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Research Paper

Unsteady Hydromagnetic Flow of Eyring-Powell Nanofluid over an Inclined Permeable Stretching Sheet with Joule Heating and Thermal Radiation

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Abstract. The present analysis deals with an unsteady magnetohydrodynamic flow of Eyring-Powell nanofluid over an inclined permeable stretching sheet. Effects of thermal radiation, Joule heating, and chemical reaction are considered. The effects of Brownian motion and thermophoresis on the flow over the permeable stretching sheet are discussed. Using Runge-Kutta fourth-order along with shooting technique, numerical and graphical results were obtained for the governing flow equations. The influence of various parameters on flow variables have been examined in detail. The results reveal that the temperature of the fluid enhanced with increasing Brownian and thermophoresis parameters. The increase of fluid velocity with the local Grashof number, the solutal Grashof number has been noticed. Further, the nanoparticles concentration decreased for a given increase in Brownian motion and chemical reaction parameters, while it increased with an increase in the thermophoresis parameter.

Keywords: Chemical reaction, Eyring-Powell nanofluid, Hartmann number, Inclined stretching sheet and Joule heating.

1. Introduction

In general, nanofluid is the fluid which contains nanosized particles within the range (1-100mm). In 1995, Choi [10] was the first one to introduce this colloidal suspension for the enhancement of thermal conductivity as well as heat transfer of base fluids by adding nanoparticles. To improve convective heat transfer, it is essential to have such fluids, which enhanced heat transfer characteristics. In this case, nanofluids are effective substitutes as compared to usual heat transportation liquids. With this objective, Waqas et al. [60] recently formulated the mixed convective Jeffrey nano liquid flow in the direction of moving stratified surface. Distinctive properties of nanofluids have made useful in industrial applications such as heat transfer, involving microelectronics, machine cooling, fuel cells, and artificial-powered motors. (Abu-Nada and Oztop [1]; Choi [11]; Ibrahim and Shankar [23]; Khanafer and Vafai [29]; Makinde and Aziz [31]; Nadeem et al. [37]; Wang [57;58]). In 2005, Buongiorno [7] has concluded that only thermophoresis and Brownian motion are essential slip mechanisms in nano fluids. Based on this factor, he proposed a non-homogeneous equilibrium model that contains two-components and four-equations to improve convective heat transport in nanofluid. Kuznetsov and Nield [30] have examined the boundary layer flow of a nanofluid over a vertical plate by considering the effects of thermophoresis and Brownian motion. Khan and Pop [28] numerically studied the problem of boundary layer flow of nanofluid over a stretching sheet. Pal and Mandal [40] studied the mixed convection on stagnation point flow of nanofluid over a shrinking/stretching sheet in the existence of heat generation, radiation, and viscous dissipation. Hayat et al. [17] have analyzed the influence of thermal radiation on hydromagnetic flow of Powell-Eyring nanofluid over a stretching cylinder. Zhu et al. [61] discussed about the effects of velocity slip and nanoparticle migration on the flow of nanofluid using Buongiorno model. Hayat et al. [22] studied heat and mass transfer characteristics of the magnetohydrodynamic (MHD) non-linear convective flow of thixotropic nanofluid over a stretching sheet with variable thickness by using homotopy analysis method.

Studies related to non-Newtonian fluids has gained a lot of attention of several researchers due to numerous applications in science and industry. For example, blood, liquid cosmetics, performance of oils, paints and greases (Mehmood et al. [34]). Rahimi et al. [46] employed collocation method for the Eyring-Powell fluid flow over a stretching sheet. Malik et al. [33] analyzed numerically, the two-dimensional mixed convection flow of nanofluid over a stretching sheet in the presence of applied magnetic field. Hayat et al. [18] investigated the boundary layer flow of Eyring-Powell fluid over a stretching sheet that having convective boundary conditions. Khan et al. [27] examined the thermal and concentration diffusion on Jeffery nanofluid using generalized Fick's and Fourier's laws over an inclined stretching sheet. Hayat et al. [16] studied the problem related to the flow, heat and mass transfer of Eyring-Powell nanofluid over an exponentially stretching sheet. Gangadhar et al. [14; 15] examined the heat transfer and axisymmetric flow of Powell-Eyring fluid over a heated disk with Newtonian heating in addition to thermal radiation by using the spectral relaxation method.

The flow and heat transfer over a stretching surface is applicable in numerous industrial processes like continuous metal casting and extrusion of polymers in metal spinning. Some investigators (Agbaje et al. [2]; Javed et al. [25]; Khan et al. [27]; Khan and Pop [28]; Makinde and Aziz [31]; Rehman et al. [48]; Sharidan et al. [51]) have analyzed the boundary layer flow and heat transfer characteristics of Newtonian/non-Newtonian fluids over a stretching surface. Nadeem et al. [37] studied the non-orthogonal stagnation point flow and heat transfer of non-Newtonian nanofluid over a stretching surface. Ibrahim and Shankar [23] examined the hydromagnetic boundary layer flow and heat transfer of nanofluid over a permeable stretching sheet in the presence of velocity, thermal and solutal slip conditions. Eegunjobi et al. [13] studied the unsteady MHD mixed convection slip flow with chemical reaction past a stretching sheet in a porous medium. Recently, Hayat et al. have negotiated the MHD flow of second-grade nanomaterials over a stretching sheet.

