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Modelling of Love Waves in Fluid Saturated Porous Viscoelastic Medium resting over an Exponentially Graded Inhomogeneous Half-space Influenced by Gravity

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Abstract. The present article is devoted to a theoretical study on Love wave vibration in a pre-stressed fluid-saturated anisotropic porous viscoelastic medium embedded over an inhomogeneous isotropic half-space influenced by gravity. The expression of dispersion has been achieved with the help of mathematical tools such as variable separable method and Whittaker's function's expansion under certain boundary conditions. After that, the obtained result has been coincided with the pre-established classical equation of Love wave, as shown in the section of particular cases and validation. The substantial influence of various affecting factors like gravity, initial stress, porosity, viscosity and inhomogeneity on dispersion curves of Love wave has been investigated extensively by means of graphical depictions and discussions accomplished by numerical results.

Keywords: Love waves; Porous; Viscoelastic; Gravity; Whittaker's function.

1. Introduction

An investigating the surface seismic waves propagation in multi-layered model is of paramount significance to know the mechanics of layer of the Earth's surface and its application. But, the exact nature and behavior of the layer structure beneath the Earth's surface may not be examined directly. Therefore, Seismologists utilize the seismic waves to perceive the structure of the Earth's interior and minimize the inevitable outcomes of the natural disaster. Since, the surface seismic waves behave differently during the earthquake when they propagate through various types of materials with different physical states such as solid, molten, semi-molten. Love waves are surface seismic waves in which the movement of the particles is horizontally perpendicular to the propagation of wave. The surface waves vibrations during the earthquake or artificial exploration in the Earth's crust are extremely dominated by the physical properties of the layer. Due to extensive applications in many fields of engineering and applied sciences such as civil engineering, mechanical engineering, soil mechanics, rock mechanics, geomechanics and earthquake science etc., the analysis of Love waves in different types of layer under different physical conditions is the topic of greatest interest to researchers of various disciplines and theoretical seismologists. Many studies have been conducted on the propagation of Love waves [1-3]. Inspired with these particulars, the authors have attempted a theoretical development on modelling of Love wave vibration in anisotropic viscoelastic porous layer lying over inhomogeneous half-space influenced by gravity and initial stress, which has not been attempted by any authors till date. A huge amount of theoretical information as well as applications regarding the analysis of surface seismic waves in multi-layered model are provided in well-established books (Achenbach [4], Love [5], Ewing et al. [6] and Pujol [7]).

The porous medium is usually beneath the surface of the Earth. The phenomenon of seismic wave propagation through anisotropic porous medium is widely used in various areas, such as land survey acquisition, petroleum exploration, rock mechanics, marine survey acquisition, seismic attribute analysis and seismic data processing. As an



example, Vosges sandstone [8] naturally found in crustal portion of the Earth behaves as anisotropic. These types of materials are one of the largest materials that oppose the weathering process on surface of the Earth. In addition, such minerals are mainly contained in the formation of rocks and authorize the destruction of water and other liquids. They have sufficient porosity to store a huge amount of liquids, which makes them precious aquifers and oil reservoirs. The crucial applications of such kind of materials are being used in different fields of engineering and applied sciences, namely material science, hydrogeology, biophysics and civil engineering etc. Biot [9, 10] established the dynamic equations of the transmission of seismic surface wave in anisotropic porous layer. In addition, Biot [11] extended the investigation in porous layers with anisotropy. Based on the basic theory of Biot, many researchers have examined the surface wave propagation through porous media with isotropic and anisotropic (Kończak [12], Wang and Zhang [13]; Son and Kang [14]; Dey and Sarkar [15]). Saha et al. [16] developed a theory on the wave propagation in an anisotropic porous layered structure and found that in the presence of initial stress and porosity in the obtained dispersion relation has noteworthy impact on torsional wave velocity. Besides that, anisotropic pre-stressed porous media has been considered for the simulation of Love waves by Pandit et al. [17], who reported the prominent effect of initially stressed factor and porosity for the Love wave propagate.

The complicated layers of the Earth's surface owing to their viscous nature significantly affect the propagation of seismic surface waves. Therefore, viscoelasticity performs an essential role in the motion and behavior of the tectonic plates, the plates floating on the Earth's mantle and moving independently are responsible for vibration, volcanoes and earthquakes etc. Mechanical materials, such as sediments, salt and coal tar, which are buried within the surface of the Earth can be constituted as viscoelastic materials. The characteristics of viscoelastic materials are the conjunction of two considerable physical properties that is to say elastic and viscous. That is why such kind of materials is of great importance in many filed (Semiologist, earthquake engineering, exploration geophysics, soil dynamics and fluid dynamics) and it receives more attention from many researchers. Carcione [18] was the first to investigate the propagation of surface wave not only by anisotropy but also by intrinsic viscosity of the media. Authors [19-21] have discussed the characteristics of viscoelastic materials when the surface seismic wave propagates through it.

It is believed that the earth's surface is formed of several inhomogeneous media with variation in the stiffness and density of the materials. From practical point of view, the variation in elastic properties such as stiffness and density of different media may be estimated as linear, quadratic, hyperbolic and exponential with respect to a certain depth. The examination of the surface wave in such inhomogeneous medium provides enormous beneficial information about the internal composition of the Earth. Wilson [22] was the first one to develop a theory on the surface waves propagate through inhomogeneous media. After that, the propagation of Love, Rayleigh, SH-type and Torsional surface waves through inhomogeneous elastic body has been accomplished by many researchers in many areas (Sato [23], Tomar and Kaur [24]; Vardoulakis [25] and Chattopadhyay et al. [26]).

The use of gravity during the study of the mechanical behavior of any layer on surface of the Earth has a wide application in many fields of Earth science. Since, the acceleration caused by gravity g is of great practical importance to discover and solve the static and dynamic problems. Biot [27] described the impact of initial stress and gravity on wave propagation generated by an earthquake or artificial exploration and explained that the wave propagation influenced by gravity and initial stress is different from the classical theory of elasticity. Recently, Maity et al. [28] explored the surface wave profile in fiber-composite layer embedded over sandy half-space under gravity and pointed out the influence of wave velocity as well as attenuation due to Biot's gravity parameter. In addition to that, numerous works on wave propagation affected by gravity have been accomplished (Sethi et al. [29], Dey and Mukherjee [30]; Pal and Ghorai [31]; and Khan et al. [32]).

The current article attempts to examine the dispersion behavior on the Love waves propagate through pre-stressed anisotropic fluid saturated porous viscoelastic layer laid over an inhomogeneous half-space affected by gravity. The dispersion relation of the adopted model has been attained by means of appropriate boundary conditions. Moreover, the obtained consequence has been employed to examine the effect of all physical parameters of the considered media such as porosity, initial stress, viscosity, inhomogeneity and gravity; and also studied the various modes of Love wave with the aid of graphical illustrations.

2. Mathematical Modelling and Governing Equations

Let us consider a physical model that is comprised of highly anisotropic pre-stressed fluid saturated porous viscoelastic medium of finite width H embedded over inhomogeneous elastic substrate under the effect of gravity. A coordinate frame is chosen in order to represent our model, where the origin O lies at the interface, z -axis is directed positively vertically downward, and x -axis is considered in the direction of wave propagation, as demonstrated in Fig .1. However, the porous medium and elastic half-space are adopted to occupy the regions $-H \leq z \leq 0$ and $z \geq 0$, respectively. The variation in stiffness and density for the inhomogeneous elastic half-space has been assumed to be exponential as $\mu = \mu_2 e^{bz}$ and $\rho = \rho_2 e^{bz}$, where b is the inhomogeneity factor.

