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Research Paper

Mechanics of 2D Elastic Stress Waves Propagation Impacted by Concentrated Point Source Disturbance in Composite Material Bars

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Abstract. Green's function, an analytical approach in inhomogeneous linear differential equations, is the impulse response, which is applied for deriving the wave equation solution in composite materials mediums. This paper investigates the study of SH wave's transmission influenced by concentrated point source disturbance in piezomagnetic material resting over heterogeneous half-space. Green function approach is used to solve differential equation and obtain the dispersion relation in determinant form and match with existing classical Love wave equation for the authenticity for the article. The properties of SH wave throughout the considered framework and their state of relying on varied geometrical and physical parameters are scrutinized. The simulated outcomes of disparate physical quantities viz., dimensionless phase velocity, elastic parameter, group velocity, initial stress, piezomagnetic/heterogeneity parameter and stress distribution of SH wave in the considered structure are investigated and used to regulate the behavior of dispersion characteristics of smart material waveguides.

Keywords: Concentrated Point Source; Mechanical Surface Wave; Initial Stress; Analytical Modeling; Piezomagneticity; Green function.

1. Introduction

Elastic stress wave Generation due to concentrated point source in composite structure has lots of application in engineering. Concentrated point source to source in the shape of a point from where stress waves are generated due to some blasting, pressure or a point from where water flows which creates the waves, etc. Therefore, it is of influential interest to the researchers, academicians and engineers to study the nature of a composite framework comprising smart/composite materials interacting with vibrations/waves generated or influenced by some concentrated source of disturbance in a point form, carrying a great impact deal of potential for industrial and commercial applications. Like, similar technique is used in smart phone when we touch the screen (means provide a stress from a concentrated point source) waves are generated (surface stress waves) which help to open an applications which is saved in a phone/mobile, is a small structure example. Large structure example is a nuclear/hydrogen bomb creating huge destructive waves, which damage the structure or building. Structural behavior during blasting depends entirely on the materials used in the construction of the building.

In the present article, an analytical study of the stress wave propagation influenced by point source disturbance in piezomagnetic (PM) material layer resting over heterogeneous material half-space is performed. Wave propagation related researches are carried out through different materials [1-8]. In such articles different types of materials are used like viscoelastic, viscous liquid, piezoelectric (PE), piezomagnetic (PM) and functionally graded piezoelectric material (FGPM) to study the characteristics of stress waves propagation. The materials with some more advance specific characteristics and properties, which generate the electric current or magnetic influence in response to the mechanical disturbance or stress, are very applicable materials. Some of them are piezoelectric, piezomagnetic, FGPM and functionally graded materials (FGM). PM materials are one of the advanced smart material in which the magnetic effect obtain by applying mechanical or physical stress. These multiferroic materials are used to create sensors, transducers, actuators, etc [9-11]. The surface acoustics wave (SAW) devices work on the principle of surface wave existence in an elastic body of free surfaces where the distribution is localize near the surface area. When PM materials are embedded in composite structures, then the structures turn to smart material structures. Therefore, transmission of the surface elastic stress wave through advance materials such as PE, PM, and FGPM has significant merits. The possible applications of multiferroic material is in the field of mechanical and electrical engineering, communications, geophysics and specially in making of surface acoustic devices (SAW) are available in [12-22].

Green's function is an analytical approach, which is very important scheme in the field of applied mathematics and solid mechanics; it helps to provide solutions to the number of differential equations. It is the base foundation of numerical or analytical techniques covering the dislocation techniques methods, singular equation and boundary element methods. Moreover, it gives the unique visual interpretations of the problems, which carry actions associated to a source of force concentrated or situated at a single point. It is a well-known concept that the wave motions in a material is not cause only by an internal force but also affected by the disturbance of an external source like blasting, mechanical stress etc. Green function approach is famous for solving the mathematical problem having the non-homogeneous boundary conditions [23-24]. Green's function plays an important role in finding the displacements of particle affected by Love-type wave transmissions in a plate. Previously, the displacement generated in a form of Love-type waves due to 2-D concentrated point source in a medium has been studied by Ghosh [25], using the Green's function technique. Green's function gives solutions to the problem of elastodynamics having disturbance caused by concentrated point source. Green's function is impose accordingly on the material characteristics, that how it behaves mechanically to an impulsive excitation force. Many academicians solved the problem of wave transmission influenced by concentrated point source in the stratified models [26-29].

Research on heterogeneous media is not explore until now. Kundu *et al.* [30] studied the characteristics and properties of SH-wave transference in an initially stressed orthotropic medium sandwiched by a homogeneous and an inhomogeneous semi-infinite media. Joly and Weder [31] proved the existence of guided waves propagating with a velocity greater than the S-wave velocity at infinity in the case of unbounded elastic media invariant under translation in one space direction and asymptotically homogeneous at infinity. Until now, no research article is available to explore the transference of stress wave influenced by a point source in piezomagnetic structure by using Green function approach. Lastly, it is also important to go through the wave transmission phenomenon through material bars, which requires the stress-strain analysis theory and physics of solids, which is available in [32–53].

The present research articles represent the study of SH-wave transference in piezomagnetic layer influenced by a point source disturbance overlying a heterogeneous substrate. A Green's function technique is used to solve the governing equations of considered structure. Dispersion relation is obtained in closed determinant form analytically and matched with obtained results when wave propagates along the direction of layer. Remarkable influence of parameters like elastic constant, piezomagnetic constant, heterogeneity parameter, initial stress and layers width are presented graphically with addition of initial stress distribution surface plot. Some numerical examples such as Cobalt Iron Oxide for the piezomagnetic material layer have been consider.

2. Mathematical and Mechanical Model of the Problem

This research article is focuses on the transmission of surface stress wave under a point source disturbance in a framework comprising of piezomagnetic layer of thickness *H*-overlying heterogeneous half-space in the Fig. 1. The mathematical and mechanical model specified by a Cartesian coordinate system having 'O' as origin on the interface. Taking $y-$axis along the direction of wave transference and $x-$axis being pointing vertically downwards. A source of disturbance (point source) is located at interface 'S'. The poling direction of piezomagnetic material is parallel to $z-$axis .

