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Magnetohydrodynamics Fluid Flow and Heat Transfer over a Permeable Shrinking Sheet with Joule dissipation: Analytical Approach

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Abstract. A laminar, two dimensional, steady boundary layer Newtonian conducting fluid flow passes over a permeable shrinking sheet in the presence of a uniform magnetic field is investigated. The governing equations have converted to ordinary nonlinear differential equations (ODE) by using appropriate similarity transformations. The main idea is to transform ODE with infinite boundary condition into other sets of variables in a way that infinite boundary condition becomes a finite boundary condition. The effects of physical parameters affecting the velocity and temperature are shown. The results show that with increasing the magnetic and suction parameters, the normal velocity component of fluid increases over the sheet whereas the tangential velocity component of fluid decreases. Moreover, when the suction parameter, the Prandtl and Eckert numbers increase, the rate of the heat transfer increases. However, when the magnetic parameter increases, the rate of heat transfer reduces. Finally, the solution shows that the results of the analytical method using a special technique have an excellent agreement with numerical solutions.

Keywords: Collocation method, Magnetohydrodynamics, Permeable shrinking sheet, Similarity transformation.

1. Introduction

Nowadays, the study of viscous fluid flow and heat transfer on a permeable stretching/shrinking sheet is a considerable problem due to the industrial applications [1-10]. Examples include in many technological processes such as the construction of the plastic film, hot rolling, and production of glass fiber. A fluid flow over a stretching sheet with constant velocity was studied and the similarity transformation was used for solving governing equations by Sakidis [11, 12]. Also, flow over a stretching sheet with suction or injection has been investigated by Gupta [13]. The sheet was assumed to move by linear velocity and has a constant temperature. Also, the effect of mass transformation was considered.

The thermal boundary layer flow over a nonlinearly stretching sheet with determined surface temperature [14] and then heat and mass transfer on an exponential continuous sheet was studied [15]. Vajravelu et al. [16] investigated the heat transfer of second-grade fluid over a stretching sheet with specific surface temperature. The numerical method was used to solve the governing equations. It was shown that by increasing the elastic parameter and source, the temperature increases and by increasing the Prandtl number, the temperature decreases. Vajraveluand Rollins [17] studied the hydromagnetic flow of second-grade fluid over a permeable stretching sheet. The numerical method has been used and the results were showed that by increasing the magnetic and the permeability parameters the dimensionless velocity

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decreases and by increasing the viscoelastic parameter, the dimensionless velocity decreases. Also, it was shown that by decreasing the viscoelastic parameter and increasing the magnetic and permeability parameters, the transverse velocity decreases.

The heat transfer of second grade conducting fluid over a permeable shrinking sheet in the presence of the uniform magnetic field has been analyzed by Cortell [18]. The fourth-order Runge-Kutta method was used for solving governing equations. It was shown that by increasing the viscoelastic parameter, the Prandtl number and the permeability parameter, the wall temperature gradient and as well, the heat transfer increases. However, by increasing the Eckert number and the magnetic parameter, the wall temperature gradient decreases. A stagnation flow toward a shrinking sheet has been studied and the numerical method was used for solving the governing equations [19]. It was shown that by increasing the shrinking rate, the boundary layer thickness increases and therefore the convective heat transfer reduces.

The heat and mass transfer of second grade viscoelastic fluid over a stretching sheet under the influence of the magnetic field was investigated [20]. It was shown that by declining the viscoelastic and magnetic parameters, the velocity increases. Also by decreasing the Prandtl number and increasing the magnetic parameter and source, the temperature increases. The slip effect and heat transfer on a third-grade non-Newtonian fluid passing over a stretching sheet were studied by Sahoo [21]. It was shown that the slip reduces the momentum boundary layer thickness and increases the thermal boundary layer thickness. Whereas third-grade fluid parameter has an opposite effect on the boundary layer thickness. Olajuwon [22] reported the mass and convective heat transfer of a hydromagnetic flow of second-grade fluid passing over a stretching sheet in the presence of the thermal radiation and heat diffusion. It was shown that by increasing of the second grade parameter of fluid and the thermal radiation parameter, the mass transfer rate decreases. Whereas, by increasing the magnetic parameter and the Schmitt number, the mass transfer rate increases. Also, with increasing the second-grade parameter of fluid and the thermal radiation and magnetic parameters, the heat transfer rate increases but with increasing the Schmitt number, the heat transfer rate decreases. Sahoo and Poncet [23] investigated the flow and heat transfer of a third-grade non-Newtonian fluid passing over a stretching sheet with the slip boundary condition. The results were shown that by increasing the third grade parameter, the momentum boundary layer thickness increases but the thermal boundary layer thickness reduces.