Love wave conditions: Let the displacement vector of liquid and solid phase of porous medium are (U_x, W_z, V_y) and $(u^{(l)}, w^{(l)}, v^{(l)})$ respectively. For the inhomogeneous elastic half-space, $(u^{(h)}, v^{(h)}, w^{(h)})$ are displacement vector in x , y , and z axis. Therefore, the characteristics of Love wave propagation is given by

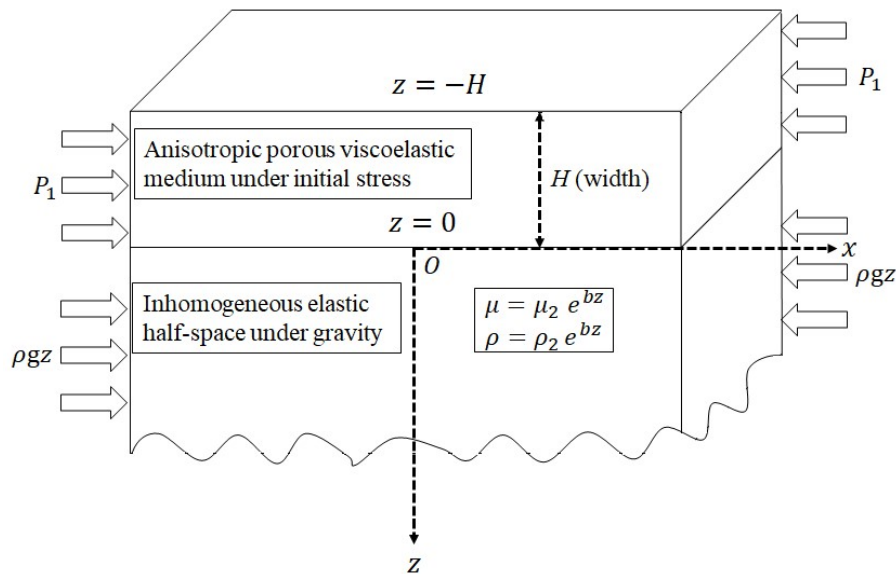


Fig. 1. Structure of the layered Earth model

$$\left. \begin{aligned} u^{(l)} = w^{(l)} = u^{(h)} = w^{(h)} = 0, v^{(l)} = v^{(l)}(x, z, t) \text{ and } v^{(h)} = v^{(h)}(x, z, t) \\ U_x = W_z = 0 \text{ and } V_y = V(x, z, t) \end{aligned} \right\} \quad (1)$$

2.1 Fluid-Saturated Porous Viscoelastic Layer ($-H \leq z \leq 0$)

The dynamical equations of motion for the upper layer under initial stress P_1 without body force are given by Biot [27] are

$$\left. \begin{aligned} \frac{\partial S_{11}}{\partial x} + \frac{\partial S_{12}}{\partial y} + \frac{\partial S_{13}}{\partial z} - P_1 \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) &= \frac{\partial^2}{\partial t^2} (\rho_{11} u^{(l)} + \rho_{12} U_x) \\ \frac{\partial S_{21}}{\partial x} + \frac{\partial S_{22}}{\partial y} + \frac{\partial S_{23}}{\partial z} - P_1 \frac{\partial \omega_z}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} v^{(l)} + \rho_{12} V_y) \\ \frac{\partial S_{31}}{\partial x} + \frac{\partial S_{32}}{\partial y} + \frac{\partial S_{33}}{\partial z} - P_1 \frac{\partial \omega_y}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} w^{(l)} + \rho_{12} W_z) \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} \frac{\partial S'}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{11} u^{(l)} + \rho_{22} U_x) \\ \frac{\partial S'}{\partial y} &= \frac{\partial^2}{\partial t^2} (\rho_{11} v^{(l)} + \rho_{22} V_y) \\ \frac{\partial S'}{\partial z} &= \frac{\partial^2}{\partial t^2} (\rho_{11} w^{(l)} + \rho_{22} W_z) \end{aligned} \right\} \quad (3)$$

where P_1 represents the initial stress in porous medium. $S_{ij}(i, j=1, 2, 3)$ are stresses of solid phase and $S' (= -f\bar{p}, f \text{ and } p \text{ are porosity and fluid pressure})$ is the stress acting liquid phase. The components of the rotational vector ω_x, ω_y and ω_z are defined as

$$\omega_x = \frac{1}{2} \left(\frac{\partial w^{(l)}}{\partial y} - \frac{\partial v^{(l)}}{\partial z} \right), \omega_y = \frac{1}{2} \left(\frac{\partial u^{(l)}}{\partial z} - \frac{\partial w^{(l)}}{\partial x} \right), \omega_z = \frac{1}{2} \left(\frac{\partial v^{(l)}}{\partial x} - \frac{\partial u^{(l)}}{\partial y} \right), \quad (4)$$

The mass coefficients ρ_{11}, ρ_{12} and ρ_{22} are concerned with total mass density ρ' of solid-liquid. If ρ_w and ρ_s are mass density of liquid and solid porous medium, respectively, then we have a relation as

$$\rho_{11} + \rho_{12} = (1-f)\rho_s, \rho_{12} + \rho_{22} = f\rho_w,$$

therefore, the total mass density is

$$\rho' = \rho_{11} + \rho_{22} + 2\rho_{12} = \rho_s + f(\rho_w - \rho_s).$$

Besides that, these mass coefficients also take into account the following inequalities:

$$\rho_{11} > 0, \rho_{22} > 0, \rho_{12} < 0, \rho_{11}\rho_{22} - \rho_{12}^2 > 0.$$

Thus, the stress-strain relation are

$$S_{23} = 2\bar{\mu}_L e_{yz} S_{12} = 2\bar{\mu}_N e_{xy}. \quad (5)$$

where $\bar{\mu}_L = L + i\omega L^*$ and $\bar{\mu}_N = N + i\omega N^*$; (L , N) are elastic constants and (L^* , N^*) are viscosity parameters. With the help of Eqs.(1), (2), (3), (4) and (5), we get

$$(\bar{\mu}_N - \frac{P_1}{2}) \frac{\partial^2 v^{(l)}}{\partial x^2} + \bar{\mu}_L \frac{\partial^2 v^{(l)}}{\partial z^2} = \frac{\partial^2}{\partial t^2} (\rho_{11} v^{(l)} + \rho_{12} V), \quad (6)$$

$$\frac{\partial^2}{\partial t^2} (\rho_{12} v^{(l)} + \rho_{22} V) = 0. \quad (7)$$

From $(\partial^2 / \partial t^2)(\rho_{12} v^{(l)} + \rho_{22} V) = 0$ and $\rho_{12} v^{(l)} + \rho_{22} V = d'$ (say), then $V = (d' - \rho_{12} v^{(l)}) / \rho_{22}$.

Now, $(\partial^2 / \partial t^2)(\rho_{11} v^{(l)} + \rho_{12} V) = d' (\partial^2 v^{(l)} / \partial t^2)$, where $d' = \rho_{11} - \rho_{12}^2 / \rho_{22}$. Therefore, Eq. (6) can be written as

$$(\bar{\mu}_N - \frac{P_1}{2}) \frac{\partial^2 v^{(l)}}{\partial x^2} + \bar{\mu}_L \frac{\partial^2 v^{(l)}}{\partial z^2} = d' \frac{\partial^2 v^{(l)}}{\partial t^2} \quad (8)$$

The velocity of Love wave for upper layer along x -axis may be written as

$$\beta = \sqrt{\frac{\bar{\mu}_N - P_1/2}{d'}} = \beta_1 \sqrt{\frac{(1 + i\omega \frac{N^*}{N}) - (P_1/2N)}{d_1}}, \quad (9)$$

where $d_1 = \gamma_{11} - \gamma_{12}^2 / \gamma_{22}$, $\beta_1 = \sqrt{N / \rho'}$, β_1 is the Love wave velocity corresponding to the non porous viscoelastic layer.

Also, $\gamma_{11} = \frac{\rho_{11}}{\rho'}$, $\gamma_{12} = \frac{\rho_{12}}{\rho'}$, $\gamma_{22} = \frac{\rho_{22}}{\rho'}$ are the dimensionless material factors of the layer as obtained by Biot [10].

Finally, it is concluded that

1. If $0 < d_1 < 1$, then the medium is poroelastic.
2. If $d_1 \rightarrow 0$, then the medium is fluid.
3. If $d_1 \rightarrow 1$, then the medium is non-porous solid.

