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Research Paper

# Stability Assessment of the Flexible System using Redundancy

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**Abstract.** In this study, dynamic behavior of a mooring line in a floating system is analyzed by probability approaches. In dynamics, most researches have shown the system model and environments by mathematical expression. We called this process as the forward dynamics. However, there is a limit to define the exact environments because of uncertainty. To consider uncertainty, we introduce the redundancy in flexible system, mooring line. For verifying the effectiveness and stability of the mooring line, criterion of axial breaking load of the mooring line is applied to joint reaction forces according to the various path of the mooring line. To cover the limits for defining the non-linearity of the environments, various responses of the mooring line along the redundancy that is used in Robotics, are derived by probability distribution. By using the Newton-Euler formulation, the inverse kinematics and the linear acceleration theorem to get joint displacements, velocities and accelerations, the joint reaction forces and moments are calculated and probability distribution of the mooring about stability and compatibility is investigated. Lastly, we simulate the flexible systems in various null motions, calculated each joint torque and force, and evaluated failure probabilities using the Monte-Carlo method.

**Keywords:** Structural Analysis, Mooring line, Dynamics, Newton-Euler formulation, Redundancy.

## 1. Introduction

Due to the global trend of fossil energy depletion and new energy generation, the development of natural resources such as oil or gas centered on the existing shallow waters is going to the deep-sea. To do this, mobile floaters and subsea plant systems, such as Floating Production Storage and Off-loading (FPSO) have been actively researched and developed [1]. In addition to fossil energy, the ocean has the advantage of being able to generate power using wave energy. In recent years, as well as wind power generators, floating wave generators have been researched and developed for commercialization [2-3]. This requires proper verification of the stability of the floats. The behavior of floating bodies is formulated as a linear problem applying a combination of sinusoidal to a system to derive a numerical solution. However, the design of mooring line has only a guideline, and the description is far from enough compared to the analysis of floating bodies undergoing various forms of external force [4]. In addition, the modeling on mooring line is formulated by the suspension line equation, many repetitive calculations are performed to check the behavior due to the limitation of interpretation of the system response obtained by the assumption of the current force component and the ignorance of the elastic effect [5]. Unlike the mooring system analysis through simple static state modeling on subsea environment in the fixed floating body analysis, the analysis conditions are different due to the action of nonlinear terms generated in FPSO and subsea system analysis. For example, local pressure changes, vortex generation, deflection and centrifugal forces that occur when mooring systems move due to uncertainty factor such as rapid flotation motions make it difficult



to predict response [6-7].

Many researches have applied the uncertainty factors of system by comparison between mathematical model and experiment result. The uncertainty was derived from not perfect condition in the mechanical processes. It could be happened by the manufacturing error, material's anisotropy and abnormal distribution. So, many researches had been conducted to evaluate the uncertainty and verify the safety with finite element method and lumped parameter method. By these efforts, it is possible to simulate nearly perfect conditions.

However, this fact does not make sense in case of the flexible system. The flexible system has well extensional, bending, torsional characteristic. It is also sensitive to external forces. Most researches have generally considered specific position where external forces are applied. And other positions were not considered because of rigid body motion. However, the flexible system is affected by small forces and each element including Coriolis and centrifugal force. Also, the environment including wave, wind, and current does not act exactly according modeling. Moreover, finite element method has a disadvantage of consuming a lot of time due to the increase in solving non-linear problem, even though it enables precise analysis. In this point, the methodology of verifying the stability should be considered.

To overcome the limitations of the forward dynamics, the inverse dynamics is considered in this paper. In the robotics, the manipulator which has more number of degrees of freedom compared to work space coordinates, can have lots of solutions corresponding to the same end-effector's position. It means that all solutions which take place in the flexible system can be considered. This concept is called as the redundancy. The redundancy can be applied and useful in the dynamics, if the system is connected by each component, similar chains. In other words, the flexible system can be modeled by links, joints. Therefore, we may get joint torque or forces, and judge system's stability by comparing joint torque or forces and reference values.

In this paper, the mooring line is applied to investigate its dynamic response and the safety. To introduce the concept, the general process of analysis and example of the mooring line system is considered. Especially, the robotics and linear acceleration theorems are used in this process. About the equations of motion, the Newton-Euler formulation is used to derive joint torques and forces. It is more efficient because of recursive calculation. Not like the Lagrange method, it shows the joint torques and forces quickly without establishing the equations of motion. From these results, we compare the values of safety criterion. Lastly, we use the Monte-Carlo method to see the safety of the mooring system.