The flow and heat transfer of second-grade fluid over a stretching sheet considering of heat source/sink, thermal radiation and viscous dissipation was studied [24]. The Viscosity and conduction heat coefficient were considered as a function of temperature. It was shown that by increasing of the Eckert number, the heat source/sink and radiation parameters, the dimensionless temperature increases but by increasing the viscoelastic parameter and the Prandtl number, the dimensionless temperature decreases. Mustafa [25] investigated the flow and heat transfer of a second-grade viscoelastic fluid over a nonlinear stretching sheet with variable temperature by using the optimal homotopy asymptotic method (OHAM). The results were shown that by increasing of nonlinearity index (n), Prandtl number and second-grade fluid parameter, dimensionless temperature decreases. The Magnetohydrodynamics (MHD) flow of a second-grade fluid over a shrinking sheet was investigated by Hayat et al. [26]. The homotopy analysis method (HPM) was used for solving the governing equations and the effects of different parameters on the fluid flow were shown. Also, the magnetohydrodynamic flow over a shrinking sheet was investigated by Fang and Zhang [27] and the governing equations were solved by the closed-form analytical solution. Bhattacharyya [28] investigated the effects of the heat source/sink on heat transfer of MHD flow passing over a shrinking sheet with the mass suction on the wall. The finite difference method was used for solving the governing equations. The results were shown that by increasing the wall suction and the magnetic field, the velocity in the boundary layer increases. Also, by increasing the Hartman number, the Prandtl number and the heat sink parameter, the temperature decreases but by increasing the heat source, the temperature increases. The effect of heat and mass transfer on MHD nonlinear flow over a shrinking sheet with mass suction was studied by Muhaimin et al. [29]. The governing equations were solved by a numerical method and the effect of different parameters on the heat and mass transfer was investigated. Mukhopadhyay [30] investigated the MHD boundary layer flow and heat transfer over an exponential shrinking sheet embedded in a thermally stratified medium with suction. The governing equations were solved by the numerical shooting method. The results were shown that by increasing the magnetic parameter, the fluid velocity decreases and the heat transfer rate at the surface increases. The boundary layer flow and heat transfer of a nanofluid over a non-isothermal stretching sheet in the presence of the magnetic field and the thermal radiation in two different cases were studied by Ibrahim and Shanker [31]. The fourth-order Runge-Kutta was used and the velocity, temperature, concentration, local Nusselt number, and Sherwood number distribution were obtained.

Considering that, a lot of research has been done in the field of analytical method such as the collocation method (CM), the least square method (LSM), the Galerkin method (GM) and etc. (Pourmehran et al. [32], Rahimi-Gorji et al. [33], Ebrahimi et al. [34], Sahebi et al. [35], Hatami et al. [36], Ghasemi et al. [37] and Rahimi et al. [38]). As it was observed in the literature, using analytical methods based on trial functions such as CM or LSM, for problems with infinite boundary condition are rare. The motivation of this study is solving the governing equations by the present method is showing the efficiency of the method's base on trial function such as Collocation Method (CM). Due to the simplicity of this method's – i.e. trial functions- in comparison with other analytical methods as the optimal homotopy perturbation method (OHPM), this method is selected. However, the main weakness of the trial function methods is in solving the problems with infinity boundary conditions. In the present study, the collocation method is used to solve the problem with infinity boundary conditions with a special technique.

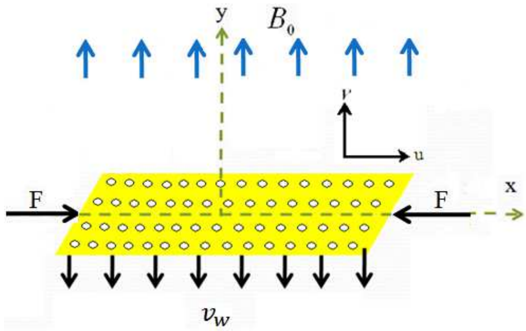


Fig. 1. Problem schematic

The main idea is using a special technique for solving the momentum and energy equations of Newtonian fluid flow passing over a permeable shrinking sheet in the presence of a uniform magnetic field by analytical Collocation Method (CM). Also, the numerical solution of the governing equations is presented to show the efficiency of the analytical method used. Finally, the results are depicted graphically and the effect of physical parameters on the flow and heat transfer were discussed.

2. Problem Statement

A laminar, two dimensional, steady boundary layer Newtonian conducting fluid flow passing over a permeable shrinking sheet in the presence of a uniform magnetic field is considered (Fig. 1). The sheet is in accordance with the y=0 surface and there is a fluid flow on $y>0$. The x and y-axes are considered along and normal to the sheet, respectively. Two equal forces in the opposite directions are applied in the x-direction axis which causes shrinking of the sheet with the linear velocity around its center point. u and v are tangential and normal velocity component, respectively. The magnetic Reynolds number is considered small, as a result, the induced magnetic field can be neglected. The fluid properties are considered constant.