2.2 Inhomogeneous Half-Space ($z \geq 0$)

From the equation (1), the non vanishing dynamical equation for inhomogeneous elastic half-space influenced by gravity and initial stress is given by Biot [27] as

$$\frac{\partial T_{12}}{\partial x} + \frac{\partial T_{22}}{\partial y} + \frac{\partial T_{23}}{\partial z} - \rho g \omega_{23} + \rho g z \frac{\partial \omega_{12}}{\partial x} - \rho g z \frac{\partial \omega_{23}}{\partial z} = \rho \frac{\partial^2 v^{(h)}}{\partial t^2} \quad (10)$$

where g is the acceleration, T_{ij} are incremental stresses, ρ is density and $\omega_{ij's}$ are rotational components. The variation in stiffness and density of the lower half-space has been considered as follows

$$\mu = \mu_2 e^{bz}, \quad \rho = \rho_2 e^{bz} \quad (11)$$

where b is inhomogeneity factor. The relationship between stress-strain are given as

$$T_{ij} = \lambda \theta \delta_{ij} + 2\mu e_{ij} \quad (12)$$

where λ and μ are Lamé's constant. With the help of Eqs. (10), (11) and (12), equation (10) is reduced to

$$\frac{\partial^2 v^{(h)}}{\partial x^2} + \frac{\partial^2 v^{(h)}}{\partial z^2} + \left(\frac{\mu_2 b - \frac{1}{2} g \rho_2}{\mu_2 - \frac{1}{2} g \rho_2 z} \right) \frac{\partial v^{(h)}}{\partial z} = \frac{\rho_2}{\mu_2 - \frac{1}{2} g \rho_2 z} \frac{\partial^2 v^{(h)}}{\partial t^2} \quad (13)$$

3. Boundary Conditions

For the propagation of Love wave in the adopted Earth model, the following boundary condition may mathematically expressed as:

(i) Upper boundary surface is stress free at $z = -H$,

$$-\mu_L \frac{\partial v^{(l)}}{\partial z} = 0 \quad (14)$$

(ii) At common interface of porous viscoelastic layer and elastic half-space, the displacements and stresses are continuous i.e., at $z = 0$,

$$-\mu_L \frac{\partial v^{(l)}}{\partial z} = \mu_2 e^{bz} \frac{\partial v^{(h)}}{\partial z} \quad (15)$$

$$v^{(l)} = v^{(h)} \quad (16)$$

4. Analytical Solution of Wave Propagation

We can take the harmonic solution for the propagation of wave in the direction of x -axis as follows

$$v^{(\kappa)}(x, z, t) = V^{(\kappa)}(z) e^{i(\omega t - kx)} \quad \kappa = l, h \quad (17)$$

where $\omega (= kc)$ is angular frequency, k is wave number; and c is speed of simple harmonic waves.

4.1 Fluid-Saturated Porous Viscoelastic Layer:

Using Eq. (17) in Eq. (8), we obtained

$$\frac{d^2 V^{(l)}}{dz^2} + k^2 \Omega_1^2 V^{(l)} = 0, \quad (18)$$

where

$$\Omega_1 = \sqrt{\gamma d_1 \left[\frac{c^2}{\beta_1^2} - \frac{(1 + i\omega \frac{N^*}{N}) - (P_1 / 2N)}{d_1} \right]}, \gamma = \frac{\bar{\mu}_N}{\mu_L}, \beta_1 = \sqrt{\frac{N}{\rho'}} \quad (19)$$

Therefore, the final displacement of the initially stressed porous viscoelastic medium takes the form

$$v^{(l)} = (D_1 \cos(k\Omega_1 z) + D_2 \sin(k\Omega_1 z)) e^{i(\omega t - kx)} \quad (20)$$

where D_1 and D_2 are constants.

4.2 Inhomogeneous Half-Space:

Substituting Eq. (17) in Eq. (13), the solution can be obtained by the following equation

$$\frac{d^2}{dz^2} V^{(h)}(z) + \frac{\beta_2^2 b + a}{\beta_2^2 + az} \frac{d}{dz} V^{(h)}(z) + k^2 \left(\frac{c^2}{\beta_2^2 + az} - 1 \right) V^{(h)}(z) = 0 \quad (21)$$

where $a = -g/2$, $\beta_2 = \sqrt{\mu_2 / \rho_2}$. Again, substituting $V^{(h)}(z) = \phi(z) / (\beta_2^2 + az)^{\frac{\beta_2^2 b + a}{2a}}$ in equation (21) to eliminate first order derivative, the following is obtained as

$$\phi''(z) + \left[\frac{a^2 - \beta_2^4 b^2}{4(\beta_2^2 + az)^2} + k^2 \left(\frac{c^2}{\beta_2^2 + az} - 1 \right) \right] \phi(z) = 0 \quad (22)$$

Taking $z_1 = -2k(\beta_2^2 + az)/a$, $s = -c^2 k/2a$ and $m = \beta_2^2 b/2a$, equation (22) changes to

$$\phi''(z_1) + \left[-\frac{1}{4} + \frac{s}{z_1} + \frac{(1/4) - m^2}{z_1^2} \right] \phi(z_1) = 0 \quad (23)$$

The solution of (23) is given by

$$\phi(z_1) = E_1 W_{-s,m}(-z_1) + E_2 W_{s,m}(z_1) \quad (24)$$

Therefore, the required solution of lower half-space in view of the condition $\phi \rightarrow 0$ as $z \rightarrow \infty$ is assumed to be

$$\phi(z_1) = E_1 W_{-s,m}(-z_1) \quad (25)$$

Hence, the final solution is

$$v^{(h)} = E_1 (\beta_2^2 + az)^{-\frac{\beta_2^2 b+a}{2a}} W_{-s,m} \left[-\left(\frac{4}{G} - 2kz \right) \right] e^{i(\omega t - kx)} \quad (26)$$

where $G = \rho_2 g / \mu_2 k$ is denoting Biot's gravity parameter; $W_{-s,m}(-z_1)$ is Whittaker's function and E_1 is constant. With the help of asymptotic expansion [33] of Whittaker's function and considering up to a second term of the expansion, the Eq. (26) can be written as

$$v^{(h)} = E_1 (\beta_2^2 + az)^{-\frac{\beta_2^2 b+a}{2a}} e^{\left(\frac{2}{G} - kz \right)} \left(2kz - \frac{4}{G} \right)^{-s} \left[1 - \frac{m^2 - \left(s + \frac{1}{2} \right)^2}{\left(\frac{4}{G} - 2kz \right)} \right] e^{i(\omega t - kx)} \quad (27)$$

which is the displacement vector for inhomogeneous half-space.

5. Dispersion Equation

Now, applying the equations (20) and (27) in the boundary conditions (14)-(16), we get the three linear and homogeneous system of equation for coefficient D_1, D_2 and E_1 . Eliminating these constant coefficients, we obtained

$$\tan(kH\Omega_1) = -\frac{\mu_2 T_2}{\mu_L \Omega_1 T_1} \quad (28)$$

where

$$T_1 = (\beta_2^2)^{-\frac{(\beta_2^2 b+a)}{2a}} \left(-\frac{4}{G} \right)^{-s} \left(1 - \frac{G}{4} \left(m^2 - \left(s + \frac{1}{2} \right)^2 \right) \right) e^{\frac{2}{G}} \quad (29)$$

and

$$T_2 = (\beta_2^2)^{-\frac{(\beta_2^2 b+a)}{2a}} \left(-\frac{4}{G} \right)^{-s} \left(1 - \frac{G}{4} \left(m^2 - \left(s + \frac{1}{2} \right)^2 \right) \right) e^{\frac{2}{G}} \left[-\frac{a \left(\frac{\beta_2^2 b+a}{2a} \right)}{\beta_2^2} - k + \frac{2skG}{4} \right] - \frac{G^2}{8} (\beta_2^2)^{-\frac{(\beta_2^2 b+a)}{2a}} \left(-\frac{4}{G} \right)^{-s} \left(m^2 - \left(s + \frac{1}{2} \right)^2 \right) e^{\frac{2}{G}} \quad (30)$$

Equation (28) is the required dispersion relation of Love wave in a pre-stressed anisotropic viscoelastic porous layer overlying inhomogeneous half-space influenced by gravity.

6. Particular Cases and Validation

Case 6.1 Anisotropic Non-Porous Viscoelastic Layer Over Inhomogeneous Half-Space with Gravity

When the bounded layer of the considered model is assumed to be non-porous viscoelastic medium with stress free i.e., $d_1 \rightarrow 1, P_1/2N \rightarrow 0$, then Eq. (28) is reduced to

