



Journal of Applied and Computational Mechanics



Research Paper

Heat Transfer Analysis of Nanofluid Flow with Porous Medium through Jeffery Hamel Diverging/Converging Channel

Akinbowale T. Akinshilo¹, Adeleke Ilegbusi², Hafiz M. Ali^{3,4}, Abdul-Jalil Surajo⁵

¹ Department of Mechanical Engineering, University of Lagos, Akoka- Yaba, Lagos, Nigeria, Email: ta.akinshilo@gmail.com

² Department of Mechanical Engineering, Yaba College of Technology, Yaba, Lagos, Nigeria, Email: illegbusi@gmail.com

³ Department of Mechanical Engineering, University of Engineering and Technology, Taxila, Pakistan, Email: h.m.ali@qmul.ac.uk

⁴ Mechanical Engineering Department, King Fahd University of Petroleum and Minerals (KFUPM), Dhahran 31261, Saudi Arabia

⁵ Department of Chemical Engineering, Ahmadu Bello University, Zaria, Kaduna, Nigeria, Email: abbasurajo@gmail.com

Received May 09 2019; Revised July 23 2019; Accepted for publication July 23 2019.

Corresponding author: A.T. Akinshilo (ta.akinshilo@gmail.com)

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& International Research Center for Mathematics & Mechanics of Complex Systems (M&MoCS)

Abstract. In this paper, flow and heat transfer of nanofluid through a converging or diverging channel with porous medium is investigated. The fluid constantly flows under the effect of magnetic field through the channel. The diverging/converging fluid motion is modeled using the momentum and energy equations. The influence of some parameters such as opening channel angle, Reynolds number and Darcy's number when the nanofluid flows through the non-parallel plates are studied. It is seen that high Reynolds number enhances the fluid viscosity while decreases velocity. Similarly, heat transfer reduces at high Darcy's number owing to decreased flow consequently internal friction reduces. The obtained results in comparison with the similar studies in the literatures show satisfactory agreement.

Keywords: Jeffery Hamel; Nanofluid; Diverging/Converging plates; Porous medium; HPM.

1. Introduction

Diverging or converging channel flow was proposed by Jeffery and Hamel within the period from 1915 to 1916. Despite a long period of discovery, not much research attention has been devoted to this issue. Nevertheless, flow through this type of medium has practical and important applications including blood flow through arteries and veins, veins and arteries are blood carrying tubes to and from the heart, respectively. As blood returns to the heart, the veins increase in size while blood leaving the heart decreases the size of arteries. This shows a convergent and divergent phenomenon. Similar transport phenomena are experienced in environmental engineering in channels such as dams and irrigations canals amongst other numerous applications. Consequently, Chen et al. [1] described composition of particles clusters as a porous medium. This shows clusters of micro size consist of pores. Practical applications of porous medium flows include geothermal systems, reservoir bed, waste disposal, catalytic converters. Darcy number defines the flow through porous medium [2-12]. Low Reynolds number within the range of one to ten proves the validity of Darcy theory as presented by Fard et al. [11].

Alfven [13] discovered conducting flowing fluid develops electromotive force which distorts velocity distribution profile. Electromotive forces developed from flows which are generated by induced current. Thus, the magneto hydrodynamic fluid flow through porous medium has found many exciting relevance in modern engineering including plasma physics, nuclear reactor, and renewable energy extraction amongst others [14-19]. Upon this pioneering work



other researchers have also conducted researches. The heat transfer and flow through stretching walls was presented by Raftari and Vajravelu [20] considering influence of magnetic field. Similarly, Hatami et al. [21] investigated flat plate nanofluid flow under constant magnetic force field. Heat transfer, entropy generation and nanofluid flow utilizing varying flow channels was investigated by Sheikholeslami et al. [22-26]. The locally heated natural convective flow was presented by Hussein and Mustafa [27] using water-copper nanofluid. Stagnation point flow of an MHD nanofluid over convecting surface was studied by Prabhakar et al. [28] considering the effects of slip and thermal radiation. Entropy generation analysis of methanol-based nanofluid was investigated in a wavy sinusoidal channel by Al-Mdallal [29]. Ganesh [30] carried out a numerical study of Newtonian fluid flow with entropy and the thermal slip of the second order. Slip effect of the second-order and heat transfer of MHD Maxwell fluid in a porous medium was analyzed by Aman et al. [31]. Ganesh et al. [32] studied thermal radiative Maragoni boundary layer flow non-linearly over a stretching sheet.

Since most real-life problems are modeled by nonlinear systems of equations. It is imperative to employ a suitable, reliable, and accurate method of solutions to analyze these problems. So, approximate analytical, semi-analytical and numerical solution methods have been adopted by researchers over time in the analysis of these problems. Methods of solutions include the variation of parameters method (VPM), the differential transform method (DTM), the differential transform-Pade method (an extension solution of DTM), collocation, least-square and Galerkin methods of weighted residuals, adomian decomposition method (ADM), homotopy perturbation method (HPM) and variational iteration method (VIM) [33-62]. The stated methods suffer from their limitations and round off errors. Out of many, the HPM has been selected as a convenient, accurate method, not dependent on the small perturbation parameter; neither linearization nor discretization has been utilized in this paper.