## 2. Mathematical Model

### 2.1 Redundancy

The Jacobian is relationship between joint space and work space [8-11]. Using the Jacobian, transformation between two spaces is easy and it is possible to derive joint or work space solutions without direct calculation. The definition of the Jacobian was derived by Equation (1). If m-th vector  $p$  is function of n-th vector  $q$ , the Jacobian is Equation (1).

$$J = \frac{\partial p}{\partial q} = \begin{bmatrix} \frac{\partial p_1}{\partial q_1} & \frac{\partial p_1}{\partial q_2} & \dots & \frac{\partial p_1}{\partial q_n} \\ \frac{\partial p_2}{\partial q_1} & \frac{\partial p_2}{\partial q_2} & \dots & \frac{\partial p_2}{\partial q_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial p_m}{\partial q_1} & \frac{\partial p_m}{\partial q_2} & \dots & \frac{\partial p_m}{\partial q_n} \end{bmatrix} \quad (1)$$

As referred, we determined the velocity in work space by using the velocity in joint space and the Jacobian. Equation (2) shows this relationship.

$$\dot{p} = \frac{dp}{dt} = \frac{\partial p}{\partial q} \frac{dq}{dt} = J\dot{q} \quad (2)$$

But, this formulation was little different to the robotics because the manipulator was composed of different joints. Equation (3) shows the general expression of the Jacobian matrix in the manipulator. Equation (4) is revised velocity relationship.

$$J_v = [J_{v1}, J_{v2}, \dots, J_{vn}] \quad (3)$$

where

$$J_{vi} = \begin{bmatrix} \begin{bmatrix} {}^0z_{i-1} \times {}^0p_{i-1,n} \\ {}^0z_{i-1} \end{bmatrix}_{revolute} \\ \begin{bmatrix} {}^0z_{i-1} \\ 0 \end{bmatrix}_{prismatic} \end{bmatrix} \quad (4)$$

$$v = \begin{bmatrix} \dot{p}_n \\ 0 \\ \omega_n \end{bmatrix} = J_v \dot{q}$$

Using the Jacobian matrix, it was possible to get solutions in work and joint space. The most important part of inverse kinematic is redundancy. A manipulator has the redundancy if it has more degrees of freedom than coordinates which are necessary to define position. Using the redundancy, there are advantages including potential to avoid singularities, obstacles, structural limitations. The general formulation of the redundancy was used, Equation (5).

$$\delta q = J^\# \delta p + (I - J^\# J)z \quad (5)$$

In which  $J^\#$  is pseudo-inverse matrix. Its basic characteristics are arranged in reference [12-14]. Equation (5) was derived by least square solution method using pseudo-inverse matrix. In other word, finding the solutions which make minimum value of  $\|\delta p - J\delta q\|$  was task.

Among all set of solutions, there was unchangeable rule. Equation (6) shows the difference between different solutions. It shows that end effector's position is not change, despite of different joint angles.

$$J(q)\delta q_1 - J(q)\delta q_2 = \emptyset \quad (6)$$

In Equation (5), all variables except  $z$  vector are same in every calculation. However,  $z$  vector is arbitrary vector. To use the redundancy, the definition of  $z$  vector is important. We defined  $z$  vector as Equation (7).

$$z = k_0 \left( \frac{\partial w(q)}{\partial q} \right)^T \quad (7)$$

The  $k_0$  coefficient was larger than 0, and  $w(q)$  was the second objective function of joint variables. The second objective function can be derived by avoiding singular points, obstacles, joint limit. In this paper, we used joint limit concept, and showed objective function of Equation (8).

$$w(q) = -\frac{1}{2n} \sum_{i=1}^n \left( \frac{q_i - \bar{q}_i}{q_{iM} - q_{im}} \right)^2 \quad (8)$$

$q_{iM}$ ,  $q_{im}$  are maximum, minimum of joint limits. And  $\bar{q}_i$  is average of joint limit.