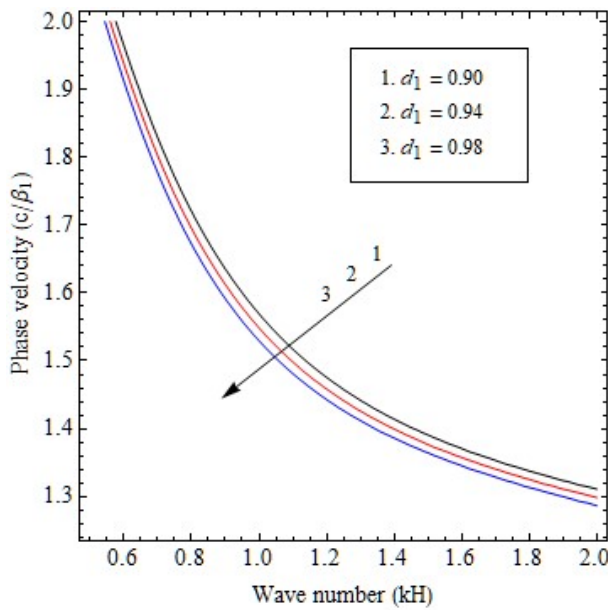


Fig. 2. Change of c / β_1 for various magnitude of d_1 against kH

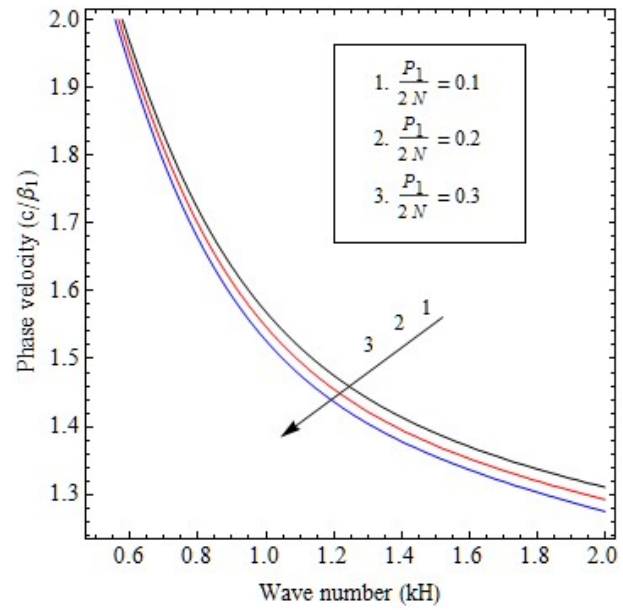


Fig. 3. Change of c / β_1 for various magnitude of $P_1 / 2N$ against kH

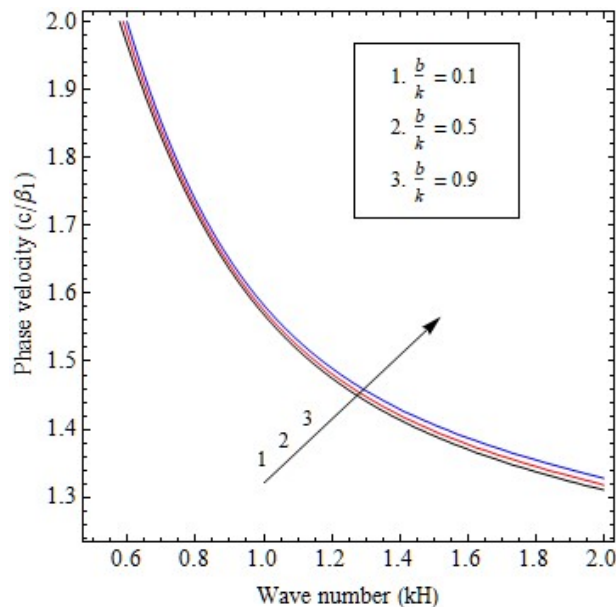


Fig. 4. Change of c / β_1 for various magnitude of b / k against kH

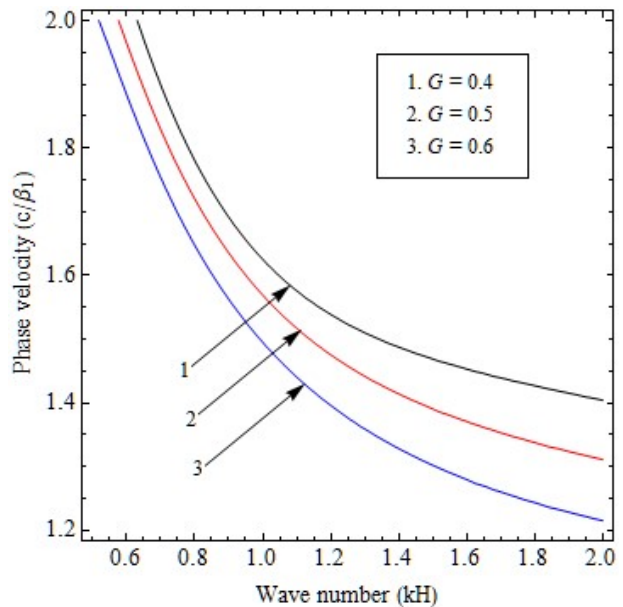


Fig. 5. Change of c / β_1 for various magnitude of G against kH

$$\tan \left(kH \frac{\bar{\mu}_N}{\bar{\mu}_L} \sqrt{\frac{c^2}{\beta_1^2} - \left(1 + i\omega \frac{N^*}{N} \right)} \right) = - \frac{\mu_2 T_2}{\bar{\mu}_N T_1 \sqrt{\frac{c^2}{\beta_1^2} - \left(1 + i\omega \frac{N^*}{N} \right)}} \quad (31)$$

Eq. (31) is dispersion equation of Love wave in anisotropic viscoelastic layer laid over an inhomogeneous half-space under the effect of gravity.

Case 6.2 Isotropic Solid Layer Over Inhomogeneous Half-Space With Gravity

When the bounded layer of the problem is considered to be a simply isotropic with viscous free i.e., $N^* \rightarrow L^* \rightarrow 0, N \rightarrow L \rightarrow \mu_1$ (rigidity), then Eq. (31) is reduced to

$$\tan \left(kH \sqrt{\frac{c^2}{\beta_1^2} - 1} \right) = - \frac{\mu_2 T_2}{\mu_1 T_1 \sqrt{\frac{c^2}{\beta_1^2} - 1}} \quad (32)$$

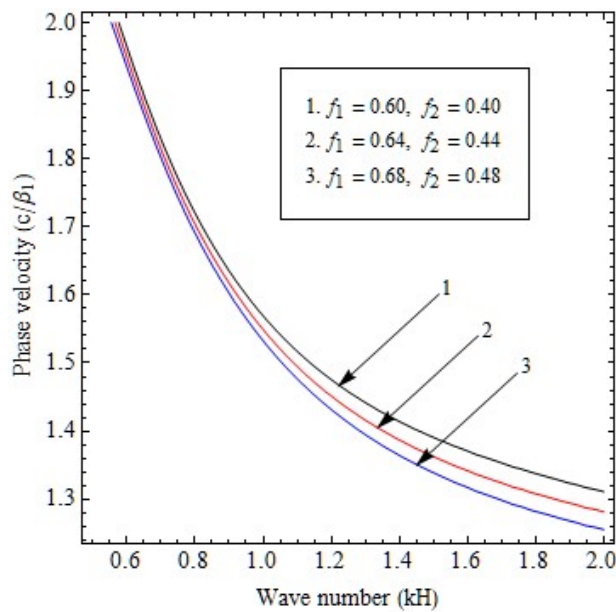


Fig. 6. Change of c / β_1 for various magnitude of (f_1, f_2) against kH

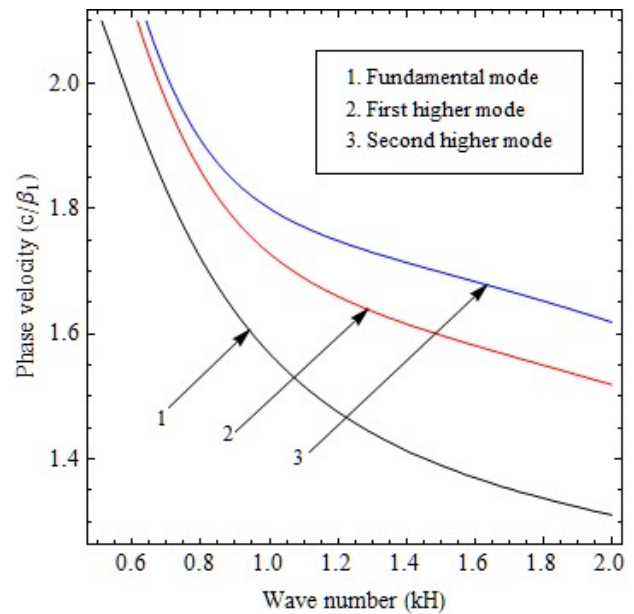


Fig. 7. Change of c / β_1 for fundamental, first higher and second higher modes against kH