In the light of past research works, this study investigates the effects of heat transfer and flow of electrically conducting magnetohydrodynamic nanofluid through a diverging or converging channel with porous medium considering internally generated heat. Since the system of equations describing this problem is nonlinear (high-order), so the homotopy perturbation method is utilized as a suitable method of solution.

2. Model Development and Problem Formulation

Here, incompressible, two dimensional, and electrically conducting, nanofluid flow is under consideration. The flow through the channel is described utilizing cylindrical co-ordinate (r, θ, z) since intersecting planes are in the z axis. Pure radial motion is assumed since there is no change in fluid parameters. The motion of fluid is therefore at axis θ and r . This is observed in the Fig.1. Viscous flow governing equation system is described by Navier Stokes with terms given in the nomenclature as presented in [22, 57]:

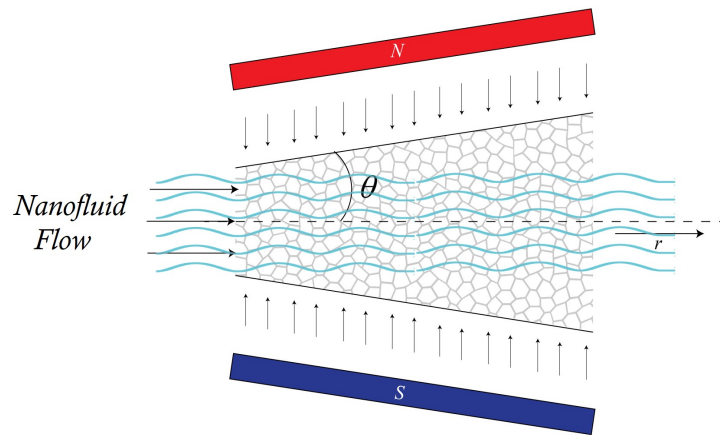


Fig. 1. Model of the problem.

$$\frac{\rho_{nf}}{r} \frac{\partial(ru(r, \theta))}{\partial r} (ru(r, \theta)) = 0 \quad (1)$$

$$u(r, \theta) \frac{\partial u(r, \theta)}{\partial r} = -\frac{1}{\rho_{nf}} \frac{\partial P}{\partial r} + \nu_{nf} \left[\frac{\partial^2 u(r, \theta)}{\partial r^2} + \frac{1}{r} \frac{\partial u(r, \theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} (r, \theta) - \frac{u(r, \theta)}{r^2} \right] - \frac{\sigma_{nf} B_0^2}{\rho_{nf} r^2} u(r, \theta) - \frac{\mu u}{K_p r^2} (r, \theta) \quad (2)$$

$$\frac{1}{r \rho_{nf}} \frac{\partial P}{\partial \theta} - \frac{2 \nu_{nf}}{r^2} \frac{\partial u}{\partial \theta} (r, \theta) = 0 \quad (3)$$

$$(\rho C_p)_{nf} u(r, \theta) \frac{\partial T}{\partial r} = k_{nf} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] + \mu_{nf} \left[2 \left(\frac{\partial u}{\partial r}(r, \theta) \right)^2 + \left(\frac{u}{r}(r, \theta) \right)^2 + \left(\frac{1}{r} \frac{\partial u}{\partial r}(r, \theta) \right)^2 \right] + \sigma B_0^2 \frac{u^2}{r}(r, \theta) + Q(T - T_w) \quad (4)$$

with appropriate boundary condition introduced as:

$$\begin{aligned} u = u_{\max}, \quad \frac{\partial u}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \theta} = 0 & \quad \text{at} \quad \theta = 0 \\ u = 0, \quad T = T_w & \quad \text{at} \quad \theta = \alpha \end{aligned} \quad (5a)$$

where the nanofluid parameter includes effective dynamic viscosity μ_{nf} , effective density ρ_{nf} and fluid conductivity σ . Effective kinematic viscosity ν_{nf} introduced as:

$$\rho_{nf} = \rho_f (1 - \phi) + \rho_s \phi, \quad \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \nu_{nf} = \frac{\mu_f}{\rho_{nf}}, \quad \frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_s}{\sigma_f} + 2 \right) - \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \quad (5b)$$

The nanoparticle volume concentration is ϕ . Since pure radial flow is assumed, velocity is given as:

$$f(\theta) = ru(r) \quad (6)$$

$\eta = \theta/\alpha$ is the similarity variable. The Dimensionless velocity and temperature are expressed as:

$$f(\eta) = \frac{f(\theta)}{f_{\max}}, \quad g(\eta) = r^2 \frac{T}{T_w} \quad (7)$$

The thermo-physical properties of the nano fluid are expressed in Table 1.

Table 1. Thermal physical properties of pure water and Copper nanoparticle [22].

