^{iT} w_{0,\psi}^i - r^{-1} N_{\psi\psi}^{iT} w_{0,\psi\psi}^i - N_{\chi\psi,\chi}^i w_{0,\psi}^i - 2N_{\chi\psi}^i w_{0,\chi\psi}^i \\ & -N_{\chi\psi,\psi}^i w_{0,\chi}^i - Q_{\chi 0}^i \sin\beta - rQ_{\chi 0,\chi}^i - Q_{\psi 0,\psi}^i - r\lambda_{zi} = +I_{0i} r \ddot{w}_0^i \end{aligned} \quad (41)$$

$$-M_{\chi\chi}^i \sin\beta - rM_{\chi\chi,\chi}^i - M_{\chi\chi}^{iT} \sin\beta - rM_{\chi\chi,\chi}^{iT} + M_{\psi\psi}^i \sin\beta + M_{\psi\psi}^{iT} \sin\beta - M_{\chi\psi,\psi}^i + rQ_{\chi 0}^i + r \frac{h_i}{2} \lambda_{\chi i} = +I_{1i} \ddot{r}u_0^i + I_{2i} r \ddot{\phi}_\chi^i \quad (42)$$

$$-M_{\psi\psi,\psi}^i - M_{\psi\psi,\psi}^{iT} - 2M_{\chi\psi}^i \sin\beta - rM_{\chi\psi,\chi}^i - Q_{\psi 1}^i \cos\beta + rQ_{\psi 0}^i + r \frac{h_i}{2} \lambda_{\psi i} = +I_{1i} r \ddot{v}_0^i + I_{2i} r \ddot{\phi}_\psi^i \quad (43)$$

$$-R_{\chi}^c \sin\beta - rR_{\chi,\chi}^c - R_{\chi}^{cT} \sin\beta - rR_{\chi,\chi}^{cT} + R_{\psi}^c \sin\beta + R_{\psi}^{cT} \sin\beta - Q_{\chi\psi,\psi}^c - r\lambda_{\chi o} + r\lambda_{\chi i} = +I_{0c} \ddot{r}u_0^c + I_{1c} \ddot{r}u_1^c + I_{2c} \ddot{r}u_2^c + I_{3c} \ddot{r}u_3^c \quad (44)$$

$$\begin{aligned} & -M_{\chi 1}^c \sin\beta - rM_{\chi 1,\chi}^c - M_{\chi 1}^{cT} \sin\beta - rM_{\chi 1,\chi}^{cT} + M_{\psi 1}^c \sin\beta + M_{\psi 1}^{cT} \sin\beta + rQ_{\chi z}^c - M_{Q1\chi\psi,\psi}^c + r \frac{h_c}{2} \lambda_{\chi o} + r \frac{h_c}{2} \lambda_{\chi i} = \\ & +I_{1c} \ddot{r}u_0^c + I_{2c} \ddot{r}u_1^c + I_{3c} \ddot{r}u_2^c + I_{4c} \ddot{r}u_3^c \end{aligned} \quad (45)$$

$$-M_{\chi^2}^c \sin \beta - rM_{\chi^2, \chi}^c - M_{\chi^2}^{cT} \sin \beta - rM_{\chi^2, \chi}^{cT} + M_{\psi^2}^c \sin \beta + M_{\psi^2}^{cT} \sin \beta + 2rM_{Q1\chi^z}^c - M_{Q2\chi\psi, \psi}^c - r\frac{h_c^2}{4}\lambda_{\chi^o} + r\frac{h_c^2}{4}\lambda_{\chi^i} =$$

$$+I_{2c}\ddot{u}_0^c + I_{3c}\ddot{u}_1^c + I_{4c}\ddot{u}_2^c + I_{5c}\ddot{u}_3^c \quad (46)$$

$$-M_{\chi^3}^c \sin \beta - rM_{\chi^3, \chi}^c - M_{\chi^3}^{cT} \sin \beta - rM_{\chi^3, \chi}^{cT} + M_{\psi^3}^c \sin \beta + M_{\psi^3}^{cT} \sin \beta + 3rM_{Q2\chi^z}^c - M_{Q3\chi\psi, \psi}^c + r\frac{h_c^3}{8}\lambda_{\chi^o} + r\frac{h_c^3}{8}\lambda_{\chi^i} =$$

$$+I_{3c}\ddot{u}_0^c + I_{4c}\ddot{u}_1^c + I_{5c}\ddot{u}_2^c + I_{6c}\ddot{u}_3^c \quad (47)$$

$$-R_{\psi, \psi}^c - R_{\psi, \psi}^{cT} - Q_{\psi z}^c \cos \beta - 2Q_{\chi\psi}^c \sin \beta - rQ_{\chi\psi, \chi}^c - r\lambda_{\psi^o} + r\lambda_{\psi^i} = +I_{0c}r\ddot{v}_0^c + I_{1c}r\ddot{v}_1^c + I_{2c}r\ddot{v}_2^c + I_{3c}r\ddot{v}_3^c \quad (48)$$

$$-M_{\psi 1, \psi}^c - M_{\psi 1, \psi}^{cT} - M_{Q1\psi z}^c \cos \beta + rQ_{\psi z}^c - 2M_{Q1\chi\psi}^c \sin \beta - rM_{Q1\chi\psi, \chi}^c + r\frac{h_c}{2}\lambda_{\psi^o} + r\frac{h_c}{2}\lambda_{\psi^i}$$

$$= +I_{1c}r\ddot{v}_0^c + I_{2c}r\ddot{v}_1^c + I_{3c}r\ddot{v}_2^c + I_{4c}r\ddot{v}_3^c \quad (49)$$

$$-M_{\psi 2, \psi}^c - M_{\psi 2, \psi}^{cT} - M_{Q2\psi z}^c \cos \beta + 2rM_{Q1\psi z}^c - 2M_{Q2\chi\psi}^c \sin \beta - rM_{Q2\chi\psi, \chi}^c - r\frac{h_c^2}{4}\lambda_{\psi^o} + r\frac{h_c^2}{4}\lambda_{\psi^i} =$$

$$+I_{2c}r\ddot{v}_0^c + I_{3c}r\ddot{v}_1^c + I_{4c}r\ddot{v}_2^c + I_{5c}r\ddot{v}_3^c \quad (50)$$

$$-M_{\psi 3, \psi}^c - M_{\psi 3, \psi}^{cT} - M_{Q3\psi z}^c \cos \beta + 3rM_{Q2\psi z}^c - 2M_{Q3\chi\psi}^c \sin \beta - rM_{Q3\chi\psi, \chi}^c + r\frac{h_c^3}{8}\lambda_{\psi^o} + r\frac{h_c^3}{8}\lambda_{\psi^i} =$$