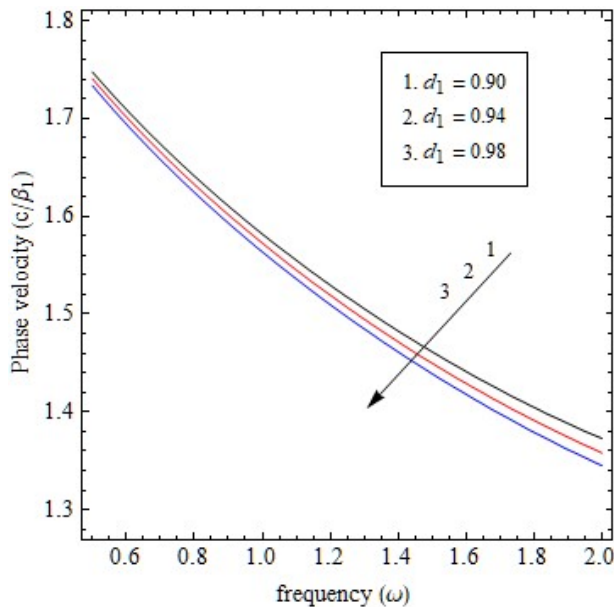


Fig. 8. Change of c / β_1 for various magnitude of d_1 against frequency ω

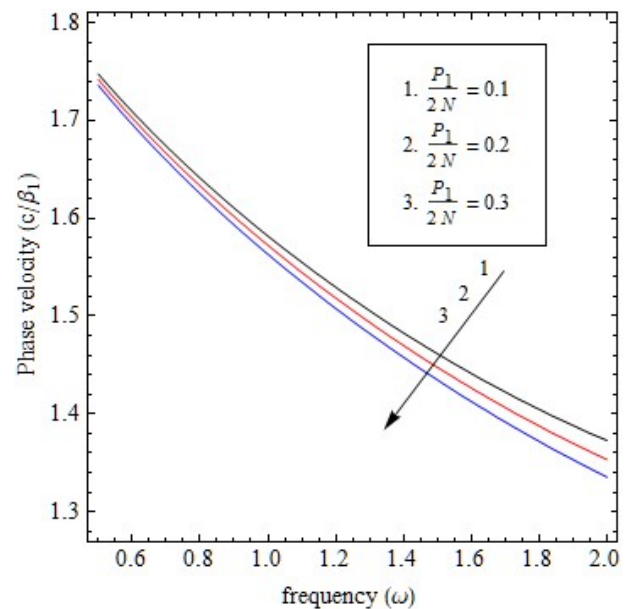


Fig. 9. Change of c / β_1 for various magnitude of $P_1 / 2N$ against frequency ω

Eq. (32) is dispersion equation of Love wave in a simply isotropic layer laid over an inhomogeneous half-space under the effect of gravity.

Case 6.3 Isotropic Layer Over Inhomogeneous Half-Space Without Gravity

When it is assumed that the lower elastic half space is free from gravity i.e., $G \rightarrow 0$, then Eq. (32) converts to

$$\tan \left(kH \sqrt{\frac{c^2}{\beta_1^2} - 1} \right) = \frac{\mu_2 \left(\frac{b}{k} + \sqrt{1 - \frac{c^2}{\beta_2^2}} \right)}{\mu_1 \sqrt{\frac{c^2}{\beta_1^2} - 1}} \quad (33)$$

Eq. (33) is dispersion equation of Love wave in a simply isotropic layer lying over an inhomogeneous half-space.

Case 6.4 Isotropic Layer Over Homogeneous Half-Space

When the lower elastic half space is considered to be homogeneous i.e., $b \rightarrow 0$, then Eq. (33) is reduced to

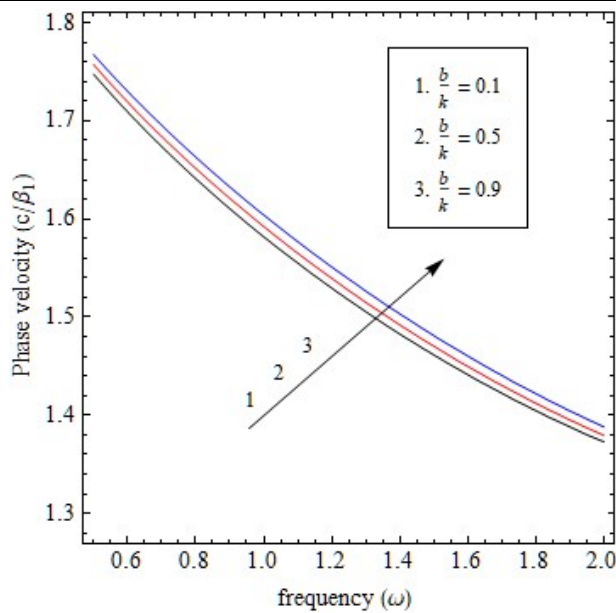


Fig. 10. Change of c/β_1 for various magnitude of b/k against frequency ω

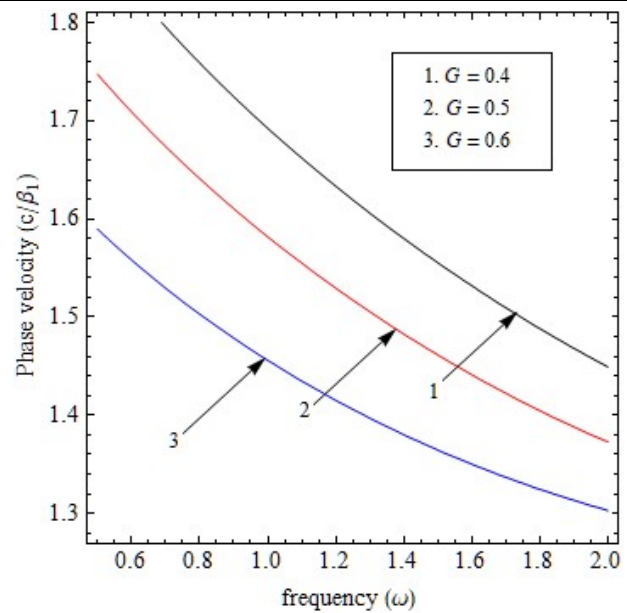


Fig. 11. Change of c/β_1 for various magnitude of G against frequency ω

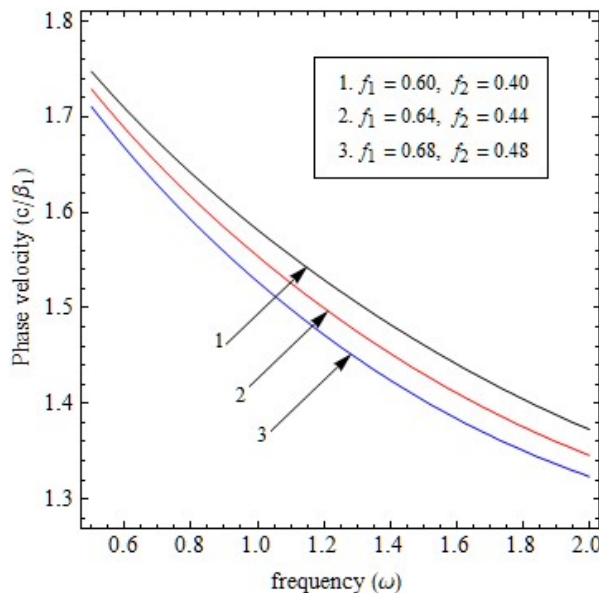


Fig. 12. Change of c/β_1 for various magnitude of (f_1, f_2) against frequency ω

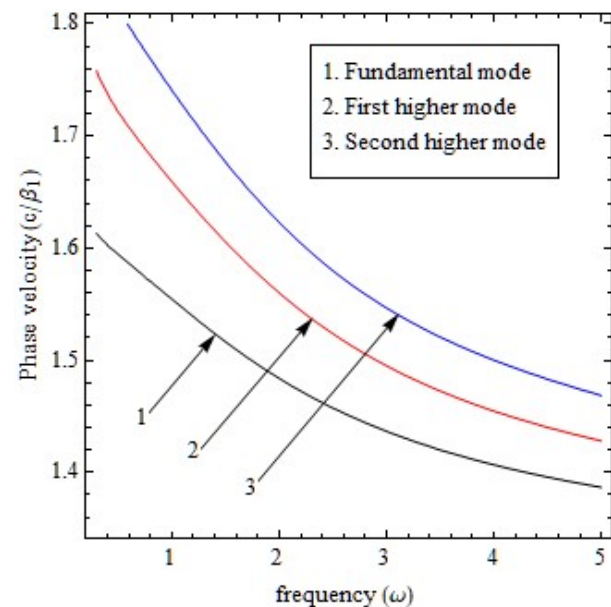


Fig. 13. Change of c/β_1 for fundamental, first higher and second higher modes against frequency ω

$$\tan \left(kH \sqrt{\frac{c^2}{\beta_1^2} - 1} \right) = \frac{\mu_2 \sqrt{1 - \frac{c^2}{\beta_2^2}}}{\mu_1 \sqrt{\frac{c^2}{\beta_1^2} - 1}} \quad (34)$$

where, $\beta_1 = \sqrt{\mu_1 / \rho'}$ and $\beta_2 = \sqrt{\mu_2 / \rho_2}$. Eq. (34) represents the well-known pre-established classical equation of Love wave in a homogeneous isotropic layer lying over homogeneous half-space which coincides the results recently obtained by Qian et al. [34], Kumari et al. [35]; and Kakar and Kakar [36].