The governing equations for explained flow by considering the boundary layer approximations are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2 u^2}{\rho c_p} \tag{3}$$

which Eqs. (1-3) are continuity, momentum and energy equations, respectively. T is temperature, ρ is the density, p is the pressure, c_p is the fluid specific heat in constant pressure, k is the thermal conductivity of the fluid, ν is the kinematic viscosity and σ is electrical conductivity coefficient of fluid.

The boundary conditions are defined as follows:

$$u=u_w=-cx,\quad v=-v_w,\quad T=T_w \quad at\ y=0,\quad c\succ 0 \tag{4}$$

$$u\to 0,\quad T\to T_\infty \quad as\ y\to\infty \tag{5}$$

where c is the sheet shrinking rate, T_∞ is free flow temperature, T_w is sheet temperature and v_w is distribution of wall mass suction. The transformation and dimensionless parameters are defined as:

$$u=cxf'(\eta),\quad v=-(c\upsilon)^{1/2}f(\eta)$$
$$\eta=\left(\frac{c}{\upsilon}\right)^{1/2}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where η is the similarity parameter. Using these parameters, the momentum and energy equations are converted to nonlinear ordinary differential equations as:

$$f'''+ff''-(f')^2-Mf'=0 \tag{7}$$

$$\theta'' + \Pr f\theta' + \Pr Ec\left(f''\right)^2 + M \Pr Ec\left(f'\right)^2 = 0 \quad (8)$$

where $M = \sigma B_0^2 / \rho c$ is the magnetic parameter, $\Pr = \mu c_p / k$ is the Prandtl number and $Ec = u_w{}^2 / c_p(T_w - T_\infty)$ is the Eckert number. Also, the boundary conditions are converting as follows:

$$f = S, \quad f' = -1, \quad \theta = 1 \quad at \quad \eta = 0 \quad (9)$$

$$f' \to 0, \quad \theta \to 0 \quad as \quad \eta \to \infty \quad (10)$$

where $S = v_w / (c\upsilon)^{1/2}$ is the mass suction parameter.

3. Mathematical Method

The basic principles of Collocation Method (CM) is introduced in this section. For understanding CM, the following function is considered [32, 33, 35]:

$$D(u(x)) - f(\mathrm{x}) = 0, \quad \mathrm{x} \in \Omega \quad (11)$$

which D is a general differential operator, u is an unknown function and $f(x)$ is a known analytical function and Ω is the function domain. To obtain approximate solutions $\tilde{u}$ can be made by some basic functions such as polynomials, which contains a considerable amount of n unknown coefficient, i.e. $c_1, c_2, \ldots, c_n$ as [35]:

$$u \cong \tilde{u} = \sum_{i=1}^{n} C_i \phi_i \quad (12)$$

The trial solution of $\tilde{u}$ is selected in a way that the boundary conditions are satisfied. R is an error function or residual. Since $\tilde{u}$ is not the exact solution of the equation, by substituting in the differential equation it can't satisfy the solution answer and create residual function [35]:

$$R(x) = D(\tilde{u}(x)) - f(\mathrm{x}) \neq 0 \quad (13)$$

R will be a function of the invariant and variable values and for this reason, the constant c_1 to c_n should be calculated in a way that the residual function remain close to zero in the range of the problem [35]:

$$\int_x R(x)\, W_i(x) = 0, \ i = 1,2,\ldots\ldots,n \quad (14)$$

Where W_i is a weighted function and the number of the weighted function W_i is exactly equal to the number of the unknown coefficients c_i in the $\tilde{u}$ function. A set of n algebraic equation with n unknown of c_i are obtained from Eq. 14. The main idea of CM is that the sum of all residuals are minimized, i.e. [35]:

$$R\left(x_i\right) = 0, \ x_i \in \Omega, \ i = 1,2,\ldots\ldots,n \quad (15)$$

To achieve the above objective, a weighted function can be considered as a Dirac delta function form in CM [35]:

$$W_i(x) = \delta(x - x_i) = \begin{cases} 1 & x = x_i \\ 0 & otherwise \end{cases} \quad (16)$$

4. CM Application with Special Technique in the Problem

The existence of an infinite boundary condition is one of the main problems in solving the boundary layer equations by analytical methods especially methods based on trial functions like Collocation Method, Least Square Method and, etc. Therefore, to solve these types of equations, an appropriate change of variable can be helpful.
Eqs. 7 and 8 that are the governing equations of flow and heat transfer have infinite boundary conditions (Eqs. 9 and 10). By defining new variable k which has a constant value, we define new variable z as:

$$z = \frac{\eta}{k} \quad (17)$$

Therefore, for large enough value of k, the $\eta \to \infty$ conditions in Eq. 10 can be substituted by following conditions [22,23]:

$$f'(k)=0, \quad \theta(k)=0 \tag{18}$$

Using transformation, the Eqs. 9 and 10 can be converted as follow:

$$g''' + k^2\left[gg'' - (g')^2 - Mg'\right] = 0 \tag{19}$$

$$h'' + k^2 \Pr gh' + \frac{\Pr Ec}{k}(g'')^2 + kM \Pr Ec (g')^2 = 0 \tag{20}$$

The prime represents derivatives with respect to $z \in [0,1]$ and h and g are defined as:

$$g(z) = \frac{f(\eta)}{k}, \quad h(z) = \frac{\theta(\eta)}{k} \tag{21}$$

Also, the boundary conditions (Eqs. 9 and 10) converted as follow:

$$g = \frac{S}{k}, \quad g' = -1, \quad h = \frac{1}{k} \quad at \quad z = 0 \tag{22}$$

$$g' \to 0, \quad h \to 0 \quad as \quad z \to 1 \tag{23}$$

An approximate solution will be obtained for Eqs. 19 and 20. The basic functions of $g(z)$ and $h(z)$ were chosen as a polynomials function in term of z which they are satisfying the boundary conditions (Eqs. 22 and 23). These functions are made so that the $g(z)$ has 8 unknown coefficients (c_0, c_1, …, c_8) and $h(z)$ has 12 unknown coefficients (b_0, b_1, …, b_8) as follows:

$$\begin{aligned} g(z) = {} & \frac{S}{k} - z + \frac{1}{2}z^2 + c_0\left(z^3 - \frac{3}{2}z^2\right) + c_1\left(z^4 - 2z^2\right) + c_2\left(z^5 - \frac{5}{2}z^2\right) + c_3\left(z^6 - 3z^2\right) + \\ & c_4\left(z^7 - \frac{7}{2}z^2\right) + c_5\left(z^8 - 4z^2\right) + c_6\left(z^9 - \frac{9}{2}z^2\right) + c_7\left(z^{10} - 5z^2\right) + c_8\left(z^{11} - \frac{11}{2}z^2\right) \end{aligned} \tag{24}$$

$$\begin{aligned} h(z) = {} & \frac{1}{k} - \frac{1}{k}z + b_0\left(z^2 - z\right) + b_1\left(z^3 - z\right) + b_2\left(z^4 - z\right) + b_3\left(z^5 - z\right) + b_4\left(z^6 - z\right) + \\ & b_5\left(z^7 - z\right) + b_6\left(z^8 - z\right) + b_7\left(z^9 - z\right) + b_8\left(z^{10} - z\right) + b_9\left(z^{11} - z\right) + b_{10}\left(z^{12} - z\right) \end{aligned} \tag{25}$$

By substituting Eqs. 24 and 25 in the differential Eqs. 19 and 20 and by choosing $k=3$, the residuals equations are obtained as follow:

$$\begin{aligned} R_1(c_0, c_1, c_2, \ldots, z) = {} & \text{-9+27M}c_0\text{z+36M}c_1\text{z+45Mz}c_2\text{+54Mz}c_3\text{+63Mz}c_4\text{+72Mz}c_5\text{+...} \\ & \text{-27M}c_0 z^2 \text{-36M}c_1 z^3 \text{-45M}c_2 z^4 \text{-54M}c_3 z^5 \text{-63M}c_4 z^6 \text{-72M}c_5 z^7 \text{-...} \\ & +90\, c_2\, z^{13}\, c_7 + 180\, c_2\, z^{14}\, c_8 - 720\, z^7\, c_7\, c_4 + \ldots - 18 c_0 z^3 - 9 c_1 z^4 = 0 \end{aligned} \tag{26}$$

$$\begin{aligned} R_2(c_0, c_1, \ldots, b_0, b_1, \ldots, z) = {} & \text{-162MPrEc}z^3 c_0 c_6 \text{-216MPrEc}z^4 c_1 c_6 \text{-270MPrEc}z^5 c_2 c_6 \text{+...} \\ & -450\Pr\, c_7\, z^{11}\, b_8 - 495\Pr\, c_7\, z^{12}\, b_9 - 540\Pr\, c_7\, z^{13}\, b_{10} + \ldots \\ & +336\text{PrEc}\, c_1 c_4\, z^7 - 88\text{PrEc}\, c_1 c_8\, z^2 - 80\text{PrEc}\, c_1 c_3\, z^4 = 0 \end{aligned} \tag{27}$$

Now, the problem is to find the approximate solutions in the range of $0 < z < 1$ and actually is setting the values of unknown coefficients so that residuals remain close to zero throughout this range:

$$R_1\left(\frac{1}{10}\right)=0, \quad R_1\left(\frac{2}{10}\right)=0, \quad R_1\left(\frac{3}{10}\right)=0, \quad \ldots, \quad R_1\left(\frac{9}{10}\right)=0 \tag{28}$$

$$R_2\left(\frac{1}{12}\right)=0, \quad R_2\left(\frac{2}{12}\right)=0, \quad R_2\left(\frac{3}{12}\right)=0, \quad \ldots, \quad R_2\left(\frac{11}{12}\right)=0 \tag{29}$$