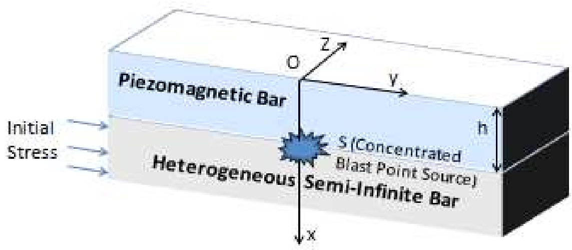

Fig. 1. Geometry of the Problem

3. Formulation of the Problem

The constitutive equation for anisotropic elastic material that possess piezomagnetic material and magneto electric effects are as

$$\sigma_{z\chi}^{pm}=c_{44}^{pm}\frac{\partial w_1^{pm}}{\partial x_\chi}+h_{15}^{pm}\frac{\partial \psi_1^{pm}}{\partial x_\chi},\ B_\chi^{pm}=h_{15}^{pm}\frac{\partial w_1^{pm}}{\partial x_\chi}-\mu_{11}^{pm}\frac{\partial \psi_1^{pm}}{\partial x_\chi} \tag{1}$$

The Strain components and Mechanical displacement are relate as

$$S_{ij}^0=\frac{1}{2}\left[w_{i,j}+w_{j,i}\right]=\frac{1}{2}\frac{\partial w_i}{\partial x_j}+\frac{1}{2}\frac{\partial w_j}{\partial x_i} \tag{2}$$

where, $i,j=1,2,3$. The relation between magnetic field and magnetic potential is given as

$$H_i^0=-\frac{\partial \psi}{\partial x_i} \tag{3}$$

where, ψ is the magnetic potential. The equation of motion, electrostatics and magneto statics in the usual quasi-static approximation are

$$\begin{aligned}&\sigma_{ij,i}^0=\rho^0\left(\partial^2 w/\partial t^2\right)\\&B_{i,i}^0=0\end{aligned} \tag{4}$$

where, w is the displacement component and ρ^0 is the mass density. Now, if we introduced the source in the form of Dirac δ function, then the expression for the displacement function is same like the value of Green's function for all system at that point. Let us assume $\Psi_1(r,t)$ is the distribution of force density in the PM layer due to a source which is exists at the common interface of the layer and the half space. Then with the help of Eq. (1) to (4), the governing equation is

$$\begin{cases}c_{44}^{pm}\left(\nabla^2 w_1^{pm}\right)+h_{15}^{pm}\left(\nabla^2\psi_1^{pm}\right)=\rho^{pm}\left(\partial^2 w_1^{pm}/\partial t^2\right)+4\pi e^{-\theta x}\Psi_1(r,t),\\ h_{15}^{pm}\left(\nabla^2 w_1^{pm}\right)-\mu_{11}^{pm}\left(\nabla^2\psi_1^{pm}\right)=0\end{cases} \tag{5}$$

where, $\nabla^2=\left(\partial_{xx}+\partial_{yy}\right)$ is the two dimensional Laplacian operator. Now by taking the Fourier transformation of Eq. (5), we obtained the following ordinary differential equation

$$\begin{aligned}&\Omega_1\left(\nabla^2 w_1^{pm}\right)=\rho^{pm}\left(\partial^2 w_1^{pm}/\partial t^2\right)++4\pi\Psi_1(r,t)\\&\text{where,}\quad \Omega_1=c_{44}^{pm}+\frac{\left(h_{15}^{pm}\right)^2}{\mu_{11}^{pm}}\end{aligned} \tag{6}$$

assuming

$$w_1^{pm}(x,y,t)=w_1^{pm}(x,y)e^{i\varpi t},\ \psi_1^{pm}(x,y,t)=\psi_1^{pm}(x,y)e^{i\varpi t},\ \Psi_1(r,t)=\Psi_1(r)e^{i\varpi t} \tag{7}$$

Equation (6) leads to

$$\begin{aligned}&\nabla^2 w_1^{pm}+P_1^2 w_1^{pm}=\frac{4\pi e\ \Psi_1(r,)}{\Omega_1}\\&\text{where,}\ P_1^2=\frac{\rho_1^{pm}\varpi^2}{\Omega_1}\ \text{with}\ \varpi=kc\end{aligned} \tag{8}$$

where, $\Psi_1(r)=\delta(y)\delta(x-h)$. The new functions $W_1^{pm}(\xi,x)$ and $\varphi_1^{pm}(\xi,x)$ are the Fourier transformation of the functions $w_1^{pm}(\xi,x)$ and $\psi_1^{pm}(\xi,x)$, which defined as

$$\begin{cases}W_1^{pm}(\xi,x)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}w_1^{pm}(x,y)e^{i\xi y}\,dy\\ \varphi_1^{pm}(\Gamma,x)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty}\psi_1^{pm}(x,y)e^{i\xi y}\,dy\end{cases} \tag{9}$$

and there inverse Fourier transformation function are

$$\begin{cases} w_1^{pm}(x,y)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} W_1^{pm}(\xi,x)e^{-i\xi y}\,d\xi \\ \psi_1^{pm}(x,y)=\dfrac{1}{2\pi}\displaystyle\int_{-\infty}^{\infty} \varphi_1^{pm}(\xi,x)e^{-i\xi y}\,d\xi \end{cases} \tag{10}$$

From second equation of eq. (9) we can write

$$\begin{gathered}\left(d_{xx}W_1^{pm}-k_1^{\,2}W_1^{pm}\right)=4\pi\Psi_1(x)\\ \text{where, } k_1^{\,2}=\xi^2-P_1^2,\ P_1^2=\frac{\rho_1^{pm}\varpi^2}{\Omega_1}\text{ and } \Psi_1(x)=\frac{\delta_1(x-h_1)}{2\pi\Omega_1}\end{gathered} \tag{11}$$

Wave analysis the heterogeneous substrate under initial half space

For the pre-stressed heterogeneous half-space, the equation of motion is (Chattopadhyay and Kar, [28])

$$\partial_y\left[\left(\mu_1-\frac{\sigma_1}{2}\right)\partial_y w_2^{hg}\right]+\partial_x\left(\mu_1\partial_x w_2^{hg}\right)=\rho_2^{hg}\partial_{tt}w_2^{hg} \tag{12}$$

where σ_1, ρ_2^{hg} and μ_1 denotes the initial stress, density and rigidity of the material respectively.
For the homogeneous half-space medium, the substrate is heterogeneous. So that its rigidity is assumed to very as linear function of depth. Then the modulus of rigidity is $\mu_1=\mu+\theta_2(x-h)$, where θ_2 is the heterogeneity parameter and μ is the rigidity of the substrate. Then after using the substitution $w_2^{hg}(x,y,t)=w_2^{hg}(x,y)e^{i\varpi t}$, and rigidity condition the Eq. (12) reduces,

$$\left(\mu-\frac{\sigma_1}{2}\right)\partial_{yy}w_2^{hg}+\left(\mu\partial_{xx}w_2^{hg}\right)-\rho_2^{hg}\partial_{tt}w_2^{hg}=-\theta_2(x-h_1)\partial_{yy}w_2^{hg}-\theta_2(x-h_1)\partial_{xx}w_2^{hg}-\theta_2\partial_x w_2^{hg} \tag{13}$$

The new function $W_2^{hg}(\xi,x)$ is the Fourier transformation of the function $w_2^{hg}(\xi,x)$, which defined as

$$W_2^{hg}(\xi,x)=\frac{1}{2\pi}\int_{-\infty}^{\infty} w_2^{hg}(x,y)e^{i\xi y}\,dy \tag{14}$$

and there inverse Fourier transformation function are

$$w_2^{hg}(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty} W_2^{hg}(\xi,x)e^{-i\xi y}\,d\xi \tag{15}$$