	Density (Kg/m ³)	Specific heat capacity (J/kgK)	Thermal conductivity (W/mk)	Electrical conductivity σ ($\Omega.m$) ⁻¹
Water	997.1	4179	0.613	0.05
Copper	8933	385	401	5.96x10 ⁷

Eq. (5) is presented into Eqs. (2-3) eliminating the pressure term. The normalized profiles for the flow and heat transfer are given as:

$$A^* = (1 - \phi) + \frac{\rho_s}{\rho_f} \phi; B^* = 1 + \frac{3 \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left(\frac{\sigma_s}{\sigma_f} + 2 \right) - \left(\frac{\sigma_s}{\sigma_f} - 1 \right) \phi} \quad (8)$$

$$g'' + 4\alpha^2 g + \frac{A^*}{B^*} 2\alpha^2 \text{Pr} fg + \frac{\text{Pr} Ec}{\text{Re} B^* (1 - \phi)^{2.5}} (4\alpha^2 f^2 + f'^2) + \frac{\text{Pr} Ec Ha}{B^*} f^2 + \zeta g = 0 \quad (9)$$

where the Re is the Reynolds number, A^* and B^* are the nano fluid parameters, Ha is the magnetic parameter, Da is the Darcy number, and ζ is the heat generation parameter introduced as:

$$(\rho C_p)_{nf} u(r, \theta) \frac{\partial T}{\partial r} = k_{nf} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} \right] + \mu_{nf} \left[2 \left(\frac{\partial u}{\partial r}(r, \theta) \right)^2 + \left(\frac{u}{r}(r, \theta) \right)^2 + \left(\frac{1}{r} \frac{\partial u}{\partial r}(r, \theta) \right)^2 \right] + \sigma B_0^2 \frac{u^2}{r}(r, \theta) + Q(T - T_w) \quad (10)$$

$$\text{Re} = \frac{f_{\max}}{\nu_f} = \frac{u_{\max}}{\nu_f} \begin{cases} \text{divergent-channel} : \alpha > 0, f_{\max} > 0 \\ \text{convergent-channel} : \alpha < 0, f_{\max} < 0 \end{cases} \quad (11)$$

$$Ha = \sqrt{\frac{\sigma_f B_o^2}{\rho_f \nu_f}} \quad (12a)$$

$$Da = \frac{\mu u}{K_p} \quad (12b)$$

$$\zeta = \frac{QA(\rho C_p)_{nf}}{k_{nf}(T - T_w)} \quad (12c)$$

Ranges of Reynolds number is selected within $1 < \text{Re} < 8$ assuming nanofluid flow through the non-parallel channel is slow, laminar, viscous. Darcys number range from $0 < Da < 1$ to be valid for laminar flows regimes [56]. Other parameters are selected to be within reasonable ranges as in Ref. [22]. The appropriate boundary condition is expressed as:

$$\begin{aligned} f(0) = 1, f'(0) = 0, g'(0) = 0 & \quad \eta = 0 \\ f(1) = 0, g(1) = 1 & \quad \eta = 1 \end{aligned} \quad (13)$$

2.1. Fundamentals of Homotopy Perturbation Method

The principles of the homotopy perturbation method are explained using the following equation [12]:

$$A(u) - f(r) = 0 \quad r \in \Omega \quad (14)$$

The boundary condition adopted read as:

$$B(u, \frac{\partial u}{\partial \eta}) = 0 \quad r \in \Gamma \quad (15)$$

The analytical function $f(r)$, the domain boundary of Γ is Ω , general operator differential is A while the boundary operator is B . The general differential operator is separated into nonlinear and linear terms N and L , respectively. Eq. (14) is reconstructed as

$$L(u) + N(u) - f(r) = 0 \quad r \in \Omega \quad (16)$$

The structure of the Homotopy perturbation takes the form of:

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + P[A(v) - f(r)] = 0 \quad (17)$$

where

$$v(r, p) : \Omega \times [0, 1] \rightarrow R \quad (18)$$

The embedding parameter is $P \in (0, 1)$ as observed in Eq. (17) and the initial term which satisfies condition at boundary is taken as u_0 . Series solution of Eq. (17) is expressed as:

$$v = v_0 + P v_1 + P^2 v_2 + P^3 v_3 + \dots \quad (19)$$

The solution takes the form of:

$$u = \lim_{p \rightarrow 1} (v_0 + P v_1 + P^2 v_2 + P^3 v_3 + \dots) \quad (20)$$

2.2. Application of the Homotopy Perturbation Method

Equation (8) is reconstructed using the homotopy, with orders expressed as:



$$P^0 : \frac{d^3 f_0}{d\eta^3} \quad (21)$$

$$P^1 : \frac{d^3 f_1}{d\eta^3} - \alpha \left[(B^* Ha(1-\phi)^{1.25}) \frac{1}{Da} - 4 \right] \frac{df_0}{d\eta} + 2A^* Re \alpha f_0 \frac{df_0}{d\eta} (1-\phi)^{2.5} \quad (22)$$

$$P^2 : \frac{d^3 f_2}{d\eta^3} - \alpha \left[(B^* Ha(1-\phi)^{1.25}) \frac{1}{Da} - 4 \right] \frac{df_1}{d\eta} + 2A^* Re \alpha f_0 * \frac{df_1}{d\eta} (1-\phi)^{2.5} + 2A^* Re \alpha f_1 \frac{df_0}{d\eta} (1-\phi)^{2.5} \quad (23)$$