By using Eqs. 28 and 29, twenty equations with twenty unknowns created. By solving them the constants for different values of *Pr, Ec, M* and *S* are obtained and substituting of them in $g(z)$ and $h(z)$ is obtained. Then by using transformation (Eq. 21), $f(\eta)$, $f'(\eta)$ and $\theta(\eta)$ can be obtained. For example, for $S = 3.5$, $M = 2.5$, $Ec = 1$, $\Pr = 0.3$, $g(z)$ and $h(z)$ and therefore and $\theta(\eta)$ is obtained as:

$$\begin{aligned} g(z) &= 1.166666667 - z + 5.789525720\, z^2 - 22.01883345\, z^3 + 60.82431415\, z^4 \\ & -126.6282370\, z^5 + 199.4690544\, z^6 - 233.6830827\, z^7 + 196.4788586\, z^8 \\ & -111.5331004 z^9 + 38.12190744 z^{10} - 5.905764530 z^{11} \end{aligned} \tag{30}$$

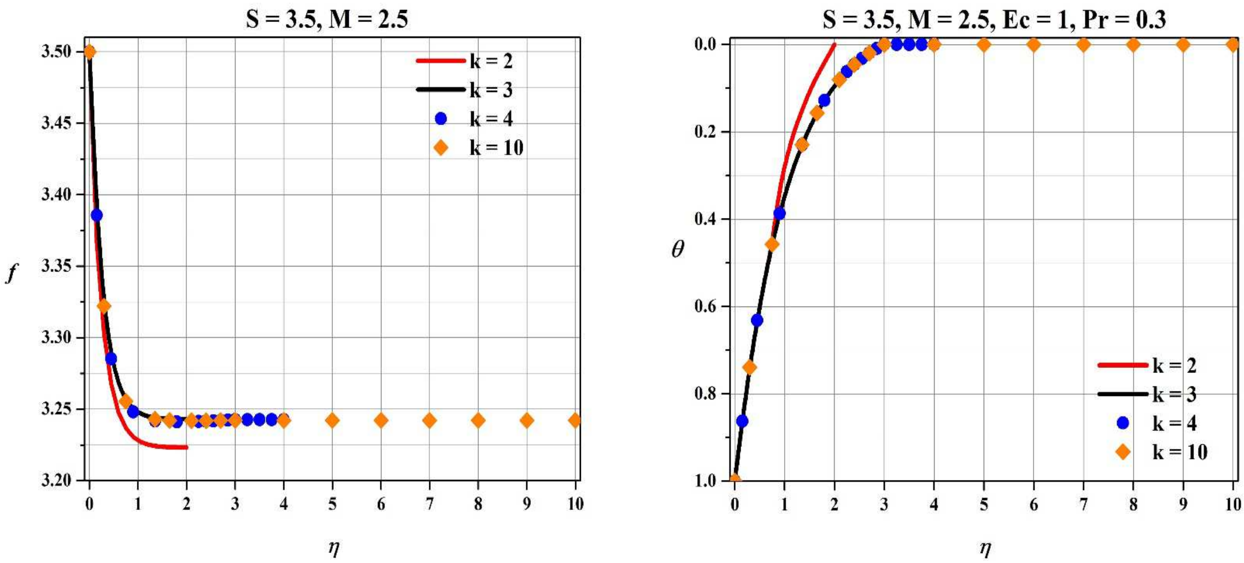
$$\begin{aligned} h(z) &= 1/3 - 0.43020369z - 5.690590675z^2 + 46.1955439z^3 - 210.6696257z^4 \\ & +666.0638052\, z^5 - 1517.844714\, z^6 + 2508.41321\, z^7 - 2976.568055\, z^8 \\ & +2496.300601\, z^9 - 1358.149942\, z^{10} + 444.5247553\, z^{11} - 65.47812461\, z^{12} \end{aligned} \tag{31}$$

$$\begin{aligned} f(\eta) &= 3.500000001 - \eta + 1.929841907\eta^2 - 2.446537050\eta^3 + 2.252752376\eta^4 - 1.563311568\eta^5 \\ & +0.8208603063\eta^6 - 0.3205529256\eta^7 + 0.08983944153\eta^8 - 0.01699940564\eta^9 \\ & +0.001936793550\eta^{10} - 0.0001000146409\eta^{11} \end{aligned} \tag{32}$$

$$\begin{aligned} \theta(\eta) &= 1 - 0.4302036900\eta + 1.896863558\eta^2 + 5.132838210\eta^3 - 7.802578728\eta^4 + 8.223009936\eta^5 \\ & -6.246274542\eta^6 + 3.440896044\eta^7 - 1.361027917\eta^8 + 0.3763604022\eta^9 \\ & -0.06900116550\eta^{10} + 0.007528065756\eta^{11} - 0.000369625929\eta^{12} \end{aligned} \tag{33}$$