The study of MHD flow has gained much consideration of several investigators because of its wide applications in engineering and industries. Such applications incorporate liquid metals for the designing of cooling machines, accelerators, pumps, MHD generators, nuclear reactors, flow meters, and energy storage. Several researchers, (Hayat et al. [17]; Misra and Sinha [35]; Srinivas et al. [54]; Su and Zheng [56]) have examined the behaviour of MHD flow over-stretching surfaces in different physical characteristics. Prasad and Vajravelu [44] have analyzed the heat transfer and MHD boundary layer flow in the power-law fluid past a stretching surface. Hayat et al. [21] discussed the unsteady stretching flow of a viscous fluid in the presence of thermal radiation, velocity, and thermal slip conditions. Srinivas et al. [55] studied the impact of thermal radiation and chemical reaction on MHD flow over an inclined stretching surface with the non-uniform heat source. Rosca and Pop [49] numerically explored the boundary layer flow and heat exchange of Eyring-Powell fluid over the contracting surface in parallel free steam. Ashraf et al. [3] discussed the flow of Eyring-Powell nanofluid by a bidirectional exponentially stretching sheet. Ramzan et al. [47] revealed the effects of double stratification and MHD flow of Eyring-Powell nanofluid past a stretched cylinder near the stagnation point. Prasad et al. [45] numerically analyzed the MHD flow and transfer of heat in a nanofluid over an elastic sheet. Athira et al. [4] investigated and noticed the effects of the induced magnetic field and non-linear convection in the viscous fluid flow over a porous plate by considering the heat source and chemical reaction.

Investigations about energy translation of non-Newtonian nanomaterials over a moving surface have significant involvement in engineering and industries. Such studies have importance in the extrusion of polymer sheet via higher radiation mechanism combined with a magnetic field through a dye. In such a system, the stretched polymer-melt is cooled through nanomaterial subject to radiation and magnetic field (Waqas et al. [59]). Joule heating is the process of producing heat when electric current flows through the conductor (Chen [9]; Kanzawa and Pfender [26]; Sahoo [50]; Sharma and Sinha [52]). Daniel et al. [12] examined the effects like Ohmic dissipation, thermal radiation and concentration stratification in the two-dimensional steady flow of electrically conducting nanofluid. Pal and Talukdar [41] theoretically analyzed the matter of an unsteady, incompressible boundary layer flow of MHD fluid past a moving vertical plate in the occurrence of viscous and Joule heating effects. Pantokratoras [43] investigated the flow along with a permeable vertical stretching sheet embedded in a Darcy−Brinkman porous medium. Hayat et al. [19] analyzed the Joule heating and viscous dissipation effects on second-grade Nano liquid over a rotating disk in the presence of Brownian motion and thermophoresis. Azim et al. [5] studied the problem of heat transfer of electrically conducting fluid past a vertical flat plate in the presence of viscous dissipation and Joule heating using an implicit finite difference scheme. Srinivas et al. [54] analyzed the hydromagnetic pulsating flow of Casson fluid in a porous channel by considering Joule heating and thermal radiation. Nayak et al. [38] presented a computational study of the magneto-hydrodynamic (MHD) non-linear convective Falkner-Skan flow over a stretching wedge by considering hyperbolic tangent fluid, a special geometric approach that has more realistic applications in the field of engineering materials. Hayat et al. [20] discussed the MHD flow of second-grade nanomaterials on the stretching sheet by considering nonlinear thermal radiation, Joule heating, and viscous dissipation.

Motivated by the above studies, the present analysis aims to examine numerically, the unsteady flow of non-Newtonian nanofluid over a permeable inclined stretching sheet accounting the effects of viscous dissipation, thermal radiation, and chemical reaction. Shooting method along with the Runge-Kutta method of order four has been employed to solve the transformed boundary layer equations. Effects of various parameters on unsteady velocity, temperature, and nanoparticles concentration have been discussed in detail.

2. Formulation of the Problem

Consider an unsteady laminar and incompressible two-dimensional flow of Eyring-Powell nanofluid over a permeable inclined stretching sheet with an acute angle α to the vertical axis. A constant magnetic field (B_0) is applied perpendicular to the sheet. The surrounding temperature and nanoparticles concentration are T_∞ and C_∞ respectively as shown in Fig. 1.

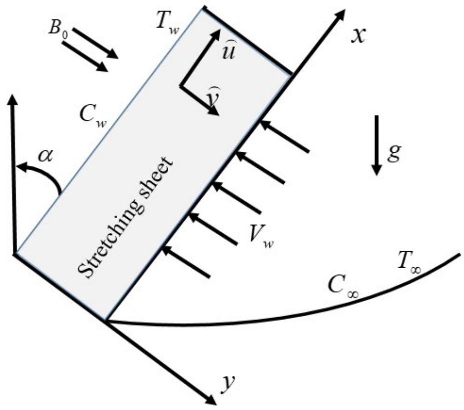

Fig. 1. Physical Model of the Problem.

For the Eyring-Powell incompressible nanofluid flow, the constitutive equation is as follows (Shawky [53])

$$\tau_{ij} = \mu \frac{\partial V_i}{\partial x_j} + \frac{1}{\beta}\sinh^{-1}\left(\frac{1}{\gamma}\frac{\partial V_i}{\partial x_j}\right) \tag{1}$$

when $|x| \ll 1, \sinh^{-1} x \approx x$, thus Eq. (1) reduced to

$$\tau_{ij} = \mu \frac{\partial V_i}{\partial x_j} + \frac{1}{\beta}\left(\frac{1}{\gamma}\frac{\partial V_i}{\partial x_j}\right) = \mu\left(1+\frac{1}{\mu\beta\gamma}\right)\frac{\partial V_i}{\partial x_j}. \tag{2}$$

Under these suppositions, based on the balance laws of mass, momentum, energy and nanoparticles concentration, the boundary layer equations for Eyring-Powell nanofluid have taken the resulting form:

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \hat{u}}{\partial x} + \hat{v}\frac{\partial \hat{u}}{\partial y} = \left(\upsilon + \frac{1}{\rho\beta\gamma}\right)\left(\frac{\partial^2 \hat{u}}{\partial y^2}\right) - \left(\frac{1}{2\rho\beta\gamma^3}\right)\left(\frac{\partial \hat{u}}{\partial y}\right)^2\left(\frac{\partial^2 \hat{u}}{\partial y^2}\right) + g\left[\beta_T^*\left(\hat{T}-T_\infty\right) + \beta_C^*\left(\hat{C}-C_\infty\right)\right]\cos\alpha - \frac{\sigma B_0^2}{\rho}\hat{u} \tag{4}$$

$$\frac{\partial \hat{T}}{\partial t} + \hat{u}\frac{\partial \hat{T}}{\partial x} + \hat{v}\frac{\partial \hat{T}}{\partial y} = \frac{\kappa}{\rho c_p}\left(\frac{\partial^2 \hat{T}}{\partial y^2}\right) + \tau\left(D_b \frac{\partial \hat{C}}{\partial y}\frac{\partial \hat{T}}{\partial y} + \frac{D_t}{T_\infty}\left(\frac{\partial \hat{T}}{\partial y}\right)^2\right) + \frac{q'''}{\rho c_p} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p}\left(1+\frac{1}{\mu\beta\gamma}\right)\left(\frac{\partial \hat{u}}{\partial y}\right)^2 + \left(\frac{\sigma B_0^2}{\rho c_p}\right)\hat{u}^2 \tag{5}$$

$$\frac{\partial \hat{C}}{\partial t} + \hat{u}\frac{\partial \hat{C}}{\partial x} + \hat{v}\frac{\partial \hat{C}}{\partial y} = D_b\left(\frac{\partial^2 \hat{C}}{\partial y^2}\right) + \frac{D_t}{T_\infty}\left(\frac{\partial^2 \hat{T}}{\partial y^2}\right) - K(t)\left(\hat{C}-C_\infty\right) \tag{6}$$

The corresponding boundary conditions are:

$$\begin{gathered} \hat{u} = U_w, \hat{v} = V_w, \hat{T} = T_w \text{ and } \left(\frac{\partial \hat{C}}{\partial y}\right)D_b + \frac{1}{T_\infty}\left(\frac{\partial \hat{T}}{\partial y}\right)D_t = 0 \text{ at } y = 0 \\ \hat{u} \to 0, \hat{T} \to T_\infty \text{ and } \hat{C} \to C_\infty \text{ as } y \to \infty \end{gathered} \tag{7}$$

where $\tau=\left(\rho c_p\right)_{np}/\left(\rho c_p\right)_f$ is the ratio of the heat capacitance of the nanoparticle material and the heat capacitance of the fluid. Let us consider $k_0(t)=k_1(1-\lambda_3 t)$ and $K(t)=k_2(1-\lambda_3 t)^{-1}$. The velocity U_w, surface temperature T_w and the nanoparticles concentration C_w are considered as (Hayat et al. [21], Misra and Sinha [35], Musa et al. [36], Srinivas et al. [55]).

$$U_w(x,t)=\frac{\lambda_1 x}{1-\lambda_3 t},\ T_w(x,t)=\frac{\lambda_2 x}{1-\lambda_3 t}+T_\infty \text{ and } C_w(x,t)=\frac{\lambda_4 x}{1-\lambda_3 t}+C_\infty, \tag{8}$$

where $\lambda_1>0$, $\lambda_2\geq 0$, $\lambda_3\geq 0$ and $\lambda_4\geq 0$ are constants and $\lambda_3 t<1$. In addition, V_w signifies the velocity of suction/injection which is given by

$$V_w=-f(0)\sqrt{\frac{\upsilon\lambda_1}{1-\lambda_3 t}}=S. \tag{9}$$

The above condition suggests that near the capillary wall, mass transfer happens with a velocity S, where injection occurs when $S>0$ and $S<0$ in the suction case. The non-uniform heat sink/source, q''' is taken as (Srinivas et al. [55])

$$q'''=\frac{(U_w)\kappa}{\upsilon x}\left[(f')(T_w-T_\infty)\Gamma+(\hat{T}-T_\infty)\xi\right]. \tag{10}$$

It shows that when $\Gamma>0; \xi>0$, generates the internal heat and when $\Gamma<0; \xi<0$, absorbs the internal heat. By using the Rosseland approximation for radiative heat flux, q_r is simplified as (Hayat et al. [21])

$$q_r=-\frac{4}{3}\left(\frac{\partial\hat{T}^4}{\partial y}\right)\left(\frac{\sigma^*}{\kappa^*}\right) \tag{11}$$

where $\hat{T}^4\cong 4\hat{T}\left(T_\infty^3\right)-3\left(T_\infty^4\right)$. (By using the Taylor's series approximation). The following self-similar transformations are then introduced as (Pantokratoras [42]; Srinivas et al. [55])

$$\begin{gathered}\eta=\left(\frac{U_w}{\upsilon x}\right)^{1/2} y\Rightarrow\eta=\left(\frac{y}{x}\right)Re_x^{1/2}\\ \psi=\left(\upsilon x U_w\right)^{1/2} f(\eta),\\ \theta(\eta)=\frac{\hat{T}-T_\infty}{T_w-T_\infty}\text{ and }\phi(\eta)=\frac{\hat{C}-C_\infty}{C_w-C_\infty}\end{gathered} \tag{12}$$

where $Re_x=U_w x/\upsilon$ and ψ is the stream function, with a definitive objective that

$$\hat{u}=\frac{\partial\psi}{\partial y}\Rightarrow\hat{u}=\left(\frac{\lambda_1 x}{1-\lambda_3 t}\right)f'(\eta) \tag{13}$$

$$\hat{v}=-\frac{\partial\psi}{\partial x}\Rightarrow\hat{v}=-\left(\frac{\upsilon\lambda_1}{1-\lambda_3 t}\right)^{1/2} f(\eta). \tag{14}$$