Equation (9) is reconstructed using the homotopy, with orders expressed as:

$$\frac{d^2 g_0}{d\eta^2} \quad (24)$$

$$\frac{d^2 g_1}{d\eta^2} + 4\alpha^2 g + 2\alpha^2 \frac{A^*}{B^*} Pr f_0 g_0 + \frac{Pr Ec}{Re B^* (1-\phi)^{2.5}} (4\alpha^2 f_0^2 + f_0'^2) + \frac{Pr Ec Ha}{B^*} f_0^2 + \zeta g_0 \quad (25)$$

$$\frac{d^2 g_2}{d\eta^2} + \frac{2Ecpr}{B Re(1-\phi)^{2.5}} \frac{df_0}{d\eta} \frac{df_1}{d\eta} + 4\alpha^2 g_1 + \frac{8Ec Pr \alpha^2 f_0 f_1}{B Re(1-\phi)^{2.5}} + \zeta g_1 + 2 \frac{A}{B} Pr \alpha^2 f_0 g_1 + 2 \frac{A}{B} Pr \alpha^2 f_1 g_0 + \frac{2EcHa Pr f_0 f_1}{B} \quad (26)$$

The relevant boundary conditions are given as:

$$\begin{aligned} f(0) = 1, f'(0) = 0, g'(0) = 0 & \quad \eta = 0 \\ f(1) = 0, g(1) = 1 & \quad \eta = 1 \end{aligned} \quad (27)$$

Applying the boundary condition (Eq. (27) to Eqs. (21-23)), yields the final form of the equation as:

$$f_0 = 1 - \eta^2 \quad (28)$$

$$f_1 = -(\eta^2 - 1) \left(60Da - 20Da\alpha\eta^2 + 5BM\alpha\eta^2(1-\phi)^{1.25} - 8ADa Re \alpha \eta^2(1-\phi)^{2.5} + 2ADa Re \alpha \eta^4(1-\phi)^{2.5} \right) / 60Da \quad (29)$$

$$\begin{aligned} f_2 = & -(\eta^2 - 1) \times (7500Da^2 - 25200Da^2\alpha\eta^2 - 5040Da^2\eta^2\alpha^2 + 3360Da^2\alpha^2\eta^4 - 315B^2M^2\alpha^2\eta^2(1-\phi)^{2.5} \\ & + 210B^2M^2\alpha^2\eta^4(1-\phi)^{2.5} - 625A^2Da^2 Re^2 \alpha^2\eta^2 + 1028A^2Da^2 Re^2 \alpha^2\eta^4 - 484A^1Da^2 Re^2 \alpha^2\eta^6 \\ & + 56A^2Da^2 Re^2 \alpha^2\eta^8 + 2520BDaM\alpha^2\eta^2(1-\phi)^{1.25} - 1680BDaM\alpha^2\eta^4(1-\phi)^{1.25} \\ & - 20160ADa^2 Re \alpha \eta^4(1-\phi)^{2.5} + 3260A^2Da^2 Re^2 \alpha^2\eta^2\phi - 5140A^2Da^2 Re^2 \alpha^2\eta^4\phi + \dots) \end{aligned} \quad (30)$$

Applying the boundary condition (Eq. (27) to Eqs. (24-26)), yields the final form of the equation as:

$$g_0 = 1 \quad (31)$$

$$\begin{aligned} g_1 = & (10Ec Pr + 30B^* Re(1-\phi)^{2.5} + 44Ec Pr \alpha^2 - 10Ec Pr \eta^4 + 60B^* Re \alpha^2(1-\phi)^{2.5} - 60Ec Re \alpha^2\eta^2 + 20Ec Pr \alpha^2\eta^4 \\ & - Ec Pr \alpha^2\eta^6 + 15B^*\zeta Re(1-\phi)^{2.5} - 60B^* Re \alpha^2\eta^2(1-\phi)^{2.5} + B^* Ec Pr Re(1-\phi)^{2.5} + 25A^* Re Pr \alpha^2(1-\phi)^{2.5} \\ & - 15B^*\zeta Re \eta^2(1-\phi)^{2.5} - 30A^* Pr Re \alpha^2\eta^2(1-\phi)^{2.5} + 5A^* Pr Re \alpha^2\eta^4(1-\phi)^{2.5} - 15EcHa Pr Re \eta^2(1-\phi)^{2.5} \\ & + 5EcHa Pr Re \eta^4(1-\phi)^{2.5} - Ec Pr Ha Re \eta^6(1-\phi)^{2.5}) / (1-\phi)^{2.5} \end{aligned} \quad (32)$$

$$\begin{aligned} g_2 = & 75600B^{*2} Da Re(1-\phi)^{2.5} + 50400B^* Da Ec Pr + 272160B^* Da Ec Pr \alpha^2 + 170400B^* Da Ec Pr \alpha^4 - 50400B^* Da Ec Pr \eta^4 \\ & + 37800B^{*2} Da \zeta Re(1-\phi)^{2.5} + 20220A^* Da Ec Pr^2 \alpha^2 + 78704A^* Da Ec Pr^2 \alpha^4 + 11760B^* Da Ec Pr \zeta \\ & + 15750B^2 Da \zeta^2 Re(1-\phi)^{2.5} + 151200B^{*2} Da Re \alpha^2(1-\phi)^{2.5} + 252000B^{*2} Da Re \alpha^4(1-\phi)^{2.5} \\ & - 352800B^* Da Ec Pr \alpha^2\eta^2 + 84000B^* Da Ec Pr \alpha^2\eta^4 \\ & - 221760B^* Da Ec Pr \alpha^4\eta^2 - 3360B^* Da Ec Pr \alpha^2\eta^6 + \dots \end{aligned} \quad (33)$$

The expressions for the flow and heat transfer profile are finally presented as:

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) \quad (34)$$

$$g(\eta) = g_0(\eta) + g_1(\eta) + g_2(\eta) \quad (35)$$

Table 2. Comparison of values for velocity distributions for diverging channel (3) when $\phi=0.04$, $Ec=0.5$, $Pr=7$, $Ha=2$, $Re=35$, $\alpha=3$, $\zeta=Da=0$.