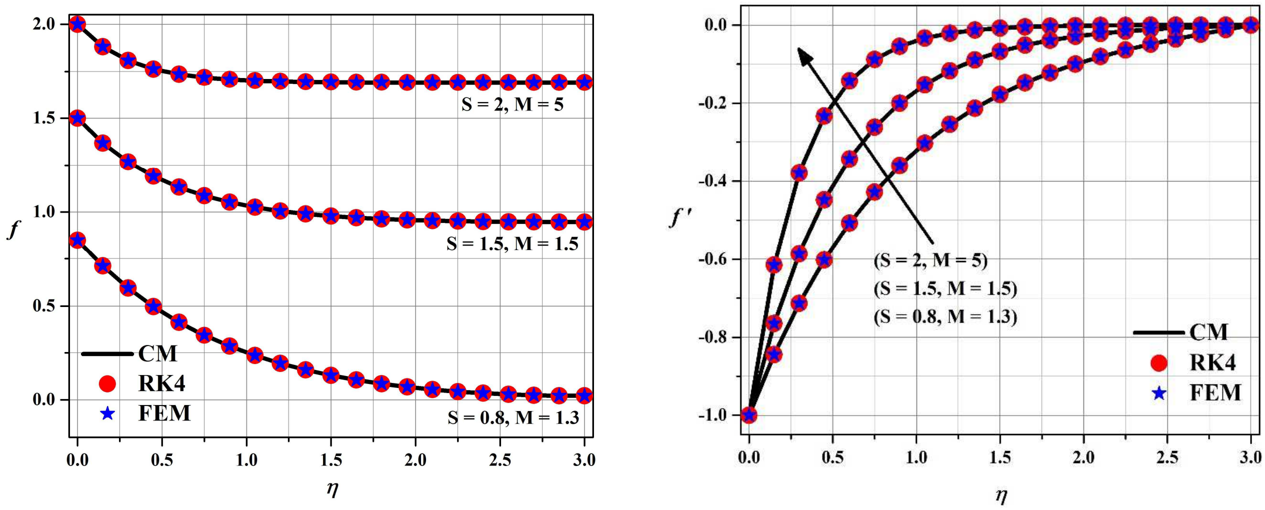
5. Results and Discussion

In this section, the effect of physical parameters on velocity distribution, temperature, and heat transfer rate are discussed. According to Eq. (17), we have to select the big enough value for *k*. For that, somewhat the independence from *k* has been provided. The solution is done for *k*=2, 3, 4 and 10, as it is shown in Fig. 2 (a and b). As can be seen, for *k* smaller than 3, the solution results will be different; however, for the value of *k* bigger than 3, there is no significant difference in the result. Therefore, *k*=3 is selected as the smallest value that can satisfy the infinity boundary condition.
Fig. 3 (a and b) and Fig. 4 (a and b) present the efficiency of the used analytical methods for different values of physical parameters in comparison with numerical methods 4th order Runge-Kutta (RK4) and Finite Element Method (FEM). The FEM is conducted by FlexPDE software which the detail explanations are available in [32-35].



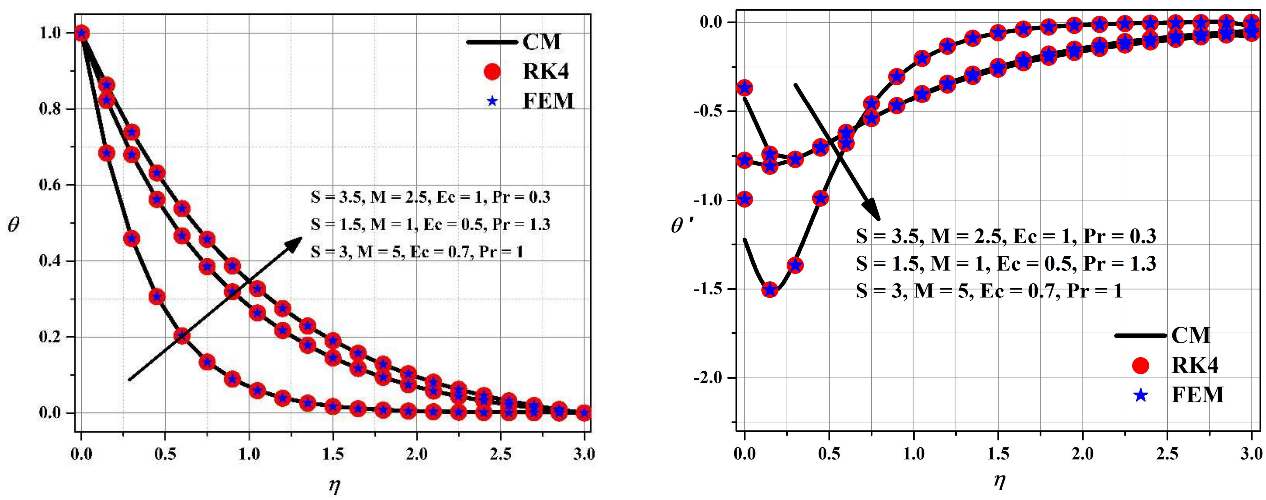
(a) (b)