Now, by using equations (8) - (14), equations (4) - (6) reduced to

$$A\left(f'+\frac{1}{2}f''\eta\right)-ff''+(f')^2=\left[(1+\varepsilon)-\varepsilon\delta(f'')^2\right]f'''+(Gr_x\cos\alpha)\theta+(Gc_x\cos\alpha)\phi-(M)f' \tag{15}$$

$$A\left(\theta+\frac{1}{2}\eta\theta'\right)-f\theta'+\theta f'=\frac{\theta''}{\Pr}+N_b\theta'\phi'+N_t(\theta')^2+\frac{1}{\Pr}(\Gamma f'+\xi\theta)+\frac{4}{3}\frac{1}{\Pr}Rd\theta''+Ec\left[M(f'(\eta))^2+(1+\varepsilon)(f''(\eta))^2\right] \tag{16}$$

$$A\left(\phi+\frac{1}{2}\eta\phi'\right)-f\phi'+\phi f'=\frac{\phi''}{\Pr Le}+\frac{1}{\Pr Le}\frac{N_t}{N_b}\theta''-k_3\phi \tag{17}$$

In addition, the resultant new boundary conditions after the transformation are as given below

$$\begin{gathered} f(0)=S, \theta(0)=1, f'(0)=1 \text{ and } N_b\phi'(0)+N_t\theta'(0)=0 \text{ at } \eta=0 \\ f'\to 0, \theta\to 0 \text{ and } \phi\to 0 \text{ at } \eta\to\infty \end{gathered} \tag{18}$$

where $A=\lambda_3/\lambda_1$, $\varepsilon=1/(\mu\beta\gamma)$, $\delta=U_w^3/(2\upsilon x\gamma^2)$, $M=\sigma B_0^2\times(1-\lambda_3 t)/(\rho\lambda_1)$ (Musa et al. [36]), $\Pr=\mu c_p/\kappa$, $Gr_x=g\beta_T^* x(T_w-T_\infty)/U_w^2$ (Bejan [6]), $Gc_x=g\beta_C^* x(C_w-C_\infty)/U_w^2$, $Rd=4\sigma^* T_\infty^3/(k^*\kappa)$, $Ec=U_w^2/[c_p(T_w-T_\infty)]$, $k_3=k_2/\lambda_1, N_b=\tau D_b(C_w-C_\infty)/\upsilon$, $N_t=(\tau/\upsilon)(D_t/T_\infty)(T_w-T_\infty)$, $Le=\alpha^*/D_b$ where $\alpha^*=\kappa/(\rho c_p)$. Further, the functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are defined to calculate the coefficient of local skin friction (C_f), local Nusselt number (Nu_x) and local Sherwood number (Sh_x) respectively (Ramzan et al. [47]).

$$\begin{gathered} C_f Re_x^{1/2}=f''(0)(1+\varepsilon)-\frac{1}{3}\varepsilon\delta\left(f''(0)\right)^3 \\ Nu_x Re_x^{-1/2}=-\left(1+\frac{4Rd}{3}\right)\theta'(0) \\ Sh_x Re_x^{-1/2}=-\phi'(0) \end{gathered} \tag{19}$$

3. Results and Discussions

The non-dimensional Eqs. (16) - (17) subjected to the boundary conditions given in Eq. (18) have been solved numerically by using the shooting technique along with the Runge-Kutta method of 4^{th} order, which has been solved with the help of the function NDSolve in MATHEMATICA, which is the current specialized computing system. NDSolve gives the solutions iteratively. To begin, NDSolve must be given proper initial or boundary conditions for the Null and their derivatives. To check the validity of the present model, the obtained results are compared with the previous investigations and these results are stored in Table 1 and Table 2.

Table 1. Comparison of $f''(0)$ between the present results and previous results when $M=Gr_x=Gc_x=Nt=Nb=\Gamma=\xi=Le=0$ and absence of slip conditions.

A	Pal [39]	Champka et al. [8]	Present
0.8	-1.26104	-1.26151	-1.26108
1.2	-1.37772	-1.37805	-1.37777

Table 2. Comparison of $\theta'(0)$ between the present results and previous results when $M=Gr_x=Gc_x=Nt=Nb=\Gamma=\xi=Le=0$ and absence of slip conditions

A	S	Pr	Ishak et al. [24]	Pal [39]	Hayat et al. [21]	Present
0	-1.5	0.72	0.4570	0.45702	0.45702	0.45702
1	-1.5	1	0.8095	0.80951	0.80951	0.80951

To understand the influence of various parameters, we have chosen $\alpha=\pi/3$, $\Pr=7$, $Rd=\Gamma=\xi=1, k_3=1$, $A=0.4, S=Ec=0.5$, $M=1.5$, $Gr_x=Gc_x=5$, $Le=2$, $\varepsilon=\delta=0.1$, and $N_b=N_t=0.3$ unless otherwise stated. For carrying out the computational study, we have made use of several dimensionless parameter ranges that have been specified in technical literatures by previous experimental/theoretical investigators (Ishak et al. [24], Hayat et al. [21], Misra and Sinha [35] and Makinde and Osalusi [32]).

$$\begin{array}{ll} A=0.1,\ 0.2,\ 0.3,\ 0.4. & Gr_x, Gc_x=1,\ 3,\ 5. \\ \alpha=0,\ \pi/4,\ \pi/3,\ \pi/2. & N_b, N_t=0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5. \\ \Gamma,\xi=-2,\ -1,\ 0,\ 1,\ 2. & Rd=0,\ 0.5,\ 1.5,\ 2. \\ M=0,\ 1,\ 1.5,\ 2. & Ec=0.1,\ 0.3,\ 0.5,\ 0.7. \\ S=0.5,\ 1,\ 1.5,\ 2. & Le=1,\ 2,\ 3,\ 4. \\ \varepsilon=0,\ 0.5,\ 0.7,\ 1. & k_3=0,\ 0.3,\ 0.6,\ 1. \end{array}$$

The influence of A, k_3, M, N_b, N_t, S and Rd on C_f, Nu_x and Sh_x are shown in Table 3. Here one can observe that C_f decreases as increase in A, k_3, M, N_t and S while it increases for a given increase in N_b and Rd. From the same table, it can be seen that there is an enhancement in local Nusselt number as the values of A, S and Rd goes higher whereas it decreases as increase in k_3, M, N_b and N_t. Further, there is an increment in local Sherwood number when k_3, M and N_t increases and it has decreased as increase in A, N_b, S and Rd.