## 2.2 Joint displacement, velocity, and acceleration [15-17]

Using the redundancy, the increase or decrease of joint displacement was derived. Next step was the arrangement of these values for calculation. Equation (9)-(11) are joint displacement, velocity, and acceleration.

$$q_{i+1} = q_i + \partial q \quad (9)$$

$$\dot{q}_{i+1} = \frac{\partial q}{\Delta t} \quad (10)$$

$$\ddot{q}_{i+1} = \frac{1}{\alpha(\Delta t)^2} + (q_{i+1} - q_i) - \frac{1}{\alpha\Delta t} \dot{q}_i - \left( \frac{1}{2\alpha} - 1 \right) \ddot{q}_i \quad (11)$$

The Joint acceleration was derived by assumption in Equation (11). The assumption was that acceleration will be same in adjoin section. The coefficient  $\alpha$  was 1/4.

## 2.3 Newton-Euler formulation

There are two method of calculating dynamics. The Lagrange equation of motion is powerful for understanding system. It divides system into each physical term including mass & inertia, Coriolis & centrifugal, and gravity terms. But, there is a big weakness, computation efficiency. In case of more than 3 degrees of freedom, it will take long time, despite of high computation capacity.

The Newton-Euler formulation is numerical method to acquire the equation of motion. Its process is recursive. From first point to end point, same formulation was repeated in the mooring model. The main purpose of this method is to calculate the joint driving force for realizing a given trajectory of joint vector  $q$ . There were two assumptions for use.

First, the present values of joint displacements  $q_i$ , joint velocities  $\dot{q}_i$ , and the desired values of joint accelerations  $\ddot{q}_i$  were given for all links. And, the force  ${}^0f_i$  and moment  ${}^0n_i$  exerted on the end link by the environment were given.

From these assumptions, we found that it is suitable for calculating the flexible system. In our paper, the joint driving force was corresponding joint force which are caused by uncertainty and external force. The basic procedure of the Newton-Euler formulation is well known. Reference shows information about the Newton-Euler formulation [18-21].



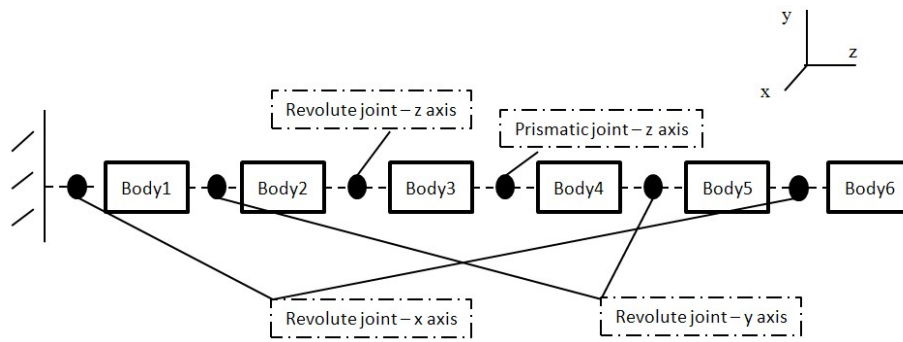
## 2.4 Mooring line

The type of mooring line model was defined as spar platform. In this paper, we used chain part of whole mooring line for local analysis [22-25]. The mooring system consisted of four part are equally spaced around the buoy. The system properties were referred from reference, shown Table 1 [26].

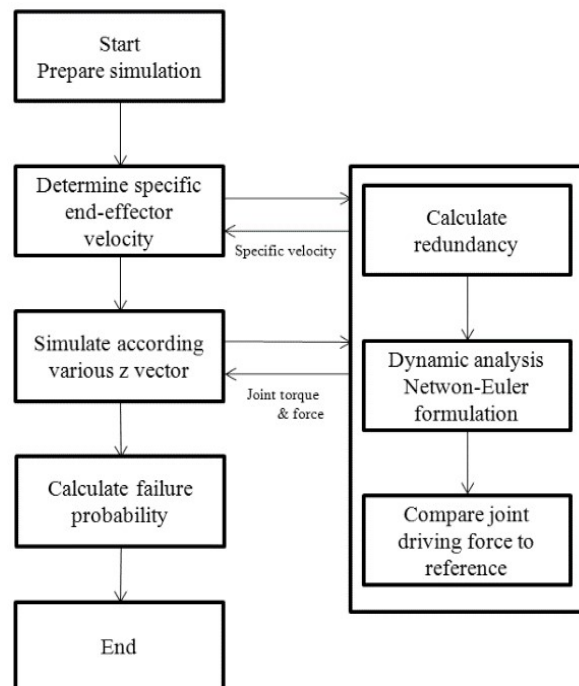
**Table 1.** T Properties of mooring lines for the spar platform.