$$+I_{3c}r\ddot{v}_0^c + I_{4c}r\ddot{v}_1^c + I_{5c}r\ddot{v}_2^c + I_{6c}r\ddot{v}_3^c \quad (51)$$

$$-R_{\chi}^c \sin \beta w_{0, \chi}^c - R_{\chi, \chi}^c r w_{0, \chi}^c - R_{\chi}^c r w_{0, \chi\chi}^c - R_{\chi}^{cT} \sin \beta w_{0, \chi}^c - R_{\chi, \chi}^{cT} r w_{0, \chi}^c - R_{\chi}^{cT} r w_{0, \chi\chi}^c + R_{\psi}^c \cos \beta - r^{-1}R_{\psi, \psi}^c w_{0, \psi}^c$$

$$-r^{-1}R_{\psi}^c w_{0, \psi\psi}^c + R_{\psi}^{cT} \cos \beta - r^{-1}R_{\psi, \psi}^{cT} w_{0, \psi}^c - r^{-1}R_{\psi}^{cT} w_{0, \psi\psi}^c - Q_{\chi^z}^c \sin \beta - rQ_{\chi^z, \chi}^c - Q_{\psi z, \psi}^c - Q_{\chi\psi, \chi}^c w_{0, \psi}^c - 2Q_{\chi\psi}^c w_{0, \chi\psi}^c$$

$$-Q_{\chi\psi, \psi}^c w_{0, \chi}^c - r\lambda_{z^o} + r\lambda_{z^i} = +I_{0c}r\ddot{w}_0^c + I_{1c}r\ddot{w}_1^c + I_{2c}r\ddot{w}_2^c + I_{3c}r\ddot{w}_3^c \quad (52)$$

$$+M_{\psi 1}^c \cos \beta + M_{\psi 1}^{cT} \cos \beta + rR_z^c + rR_z^{cT} - M_{Q1\chi^z}^c \sin \beta - rM_{Q1\chi^z, \chi}^c - M_{Q1\psi z, \psi}^c + r\frac{h_c}{2}\lambda_{z^o} + r\frac{h_c}{2}\lambda_{z^i} =$$

$$+I_{1c}r\ddot{w}_0^c + I_{2c}r\ddot{w}_1^c + I_{3c}r\ddot{w}_2^c + I_{4c}r\ddot{w}_3^c \quad (53)$$

$$+M_{\psi 2}^c \cos \beta + M_{\psi 2}^{cT} \cos \beta + 2rM_{z1}^c + 2rM_{z1}^{cT} - M_{Q2\chi^z}^c \sin \beta - rM_{Q2\chi^z, \chi}^c - M_{Q2\psi z, \psi}^c - r\frac{h_c^2}{4}\lambda_{z^o} + r\frac{h_c^2}{4}\lambda_{z^i} =$$

$$+I_{2c}r\ddot{w}_0^c + I_{3c}r\ddot{w}_1^c + I_{4c}r\ddot{w}_2^c + I_{5c}r\ddot{w}_3^c \quad (54)$$

$$+M_{\psi 3}^c \cos \beta + M_{\psi 3}^{cT} \cos \beta + 3rM_{z2}^c + 3rM_{z2}^{cT} - M_{Q3\chi^z}^c \sin \beta - rM_{Q3\chi^z, \chi}^c - M_{Q3\psi z, \psi}^c + r\frac{h_c^3}{8}\lambda_{z^o} + r\frac{h_c^3}{8}\lambda_{z^i} =$$

$$+I_{3c}r\ddot{w}_0^c + I_{4c}r\ddot{w}_1^c + I_{5c}r\ddot{w}_2^c + I_{6c}r\ddot{w}_3^c \quad (55)$$

$$u_{0o} - \frac{h_o}{2}\phi_{\chi}^o - u_{0c} + \frac{h_c}{2}u_{1c} - \frac{h_c^2}{4}u_{2c} + \frac{h_c^3}{8}u_{3c} = 0 \quad (56)$$

$$v_{0o} + \frac{h_o}{2}\phi_{\psi}^o - v_{0c} + \frac{h_c}{2}v_{1c} - \frac{h_c^2}{4}v_{2c} + \frac{h_c^3}{8}v_{3c} = 0 \quad (57)$$

$$w_{0o} - w_{0c} + \frac{h_c}{2}w_{1c} - \frac{h_c^2}{4}w_{2c} + \frac{h_c^3}{4}w_{3c} = 0 \quad (58)$$

$$u_{0c} + \frac{h_c}{2}u_{1c} + \frac{h_c^2}{4}u_{2c} + \frac{h_c^3}{8}u_{3c} - u_{0i} + \frac{h_i}{2}\phi_{\chi}^i = 0 \quad (59)$$

$$v_{0c} + \frac{h_c}{2}v_{1c} + \frac{h_c^2}{4}v_{2c} + \frac{h_c^3}{8}v_{3c} - v_{0i} + \frac{h_i}{2}\phi_{\psi}^i = 0 \quad (60)$$

$$w_{0c} + \frac{h_c}{2}w_{1c} + \frac{h_c^2}{4}w_{2c} + \frac{h_c^3}{4}w_{3c} - w_{0i} = 0 \quad (61)$$

The high order thermal stress resultants of the face sheets and the core that appear in the equations are defined as follows:

$$\begin{aligned} \begin{Bmatrix} N_{\chi\chi}^{jT} \\ M_{\chi\chi}^{jT} \end{Bmatrix} &= \begin{Bmatrix} N_{\psi\psi}^{jT} \\ M_{\psi\psi}^{jT} \end{Bmatrix} = \int_{-hj/2}^{hj/2} \left(\frac{E_j}{1-\nu_j} \alpha_j T_j \right) \begin{Bmatrix} 1 \\ z_j \end{Bmatrix} dz_j, \quad j = (o, i) \\ \begin{Bmatrix} R_{\chi\chi}^{cT} \\ M_{\chi\chi}^{cT} \\ M_{\chi\chi 1}^{cT} \\ M_{\chi\chi 2}^{cT} \\ M_{\chi\chi 3}^{cT} \end{Bmatrix} &= \begin{Bmatrix} R_{\psi\psi}^{cT} \\ M_{\psi\psi}^{cT} \\ M_{\psi\psi 1}^{cT} \\ M_{\psi\psi 2}^{cT} \\ M_{\psi\psi 3}^{cT} \end{Bmatrix} = \begin{Bmatrix} R_{zz}^{cT} \\ M_{z1}^{cT} \\ M_{z2}^{cT} \end{Bmatrix} = \int_{-hc/2}^{hc/2} \left(\frac{E_c}{1-\nu_c} \alpha_c T_c \right) \begin{Bmatrix} 1 \\ z_c \\ z_c^2 \\ z_c^3 \end{Bmatrix} dz_c \end{aligned} \quad (62)$$