7. Numerical Computation and Discussions

Numerical calculations based on the derived dispersion equation have been presented to investigate the variation of all affecting parameters such as porosity (d_1), initial stress ($P_1 / 2N$), inhomogeneity (b/k), viscosity ($f_1 = N^* / N$, $f_2 = L^* / L$) and gravity (G) for the Love wave propagation in adopted layered model. The relevant materials data, used in numerical computation, are taken from Gubbins [37], which are listed in Table 1.



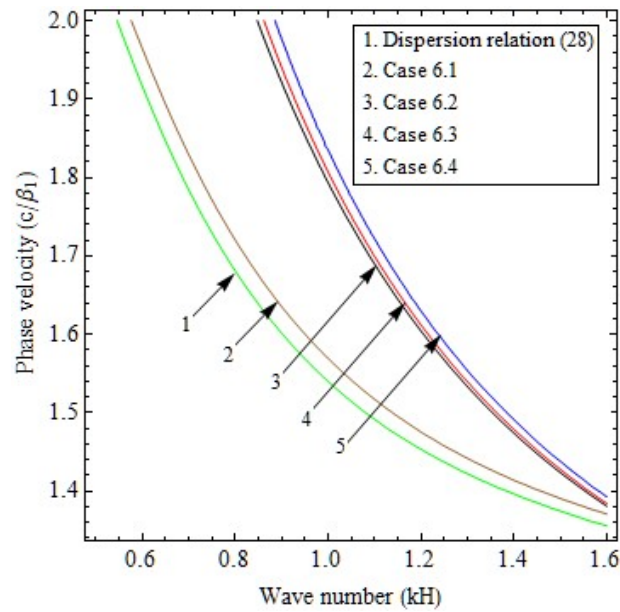


Fig. 14. Comparison of phase velocity with respect to wave number

Table 1. Materials properties for medium.

Medium	Material Properties (Unit: $N, L, \mu_2 : N / m^2; \rho_{11}, \rho_{12}, \rho_{22}, \rho_2 : Kg / m^3$)
Porous viscoelastic Layer	$N = 0.2774 \times 10^{10}, L = 0.1387 \times 10^{10},$ $\rho_{11} = 1.926137 \times 10^3, \rho_{12} = -0.002137 \times 10^3,$ $\rho_{22} = 0.215337 \times 10^3, f = 0.26.$
Inhomogeneous Half-Space	$\mu_2 = 7.84 \times 10^{10}, \rho_2 = 3535.$

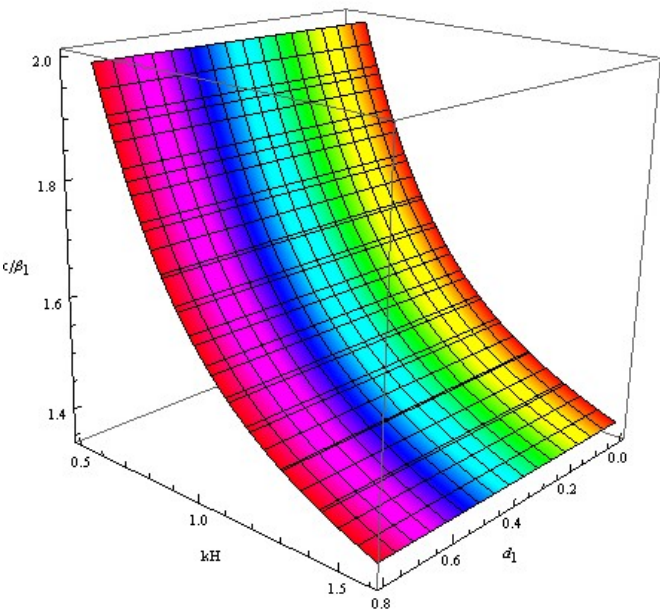


Fig. 15. Surface plots showing combined variation of c / β_1 against kH and d_1

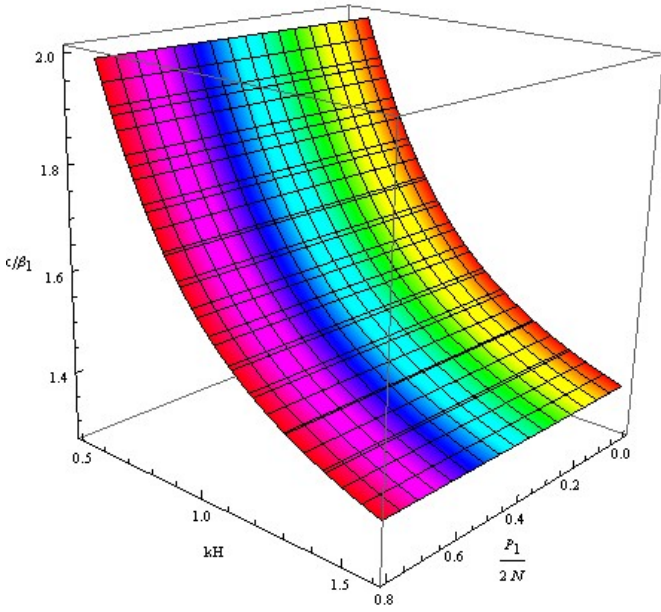


Fig. 16. Surface plots showing combined variation of c / β_1 against kH and $P_1 / 2N$

MATHEMATICA 9.0 software is employed to study the influence of dispersion curve on the vibration of Love wave and obtained results are elucidated graphically. In this section, all the parameters such as wave number, phase velocity, porosity, initial stress, inhomogeneity and gravity have been done under the dimensionless process. The detailed discussions on figures are as follows:

Figure 2 describes the influence of porosity concerned with the medium on Love wave velocity with the change in wave number. Increment magnitude of porosity d_1 for curves 1 2 and 3 are taken as 0.90, 0.94 and 0.98. By means of figure 2, with an enhancement in porosity of the medium and wave number, the tendency of the phase velocity is reduced. The

phase velocity of Love surface wave is uniformly declined with the increment of porosity. Thus, we conclude that the presence of porosity in the given media, the Love waves travel slowly through considered structure. In Figure 3, the variational effect of the phase velocity against wave number is established for the various magnitude of initial tension. The dispersion curves 1, 2 and 3 have been represented for distinct values of initial stress $P_1/2N$ as 0.1, 0.2 and 0.3 respectively. It can be remarked from figure 3 that if we rise the initial tension, the phase velocity is reduced. More expressively, the Love wave velocity has inverse effect on the growth of initial stress. The noticeable effects of the initial stress have been found in a higher region of the wave number instead of a lower region of the wave number.