Table 3. The numerical values of $C_f Re_x^{1/2}$, $Nu_x Re_x^{-1/2}$ and $Sh_x Re_x^{-1/2}$ at different values of parameters.

Parameter		$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$
A	0.5	-1.30383	0.46758	-0.46758
	0.6	-1.34395	0.554226	-0.554226
	0.7	-1.38356	0.637453	-0.637453
α	0	-0.199164	0.755741	-0.323889
	$\pi/4$	-0.822805	0.861934	-0.3694
	$\pi/2$	-2.25514	0.482478	-0.206776
k_3	0	-1.20364	1.09649	-0.469926
	0.5	-1.21139	1.09056	-0.467381
	1	-1.21474	1.08612	-0.465481
M	0	-0.364644	2.7241	-1.16747
	1	-0.774149	1.91624	-0.821247
	2	-1.73428	0.149827	-0.0642117
N_b	0.1	-1.38847	-0.176741	0.227238
	0.3	-1.06518	-0.197609	0.0846896
	0.5	-1.00237	-0.204528	0.052593
N_t	0.1	-0.970192	-0.174492	0.0249274
	0.3	-1.06518	-0.197609	0.0846896
	0.5	-1.16383	-0.220693	0.157638
S	0.5	-1.21474	1.08612	-0.465481
	1	-1.53871	1.84973	-0.792743
	1.5	-1.91912	2.71724	-1.16453
Rd	0	-1.30383	0.46758	-0.46758
	0.5	-1.24891	0.797551	-0.478531
	1	-1.21474	1.08612	-0.465481

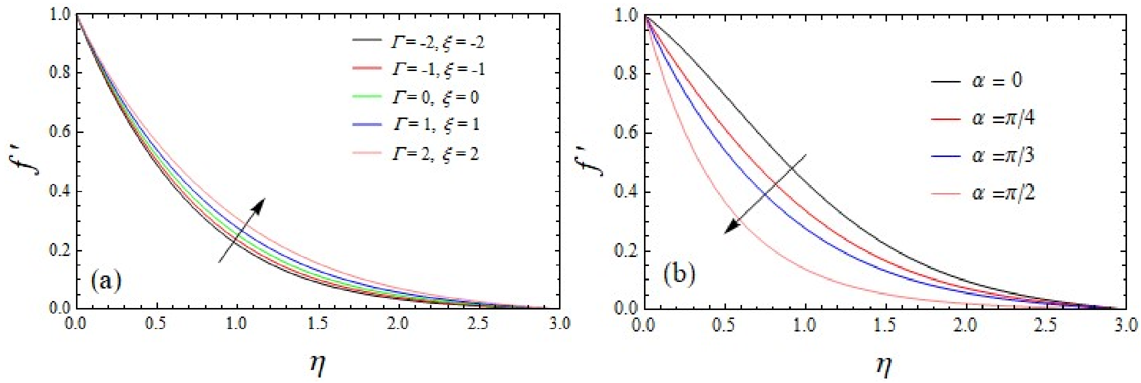

Fig. 2. (a) Effect of Γ and ξ on $f'(\eta)$ (b) Effect of α on $f'(\eta)$.

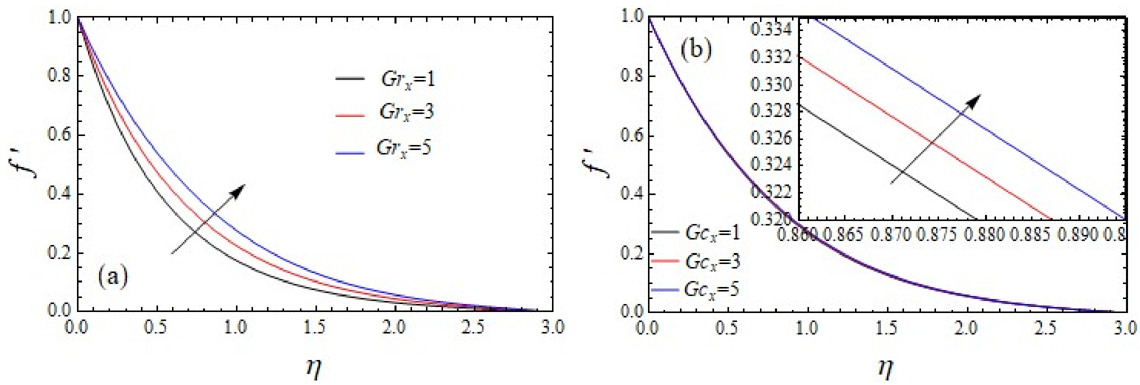

Fig. 3. (a) Effect of Gr_x on $f'(\eta)$ (b) Effect of Gc_x on $f'(\eta)$.

The influence of various parameters on unsteady velocity (f'), temperature (θ) and nanoparticles concentration (ϕ) profiles are shown graphically from Figures 2 - 9. Figure 2(a) shows that f' increases with increasing Γ and ξ because increasing heat sink/source causes increment in momentum boundary layer. From Figure 2(b), the velocity reduces as the inclination angle (α) increases.