| Design                 | Quantity          | Unit |
|------------------------|-------------------|------|
| Length                 | 6                 | m    |
| Diameter               | 245               | mm   |
| Distributed mass       | 287.8             | kg/m |
| Wet weight             | 2,485             | N/m  |
| Stiffness AE           | $1.03 \cdot 10^6$ | kN   |
| Minimum breaking load  | $11.8 \cdot 10^3$ | kN   |
| Distributed added mass | 37.4              | kg/m |
| Drag force coefficient | 2.45              | -    |

To define chain, the cylinder shaped was used. The inertia matrix was determined by diameter, mass, and cylinder geometry. About stiffness, the stretching stiffness was only considered. The bending and torsional stiffness are not generally included in the chain. If the chain is fixed rigidly, its consideration should be reminded. However, it was negligible comparing stretching stiffness in this paper. Using the link-joint concept, we simplified mooring model in Fig. 1.



**Fig. 1.** One-element mooring line model



**Fig. 2.** MATLAB Algorithm

For detail, the two or two element mooring line model can be modeled by adding one-element to the end of original element. About the environments, each chain took drag force from current. And they have gravity force and self-buoyancy simultaneously. Equations (12)-(14) show each force formulation.

$$F_{CR} = \frac{1}{2} C_{CR} \rho V_{CR}^2 A_{UP} \quad (12)$$

$$F_{GR} = mg \quad (13)$$

$$F_{BY} = \rho_{water} g V \quad (14)$$

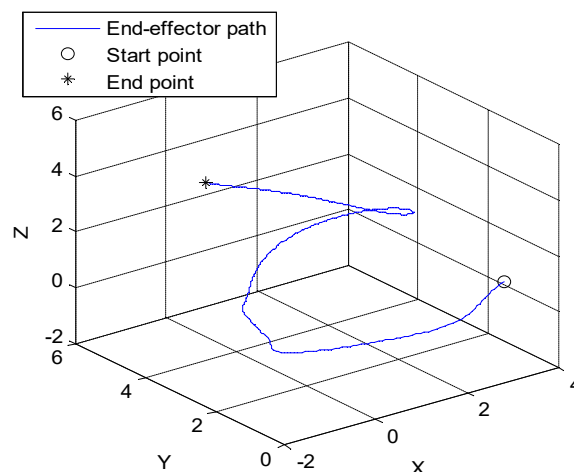
In Equation (12), the drag force always acted on normal surface. The average current velocity was 0.0341 m/s. Not rigid body motion, each element of mooring line moved 6 directions freely. So, they took variable forces immediately. To apply this movement, we calculated orientation according Z-Y-X Euler angle in the every iteration. These angles were applied into Equation (12) about x, y, z direction. The average external force including current applied to all simulations.

## 2.5 Simulation

To analyze the mooring line, the MATLAB code was established. The MATLAB code was composed of these theorems in regular sequence. Fig. 2 is the MATLAB algorithm. From simulation, the specific end-effector's velocity and failure probability were acquired. The specific end-effector's velocity was velocity of convergence to target point. If the velocity is high, flexible system will have large joint driving force. It means that system takes risk compared to low velocity. This situation can be sufficiently happened in case of mobile system including floating, barge system. Second, the mooring line of specific end-effector velocity and arbitrary z vector was considered. In Robotics, the larger  $k_0$  and  $\delta_p$  in z vector, the faster the convergence speed, but the joint reaction force diverges beyond the threshold or fails to reach the target point. This fact allows the two variables to correspond to the environment in the deep-sea and the motility of the floating body. To increase the effectiveness of the analysis,  $k_0$  and  $\delta_p$  were applied to random numbers with a fixed size range, and then arranged into a probability distribution. Based on the concept, the joint driving force concerning the uncertainty was acquired. And, the failure probability was calculated to show stability of system using the Monte-Carlo method. Table 2 shows simulation parameters that is correspond to  $k_0$  and  $\delta_p$ .