where "E", "ν" and "α" are the Young's modulus, the Poisson's ratio and the thermal expansion coefficient, respectively. The inertia terms of the FG face sheets and the core are calculated as follows:

$$(I_{0j}, I_{1j}, I_{2j}) = \int_{-hj/2}^{hj/2} \rho_j (1, z_j, z_j^2) dz_j, \quad (j = o, i) \quad (63)$$

$$(I_{0c}, I_{1c}, I_{2c}, I_{3c}, I_{4c}, I_{5c}, I_{6c}) = \int_{-hc/2}^{hc/2} \rho_c (1, z_c, z_c^2, z_c^3, z_c^4, z_c^5, z_c^6) dz_c \quad (64)$$

Also, the out-of-plane and in plane stresses of the core leads to the high order resultants which are calculated as:

$$Q_{\chi c}, M_{Q1\chi z}, M_{Q2\chi z}, M_{Q3\chi z} = \int_{-hc/2}^{hc/2} (1, z_c, z_c^2, z_c^3) \tau_{\chi z}^c dz_c \quad (65)$$

$$Q_{\psi z}, M_{Q1\psi z}, M_{Q2\psi z}, M_{Q3\psi z} = \int_{-hc/2}^{hc/2} (1, z_c, z_c^2, z_c^3) \tau_{\psi z}^c dz_c \quad (66)$$

$$R_z, M_{z1}, M_{z2} = \int_{-hc/2}^{hc/2} (1, z_c) \sigma_{zz}^c dz_c \quad (67)$$

$$Q_{\chi\psi}^c, M_{Q1\chi\psi}^c, M_{Q2\chi\psi}^c, M_{Q3\chi\psi}^c = \int_{-hc/2}^{hc/2} (1, z_c, z_c^2, z_c^3) \tau_{\chi\psi}^c dz_c \quad (68)$$

$$R_{\chi}^c, M_{\chi 1}^c, M_{\chi 2}^c, M_{\chi 3}^c = \int_{-hc/2}^{hc/2} (1, z_c, z_c^2, z_c^3) \sigma_{\chi\chi}^c dz_c \quad (69)$$

$$R_{\psi}^c, M_{\psi 1}^c, M_{\psi 2}^c, M_{\psi 3}^c = \int_{-hc/2}^{hc/2} (1, z_c, z_c^2, z_c^3) \sigma_{\psi\psi}^c dz_c \quad (70)$$

Finally, by substituting the high order stress resultants in the governing equations in terms of the displacement components, the equations are derived in terms of the twenty eight unknowns. In the following, vibration problem of two types of truncated conical sandwich shells with simply support boundary conditions are solved by a Galerkin method in this study.

3. Simply supported truncated conical sandwich shells

Figure 1 shows a schematic of a truncated conical sandwich shell in a curve linear coordination. The semi-vertex angle of the cone is depicted by "γ". "h_o", "h_i" and "h_c" are the thicknesses of the layers. "R₁" and "R₂" show the small and large ends radii of the cone, respectively. Radius variation of the truncated conical shell is as follows:

$$R(\chi) = R_1 + \chi \sin \gamma \quad (71)$$

Galerkin method is used to solve the governing equations of truncated conical sandwich shells, with twenty eight trigonometric shape functions, which satisfy the boundary conditions. The shape functions can be expressed as:

$$u_{0k} = C_{uk} \cos \left(\frac{m\pi\chi}{L} \right) \cos(n\psi); \quad k = (o, i, c) \quad (72)$$

$$\nu_{0k} = C_{\nu k} \sin\left(\frac{m\pi\chi}{L}\right) \sin(n\psi) \quad (73)$$

$$w_{0k} = C_{wk} \sin\left(\frac{m\pi\chi}{L}\right) \cos(n\psi) \quad (74)$$

$$\phi_{\chi}^j = C_{\phi_{\chi}j} \cos\left(\frac{m\pi\chi}{L}\right) \cos(n\psi); \quad j = (o, i) \quad (75)$$

$$\phi_{\psi}^j = C_{\phi_{\psi}j} \cos\left(\frac{m\pi\chi}{L}\right) \cos(n\psi) \quad (76)$$

$$\lambda_{\chi j} = C_{\lambda_{\chi}j} \cos\left(\frac{m\pi\chi}{L}\right) \cos(n\psi) \quad (77)$$

$$\lambda_{\psi j} = C_{\lambda_{\psi}j} \sin\left(\frac{m\pi\chi}{L}\right) \sin(n\psi) \quad (78)$$

$$\lambda_{zj} = C_{\lambda_{zj}} \sin\left(\frac{m\pi\chi}{L}\right) \cos(n\psi) \quad (79)$$

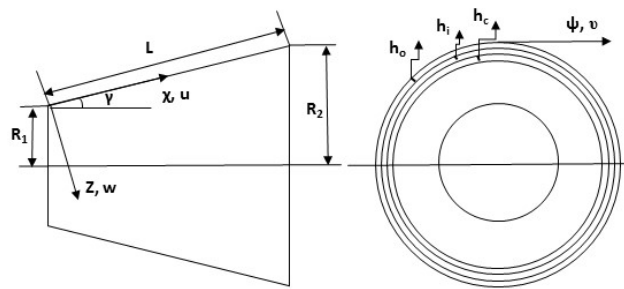


Fig. 1. Schematic of truncated conical sandwich shell.

where " m " and " n " are the wave numbers; and " C "s are the twenty eight unknown constants of the shape functions. These twenty eight equations are not independent and a number of them can be reduced by a reduction approach. Lagrange constants can be isolated as the expression of the face sheets. It's seen that based on the compatibility conditions, the unknown constants of the faces are dependent to the core constants. Eventually, the number of the equations are reduced to sixteen in terms of the core and rotations of the normal to the middle surface of the faces unknown constants. These sixteen equations can be written in a 16×16 matrix which include the mass, " M^* ", and stiffness, " K^* ", matrices as follows:

$$(K_{mn}^* - \omega_{mn}^2 M_{mn}^*) C_{mn} = 0 \quad (80)$$

In eq. (80), " ω_{mn} " is the natural frequency; and " C_{mn} " is the eigen vector which contains sixteen unknown constants of the core and faces.