Then from Eq. (13) and (15), we get

$$\left(d_{xx}W_2^{hg}-k_2^{\,2}W_1^{hg}\right)=4\pi\Psi_2(x)=\frac{-\theta_1(x-h_1)d_{xx}W_2^{hg}}{\mu}+\frac{\theta_1(x-h_1)\xi^2W_2^{hg}}{\mu}-\frac{\theta_1 d_x W_2^{hg}}{\mu} \tag{16a}$$

where,

$$k_2^2=\left(1-\frac{\sigma_1}{2\mu}\right)\xi^2-\frac{\rho_2^{hg}\varpi^2}{\mu},\quad 4\pi\psi_2(x)=\frac{-\theta_2(x-h_1)d_{xx}W_2^{hg}}{\mu}+\frac{\theta_2(x-h_1)\xi^2W_2^{hg}}{\mu}-\frac{\theta_2 d_x W_2^{hg}}{\mu} \tag{16b}$$

4. Boundary Condition

The aim of this section is to find the solution of given eq. (11) and (16) by using Green's Function, so it has to satisfy the following boundary condition, which are given below.
The Mechanical Traction free condition electrical boundary condition at the free surface at $x=0$ is

$$(a)\,\sigma_{zx}^{pm}(x)=0,\ \ (b)\,B_x^{pm}(\tilde{x})=0,\ \ (c)\,\psi_1^{pm}(x)=0, \tag{17}$$

The continuity condition at the common interface of the upper layer and substrate at $x=h$

$$(e)\sigma_{zx}^{pm}(x)=\sigma_{zx}^{hg}(x),\ (f)B_x^{pm}(x)=0,\\ (g)\psi_1^{pm}(x)=0,\qquad (h)W_1^{pm}(x)=W_2^{hg}(x), \tag{18a}$$

Next, the aim is deduce the given expression for the displacements in the piezomagnetic layer as well as in the heterogeneous half space, combined the (a), (b), (e) and (f).

$$\partial_x W_1^{pm}\ \text{at}\ x=0\ \ \text{and}\ \ c_{44}^{pm}\partial_x W_1^{pm}=\mu\partial_x W_2^{hg}\ \ \text{at}\ \ x=h. \tag{18b}$$

Let us assume $G_1(x/x_\circ)$ be the Green's function of the upper PM layer that satisfies the condition $d_x G_1(x/x_\circ)=0$ at $x=0$ and h, where $x_\circ$ is the any arbitrary point of upper layer. Then, the $G_1(x/x_\circ)$ will satisfy the following given equation

$$d_{xx}G_1(x/x_\circ)-k_1^2G_1(x/x_\circ)=\delta_1(x-x_\circ). \tag{19}$$

Now, from Eqs. (13) and (19), multiply by $G_1(x/x_\circ)$ and W_1^{pm} respectively and then subtract. After integration with respect to x between the limit $x=0$ and $h,$ we get the following equation

$$\int_{x=0}^{x=h}\left[G_1(x/x_\circ)d_{xx}W_1^{pm}-W_1^{pm}d_{xx}G_1(x/x_\circ)\right]dx=\int_{x=0}^{x=h}\left[4\pi\Psi_1(x)G_1(x/x_\circ)-\delta_1(x-x_\circ)W_1^{pm}\right]dx \tag{20}$$

Then with the help of condition $d_x G_1(x/x_\circ)=0$ at $x=0$ and h, we get

$$G_1(h_1/x_\circ)\left(\partial_x W_1^{pm}\right)_{x=h}=\frac{2}{\Omega_1}G_1(h_1/x_\circ)-W_1^{pm}(x_\circ) \tag{21}$$

Now, we known that $G_1(h/x)=G_1(x/h)$, so replacing $x_\circ$ by x, then

$$W_1^{pm}(x)=\frac{2}{\Omega_1}G_1(x/h_1)-G_1(x/h_1)\left(\partial_x W_1^{pm}\right)_{x=h} \tag{22}$$

Let us assume that $G_2(x/x_\circ)$ be the Green's function of lower heterogeneous half space, which satisfies the condition $d_x G_2(x/x_\circ)=0$ at $x=h$ and approaches 0 as $x\to\infty$. Where $x_\circ$ is the any point of half space. Then, $G_2(x/x_\circ)$ will satisfy the following given equation

$$d_{xx}G_2(x/x_\circ)-k_2^2G_2(x/x_\circ)=\delta(x-x_\circ). \tag{23}$$

From Eqs. (19) and (23), multiply by $G_2(x/x_\circ)$ and W_2^{hg} respectively and then subtract. Applying integration with respect to x from 0 to ∞ results in the following equation

$$\int_{x=h}^{x=\infty}\left[G_2(x/x_\circ)d_{xx}W_2^{hg}-W_2^{hg}d_{xx}G_2(x/x_\circ)\right]dx=\int_{x=h}^{x=\infty}\left[4\pi\Psi_2(x)G_2(x/x_\circ)-\delta(x-x_\circ)W_2^{hg}\right]dx \tag{24}$$

which is reduced to

$$-G_2(h/x_\circ)d_{xx}W_2^{hg}=\int_{x=h_1}^{x=\infty}4\pi\Psi_2(x)G_2(x/x_\circ)dx-W_2^{hg}(x_\circ) \tag{25a}$$

Replacing x and $x_\circ$ in the right hand side of Eq. (25), results in

$$W_2^{hg}(x)=\int_{x=h}^{x=\infty}4\pi\Psi_2(x_\circ)G_2(x/x_\circ)dx+G_2(x/h_1)\left(d_x W_2^{hg}\right)_{x=h} \tag{25b}$$

with, $G_2(h/x)=G_2(x/h)$ and $G_2(x_\circ/x)=G_2(x/x_\circ)$.

Now, from the boundary condition (h) and equality $c_{44}^{pm}\left(\partial_x W_1^{pm}\right)=\mu\partial_x W_2^{hg}$ at $x=h.$

$$\left(\partial_x W_1^{pm}\right)=\frac{2}{\Omega_1}\frac{G_1(h/h)}{G_1(h/h)+\frac{c_{44}^{pm}}{\mu}G_2(h/h)}-\frac{\int_{x_\circ=h}^{x_\circ=\infty}4\pi\Psi_2(x_\circ)G_2(h/x_\circ)dx_\circ}{G_1(h/h)+\frac{c_{44}^{pm}}{\mu}G_2(h/h)} \tag{26}$$

Putting the value of Eq. (26) in Eq. (22) and the using the value of $4\pi\Psi_2(x)$, We get

$$\begin{aligned}W_1^{pm}&=\frac{2c_{44}^{pm}}{\Omega_1}\frac{G_1(x/h)G_2(h_1/h_1)}{\mu G_1(h/h)+c_{44}^{pm}G_2(h/h)}-\frac{\theta_2 G_1(\tilde{x}/h_1)}{\mu G_1(h/h)+c_{44}^{pm}G_2(h/h)}*\\&\int_{x_\circ=h}^{x_\circ=\infty}\left\{\theta_2(x_\circ-h_1)d_{xx}W_2^{hg}-\theta_2(x_\circ-h_1)\xi^2W_2^{hg}+\theta_1 d_x W_2^{hg}\right\}G_2(h_1/x_\circ)dx_\circ\end{aligned} \tag{27}$$