The effects of Gr_x and Gc_x on f' are depicted in Fig. 3(a) and Fig. 3(b) respectively. One can observe that there is a rise in velocity for a given increase in Gr_x and Gc_x. There is a rise in velocity as the Eyring-Powell fluid parameter (ε) increases which has shown in Fig. 4(a). Figure 4(b) illustrates the effect of M against velocity of the fluid. Here, the increase in M results the decrement in velocity. This is because the magnetic field produces retarding forces, which act as resistive drag forces opposite to the flow direction. The same behaviour can notice with an increase in S from Fig. 4(c).

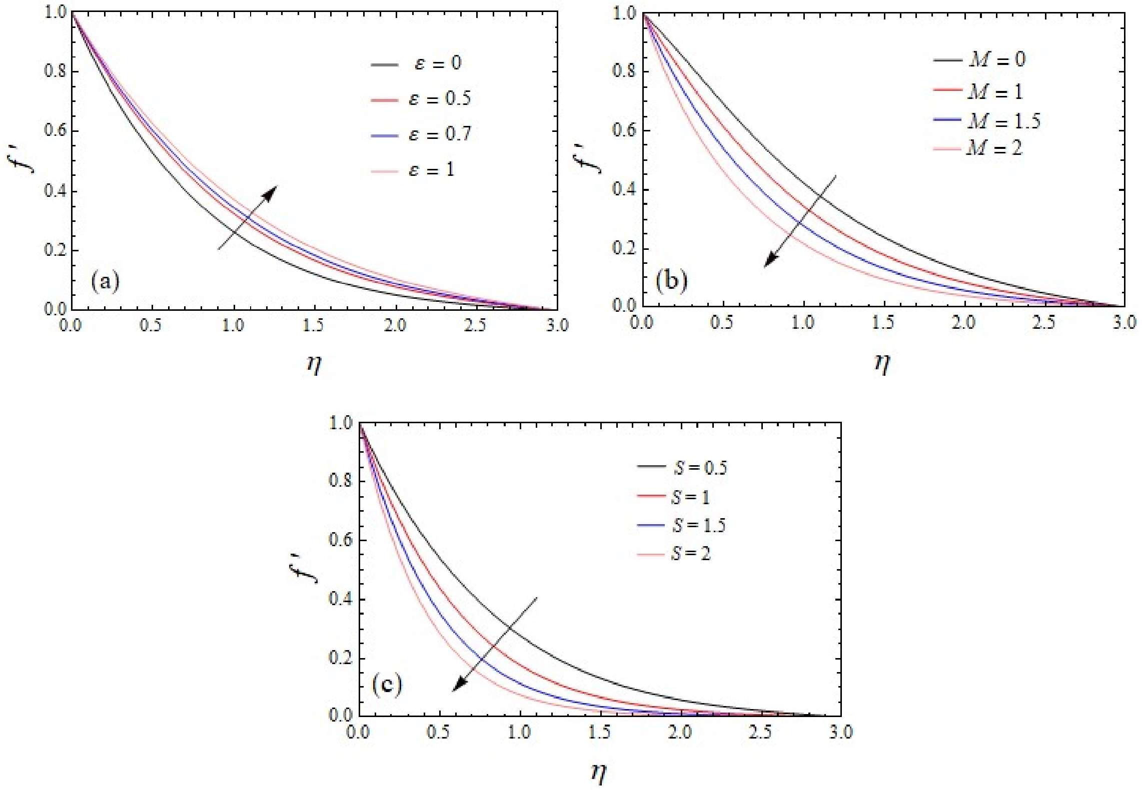

Fig. 4. (a) Effect of ε on $f'(\eta)$ (b) Effect of M on $f'(\eta)$ (c) Effect of S on $f'(\eta)$.