4. Verification and Numerical Results

To verify the approach of this study, present results are compared with finite element method (FEM) results of Abaqus software and in a special case are compared with results of literatures [42] and [43]. Consider an isotropic conical shell which made of aluminum with structural parameters such as " $h = 0.004m$ ", " $h/R_2 = 0.01$ ", " $L \sin(\beta)/R_2 = 0.25$ ". These comparisons are shown in Table 1 for three values of " γ ".

Table 1. Comparisons of $\tilde{\omega}$ results in isotropic conical shell.

γ	(Lam and Li, 1999)	(Li et al., 2009)	Present study	FEM
30	0.8420	0.8431	0.886163	0.8579
45	0.7655	0.7642	0.760881	0.7648
60	0.6348	0.6342	0.615840	0.6295

Because, theory and solution method of the present analysis are different with references [42] and [43], a discrepancy is found in the results. In this part, another numerical problem will be discussed for more investigation of the present approach. Consider two kinds of simply supported FG truncated sandwich shells. In type-I, the face sheets interior planes and the core are made of the zirconium dioxide and the outer planes of the faces are made of silicon nitride. In type-II, the interior plane of the core and inner face sheet are made of the zirconium dioxide and the outer plane of the core and outer face sheet are made of silicon nitride. The properties of these materials are available in reference [44]. Variation of the material properties in each FG layer is correspond to the modified power-law function. To validate the present method, numerical examples are simulated by Abaqus software, version 6.13. The continuum with three dimensional and eight nodes hexagonal with the effect of thermal elements (C3D8T) are used to mesh the samples as shown in Fig. 2. In order to simulate the FG face-sheets and FG core in Abaqus, all FG layers are divided to 20 sub-layers and each sublayer has different properties according to the power law function. Also, the number of the elements in FEM are dependent to the convergence of the results. As shown in Fig. 3, first 4000 elements are considered. By increasing the number of elements, it is seen that the variation of the results are high. But, after 12000 elements, there is a convergence between the results. Hence, it can be said that 12000 elements are proper and increasing the elements more than 12000, just increases the time of the solving and does not have any important effect on the results. Also, explicit solution is used to solve the problem. In this study, results are shown by a non-dimensional parameter, named, fundamental frequency parameter which is defined as follows:

$$\tilde{\omega} = \omega h \sqrt{\rho_0 / E_0} \quad (81)$$

where " $\tilde{\omega}$ " is the non-dimensional fundamental frequency parameter, h is the total thickness of sandwich shell which consists two FG faces as well as core; ρ_0 is density and equal to 1000 kg/m^3 ; and E_0 is the young module equal to 1 MPa .



Fig. 2. FE model simulated by ABAQUS

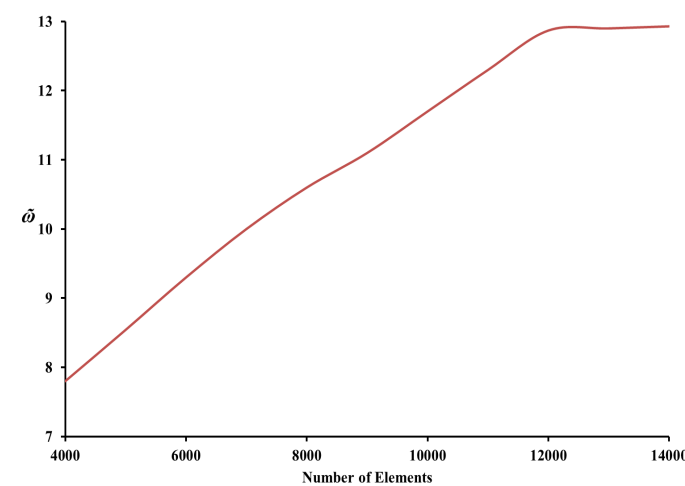


Fig. 3. Convergence of Abaqus FEM for frequency of the conical shell versus number of elements

In Table 2 fundamental frequency parameters of this approach are compared with the FEM results by Abaqus software in the temperature of the room and for different power law indices in the case of 2-1-2 sandwich. It should be noted that in 2-1-2 sandwich, every face sheet thicknesses is two times of the core thickness and the structure is symmetric. In Table 2, the discrepancies between the present results and FEM results are due to simulation method of FG layers in Abaqus software. In order to simulate the FG face-sheets and FG core in Abaqus, all FG layers are divided to 20 sublayers and each sublayer has different properties according to the power law function. There is a good agreement between the present study results and the FEM results obtained by Abaqus.

Table 2. Comparison of the $\tilde{\omega}$ results with FEM results in 2-1-2 sandwiches.

	q	Present method	Abaqus	Discrepancy (%)
Type-I	0	12.898	12.106	6.54
	0.2	12.229	11.266	8.54
	1	10.907	10.280	6.09
	2	10.247	9.525	7.58
Type-II	0	11.864	11.034	7.52
	0.2	11.776	10.902	8.01
	1	11.350	10.546	7.62
	2	11.219	10.354	8.35

Table 3. Effect of temperature variation on the Young modulus in metal and ceramics.

T	Silicon Nitride	Zirconium dioxide
300 (K)	322.27 (GPa)	168.06 (GPa)
1500 (K)	252.14 (GPa)	105.68 (GPa)
variation	21.76%	37.11%

The frequency of the structures are dependent to the temperature variation. The effect of the uniform temperature distribution on the fundamental frequency parameter is depicted in Fig. 4 for two types of 2-1-2 simply supported truncated conical FG sandwich shells in different power law indices. As shown in Fig. 4, while the temperature is increased, the fundamental frequency parameter decreases. According to Eq. (1), temperature rising reduces the strength of the material. To clarify this phenomena, in Table 3 the effect of temperature on the Young's modulus of ceramic and metal is indicated. With increasing the temperature, modulus of metal and ceramic decrease, but due to the microstructural reasons, decreasing the module of metal is more. So, increasing the temperature reduces the Young modulus of the material which is one of the most important cause of the frequency reduction in high temperatures.

Moreover, Fig. 4 depicts that in a constant temperature, the fundamental frequency decreases by increasing the power law indices. With increasing the power-law index, the properties of the FG layers are tending to metal and the total strength of the structure decreases. It is obvious in Fig. 4 that in the lower power law indices, the values of the fundamental frequency parameters in type- I are more than type-II. The slope of frequency changing in all power law indices in type-I is more than type-II. But, in larger power law indices the values of frequency in type-II is more. Also, in the larger temperature, the values of the frequency in type-II is higher than the type-I. It is concluded that the sandwiches with FG core is proper than the ones with FG face sheets, in the thermal conditions overall.