**Table 2.** Simulation configuration.

| Parameter                 | Description                                                         |
|---------------------------|---------------------------------------------------------------------|
| $p_{initial}, p_{target}$ | The initial and target position of the end-effector of mooring line |
| $\varepsilon$             | $\ p_{target} - p_{initial}\ $                                      |
| $t$                       | Time = 0 : $t_{end}$                                                |
| $k_0$                     | Coefficient of the z-vector                                         |
| $\dot{p}$                 | $\frac{p_{target} - p_{initial}}{dt}$<br>(where $dt$ = speed ratio) |



**Fig. 3.** Path of end-effector

## 3. Results and Discussion

### 3.1 Example of mooring line's stability

The numerical solutions were obtained as the result of dynamic analysis. For this simulation, the end-effector of mooring line was moved from  $p_{initial} = [3.9948, 1.5370, 0.1228]$  m to  $p_{target} = [0, 4.2426, 4.2426]$  m. The path of end-effector is shown Fig.3. And the prismatic joint 4 force is shown in Fig. 4. Other joint torque was various according

conditions. However, mooring line didn't have bending and torsional movement. According this fact, we considered only the prismatic joint in this case of movement. In this example, the specific end-effector's velocity was determined by difference between current position and target position. And this value was multiplied by total step and speed ratio 10. If the speed ratio is high, convergence time will be short. By comparing critical breaking load  $11.8 \cdot 10^3$  kN of the internal force of the mooring line, the joint force exceeded breaking load about 0.8 sec. From this case, we concluded that end-effector's velocity should be lower to verify the safety.

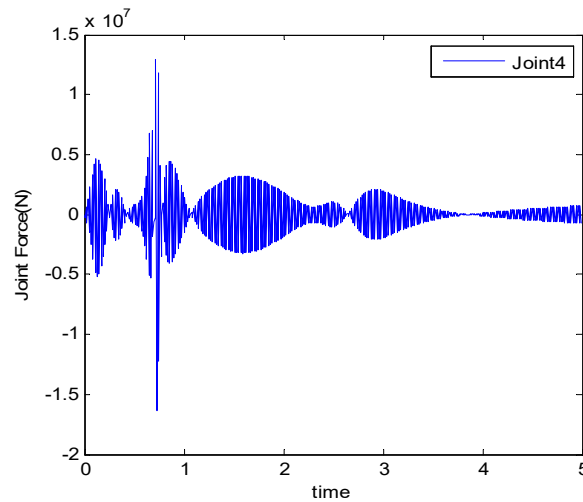


Fig. 4. Prismatic joint force in one-element mooring line

Second, 2-element mooring line was simulated. The initial, target of end-effector were  $[0.0195, 4.6104, 3.9274]$  m,  $[0, 4.2426, 4.2426]$  m. And  $dt$  was increased that is double that of previous simulation. The prismatic joint's force is shown in Fig. 5. As shown, all joint torques satisfied critical breaking load. But, the mooring line didn't reach target position. Its final position was  $[0.0006, 2.2245, 2.0045]$ . From these examples, we found selection of specific end-effector's velocity should be considered exactly. In other words, the external force regarding environmental factors were critical to interrupt the movement of the mooring line.

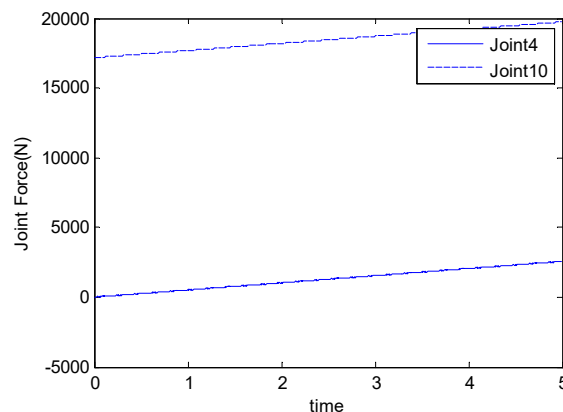


Fig. 5. Prismatic joint forces in two-element mooring line

### 3.2 Specific end-effector's velocity

The criterion of end-effector's velocity  $\dot{p}$  satisfied safe region and convergence to target. In this section, the end-effector's velocity according accuracy and safety was considered. Using target position and critical breaking load, the accuracy and safety was transformed into digitization. The two-element of mooring line model was used. The initial and target position were  $[0.0195, 4.6104, 3.9274]$  m and  $[0, 4.2426, 4.2426]$  m respectively. And the range of speed ratio  $dt$  was from  $1.0 \cdot 10^{-3}$  to  $10^5$ . The speed ratio was divided by log scale to apply random values.