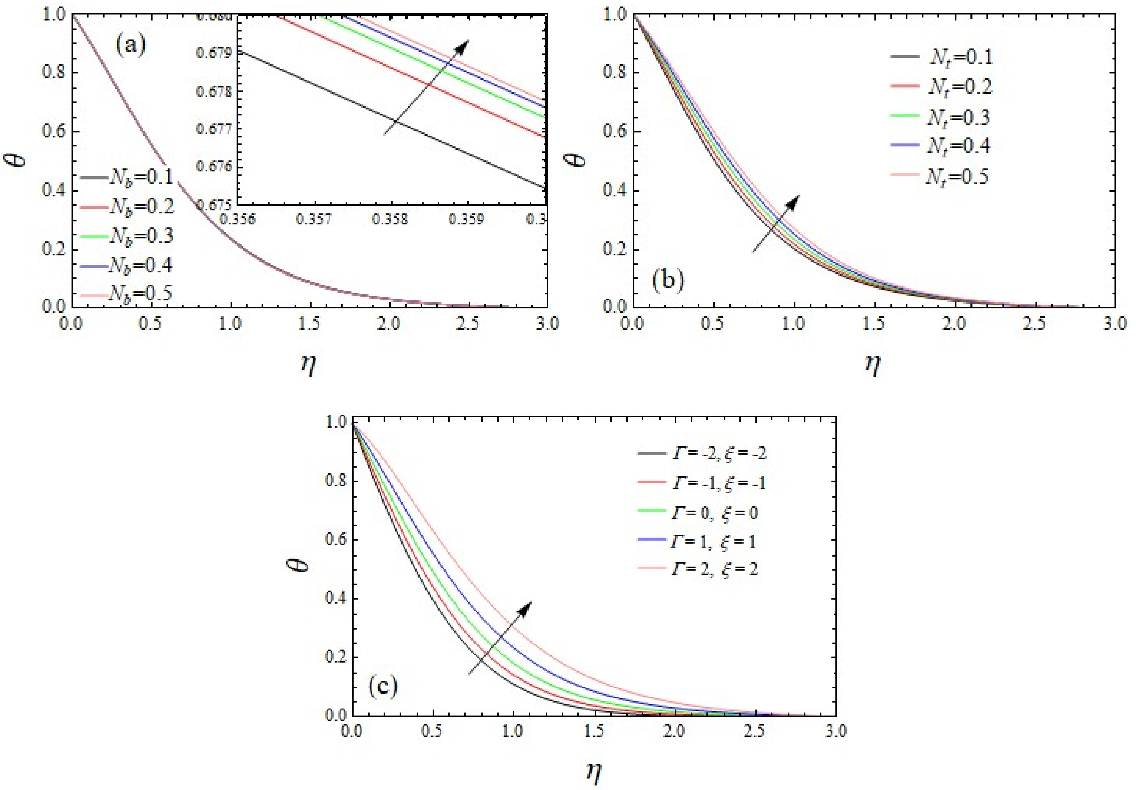

Fig. 5. (a) Effect of N_b on θ (b) Effect of N_t on θ (c) Effect of Γ and ξ on θ.

The variations of Γ, ξ, N_b, N_t, Rd, Ec, M and A on θ are presented in Figs. 5 - 7. From Fig. 5(a) and Fig. 5(b), it is observed that there is an enhancement in temperature with increase in N_b and N_t respectively. Clearly, the thermal boundary layer increases with an increase in Γ and ξ in Fig. 5(c). Figure 6(a) represents the effect of Rd on θ. The known physical statement is that as the radiation increases, there is a growth in thermal boundary layer thickness. Hence, there is a rise in temperature. Figure 6(b) elucidates the variation of Ec on θ. It is clear that there is an enhancement in wall temperature of the sheet as the Ec value increases and the thermal boundary layer thickness increases. This is because the rate of heat transfer is low at the surface when there is higher value of Ec. Indeed, there is a rise in temperature due to internal friction of molecules, which transforms mechanical energy into heat energy.

Increasing the frictional drag because of the Lorentz force is liable for the enhancement of thermal boundary layer thickness and hence the temperature increases as the M increases that has been shown in the Fig. 7(a). From Fig. 7(b), it can be seen that the temperature decreases for the higher values of A.

Further, the variations in ϕ for various values of k_3, N_b, N_t and Le are shown from Fig. 8(a) – 9(b). Figure 8(a) illustrates that an increase in the value of N_b there is a decrement in ϕ. Figure 8(b) describes that ϕ increases as increase in N_t. For higher values N_t, more particles are pushed away from the hot surface, resulting in increased concentration.

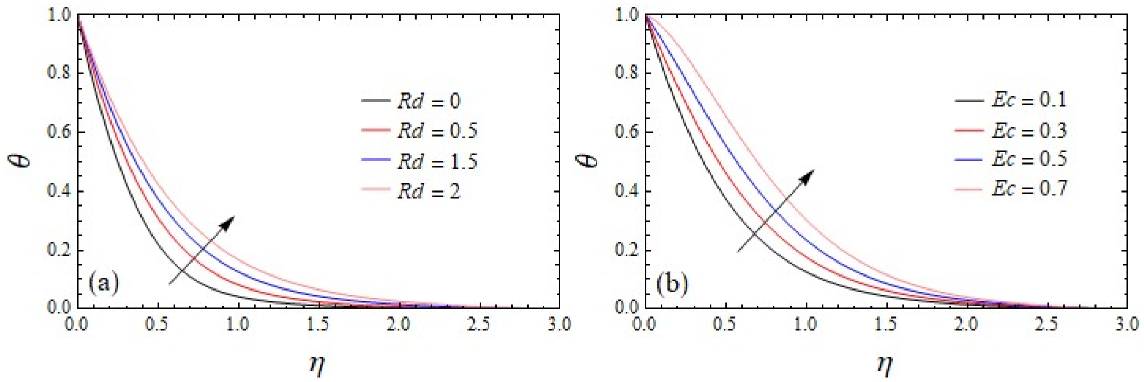

Fig. 6. (a) Effect of Rd on θ (b) Effect of Ec on θ.

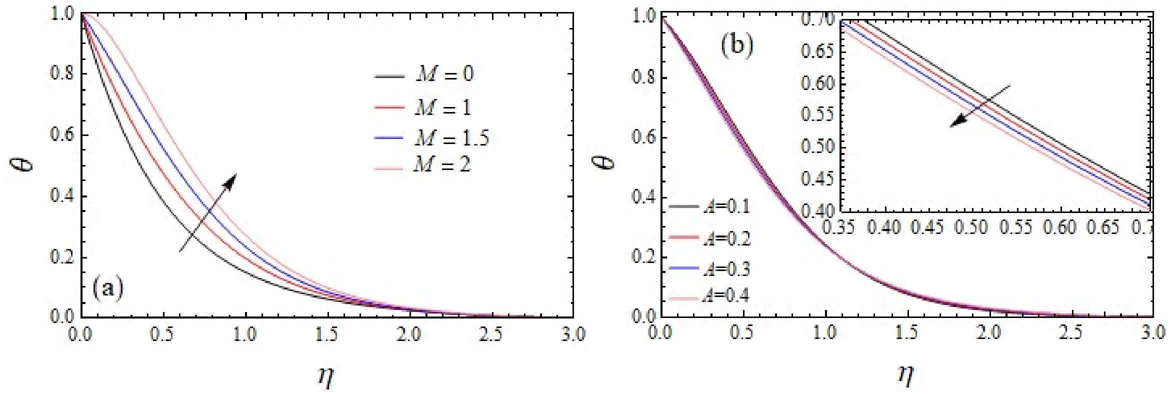

Fig. 7. (a) Effect of M on θ (b) Effect of A on θ.