By using Eq. (25) and the expression $c_{44}^{pm}\left(\partial_x W_1^{pm}\right)=\mu\partial_x W_2^{hg}$ and neglecting the higher power of $\Im$, the following expression is obtained

$$W_2^{hg}=\frac{2c_{44}^{pm}}{\Omega_1}\frac{G_2(x/h)G_1(h/h)}{\mu G_1(h/h)+c_{44}^{pm}G_2(h/h)}. \tag{28}$$

Using Eq. (28) in Eq. (27), we get the following equation

$$\begin{aligned}W_1^{pm}&=\frac{2c_{44}^{pm}}{\Omega_1}\frac{G_1(x/h)G_2(h_1/h_1)}{\mu G_1(h/h)+c_{44}^{pm}G_2(h/h)}-\frac{2c_{44}^{pm}}{\Omega_1}\frac{\theta_2 G_1(x_1/h)G_1(h/h)}{\left(\mu G_1(h/h)+c_{44}^{pm}G_2(h/h)\right)^2}*\\&\int_{x_\circ=h}^{x_\circ=\infty}\left\{(x_\circ-h)d_{xx}G_2(x_\circ/h)-(x_\circ-h)\xi^2G_2(x_\circ/h)+d_xG_2(x_\circ/h)\right\}G_2(h/x_\circ)dx_\circ\end{aligned} \tag{29}$$

Because the displacement functions are the function of G_i , then for obtaining the values of displacement it is necessary to find the values of Green's function. For obtaining the solution of Eq. (19), let us assume the Q_1 and Q_2 are two independent solution of Eq. (19), then

$$d_{xx}Q-k_1^2Q=0 \tag{30}$$

which is vanishing at $x=-\infty$ and ∞ respectively are $Q_1(x)=e^{k_1x}$ and $Q_2(x)=e^{-k_1x}$. The solution of Eq. (31) for an infinite medium is

$$\begin{cases}\dfrac{Q_1(x)Q_2(x_\circ)}{\omega} & \text{for } x<x_\circ\\[2ex] \dfrac{Q_1(x_\circ)Q_2(x)}{\omega} & \text{for } x>x_\circ\end{cases}\quad \text{where } \omega=Q_1(x)Q_2'(x)-Q_2(x)Q_1'(x)=-2k_1 \tag{31}$$

Hence the solution of Eq. (19) for infinite medium is $-e^{-k_1|x-x_\circ|}/2k_1$. Since $G_1(x/x_\circ)$ satisfies the condition $d_xG_1(x/x_\circ)=0$ at $x=0$ and h_1 . Then after using these two condition

$$\begin{cases}G_1(x/x_\circ)=\dfrac{1}{2k_1}\left(e^{-k_1|x-x_\circ|}\right)+\dfrac{1}{2k_1}\left[\dfrac{\left(e^{k_1x}\left\{e^{-k_1(h_1+x_\circ)}+e^{-k_1(h_1-x_\circ)}\right\}+e^{-h_1x}\left\{e^{k_1(h_1-x_\circ)}+e^{-k_1(h_1-x_\circ)}\right\}\right)}{e^{k_1h_1}-e^{-k_1h_1}}\right]\\ \text{and}\\ G_2(x/x_\circ)=\dfrac{1}{2k_2}\left[e^{-k_2|x-x_\circ|}+e^{-k_2(x+x_\circ-2h_1)}\right]\end{cases} \tag{32}$$

and substituting the values of Eq. (32) in Eq. (28) and (29), we get the following equation after solving it by integration and differentiation with variable separable method

$$W_1^{pm} = \frac{-2c_{44}^{pm}\left(e^{-k_1x}+e^{k_1x}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)\Omega_1} + \frac{2\theta_2 c_{44}^{pm}\left(\frac{1}{4}+\frac{\xi^2}{4(k_2)^2}\right)\left(e^{-k_1x}+e^{k_1x}\right)\left(e^{k_1h}+e^{-k_1h}\right)}{\Omega_1\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)} \tag{33a}$$

Simplified form of Eq. (33a) is

$$W_1^{\Delta p} = \frac{-2\left(e^{-k_1x}+e^{k_1x}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}\frac{c_{44}^{pm}}{\Omega_1}\left[1-\frac{\theta_2\left(\frac{1}{4}+\frac{\xi^2}{4(k_2)^2}\right)\left(e^{k_1h}+e^{-k_1h}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}\right] \tag{33b}$$

Further simplification with the some more refinements and separation of variables leads to

$$W_1^{pm} = \frac{-2\left(e^{-k_1x}+e^{k_1x}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}\frac{c_{44}^{pm}}{\Omega_1}\left[\frac{1}{1+\frac{\theta_2\left(\frac{1}{4}+\frac{\xi^2}{4(k_2)^2}\right)\left(e^{k_1h}+e^{-k_1h}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}}\right] \tag{34}$$

Equation (34) contained the parameter of heterogeneity, for the small value of θ_2, we neglect the higher power term. So by using inverse Fourier transformation, the displacement at arbitrary point for upper the piezomagnetic layer is

$$w_1^{pm} = \left(\frac{-2c_{44}^{pm}}{\Omega_1}\right)\int_{-\infty}^{\infty}\left(\frac{\left(e^{-k_1x}+e^{k_1x}\right)}{\left(e^{k_1h_1}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h_1}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)+\theta_2\left(\frac{1}{2}+\frac{\xi^2}{2(k_2)^2}\right)\mathrm{Cosh}\left(hk_1\right)}\right)e^{-i\xi x}\,d\xi \tag{35}$$

where the factor of time is omitted, because guiding layer is piezomagnetic, so now our purpose to find the magnetic potential for upper layer. Let us assume that the equation is in the following form

$$\Psi(x,y,t) = \varphi_1^{pm}(x,y,t) - \frac{e_{15}^{pm}}{\mu_{11}^{pm}}W_1^{pm}(x,y,\tilde{t}). \tag{36a}$$

then

$$\varphi_1^{pm}(\tilde{x},\tilde{y},\tilde{t}) = A_3e^{-\xi x}+A_4e^{\xi x}+\frac{h_{15}^{pm}}{\mu_{11}^{pm}}\frac{2\left(e^{-k_1x}+e^{k_1x}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}* \left[1-\frac{\theta_2\left(\frac{1}{4}+\frac{\xi^2}{4(k_2)^2}\right)\left(e^{k_1h}+e^{-k_1h}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}\right] \tag{36b}$$

For the further results, we use boundary condition (c) and (g), then we have

$$\varphi_1^{pm}(x,y,t) = \frac{-2h_{15}^{pm}c_{44}^{pm}}{\mu_{11}^{pm}\mathrm{Sinh}(\xi_1 h)\Omega_1}\left[1-\frac{\theta_2\left(\frac{1}{2}+\frac{\xi^2}{2(\nu_2)^2}\right)\left(e^{k_1h}+e^{-k_1h}\right)}{\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)}\right]* \left[2\left(\mathrm{e}^{\xi(\mathrm{x-h})}-\mathrm{e}^{-\xi(\mathrm{x-h})}\right)-\left(e^{(k_1h+\xi x)}-e^{-(k_1h+\xi x)}\right)-\left(e^{(-k_1h+\xi x)}-e^{-(-k_1h+\xi x)}\right)+\left(e^{k_1x}+e^{-k_1x}\right)\right] \tag{37}$$