$f(\eta)$					
η	Numerical [57]	DTM [57]	HPM	Error in DTM	Error in HPM
0	1.000000000000	1.000000000000	1.0000	0.00000000	0.00000000
0.25	0.918278442113	0.918278465827	0.9183	2.371×10^{-8}	2.15579E-05
0.50	0.693337998062	0.693338088015	0.6933	8.995×10^{-8}	3.79981E-05
0.75	0.372274150248	0.372274341541	0.3722	1.912×10^{-8}	7.41502E-05
1.00	0.000000000000	0.000000000000	0.0000	0.00000000	0.00000000

Table 3. Comparing the values of velocity distributions for converging channel (-3) when $\phi=0.04$, $Ec=0.5$, $Pr=7$, $Ha=2$, $Re=35$, $\alpha=3$, $\zeta=Da=0$.

$f(\eta)$					
η	Numerical [57]	DTM [57]	HPM	Error in DTM	Error in HPM
0	1.000000000000	1.000000000000	1.0000	0.00000000	0.00000000
0.25	0.95228142154	0.95228142173	0.9523	1.855×10^{-10}	1.85785E-05
0.50	0.79642744758	0.79642743351	0.7965	1.406×10^{-8}	7.25524E-05
0.75	0.49577392894	0.49577388510	0.4958	4.383×10^{-8}	2.60711E-05
1.00	0.000000000000	0.000000000000	0.0000	0.00000000	0.00000000

Table 4. Comparison of values for temperature distributions for diverging channel (3) when $\phi=0.04$, $Ec=0.5$, $Pr=7$, $Ha=2$, $Re=35$, $\alpha=3$, $\zeta=Da=0$.

$g(\eta)$					
η	Numerical [57]	DTM [57]	HPM	Error in DTM	Error in HPM
0	3.249281714484	3.249286824591	3.2450	5.11×10^{-8}	0.004281714
0.25	3.055297830368	3.055302930218	3.0554	5.099×10^{-8}	0.00010217
0.50	2.529370818618	2.529375875056	2.5295	5.056×10^{-8}	0.000129181
0.75	1.802440025864	1.802444984180	1.8025	4.958×10^{-8}	5.99741E-05
1.00	1.000000000000	1.000000000000	1.0000	0.00000000	0.00000000

Table 5. Comparing the values of temperature distributions for converging channel (-3). When $\phi=0.04$, $Ec=0.5$, $Pr=7$, $Ha=2$, $Re=35$, $\alpha=3$, $\zeta=Da=0$.

$g(\eta)$					
η	Numerical [57]	DTM [57]	HPM	Error in DTM	Error in HPM
0	3.4846082046	3.249286824591	3.2494	4.139×10^{-8}	0.235208205
0.25	3.2881642665	3.055302930218	3.0553	4.233×10^{-8}	0.232864267
0.50	2.7347677983	2.529375875056	2.5295	4.490×10^{-8}	0.205267798
0.75	1.9268569396	1.802444984180	1.8026	4.805×10^{-8}	0.12425694
1.00	1.000000000000	1.000000000000	1.0000	0.00000000	0.00000000

3. Discussion of Results

This section discusses the results obtained from the analytical investigation of the flow and heat transfer. Here the flow and heat transfer through the diverging/converging channel with porous medium is presented at constant parametric values of $Da=0.1$, $\alpha=3$, $Re=5$, $Ha=1$, $\phi=0.03$, $Ec=0.5$, $Pr=4$, and $\zeta=1$. The Table 2-4 shows the comparison between the numerical solution obtained by fourth-order Runge-Kutta and the semi-analytic solution obtained by differential transform method against the approximate solution obtained adopting homotopy perturbation method (HPM). As observed from Tables 2-4, a satisfactory conclusion is reached comparing the analytic variations. This proves the validity of this work. Figure 2 demonstrates the effect of the Reynolds number on the velocity distribution. This can be explained physically owing to increased viscosity leading to fluid motion resistance at the boundary, thus momentum boundary layer increases. The solid particle concentration effects on velocity is shown in Fig. 3, this illustrates a gradual decrease in velocity profile as concentration increases. As depicted from the plot, when $\phi=0$ it shows the transport of the fluid through the channel without nanoparticle concentration. Nanoparticle effect on the fluid reduces momentum boundary layer thickness due to the high energy exchange rate as fluid molecules move through the non-parallel channel.