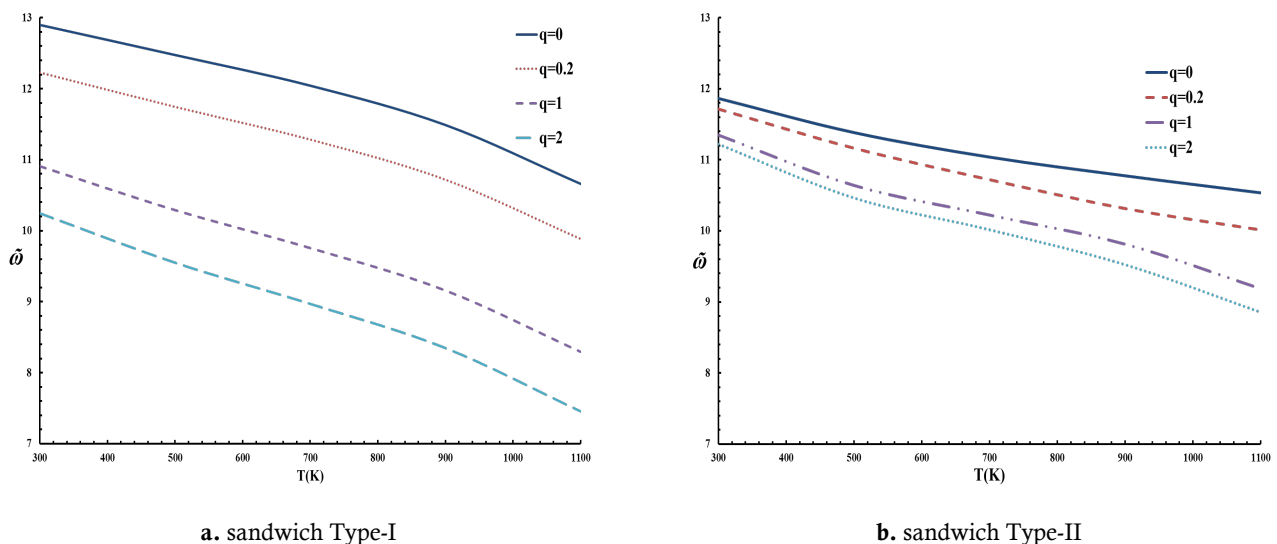
**Fig. 4.** Fundamental frequency versus temperature in various power law indices.

Figure 5 shows the effect of length to thickness ratio on the fundamental frequency parameter for two types of 2-1-2 truncated conical FG sandwich shells. According to Fig. 5, variation of frequency in a constant length to thickness ratio in type-I is more than type-II. This means that the power law index variation effect on the frequency in type-II is lower than type-I. This figure implies that when ratios are increased in a constant "q", the fundamental frequency parameter decreases. With increasing of this ratio, the stability of the structure reduces, but generally in high ratio, the type-II is better to apply in the equipment. However, it is important that long length is not proper for the truncated conical sandwich shells.

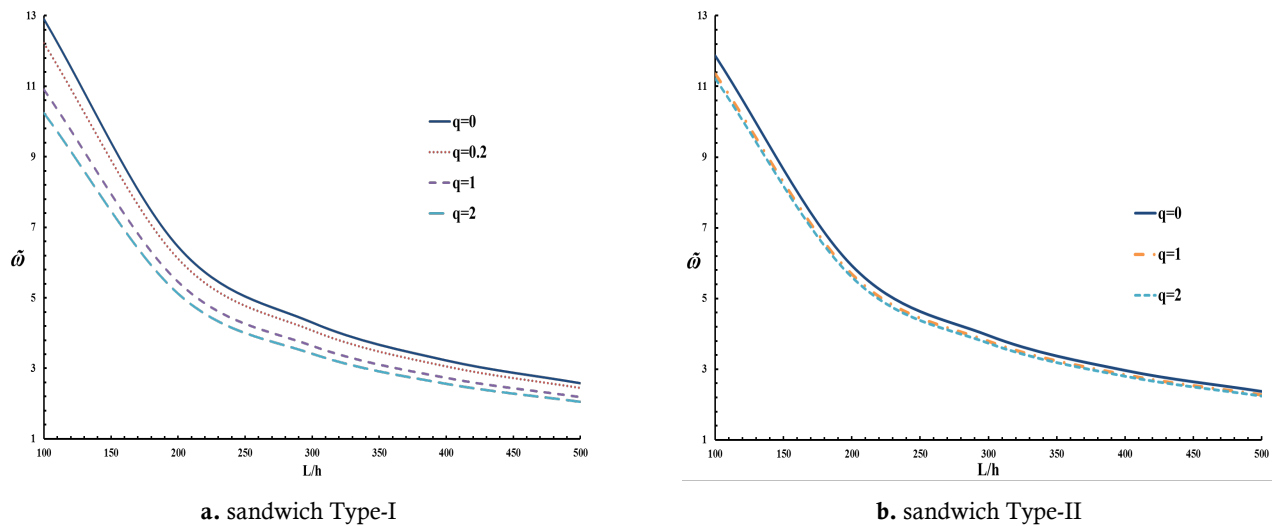


Fig. 5. Variation of the fundamental frequency with length to thickness ratio.