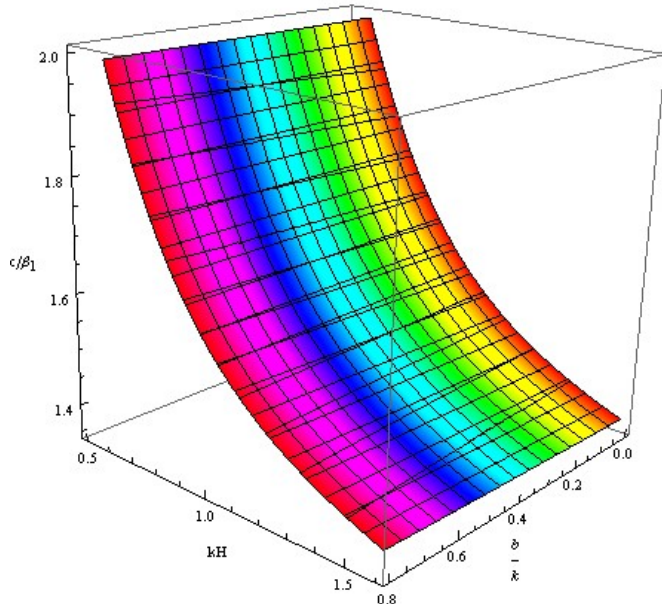


Fig. 17. Surface plots showing combined variation of c/β_1 against kH and b/k

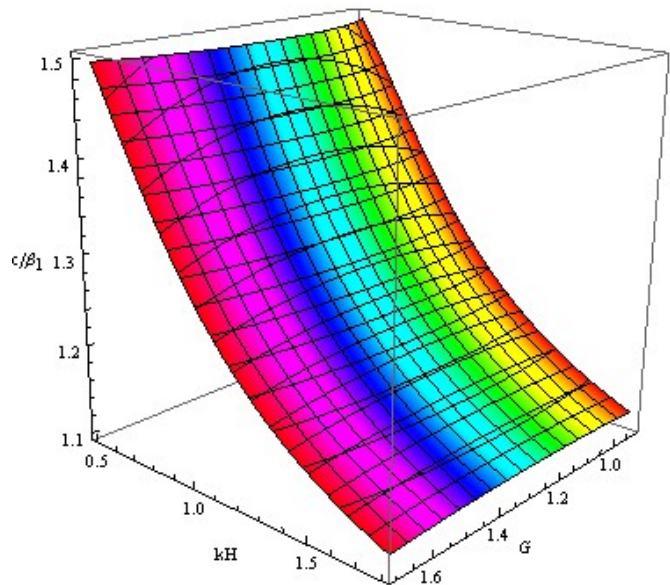


Fig. 18. Surface plots showing combined variation of c/β_1 against kH and G

Figure 4 shows the increasing variation of the inhomogeneity factor on the dispersion curves for the propagation behavior of Love wave in viscoelastic porous layer overlying gravitating half-space. The various magnitude of inhomogeneity b/k is considered as 0.1, 0.5 and 0.9 for dispersion curves 1, 2 and 3 respectively. It is concluded that the increasing rate of phase velocity versus wave number is found for rising the magnitude of inhomogeneity factor. However, the phase velocity does not depend to a great extent on the presence of inhomogeneity in the considered layer model of Love wave. The curves in Figure 5 reflect the change in phase velocity versus wave number for increasing magnitude of gravity. The increasing magnitude of gravity G has been contracted as 0.4, 0.5 and 0.6 for curves 1, 2 and 3. In this figure, the phase velocity of the Love wave diminishes with the rise in magnitude of gravity. In addition, the meticulous analysis shows that the gravity parameter has a significant impact in higher zone of wave number than lower zone of wave number.

Fig. 6 elucidates the impact of coupled viscosity factors contained in the upper bounded layer on phase velocity against wave number. Distinct values of viscosity (f_1, f_2) are taken as (0.60, 0.40), (0.64, 0.44) and (0.68, 0.48) related to curves 1, 2 and 3 accordingly. It can be noticeable based on figure 6 that the phase velocity of Love wave follows a decreasing trend with increasing values of coupled viscosity parameters. Apart from this, it has also found that the noteworthy effect of viscosity on Love wave velocity is greater in the higher range of wave number.

The behavior of the fundamental, first higher and second higher modes on Love wave velocity against wave number have been elucidated in Fig. 7. The smallest wavelength indicates the fundamental mode and the increasing wavelength sequentially represents the first higher and the second higher modes. Based on Fig. 7, the phase velocity increases with the growing amplitude of wavelength and is greater in the second higher mode than the fundamental mode.

Figs. 8 and 9 reveal the variational effect in phase velocity against frequency for the increasing values of porosity and initial stress respectively, contained in the upper bounded layer. It can be seen that the nature of curves of both figures is the same pattern for the different values of these parameters. From Fig. 8, it is quite evident that the increasing magnitude of the porosity has an inverse effect on phase velocity. In Fig. 9, as we increase the magnitude of initial stress, the phase velocity lifts downward. The influence of phase velocity of Love wave with respect to frequency for distinct magnitude of inhomogeneity and gravity parameters associated with the lower half-space have been manifested in Figs 10 and 11, respectively. As we increase the frequency of Love wave, the phase velocity of both figures diminishes. The phase velocity increases with the different mounting values of inhomogeneity, while it reduces with the increment in the magnitude of gravity. Moreover, the influence of gravity has been found more notable as compared to inhomogeneity. Figure 12 reveals the change in phase velocity against frequency for various values of coupled viscosity parameters (f_1, f_2) of viscoelastic porous layer. It is very clear from Fig. 12 the viscosity as well as frequency enhances the Love

wave velocity left downward. More expressly, the presence of viscosity in the anisotropic porous layer the frequency declines phase velocity i.e., the Love waves propagate slowly in the media of viscoelastic.

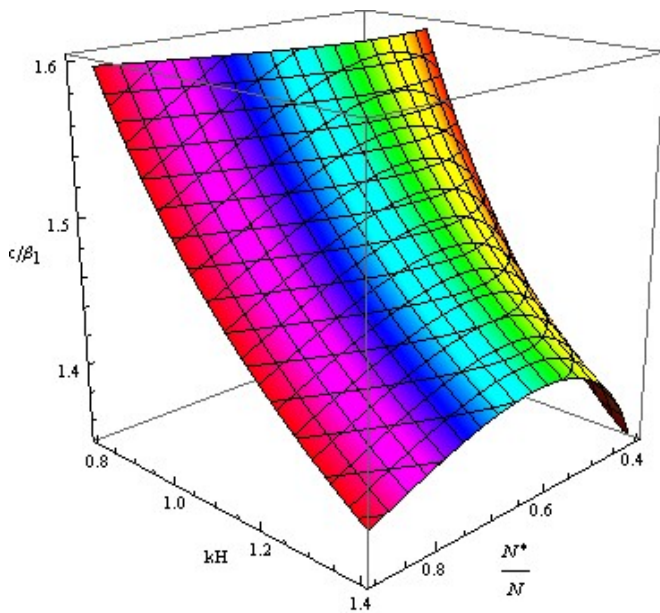


Fig. 19. Surface plots showing combined variation of c / β_1 against kH and $f_1(N^* / N)$

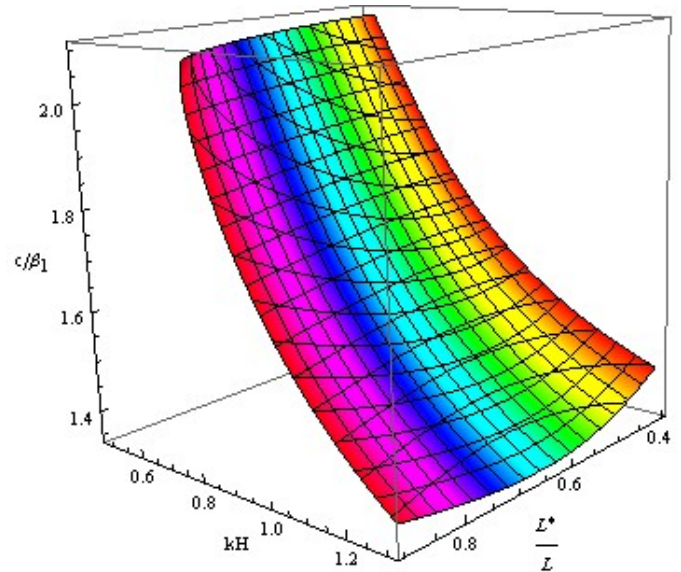


Fig. 20. Surface plots showing combined variation of c / β_1 against kH and $f_2(L^* / L)$

Fig. 13 demonstrates the behavior of various modes such as fundamental, first higher and second higher modes on the Love wave velocity against frequency. From Fig. 13, it can be clearly seen that the phase velocity curve increases rapidly with the growing amplitude of wavelength for the entire range of frequency. However, the smallest wavelength indicates the fundamental mode and the increasing wavelength sequentially represents the first higher and the second higher modes. Moreover, the meticulous inspection of this figure expresses that the speed of Love wave is larger in the case of second higher mode than both the fundamental mode and first higher mode.

The curves are drawn to reveal the particular cases separately with respect to wave number and phase velocity of Love wave through Fig. 14. The curve 1 is plotted for the dispersion equation of Love wave affected by gravity, while the curves 2, 3, 4 and 5 are plotted for particular cases 6.1, 6.2, 6.3 and 6.4 respectively. It can be seen that the phase velocity is higher for curve 5 (case 6.4) compared to the other curves, that is, the Love wave propagates faster in a homogeneous isotropic layered structure than the assumed layered model.

The surface plots in Figs. 15-20 express the combined variation in the dimensionless phase velocity against dimensionless wave number and various influencing parameters like, porosity, initial stress, inhomogeneity, gravity and viscosity. Figs. 15 and 16 represent the joint influence of porosity and initial stress concerned with porous layer respectively, with respect to the phase velocity and wave number for the propagation of Love surface wave in considered layered Earth model subjected to gravity. Figs. 17 and 18 perform the joint impact of inhomogeneity and gravity corresponding to the gravitational substrate for the propagation characteristics of Love surface waves. Moreover, Figs. 19 and 20 display the integrate effect of internal friction (viscosity) contained in viscoelastic porous layer against phase velocity and wave number of Love wave.