Fig. 2. The value of *k*



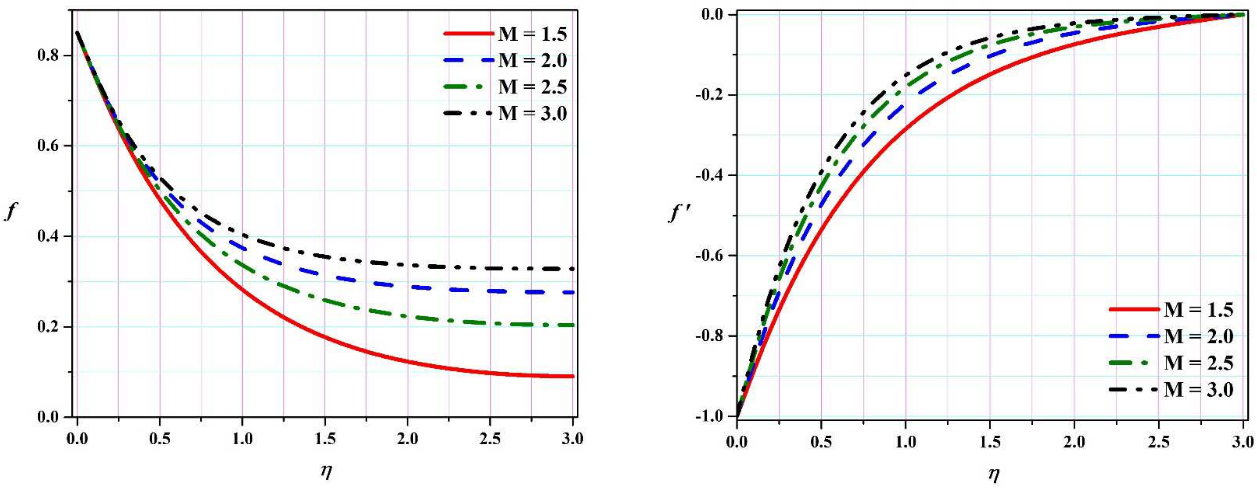
(a) (b)

Fig. 3. Comparison of CM solutions with numerical results for $f(\eta)$ and $f'(\eta)$



(a) (b)

Fig. 4. Comparison of CM solutions with numerical results for $\theta(\eta)$ and $\theta'(\eta)$



(a) (b)

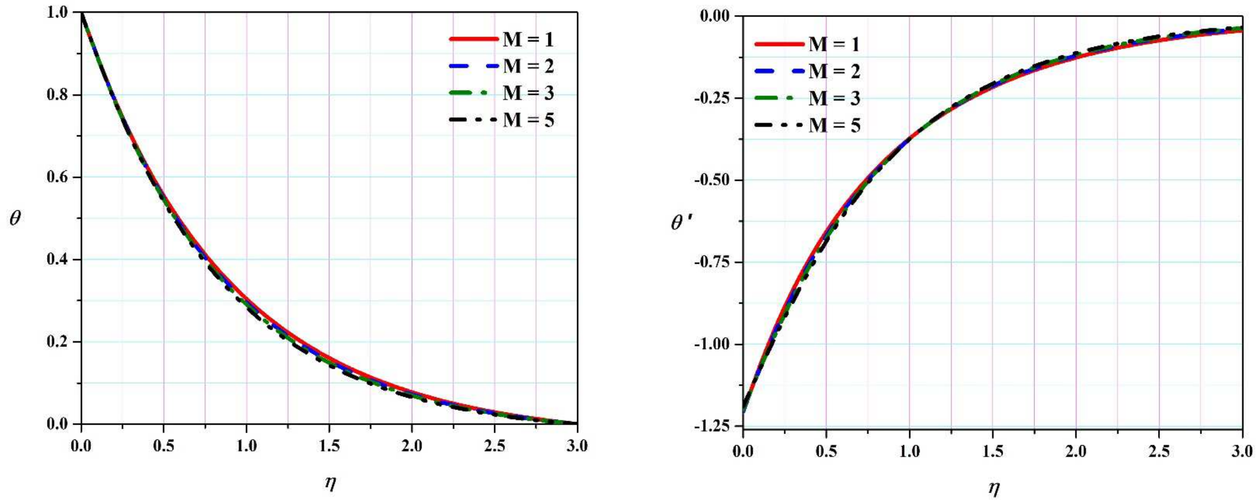
Fig. 5. Effect of the magnetic parameter on $f(\eta)$ and $f'(\eta)$ (S=0.85)

Table 1. CM and Numerical RK4 results for $f(\eta)$ (M=1.5, S=1.5)