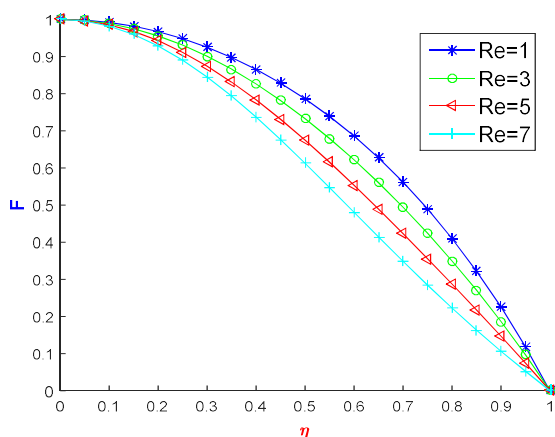


Fig. 2. Effect of Reynolds number (Re) on the velocity profile.

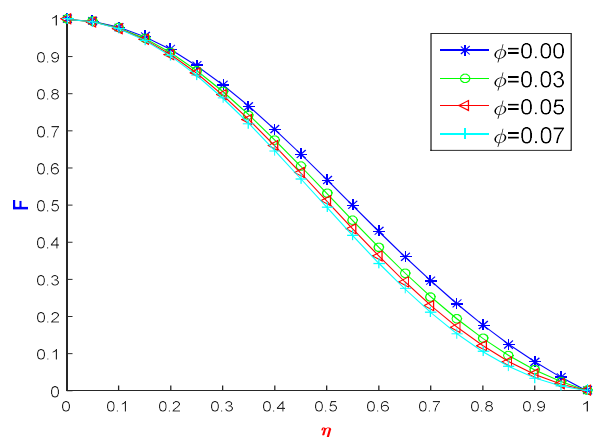


Fig. 3. Effect of nanoparticle concentration (ϕ) on the velocity profile.

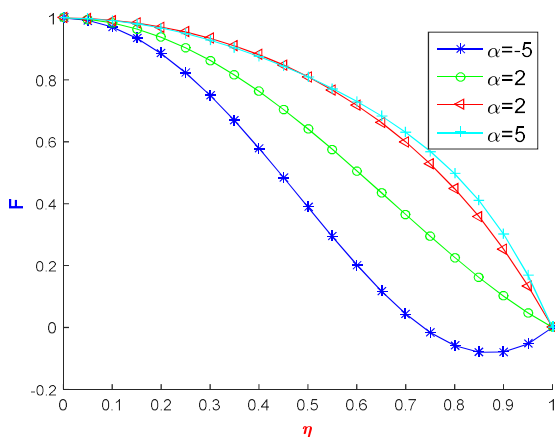


Fig. 4. Effect of inclination angle (α) on the velocity profile.

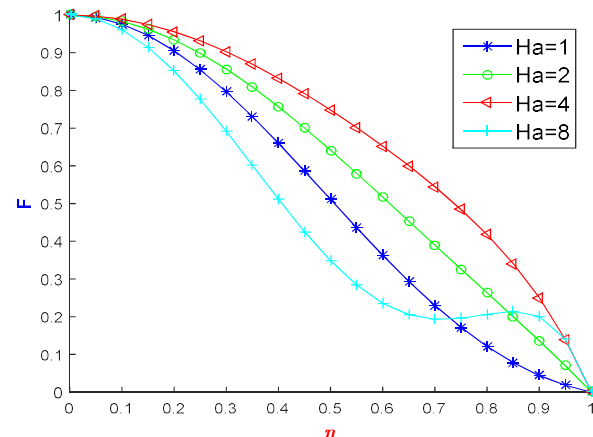


Fig. 5. Effect of Magnetic parameter (Ha) on the velocity profile.

Channel opening angle's effect on the divergent/convergent plate is observed in Fig. 4. To eliminate the occurrence of fluid backflow, relatively large open channel angles are utilized. The backflow occurrence is eliminated for the converging channel, but may occur in the diverging channel. High Reynolds number value in the presence of high magnetic field intensity eliminates backflow phenomenon occurrence. As depicted in Fig. 4, quantitative increase in the channel's angle illustrates a significant decrease in the velocity profile. Magnetic field influence on flow is seen in Fig. 5 which shows magnetic field intensity depicts fluid flow decrease through the non-parallel channel. As observed from the plot absolute velocity reduces, this can be explained physically due to the presence of resistive forces at the boundary of the channel due to boundary layer thickness increase resulting in retarding force on the velocity field.

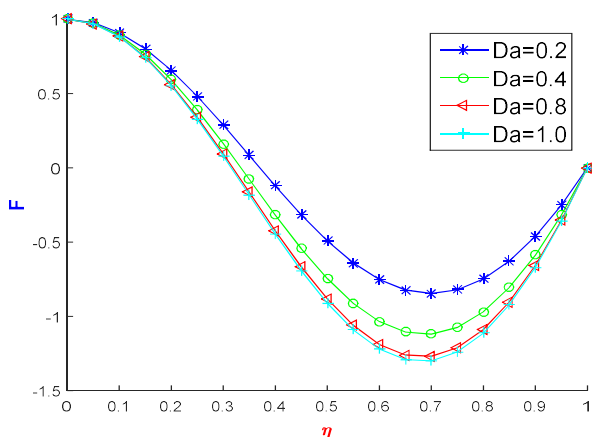


Fig. 5. Effect of Darcy parameter (Da) on velocity profile.