From iterations, the criterion indicators were calculated. Equation (15) shows the criterion indicator. From this equation, the maximum value of criterion indicator was acquired to show optimized speed ratio. In this example, speed ratio 6.6347 was determined. The maximum axial forces was  $5.577 \cdot 10^2$  kN and the difference of position was 0.5450 m about the specific end-effector's velocity.

$$\left( \frac{\text{Safety}}{\text{Accuracy}} \right) = \left( \frac{(\text{Failure load} - \max(\text{Axial force}))}{(\text{Goal position} - \text{Current position})} \right) \quad (15)$$

### 3.3 Failure probability [27-29]

The solutions of redundancy are different according objective function of redundancy. In this section, the joint limit concept was used to predict the motion of the mooring line. This assumption was confirmed from the fact that the mooring line are fixed to both ways. Not only path of the mooring line, but also the joint driving force was important to determine the stability. To see the stability, the Monte-Carlo method was applied in the simulation including various objective functions. About the environment variables, the specific speed ratio  $dt$  which was acquired in previous was used. Also, all terms were same, except the objective functions. The objective functions were different by multiplying different  $k_0$  value. The  $k_0$  was coefficient of objective function. Its range was from  $1.0 \cdot 10^{-5}$  to  $10^2$ .

The criterion was that the joint driving force should not exceed 20% of critical breaking load. The values which exceed 20% were considered as failure. In dynamic loading situation, the safety factor is 5 according experiments. And the results which the length of end-effector position exceed length of the mooring line were not considered in the probability calculation. From this assumption, the failure probability was determined. In this example, 41.3 % was fail. Despite of the specific speed ratio, the difference of target position and acquired position was big. About the safety, the joint force always satisfied to the minimum breaking load. Though these facts, the position or orientation of flexible system were more sensitive to the arbitrary  $z$  vector. To make realistic motion with the uncertainty, the well-designed  $z$  vector that is correspond to environment condition should be necessary.

## 4. Conclusion

A mathematical model of a mooring line was proposed to predict whole motion. The model verified by path of the end-effector. The path of end-effector predicted by the model was very different to the previous path. This phenomenon came from including the uncertainty. The model took into account the uncertainty caused by flexible system and environment, deep sea. Not only the path of the end-effector, but also the joint driving force corresponding body force was considered to determine the safety and accuracy. Each mooring body was dependent on the adjacent bodies because they were affected by the Coriolis and centrifugal forces. The equations of motion of the mooring line were solved to obtain the transient response of the structures. Especially, the Newton-Euler formulation was used by using the joint displacement, velocity, and acceleration information. The maximum joint displacement was applied to the body which is near to the top. To satisfy the target position exactly, it was better than moving base part of the mooring line. The mooring line model was sensitive to the end-effector's velocity and arbitrary redundancy  $z$  vector. The end-effector's velocity was examined by substitution specific range of speed ratio. The high speed ratio gave good convergence to the target position. However, it also took high joint driving force which occur failure of mooring line. From this reason, the good speed ratio value was derived. The criterion considered safety and accuracy of mooring line's movement. From the specific speed ratio value, the failure probability of mooring line was derived by arbitrary  $z$  vector. The  $z$  vector was acquired by kind of objective function. In this paper, joint limit concept was used to define the redundancy. The solutions were different according to  $z$  vector. From these results, the safety criterion was applied to figure out the failure results. To calculate the failure probability, the Monte-Carlo method was used. In the specific speed ratio, failure probability was acquired. This concept of flexible model can be applied to expect the detailed model. The high-probability of motion was predicted by changing the objective function of redundancy and the uncertainty can be included and expressed by difference of forward dynamics and redundancy based dynamics.

## Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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## Nomenclature

|                              |                                            |           |                                          |
|------------------------------|--------------------------------------------|-----------|------------------------------------------|
| $J$                          | Jacobian                                   | ${}^0n_i$ | The moment exerted by link i-1 on link i |
| $J^\#$                       | Pseudo-inverse Jacobian                    | $\rho$    | Water density                            |
| $I$                          | Identity matrix                            | $C_{CR}$  | Drag coefficient                         |
| $z$                          | Arbitrary vector for redundancy            | $V_{CR}$  | Current velocity                         |
| $w$                          | Second objective function                  | $A_{UP}$  | Normal surface                           |
| $k_0$                        | Coefficient of the objective function      | $m$       | Mass                                     |
| $\dot{q}_i, q_i, \ddot{q}_i$ | Joint displacement, velocity, acceleration | $G$       | Gravity acceleration                     |
| ${}^0f_i$                    | The force exerted by link i-1 on link i    | $V$       | Volume                                   |

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