So by using inverse Fourier transformation, the magnetic potential at arbitrary point for upper layer is

$$\psi_1^{pm} = \frac{-2e_{15}^{pm}c_{44}^{pm}}{\mu_{11}^{pm}\Omega_1}\int_{-\infty}^{\infty}\frac{\begin{bmatrix}2\left(e^{\xi(x-h)}-e^{-\xi(x-h)}\right)-\left(e^{(k_1h+\xi x)}-e^{-(k_1h+\xi x)}\right)\\ -\left(e^{(-k_1h+\xi x)}-e^{-(-k_1h+\xi x)}\right)+\left(e^{k_1x}+e^{-k_1x}\right)\end{bmatrix}e^{-i\xi x}}{\begin{bmatrix}\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)\\ -\theta_2\left(\frac{1}{2}+\frac{\xi^2}{2(k_2)^2}\right)\text{Cosh}(hk_1)\end{bmatrix}\text{Sinh}(\xi h)}d\xi \tag{38}$$

By solving the integral for displacement and electric potentials for poles, we get the following Eq. (39). Equation (39) is the dispersion relation of SH wave propagating in the composite structure under the influence of point source which is lying over a heterogeneous half space.

$$\left(e^{k_1h}\left\{\mu k_2+k_1c_{44}^{pm}\right\}-e^{-k_1h}\left\{-\mu k_2+k_1c_{44}^{pm}\right\}\right)+\theta_2\left(\frac{1}{2}+\frac{\xi^2}{2(k_2)^2}\right)\text{Cosh}(hk_1)=0 \tag{39}$$

Replacing k_1 by ik_1 and ξ by k, then achieved the required equation

$$\tan(k_1h)=\frac{\mu\sqrt{\left(1-\frac{\sigma_1}{2\mu}\right)-\frac{c^2}{\beta_2^2}}}{c_{44}^{pm}\sqrt{\frac{c^2}{\beta_1^2}-1}}-\frac{\theta_2}{2k\left(c_{44}^{pm}\right)\sqrt{\frac{c^2}{\beta_1^2}-1}}\left(\frac{1}{2}+\frac{k^2}{2\mu k^2\left(\left(1-\frac{\sigma_1}{2\mu}\right)-\frac{c^2}{\beta_2^2}\right)}\right)$$
$$ik_1=k_1^0=\sqrt{\left(1-\frac{\sigma_1}{2\mu}\right)-\frac{c^2}{\beta_2^2}},\ \beta_1^2=\frac{1}{\rho_1^{pm}}\left(c_{44}^{pm}+\frac{\left(h_{15}^{pm}\right)^2}{\mu_{11}^{pm}}\right),\ \beta_2^2=\frac{\mu}{\rho_2^{hg}} \tag{40}$$

Special Case (Validation of the problem)

When the considered geometry of the problem reduces to the homogeneous isotropic i.e., $\theta_2=0,\ h_{15}^{pm}=0,\ \sigma_1=0.$

$$\tan\left(k\sqrt{\frac{c^2}{\beta_1^2}-1}h\right)=\frac{\mu\sqrt{1-\frac{c^2}{\beta_2^2}}}{\mu_1\sqrt{\frac{c^2}{\beta_1^2}-1}},\ \beta_1^2=\frac{\mu_1}{\varsigma_1},\ \beta_2^2=\frac{\mu}{\varsigma_2}\ . \tag{41}$$

which is a required classical Love-wave equation.

5. Numerical Examples with Computational Results and Discussions

Following the dispersion relations derived in section 4, some numerical examples are considered to illustrate the nature of stress waves. The material coefficients for the PM material are given in table 1. The material properties for the PM layer at the surface are taken same as those of the Cobalt Iron Oxide.

5.1. Influence of PM and elastic coefficient on the phase velocity

Figure 2 represents the dimensionless phase velocity curves under the effect of PM coefficients against dimensionless wave number. We can observe that from the figure 2 the influence of $(h_{15})^{pm}$ on the dimensionless phase velocity profile is quite attractive. As we increase the value of PM coefficient $(h_{15})^{pm}$ in a fashion of arithmetic progression with common difference 1, the phase velocity decreases this is due to the point source considered in the interface between piezomagnetic layer and heterogeneous half-space. Also, as the phase velocity decreases the angular frequency of the considered stress wave decreases. In the considered geometry, for the piezomagnetic material we consider the numerical data for the antiferromagetic crystal (Cobalt Iron Oxide). From Fig. 3 it is depicted that as we increase the value of elastic constant in the manner of arithmetic progression with common difference of $1\ N/m^2$ which causes the decrement in the phase velocity of SH wave. The out coming from Fig. 2 and Fig. 3 is very significant in the field of mechanical and electrical engineering under the influence concentrated point source.

Table 1. Material coefficients for the PM layer (Ezzin *et al.* [9])

Materials	$c_{44}^{pm}\left(10^9\,N/m^2\right)$	$\rho_1^{pm}\left(10^3\,kg/m^3\right)$	$\mu_{11}^{pm}\left(10^{-6}\,Ns^2/C^2\right)$	$h_{15}^{pm}\left(N/Am\right)$
$CoFe_2O_4$	45.3	5.3	157	550

Table 2. Material coefficient for heterogeneous half space Chattopadhyay and Kar [28]

Medium	Isotropic heterogeneous half pace
$\mu/\left(N.m^{-2}\right)$	23.24×10^{10}
$\rho_2^{hg}/\left(kg{\cdot}m^{-3}\right)$	5.008×10^3

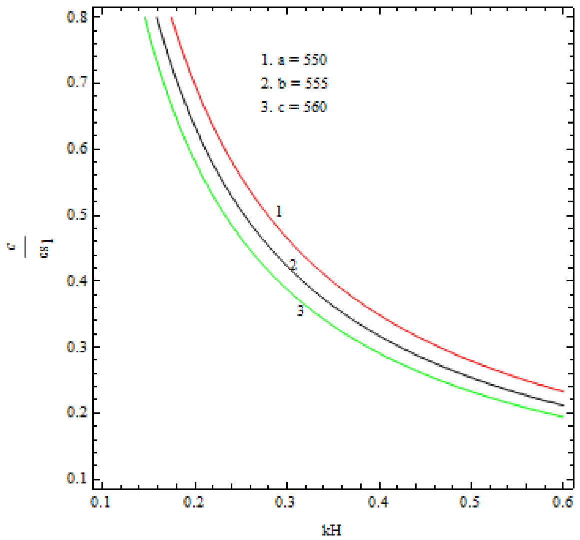

Fig. 2. Variation of dimensionless phase velocity ($cs_1 = \beta_1$) against dimensionless wave number for different values piezomagnetic coefficient $\left(a=b=c=h_{15}\right)^{pm}$.