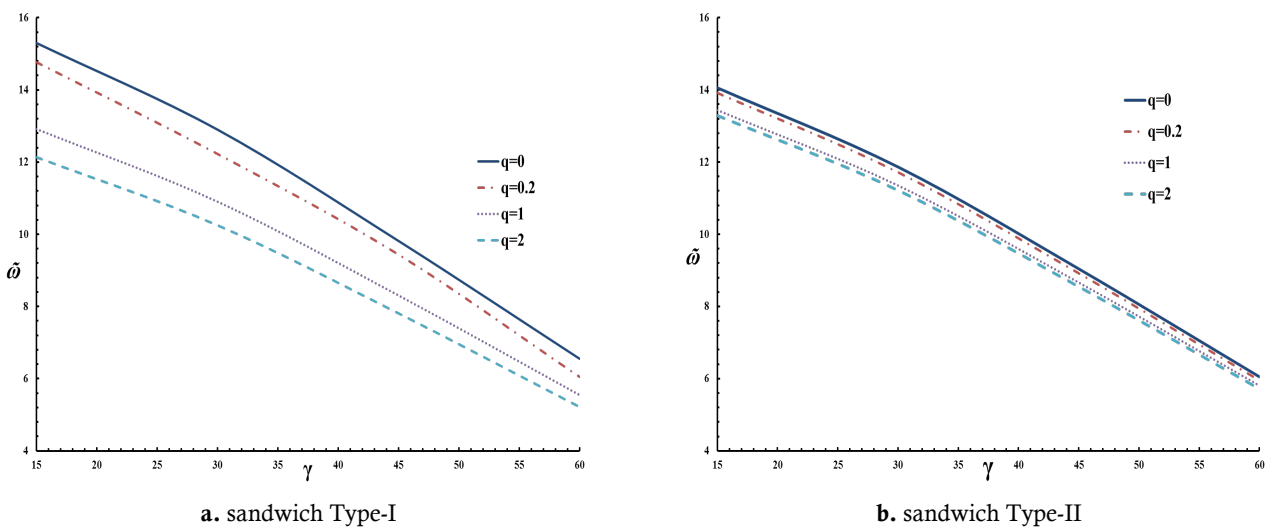


Fig. 6. Variation of the fundamental frequency with semi vertex angle.

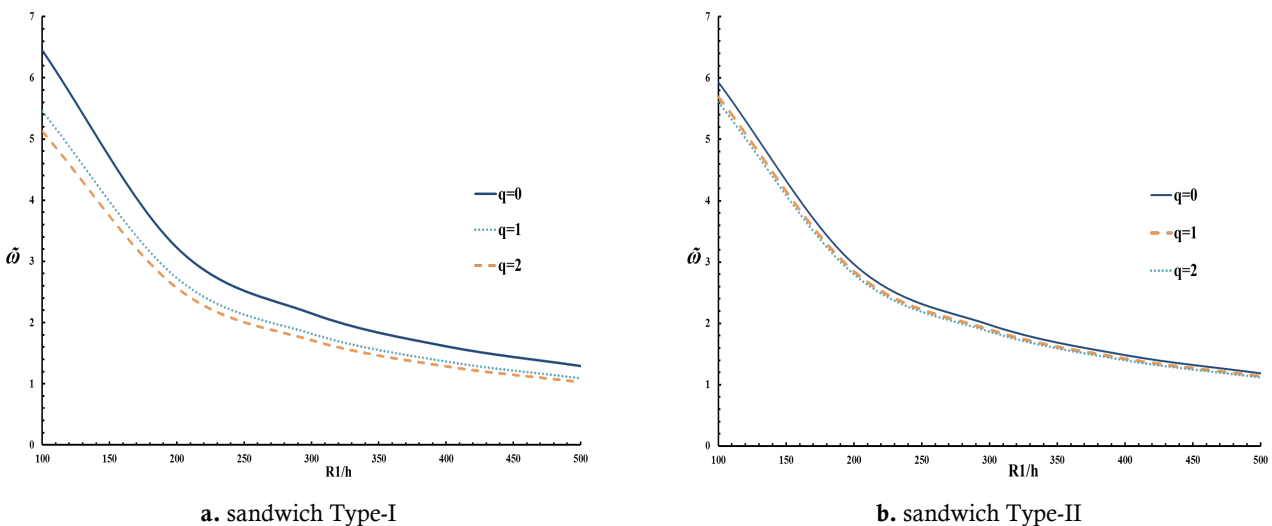


Fig. 7. Variation of the fundamental frequency with radius to thickness ratio.

Variation of the semi vertex angle is one of the most important geometrical effects in the cone. In Fig. 6, the effect of the semi vertex angle on fundamental frequency parameter for 2-1-2 conical FG sandwich shells is investigated. This figure implies that with increasing the angle in a constant power law index, the fundamental frequency parameter decreases. In smaller angles, the frequency values of type-I are more than type-II but in larger angles, and especially in larger power law indices, there are little discrepancies between two types of sandwiches. As shown in Fig. 6, the results of type-II for different power law indices are close to each other, that shows the effects of the material properties variation of FG layer

in type-II is less than type-I.

Figure 7 shows the effect of radius to thickness ratio on fundamental frequency parameter. This figure depicts that when ratio increases in constant power law index, the fundamental frequency parameters decrease. As depicted in Fig. 7, for smaller ratios with lower power law indices, the frequencies of type-I are more than type-II, but for high power law indices with higher ratios, the frequencies of type-II are higher than type-I that shows power law effects on the FG faces sandwiches are more than FG core sandwiches. In large ratios, variation of frequencies decreases so that almost frequencies of both types are close to each other.

Figure 8 depicts variation of the core to face sheet thickness ratio, h_c/h_o , on the fundamental frequency parameter in various power law indices and constant total thickness. In type-II when the power law index is zero, by increasing the ratio, the amount of ceramic is increased and the structure becomes stiffer, so, the fundamental frequency parameter increases. But, in type-I, in the zero power law index, by increasing the ratio in a constant total thickness, the amount of metal increases and the structure becomes softer. Therefore, the fundamental frequency decreases. When the power law index is increased in both types, ceramic will decrease, so by increasing the ratio, the fundamental frequency decreases. Frequency reduction in type-I is more than type-II. Also, Figure 9 shows the variation of core to face sheet thickness ratio versus fundamental frequency parameter in different temperatures. It can be observed that with increasing the ratio, the fundamental frequency parameters decrease. In the room temperature, variation in type-II is very low. Because type-II is stiffer than type-I, frequency variation in type-II in higher temperature is lower, so, the FG core efficiency is much more than the FG faces sandwiches.