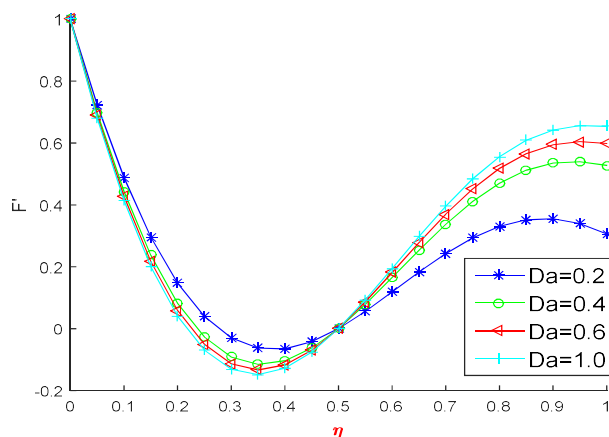


Fig. 6. Effect of Darcy Parameter on the Skin friction.

Darcy parameter effect which accounts for the porous medium in the fluid is observed in Fig. 5. As illustrated, increasing porous medium in the fluid leads to a slight decrease in fluid flow. As shown in the plot, fluid pressure drops due to

increasing dynamic viscosity of fluid transported through the non-parallel plates. Component of radial velocity on flow is shown in Fig. 6, this demonstrates the momentum boundary layer is less compared to the upper plate. The flow approaching the upper plate depicts the decreasing velocity profile. However, the reverse trend is observed at $\eta=0.53$ (not measured accurately). Eckert parameter's (Ec) effect on heat transfer is observed in Fig. 7. From the figure it is observed that Ec improves heat transfer. This is explained physically as an increase in velocity enhances fluid internal friction due to viscous dissipation effects, which increases the thermal boundary. As shown in the figure, increasing Ec has the effect of increasing the temperature distribution.

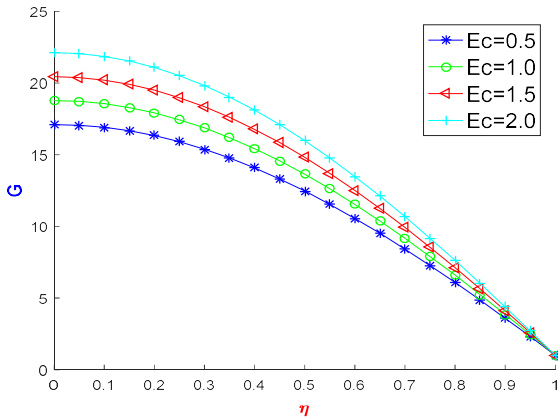


Fig. 7. Effect of Eckert number (Ec) on the temperature profile.

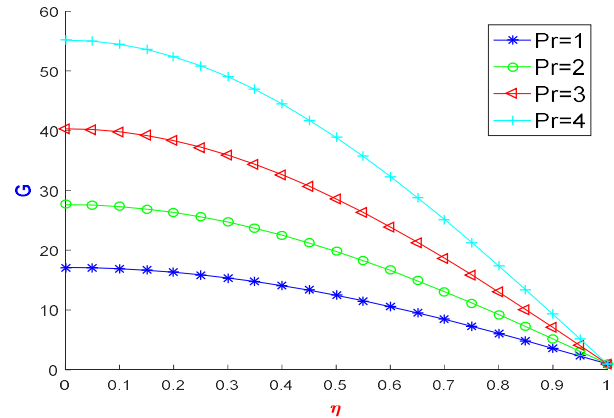


Fig. 8. Effect of Prandtl number (Pr) on the temperature profile.

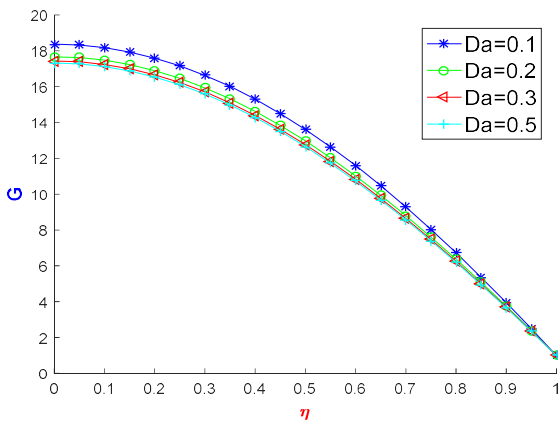


Fig. 9. Effect of Darcy number (Da) on the temperature profile.

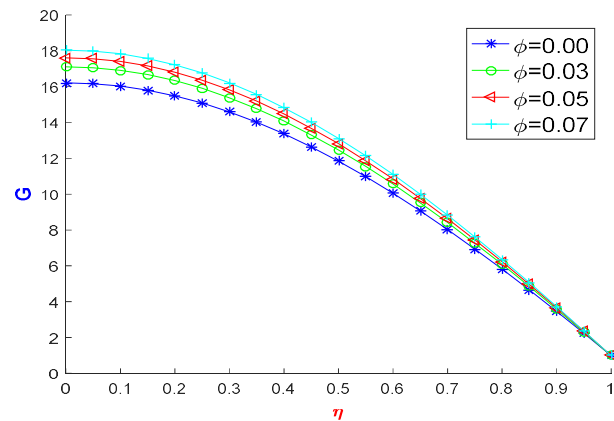


Fig. 10. Effect of nanoparticles concentration (ϕ) on the temperature profile.