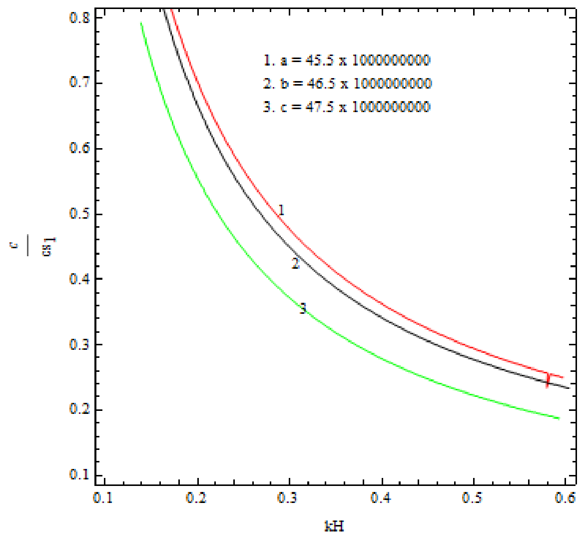

Fig. 3. Variation of dimensionless phase velocity ($cs_1 = \beta_1$) against dimensionless wave number for different values elastic constant.

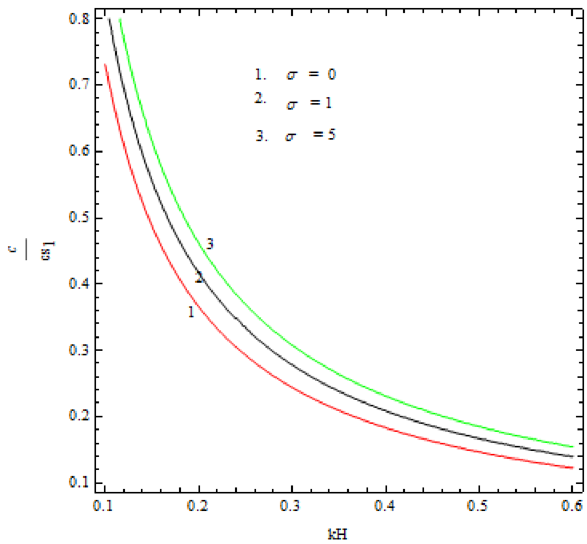

Fig. 4. Variation of dimensionless phase velocity ($cs_1 = \beta_1$) against dimensionless wave number for different values initial stress.

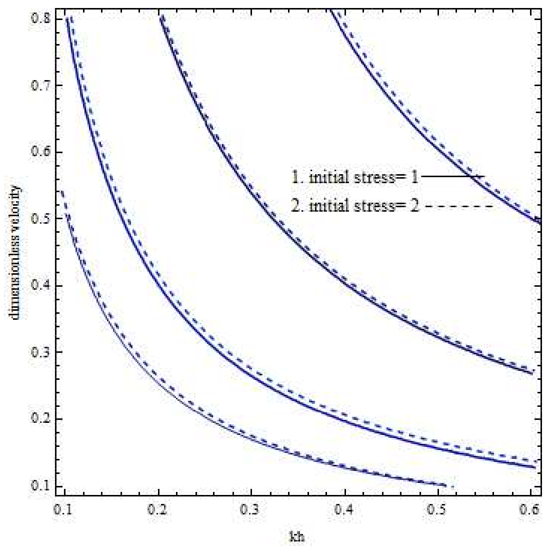

Fig. 5. Variation of dimensionless phase velocity against dimensionless wave number for different values piezomagnetic layer width.

5.2. Influence of initial stress on phase and group velocity

Figures 4 shows the prominent influence of initial stress parameter of piezomagnetic material layer on dimensional phase velocity and group velocity of SH stress waves respectively. It is depicted from Figure 4 that increment in initial stress parameter decreases the dimensionless phase velocity of SH waves. Also, from Fig. 4 we observe that in absence of initial stress the phase velocity of SH stress waves decreases as similar to the fashion when we increase the value of initial stress parameter. Also as the phase velocity decreases the angular frequency of the considered stress wave decreases. So, Fig. 5 governs that if we talk about the group velocity i.e., directly proportional as well as inversely proportional the initial stress at an instant point under the concentrated point source scheme. Moreover, the nature of phase velocity curves suggests for selection of a significantly piezomagnetic material in devices like transducers, rotating sensors and SAW device in order to prevent from the brittle fracture.

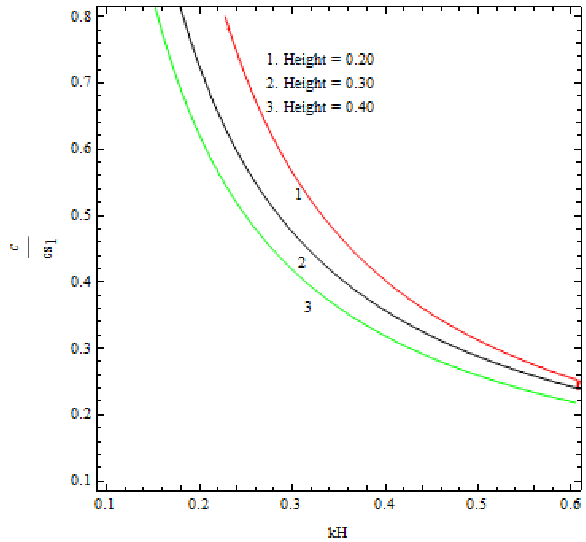

Fig. 6. Variation of dimensionless phase velocity against dimensionless wave number for different values piezomagnetic layer width.

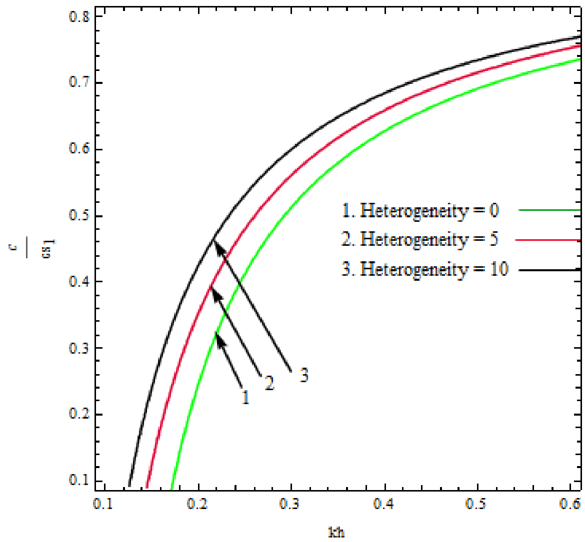

Fig. 7. Variation of dimensionless phase velocity against dimensionless wave number for different values heterogeneity.

5.3. Effect of piezomagnetic material layer width on phase velocity of stress wave

The substantial influence of the thickness of the piezomagnetic material layer has been shown in Figs. 6 with dimensionless phase velocity. Now, there is a remarkable finding as we increase the width of the piezomagnetic layer in arithmetic progression with common difference 5, the dimensionless phase velocity of SH wave increases under concentrated point source disturbance scheme and the same influence recorded on the angular frequency of the considered wave. The outcomes of Fig. 6 are very significant tell us that the width of the piezomagnetic layer is directly proportional to the dimensionless phase velocity in the considered geometry.