8. Conclusions

In this paper, we attempted to conduct a theoretical investigation of the Love surface wave vibration in a pre-stressed anisotropic fluid-saturated porous viscoelastic medium lying over inhomogeneous half-space affected by gravity. Desired expression of dispersion has been attained in compact form with the aid of mathematical tools such as separation of variable and Whittaker's function expansion. In addition, the remarkable effects of all influencing factors on the phase velocity of Love surface waves have been accomplished. Based on the overall study, the following outcomes may be encapsulated as:

- In the section of particular cases and validation, when the layered structure of the problem becomes isotropic elastic medium on homogeneous semi-infinite medium, the obtained dispersion relation reduces to the standard result of Love wave.
- The significant influence of phase velocity on both the wave number and frequency for the propagation of Love surface wave has been reported with all influencing factors.
- Phase velocity of the propagation of Love wave is minimum when the adopted model is in the presence of porosity and initial stress. The porosity and initial stress contained in the bounded layer disfavor the phase velocity.
- Due to the presence of viscoelasticity in the upper layer, it produces a low velocity of Love waves. However, the

impact of viscoelasticity on the phase velocity is more notable at the higher zone of wave number instead of lower zone of wave number.

- Increasing the magnitude of inhomogeneity of the half-space increases the phase velocity of Love wave. However, as both the wave number and frequency of Love wave increase, the phase velocity decreases monotonously.
- The Love wave slowly propagates due to the presence of gravity in the considered layered model. The influence of gravity associated with elastic half-space dominates the phase velocity of Love wave.
- The second higher mode is the one that dominates most for the increasing value of both the wave number and the frequency.

Observations made in the present theoretical framework may suggest a significant contribution to the problems of wave propagation and vibration through the Earth layered model with different material properties. Moreover, this result may be of great importance for studying the deformation behavior of solids as well as liquids due to the earthquakes and artificial explosions.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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References

- [1] Ke, L.L., Wang, Y.S., Zhang, Z.M., Propagation of Love waves in an inhomogeneous fluid saturated porous layered half-space with properties varying exponentially, *Journal of Engineering Mechanics*, 131(12), 2005, 1322-1328.
- [2] Ke, L.L., Wang, Y.S., Zhang, Z.M., Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties, *Soil Dynamics and Earthquake Engineering*, 26(6-7), 2006, 574-581.
- [3] Dey, S., Mukherjee, S.P., Love waves in granular medium over a half-space under gravity, *The Journal of the Acoustical Society of America*, 72(5), 1982, 1626-1630.
- [4] Achenbach, J.D., *Wave propagation in elastic solids*, North Holland Publishing Company, Amsterdam, 1973.
- [5] Love, A.E.H., *Mathematical theory of elasticity*, Cambridge University Press, Cambridge, 1920.
- [6] Ewing, W.M., Jardetzky, W.S., Press, F., *Elastic wave in layered media*, McGraw-Hill, New York, 1957.
- [7] Pujol, J., *Elastic wave propagation and generation in seismology*, Cambridge University Press, Cambridge, 2003.
- [8] Chattopadhyay, A., Wave reflection in triclinic crystalline medium, *Archive of Applied Mechanics*, 76(1-2), 2006, 65-74.
- [9] Biot, M.A., Generalized theory of acoustic propagation in porous dissipative media, *The Journal of the Acoustical Society of America*, 34(9A), 1962a, 1254-1264.
- [10] Biot, M.A., Mechanics of deformation and acoustic propagation in porous media, *Journal of Applied Physics*, 33(4), 1962b, 1482-1498.
- [11] Biot, M.A., Theory of elasticity and consolidation for a porous anisotropic solid, *Journal of Applied Physics*, 26(2), 1955, 182-185.
- [12] Kończak, Z., The propagation of Love waves in a fluid-saturated porous anisotropic layer, *Acta Mechanica*, 79(3-4), 1989, 155-168.
- [13] Wang, Y.S., Zhang, Z.M., Propagation of Love waves in a transversely isotropic fluid-saturated porous layered half-space, *The Journal of the Acoustical Society of America*, 103(2), 1998, 695-701.
- [14] Son, M.S., Kang, Y.J., Shear wave propagation in a layered poroelastic structure, *Wave Motion*, 49(4), 2012, 490-500.
- [15] Dey, S., Sarkar, M.G., Torsional surface waves in an initially stressed anisotropic porous medium, *Journal of Engineering Mechanics*, 128(2), 2002, 184-189.
- [16] Saha, A., Kundu, S., Gupta, S., Vaishnav, P.K., Effect of irregularity on torsional surface waves in an initially stressed anisotropic porous layer sandwiched between homogeneous and non-homogeneous half-space, *Journal of Earth System Science*, 125(4), 2016, 885-895.
- [17] Pandit, D.K., Kundu, S., Gupta, S., Propagation of Love waves in a prestressed Voigt-type viscoelastic orthotropic functionally graded layer over a porous half-space, *Acta Mechanica*, 228(3), 2017, 871-880.
- [18] Carcione, J.M., Wave propagation in anisotropic linear viscoelastic media: theory and simulated wavefields, *Geophysical Journal International*, 101(3), 1990, 739-750.
- [19] Borchardt, R.D., Rayleigh-type surface wave on a linear viscoelastic half-space, *The Journal of the Acoustical Society of America*, 54(6), 1973, 1651-1653.
- [20] Kakar, R., Kakar, S., Love wave in a Voigt-type viscoelastic heterogeneous layer overlying heterogeneous viscoelastic half-space, *International Journal of Geomechanics*, 17(1), 2016, 06016009.
- [21] Chattopadhyay, A., Gupta, S., Kumari, P., Sharma, V.K., Effect of point source and heterogeneity on the

- propagation of SH-waves in a viscoelastic layer over a viscoelastic half space, *Acta Geophysica*, 60(1), 2012, 119-139.
- [22] Wilson, J.T., Surface waves in a heterogeneous medium, *Bulletin of the Seismological Society of America*, 32(4), 1942, 297-304.
- [23] Sato, Y., Love waves propagated upon heterogeneous medium, *Bull. Earthq. Res. Inst. Univ. Tokyo*, 30, 1952, 1-12.
- [24] Tomar, S.K., Kaur, J., Reflection and transmission of SH-waves at a corrugated interface between two laterally and vertically heterogeneous anisotropic elastic solid half-spaces, *Earth, Planets and Space*, 55(9), 2003, 531-547.
- [25] Vardoulakis, I., Torsional surface waves in inhomogeneous elastic media, *International Journal for Numerical and Analytical Methods in Geomechanics*, 8(3), 1984, 287-296.
- [26] Chattopadhyay, A., Gupta, S., Kumari, P., Sharma, V.K., Propagation of torsional waves in an inhomogeneous layer over an inhomogeneous half-space, *Meccanica*, 46(4), 2011, 671-680.
- [27] Biot, M.A., *Mechanics of Incremental Deformations*, John Wiley and Sons Inc, New York, 1965.
- [28] Maity, M., Kundu, S., Pandit, D.K., Gupta, S., Characteristics of Torsional Wave Profiles in a Viscous Fiber-Reinforced Layer Resting over a Sandy Half-Space under Gravity, *International Journal of Geomechanics*, 18(7), 2018, 06018015.
- [29] Sethi, M., Gupta, K.C., Gupta, D., Surface waves in fibre-reinforced anisotropic solid elastic media under the influence of gravity, *International Journal of Applied Mechanics and Engineering*, 18(1), 2013, 177-188.
- [30] Dey, S., Mukherjee, S.P., Rayleigh waves in an initially stressed layer over a half-space under gravity, *Acta Geophysica Polonica*, 32(1), 1984, 81-90.
- [31] Pal, J., Ghorai, A.P., Propagation of Love wave in sandy layer under initial stress above anisotropic porous half-space under gravity, *Transport in Porous Media*, 109(2), 2015, 297-316.
- [32] Khan, A., Abo-Dahab, S.M., Abd-Alla, A.M., Gravitational effect on surface waves in a homogeneous fibre-reinforced anisotropic general viscoelastic media of higher and fractional order with voids, *International Journal of Physical Sciences*, 10(24), 2015, 604-613.
- [33] Whittaker, E.T., Watson, G.N., *A course of modern analysis*, Cambridge University Press, Cambridge, 1991.
- [34] Qian, Z.H., Jin, F., Lu, T., Kishimoto, K., Hirose, S., Effect of initial stress on Love waves in a piezoelectric structure carrying a functionally graded material layer, *Ultrasonics*, 50(1), 2010, 84-90.
- [35] Kumari, P., Sharma, V.K., Modi, C., Modeling of magnetoelastic shear waves due to point source in a viscoelastic crustal layer over an inhomogeneous viscoelastic half space, *Waves in Random and Complex Media*, 26(2), 2016, 101-120.
- [36] Kakar, R., Kakar, S., Love-Type Surface Wave in an Isotropic Layer Bounded between Orthotropic and Heterogeneous Half-Spaces under Initial Stresses, *International Journal of Geomechanics*, 17(3), 2016, 04016083.
- [37] Gubbins, D., *Seismology and plate tectonics*, Cambridge University Press, Cambridge, 1990.

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