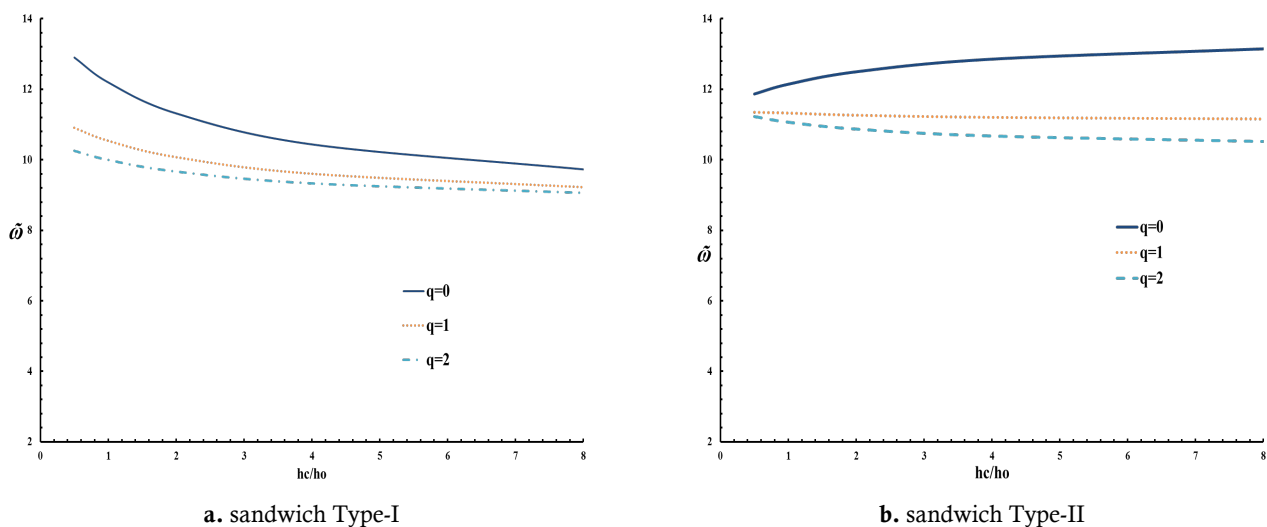


Fig 8. Variation of the fundamental frequency with core to face sheet thickness ratio for different power law indices.

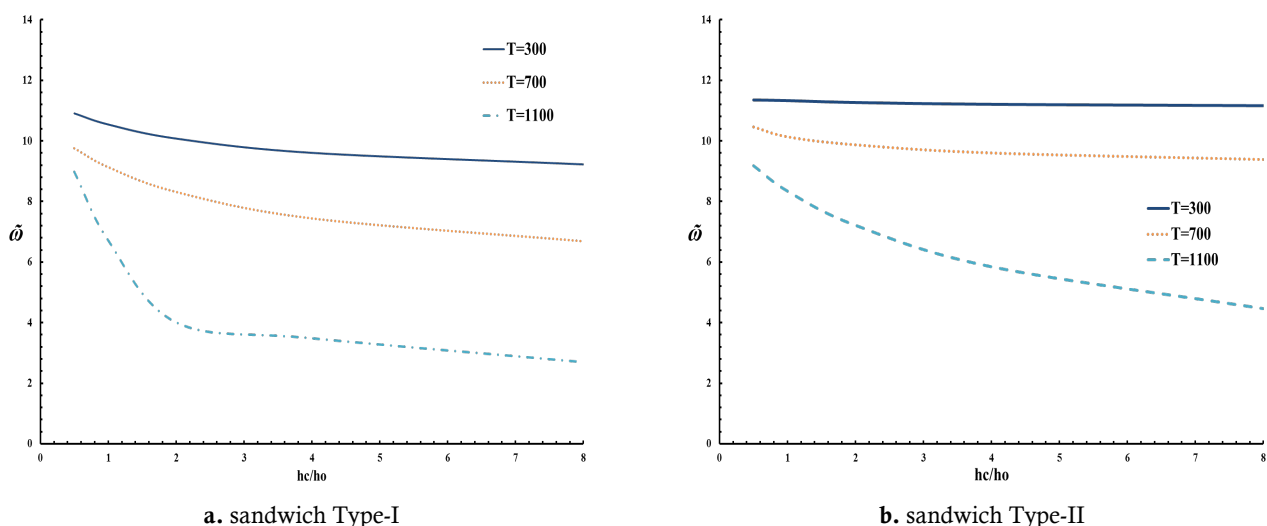


Fig 9. Variation of the fundamental frequency with core to face sheet thickness ratio for different power law indices.

In order to clearly understand the porosity influence, Figures 10 and 11 show the effect of even and uneven porosity distributions on the frequency of the sandwich shells, respectively. As depicted in these figures, in sandwiches type-I, both porosity distributions are dependent of power law indices. By increasing the porosity volume fraction, the fundamental frequency parameter first increases at lower gradient indices, but from a certain value of the power law index, increasing porosity volume leads to decreasing the fundamental frequency parameter. These increasing and

decreasing are stronger in the case of even porosity distribution in FG faces sandwiches. In even distributions, porosities occur all over the cross-section of FG layer. While, in uneven distribution, porosities are available at middle zone of cross section. In addition, in Figs. 10 and 11 in sandwiches type-II, with increasing the porosity volume fraction, the fundamental frequency parameters increase with an almost constant slope for all power law indices that shows another important different behavior of these two kinds of sandwiches.

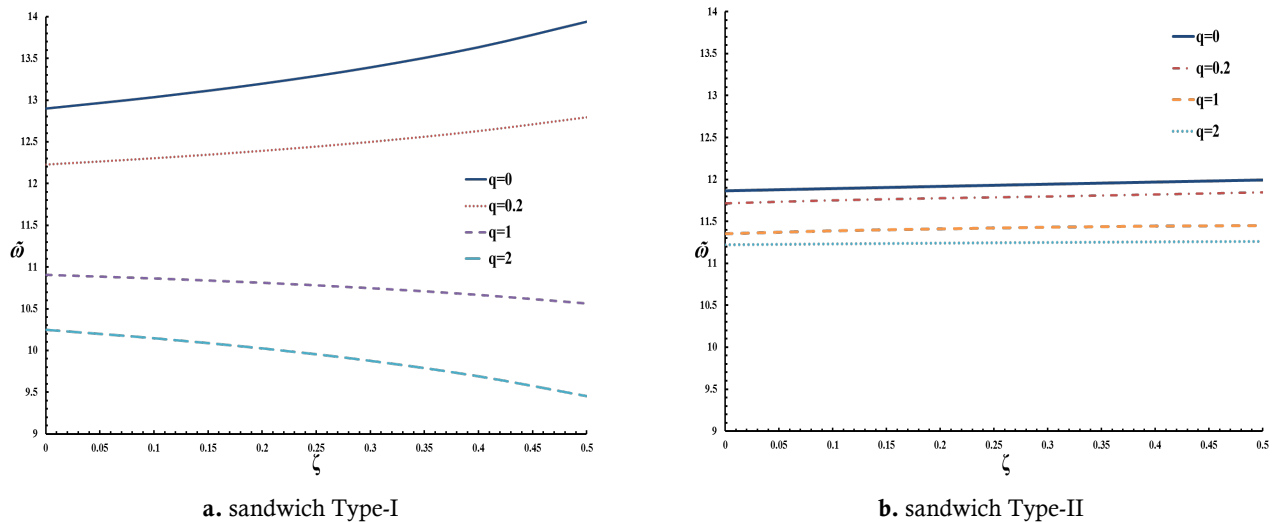


Fig 10. Variation of the fundamental frequency with even porosity distribution for different power law indices.