5.4. Influence of heterogeneity on phase velocity

Figure 7 reveals the influence of heterogeneity parameter (γ) associated with the heterogeneous half-space on the dimensionless phase velocity of SH stress wave against dimensionless wave number. It is concluded from Fig. 7 that the dimensionless phase velocity decreases monotonically with increase in the value of heterogeneity parameter (γ). We increase the value of heterogeneity parameter with difference of 5, which shows that the decrement in the phase velocity quite constantly.

6. Conclusions

The influence of heterogeneity parameter, piezomagnetic coefficient, initial stress, layers width and elastic constants on the dispersion of SH-type stress wave in a piezomagnetic layer overlying heterogeneous substrate under concentrated point source disturbance was investigated. Numerical computation was done to observe the influence of affecting parameters on dispersion curves were illustrated by means of graphs. The outcomes of the study are as follows:

- Phase velocity is directly proportional to the piezomagnetic material layers width.
- Concentrated point source disturbance is very significant in engineering which influences the stress wave phase velocity remarkably. According to the need of the device, point source disturbance affects the phase velocity by maintaining the materials width, their materials coefficients, elastic coefficients and stresses applied on it.
- Obtained dispersion relation is matched with the classical Love wave equation which gives the authenticity of the present problem.
- Influence of heterogeneity parameter, piezoelectric and elastic constants may be significant in high frequency range devices.
- As the group velocity under the influence of initial stress and layers width is discussed, it must be concluded that there is a point where the group velocity suddenly changes under concentrated point source disturbance.
- Nature of phase velocity curves and group velocity curves suggests for selection of a significantly pre-stressed piezomagnetic material in devices like transducers, rotating sensors and SAW device in order to prevent from the brittle fracture.

Author Contributions

In this paper, Abhinav Singhal performed the mathematical calculations and wrote the manuscript. Soniya Chaudhary completed the total graphical part and their interpretation.

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Conflict of Interest

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References

[1] B. Ahonsi, J.J. Harrigan, and M. Aleyaasin, On the propagation coefficient of longitudinal stress waves in viscoelastic bars, *International Journal of Impact Engineering*, 45, 2012, 39-51.

[2] H. Zhao, and G. Gary, A three-dimensional analytical solution of the longitudinal wave propagation in an infinite linear viscoelastic cylindrical bar, Application to experimental techniques, *Journal of the Mechanics and Physics of Solids*, 43, 1995, 1335-48.

[3] V.R. Feldgun, Y.S. Karinski, and D.Z. Yankelevsky, A two-phase model to simulate the 1-D shock wave propagation in porous metal foam, *International Journal of Impact Engineering*, 182, 2015, 113-29.

[4] Q.M. Li, and S.R. Reid, About one-dimensional shock propagation in a cellular material, *International Journal of Impact Engineering*, 32(11), 2006, 1898-906.

[5] Y.S. Karinski, V.R.Feldgun, and D.Z. Yankelevsky, Shock waves interaction with a single inclusion buried in a soil, *International Journal of Impact Engineering*, 45, 2012, 60-73.

[6] Singhal, S.A. Sahu, and S. Chaudhary, Liuoville-Green approximation: An analytical approach to study elastic waves vibrations in composite structure of Piezo material, *Composite Structures*, 184, 2018, 714-27.

[7] Singhal, S.A. Sahu, and S. Chaudhary, Approximation of surface wave frequency in Piezo-composite structure, *Composite Part B: Engineering*, 144, 2018, 19-28.

[8] S.A. Sahu, A. Singhal, and S. Chaudhary, Surface wave propagation in functionally graded piezoelectric material: An analytical approach, *Journal of Intelligent Material System and Structure*, 29(3), 2017, 423-37.

[9] H. Ezzin, M.B. Amor, and M.H.B Ghozlen, Love wave propagation in transversely isotropic piezoelectric layer on piezomagnetic half-space, *Ultrasonics*, 69, 2016, 83-89.

[10] H. Ezzin, M.B. Amor, and M.H.B Ghozlen, Lamb wave propagation in piezoelectric/piezomagnetic plates, *Ultrasonics*, 76, 2017, 63-9.

[11] L. Li, and P.J. Wei, Propagation of surface waves in a homogeneous layer of finite thickness over an initially stressed functionally graded magneto-electric-elastic half-space, *Journal of Theoretical and Applied Mechanics*, 45, 2015, 69-86.

[12] Singhal, S.A. Sahu, and S.Chaudhary. Approximation of surface wave frequency in Piezo-composite structure, *Composite Part B: Engineering*, 144, 2018, 19-28.

[13] S. Chaudhary, S.A. Sahu, N. Dewangan and A. Singhal, Stresses produced due to a moving load prestressed piezoelectric substrate, *Mechanics of Advanced Materials and Structures,* 26(12), 2019, 1028-1041.

[14] Othmani, F. Takali, A. Njeh, and M.H.B. Ghozlen, Numerical simulation of Lamb waves propagation in a functionally graded piezoelectric plate composed of GaAs-AlAs materials using Legendre polynomial approach, *Optik*, 142, 2017, 401-411.

[15] Othmani, F. Takali, and A. Njeh, Investigating and modeling of effect of piezoelectric material parameters on shear horizontal (SH) waves propagation in PZT-5H, PMN-0.33 PT and PMN-0.29 PT plates, *Optik*, 148, 2017, 63-75.

[16] I.A. Borodina, B.D. Zaitsev, I.E. Kuznetsova, and A.A. Teplykh, Acoustics wave in a structure carrying two piezoelectric plates separated by an air (vaccum) gap, *IEEE Trans Ultrasonic Ferroelectric Frequency and Control*, 60(12), 2013, 2677-2681.

[17] I.E. Kuznetsova, B.D. Zaitsev, and S.G. Joshi, Investigation of acoustic waves in thin plates of lithium niobate and lithium tantanate, *IEEE Trans Ultrasonic Ferroelectric Frequency and Control*, 48(1), 2001, 322-28.

[18] S. Chaudhary, S.A. Sahu and A. Singhal, On secular equation of SH waves propagating in pre-stressed and rotating Piezo-composite structure with imperfect interface, *Journal of Intelligent Material Systems and Structures*, 29(10), 2018, 2223-2235.