Prandtl's number (Pr) effect on temperature is shown in Fig. 8, as observed from the figure the effect of Pr on heat transfer is significant. An increase in the Pr parameter makes the kinematic viscosity more dominant to the thermal diffusivity; this connotes momentum boundary layer which is larger than thermal boundary layer. Effect of Darcy number (Da) on temperature is shown in Fig. 9; temperature distribution reduces as Da increases. Internal generated heat reduces due to reduction in flow through the Jeffery Hamel channel as flow with porous media is impeded due to porosity increase. As illustrated from the figure temperature distribution decreases as Da decreases. Nanoparticle concentration (ϕ) effect on temperature distribution is illustrated in Fig. 10. This shows the addition of nanoparticles to fluid enhances the temperature. The heat transfer effect without nanoparticle through the Jeffery Hamel channel is observed from the plot when ϕ is zero. Increasing ϕ depicts enhancement in the temperature distribution, as seen from Fig. 10.

Consequently, the rate of approximation for the convergence of the HPM is observed from Table 6. Varying order of the approximate solution obtained using the HPM is validated using numerical solution obtained by Runge-Kutta fourth order. From observation it is depicted that the HPM solution is enhanced at increasing order of approximation. The solution for the convergence of the HPM is proven in [58].

Table 6. Comparing the values of velocity distributions when $Ha=2$, $Re=5$, $\phi=0.03$, $\alpha=3$, $\zeta=0$ and $Da=1$.

η	$v(\eta)$					
	First Approximation		Second Approximation		Third Approximation	
	HPM	Error	HPM	Error	HPM	Error
0.0	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000
0.1	0.9901	0.0001	0.9964	0.0000	0.9856	0.0000
0.2	0.9601	0.0001	0.9851	0.0000	0.9432	0.0000
0.3	0.9101	0.0001	0.9643	0.0000	0.8758	0.0000
0.4	0.8401	0.0001	0.9310	0.0000	0.7875	0.0000
0.5	0.7501	0.0001	0.8802	0.0000	0.6838	0.0000
0.6	0.6401	0.0002	0.8049	0.0000	0.5700	0.0000
0.7	0.5101	0.0001	0.6951	0.0000	0.4499	0.0000
0.8	0.3601	0.0001	0.5373	0.0000	0.3231	0.0000
0.9	0.1901	0.0001	0.3135	0.0000	0.1810	0.0000
1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Absolute Error		0.00009091		0.0000		0.0000

4. Conclusion

This study investigated the effect of nanofluid flow through converging/diverging channel with porous medium considering internally generated heat. The flow and heat transfer was formulated using the momentum and energy equations. Since most practical problems were nonlinear, the homotopy perturbation method (HPM) was adopted as the suitable method of analysis. The analytical results obtained from the study compared against numerical Runge-Kutta Fehlberg method of the fourth-order shows satisfactory agreement. Solutions obtained from analysis were used to study the effect of fluid rheological effect on transport performance. It is deduced from obtained results that:

- Reynolds number variation has significant effect on fluid flow. High Reynolds number reduces velocity of fluid transport.
- As channel angle increases at the entrance, this reduces flow at the exit.
- The effect of Darcy number on radial fluid velocity illustrates a decrease in the velocity of fluid, but as fluid approaches center plate velocity increases.
- It is observed from the analysis of heat transfer that temperature reduces due to reduced flow through the channel when Darcy number is increased.

Practical applications relevant to the study includes blood flow through arteries and veins, environmental flows through canals, molten polymer extrusion and cold drawing operation through converging dies.

Acknowledgement

The authors wish to acknowledge the reviewers whose comments helped improve this manuscript. Also, the authors appreciate Mrs. Abimbola E. Akinshilo who helped with proof reading this paper.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The authors received no financial support for the research, authorship and publication of this article.

Nomenclature

A^*	Constant parameter
B_0	Magnetic field (wb.m ⁻²)
C_p	Specific heat capacity (J/Kg. K)
$f(\eta)$	Dimensionless velocity
Ec	Eckert number
Ha	Hartmann number
K_p	Porous medium permeability
P	Pressure term (N/m ⁻²)
Pr	Prandtl number
Q	Heat generation parameter

Greek symbols	
α	Channel angle
β	Constant
ϕ	Nanoparticle concentration
η	Dimensionless angle
μ	Dynamic viscosity (N/m ⁻²)
θ	Any angle
ρ	Density
σ	Electrical conductivity (Ωm) ⁻¹
ν	Kinematic viscosity (m ² /s)

Re	Reynolds number	ζ	Dimensionless heat generation parameter
r, θ	Cylindrical co-ordinates	Subscripts	
T	Temperature	∞	Condition at infinity
$U_{\max}$	Maximum velocity value	nf	Nanofluid
u, v	Components of velocity along x, y axes	f	Basefluid
		s	Nano-solid particles

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ORCID iD

Akinbowale T. Akinshilo  <https://orcid.org/000-0002-6436-3420>

Adeleke Ilegbusi  <https://orcid.org/000-0001-8421-1873>

Hafiz M. Ali  <https://orcid.org/000-0001-9266-408X>

Abdul-Jalil Surajo  <https://orcid.org/000-0002-3599-9008>



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