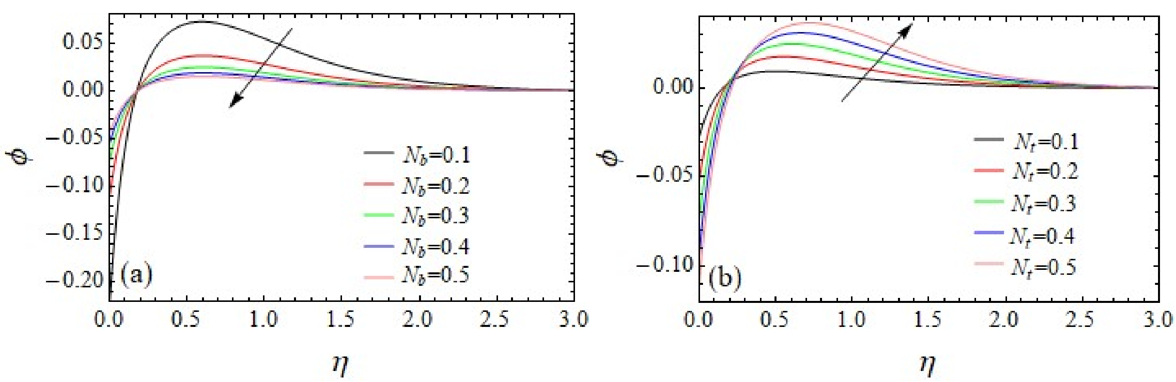

Fig. 8. (a) Effect of N_b on ϕ (b) Effect of N_t on ϕ.

Figure 9(a) depicts the influence of k_3 on ϕ. Increasing k_3 yields a decrease in the species concentration. Hence, there is a decrement in nanoparticles concentration profile. From Fig. 9(b), one can observe that, due to the lesser molecular diffusivity, the nanoparticles concentration profile decreases as Le increases.

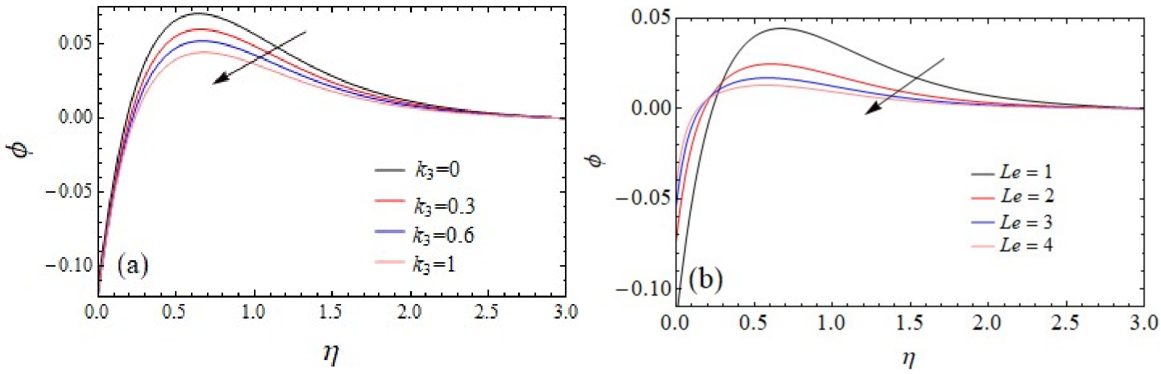

Fig. 9. (a) Effect of k_3 on ϕ (b) Effect of Le on ϕ.

4. Conclusion

The unsteady MHD flow of Eyring-Powell nanofluid over an inclined permeable shrinking/stretching sheet with viscous dissipation, Joule heating, and non-uniform heat sink/source chemical reactions numerically has been investigated. It is observed that f' decreases as an increase in α, M and S on the other hand there is an increment of f' in the case of ε, Gr_x, Γ and ξ. It is noticed that, θ enhances for a given increase in M, N_b, N_t, Γ and ξ. It is concluded that ϕ decreases with an increase in k_3, N_b & Le and there is an increment in the case of N_t. It is noticed that as increasing N_b and Rd there is an increment in C_f. Also, Nu_x increasing for the higher values of A, S and Rd, whereas Sh_x has decreased in this case. The comparison from Tables 1 and 2 show that there is an excellent agreement between the present and previous results.

Nomenclature

$\hat{u}$	velocity along x - direction
$\hat{v}$	velocity along y - direction
t	time
c_p	specific heat at constant pressure
q_r	radiative heat flux
g	acceleration due to gravity
D_b	Brownian diffusion coefficient
D_t	thermophoresis diffusion coefficient
A	unsteadiness parameter
Gr_x	local Grashof number
Gc_x	solutal Grashof number
M	magnetic parameter
Pr	Prandtl number
Ec	Eckert number
Le	Lewis number
Rd	radiation parameter
Re_x	local Reynolds number
V_i	velocity components of the fluid
S	suction/injection parameter
$K(t)$	1st order chemical reaction
$k_0(t)$	time dependent permeability
k_1	permeability constant
k_2	chemical reaction constant
k_3	chemical reaction parameter
$\hat{T}$	fluid temperature
$\hat{C}$	nanoparticles concentration
N_b	Brownian motion parameter
N_t	thermophoresis parameter

Greek Letters

υ	kinematic viscosity
α	angle of inclination
β, γ	characteristic properties of the fluid
ε, δ	Eyring-Powell fluid parameters
Γ	coefficient of space dependent heat source/sink
ξ	coefficient of temperature dependent heat source/sink
ρ	density of the fluid
μ	dynamic viscosity of the fluid
σ	electrical conductivity
σ^*	Stefan-Boltzmann constant
k^*	mean absorption coefficient
α^*	thermal diffusivity
κ	thermal diffusivity coefficient
β_T^*	thermal expansion coefficient
β_C^*	coefficient of expansion with concentration
τ_{ij}	stress tensor of the fluid

B_0 constant magnetic field strength

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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