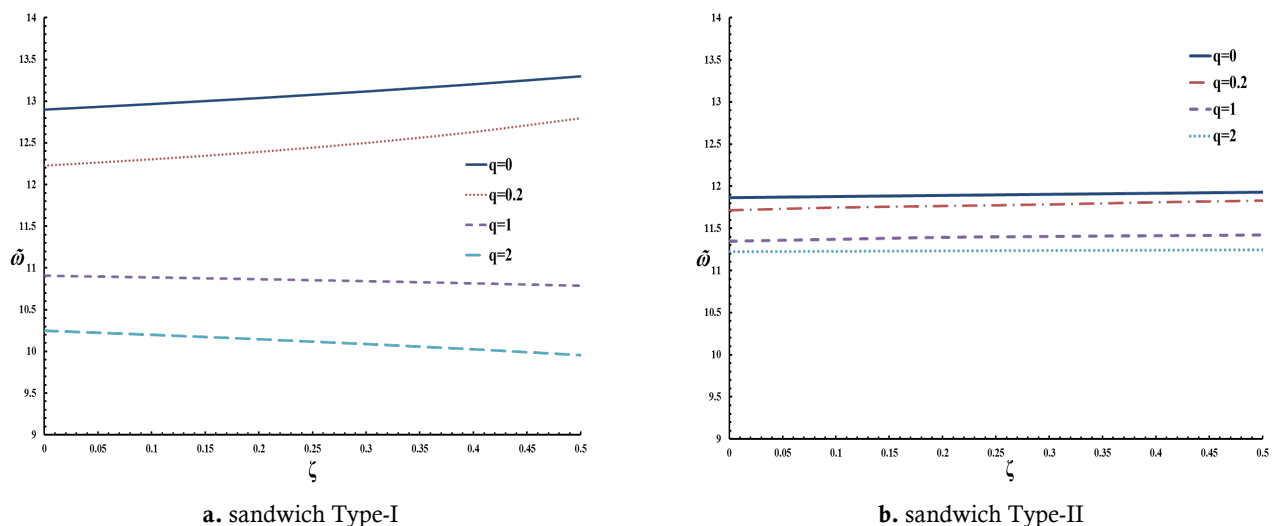


Fig 11. Variation of the fundamental frequency with uneven porosity distribution for different power law indices.

As mentioned above, three types of temperature distributions have been considered in this paper. The uniform temperature variations have been shown in Fig. 4 and Fig. 9. Table 4 shows the fundamental frequency parameters in the linear and nonlinear temperature distributions for two types of 2-1-2 FG truncated sandwich shells. The temperature of inner surface of sandwiches, T_i , is constant and equal to 300 (K) and the outer surface of sandwiches, T_o , is variable. Temperature between these two surfaces changes linearly based on Eqs. (5-7) and nonlinearly based on Eqs. (10-12).

Table 4. The fundamental frequency parameters in linear and nonlinear temperature distributions

	T_o	Linear	Non-linear
Type-I	300	12.89876394	12.89876394
	500	12.68775051	12.75699513
	700	12.48824196	12.6071643
	900	12.26908197	12.43194221
Type-II	300	11.86419863	11.86419863
	500	11.66005164	11.75321576
	700	11.47763944	11.61237865
	900	11.3086927	11.48369425

5. Conclusion

Vibration behavior of two types of truncated conical porous FG sandwich shells which were temperature dependent, was investigated in this paper. For the solution, a modified high order sandwich shell theory was applied considering the high order stress resultants and thermal stress resultants, in plane stresses and thermal stresses and nonlinear strains for both mechanical and thermal stresses in face sheets and core. The equations of the motion were obtained by Hamilton's principal and solved by using Galerkin method. Also, reducing order approach was used to reduce the equations from 28 to 16 equations. In order to confirm the capabilities of this model for vibration analysis of two types of truncated conical sandwich shells with simply supported boundary condition, the results were verified by FEM results and in a special case by literature results. A good agreement between the results was seen. Effects of temperature, power law index, semi-vertex angle, thicknesses, radius, length, porosities and three temperature distributions on the frequency were discussed. It was found that the sandwiches with FG core in high temperature have more efficiency than sandwiches with FG face sheets. Effects of power law index variations on FG faces sandwiches are more than FG core sandwiches. Generally in the larger length to thickness ratios, the type-II is better to apply in the equipment. In smaller angles, the frequency values of type-I are more than type-II but in larger angles, there are little discrepancies between two types of sandwiches. In larger radius to thickness ratios, variation of frequencies decreases so that frequencies of both types are close to each other. In the lower power law indices in type-I, with increasing the porosity volume fraction, the fundamental frequency parameter increases, but with increasing the power law index, this becomes vice versa and the fundamental frequency parameter decreases. These increasing and decreasing are more in the case of even porosity distribution. However, in type-II for all power law indices, with increasing the porosity volume fraction, the fundamental frequency parameters increase. With enlarging the core to face-sheet thickness ratio in different temperatures and different power law indices, the fundamental frequency parameters decrease, except for the zero power in type-II which with increasing the ratio the frequency increases. Frequencies in the case of non-linear distribution of temperature are higher than the linear one.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

R_1, R_2	Small and Large radius [m]	h	Total thickness [m]
T	Temperature [K]	u_{0j}, v_{0j}, w_{0j}	Displacements of mid-plane of layers
t	Time [s]	m, n	Wave numbers
t_1	Start time [s]	ΔT	Temperature difference
t_2	End time [s]	$P_0, p_{-1}, p_1, p_2, p_3$	Coefficients of temperature
L	Length [m]	u, v, w	Displacement fields
R	Variable radius [m]		
<i>Greek signs</i>			
ρ	Density [kgm ⁻³]	$\epsilon_x, \epsilon_\psi, \epsilon_z$	Normal strains in face sheets and core
χ, ψ	Curve-linear coordination component	$\gamma_{\chi\psi}, \gamma_{\psi z}, \gamma_{\chi z}$	Shear strains in face sheets and core
γ	Semi-angle vertex of the cone	ζ	Porosity volume fraction
$\varphi_\chi, \varphi_\psi$	Rotation of normal to mid-plane vector	$\lambda_\chi, \lambda_\psi, \lambda_z$	Lagrange multipliers
ν	Poisson ratio		
ω	Frequency [Hz]		
<i>superscripts</i>		<i>subscripts</i>	
T	Related to thermal resultant	$j=i, o, c$	Related to inner and outer face-sheets and core

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