[19] J. Baroi, S.A. Sahu and M.K. Singh, Dispersion of polarized shear waves in viscous liquid over a porous piezoelectric substrate, *Journal of Intelligent Material Systems and Structures*, 29(9), 2018, 2040-2048.
[20] A.G. Arani, M. Jamali, M. Mosayyebi, and R. Kolahchi, Wave propagation in FG-CNT-reinforced piezoelectric composite micro plates using viscoelastic quasi-3D sinusoidal shear deformation theory, *Composites Part B: Engineering*, 95, 2016, 209-24.
[21] A.G. Arani, R. Kolahchi, M.S. Zarei, Visco-surface-nonlocal piezielectricity effects on nonlinear dynamic stability of graphene sheets integrated with sensors and actuators using refined zigzag theory, *Composite Structures*, 132, 2015, 506-26.
[22] R. Kolahchi, M. Hosseini, and M. Esmailpour, Differential cubature and quadrature-Bolotin methods for dynamic stability of embedded piezoelectric nanoplates based on visco-nonlocal-piezoelasticity theories, *Composite Structures*, 157, 2016, 174-86.
[23] V. Kumar, R. Jiwari, and G.R. Kumar, Numerical simulation of two dimensional quasilinear hyperbolic equations by polynomial differential quadrature method, *Engineering Computations*, 30(7), 2013, 892–909.
[24] Verma, and R. Jiwari, Cosine expansion based differential quadrature algorithm for numerical simulation of two dimensional hyperbolic equations with variable coefficients, *International Journal of Numerical Methods for Heat & Fluid Flow*, 25(7), 2015, 1574–89.
[25] M.L. Ghosh, On Love waves across the ocean, *Geophysics Journal International*, 7(3), 1963, 350–360.
[26] Chattopadhyay, and A.K. Singh, Effect of point source and heterogeneity on the propagation of magnetoelastic shear wave in a monoclinic medium, *International Journal of Engineering Science and Technology*, 3(2), 2011, 68–83.
[27] Chattopadhyay, S. Gupta, and P. Kumari, Effect of point source and heterogeneity on the propagation of SH-Waves in a viscoelastic layer over a viscoelastic half space, *Acta Geophysica*, 60(1), 2012, 119-39.
[28] Chattopadhyay, and B.K. Kar, Love waves due to a point source in an isotropic elastic medium under initial stress, *International Journal of Non-Linear Mechanics*, 16(3), 1981, 247–58.
[29] A.K. Singh, A. Das, A. Ray and A. Chattopadhyay, On point source influencing Love-type wave propagation in a functionally graded piezoelectric structure: A Green function approach, *Journal of Intelligent Material Systems and Structures*, 29(9), 2018, 1928-1940.
[30] S. Kundu, S. Manna, and S. Gupta, Propagation of SH-wave in an initially stressed orthotropic medium sandwiched by a homogeneous and an inhomogeneous semi-infinite media, *Mathematical Methods in the Applied Sciences*, 38(9), 2015, 1926–36.
[31] P. Joly, and R. Weder, New results for guided waves in heterogeneous elastic media, *Mathematical Methods in the Applied Sciences*, 15(6), 1992, 395-409.
[32] T. Børvik, A.G. Hanseen, M. Langseth, and L. Olovsson, Response of structure to planar blast loads- A finite element engineering approach, *Computer & Structures*, 87, 2009, 507-20.
[33] L. Olovsson, A.G. Hanseen, T. Børvik, and M. Langseth. A particle-based approach to close-range blast loading, *European Journal of Mechanics-A/Solids*, 29(1), 2010, 1-6.
[34] G. Gruben, T. Hopperstad, and T. Børvik, Simulation of ductile crack propagation in dual phase steel, *International Journal of Fracture*, 180(1), 2013, 1-22.
[35] H.M. Sedighi, Size-dependent dynamic pull-in instability of vibrating electrically actuated microbeams based on the strain gradient elasticity theory, *Acta Astronautica*, 95, 2014, 11-23.
[36] H.M. Sedighi, The influence of small scale on the pull-in behavior of nonlocal nanobridges considering surface effect, Casimir and Van der Waals attractions, *International Journal of Applied Mechanics*, 6(3), 2014, 1450030.
[37] F. Ebrahimi, A. Rastgo, An analytical study on the free vibration of smart circular thin FGM plate based on classical plate theory, *Thin-Walled Structures*, 46(12), 2008, 1402-08.
[38] F. Ebrahimi, M.R. Barati, and A. Dabbagh, A nonlocal strain gradient theory for wave propagation analysis in temperature-dependent inhomogeneous nanoplates, *International Journal of Engineering Sciences*, 107, 2016, 169-82.
[39] F. Ebrahimi, and E. Salari, Thermo-mechanical vibration analysis of nonlocal temperature-dependent FG nanobeams with various boundary conditions, *Composites Part B: Engineering*, 78, 2015, 272-90.
[40] M. Morvaridi, G. Carta, and M. Brun, Platonic crystal with low-frequency locally-resonant spiral structures: wave trapping, transmission amplification, shielding and edge waves, *Journal of the Mechanics and Physics of Solids*, 121, 2018, 496-516.
[41] T. Ampatzidis, R.K. Leach, C.J. Tuck, and D. Chronopoulos, Band gap behaviour of optimal one-dimensional composite structures with an additive manufactured stiffener, *Composites Part B: Engineering*, 153, 2018, 26-35.
[42] Beli, J.R.F. Arruda, and M. Ruzzene, Wave propagation in elastic metamaterial beams and plates with interconnected resonators, *International Journal of Solids and Structures*, 139, 2018, 105-120.
[43] T. Ampatzidis, and D. Chronopoulos, Mid-frequency band gap performance of sandwich composites with unconventional core geometries, *Composite Structures*, 222, 2019, 110914.
[44] M. R. Barati, On wave propagation in nanoporous materials, *International Journal of Engineering Sciences*, 116, 2017, 1-11.
[45] Ebrahimi, and M. R. Barati, Scale-dependent effects on wave propagation in magnetically affected single/double-layered compositionally graded nanosize beams, *Waves in Random and Complex Media*, 28(2), 2018, 326-342.
[46] Ebrahimi, and M. R. Barati, Propagation of waves in nonlocal porous multi-phase nanocrystalline nanobeams under longitudinal magnetic field, *Waves in Random and Complex Media*, 2018, 1-20. DOI:

10.1080/17455030.2018.1506596

[47] Ebrahimi, and M. R. Barati, Vibration analysis of smart piezoelectrically actuated nanobeams subjected to magneto-electrical field in thermal environment, *Journal of Vibration and Control*, 24(3), 2018, 549-564.

[48] F. Ebrahimi, and M. R. Barati, Buckling analysis of nonlocal third-order shear deformable functionally graded piezoelectric nanobeams embedded in elastic medium, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 39(3), 2017, 937-952.

[49] F. Ebrahimi, M.R. Barati, and A. Dabbagh, A nonlocal strain gradient theory for wave propagation analysis in temperature-dependent inhomogeneous nanoplates, *International Journal of Engineering Sciences*, 107, 2016, 169-182.

[50] F. Ebrahimi, and M. R. Barati, A nonlocal higher-order shear deformation beam theory for vibration analysis of size-dependent functionally graded nanobeams, *Arabian Journal of Science and Engineering*, 41(5), 2016, 1679-1690.

[51] F. Ebrahimi, and M. R. Barati, Wave propagation analysis of quasi-3D FG nanobeams in thermal environment based on nonlocal strain gradient theory, *Applied Physics A*, 122(9), 2016, 843.

[52] F. Ebrahimi, and M. R. Barati, Flexural wave propagation analysis of embedded S-FGM nanobeams under longitudinal magnetic field based on nonlocal strain gradient theory, *Arabian Journal of Science and Engineering*, 42(5), 2017, 1715-1726.

[53] F. Ebrahimi, M. R. Barati, and A. Dabbagh, Wave dispersion characteristics of axially loaded magneto-electro-elastic nanobeams, *Applied Physics A*, 122(11), 2016, 949.

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