η	Collocation	RK4	Error
0.00	1.50000000	1.50000000	$0.00000E+00$
0.15	1.36838060	1.36838102	$4.21421E-07$
0.30	1.26765033	1.26765131	$9.83431E-07$
0.45	1.19057031	1.19057170	$1.39035E-06$
0.60	1.13159870	1.13160037	$1.67353E-06$
0.75	1.08649329	1.08649516	$1.87832E-06$
0.90	1.05200689	1.05200889	$2.00789E-06$
1.05	1.02565421	1.02565627	$2.06887E-06$
1.20	1.00553331	1.00553538	$2.07819E-06$
1.35	0.99018895	0.99019098	$2.03856E-06$
1.50	0.97850798	0.97850992	$1.95487E-06$
1.65	0.96963924	0.96964105	$1.82965E-06$
1.80	0.96293229	0.96293394	$1.66316E-06$
1.95	0.95789051	0.95789195	$1.45649E-06$
2.10	0.95413516	0.95413634	$1.20372E-06$
2.25	0.95137799	0.95137887	$9.02002E-07$
2.40	0.94940019	0.94940071	$5.54759E-07$
2.55	0.94803638	0.94803650	$1.58044E-07$
2.70	0.94716235	0.94716202	$3.02691E-07$
2.85	0.94668579	0.94668490	$8.32750E-07$
3.00	0.94653878	0.94653752	$1.21989E-06$

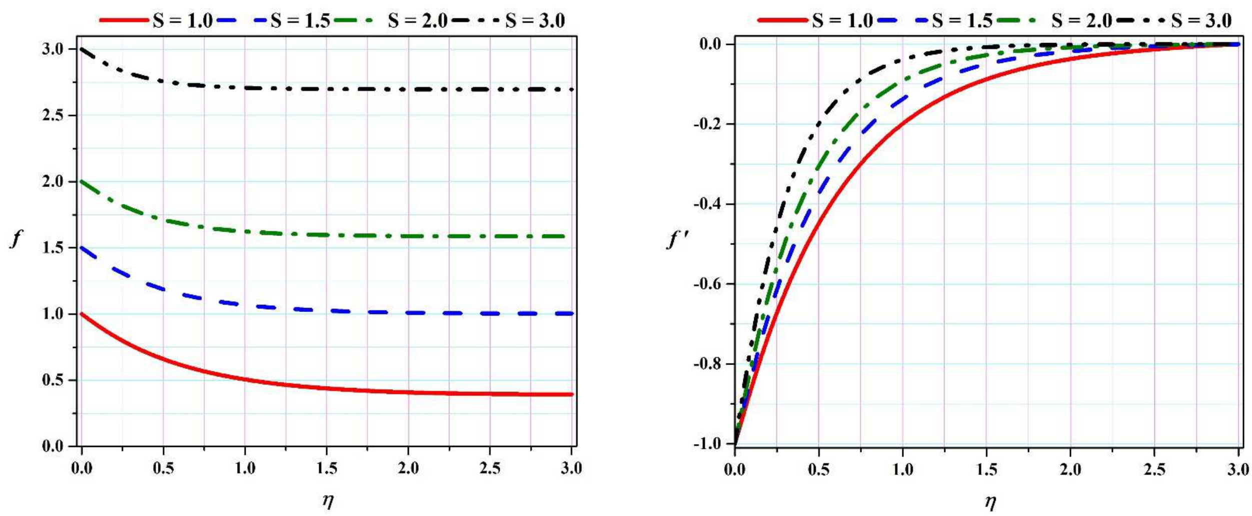
Table 2. CM and RK4 results for $\theta(\eta)$ (M=2.5, S=3.5, Ec=1, Pr=0.3)

η	Collocation	RK4	Error
0.00	1.00000000	1.00000000	$0.00000E-00$
0.15	0.90681702	0.90950383	$2.68681E-03$
0.30	0.79189650	0.79393546	$2.03896E-03$
0.45	0.68104523	0.68260739	$1.56216E-03$
0.60	0.58169348	0.58290727	$1.21379E-03$
0.75	0.49478674	0.49569412	$9.07380E-04$
0.90	0.41936874	0.42001268	$6.43946E-04$
1.05	0.35409847	0.35451876	$4.20290E-04$
1.20	0.29766687	0.29789379	$2.26918E-04$
1.35	0.24889226	0.24895166	$5.94055E-05$
1.50	0.20673831	0.20665385	$8.44585E-05$
1.65	0.17030802	0.17009919	$2.08828E-04$
1.80	0.13882583	0.13850797	$3.17863E-04$
1.95	0.11161893	0.11120619	$4.12746E-04$
2.10	0.08810639	0.08761137	$4.95022E-04$
2.25	0.06779042	0.06722016	$5.70253E-04$
2.40	0.05024179	0.04959757	$6.44220E-04$
2.55	0.03508452	0.03436767	$7.16849E-04$
2.70	0.02197657	0.02120560	$7.70975E-04$
2.85	0.01056010	0.00983058	$7.29516E-04$
3.00	−0.00000280	0.00000000	$2.80000E-06$



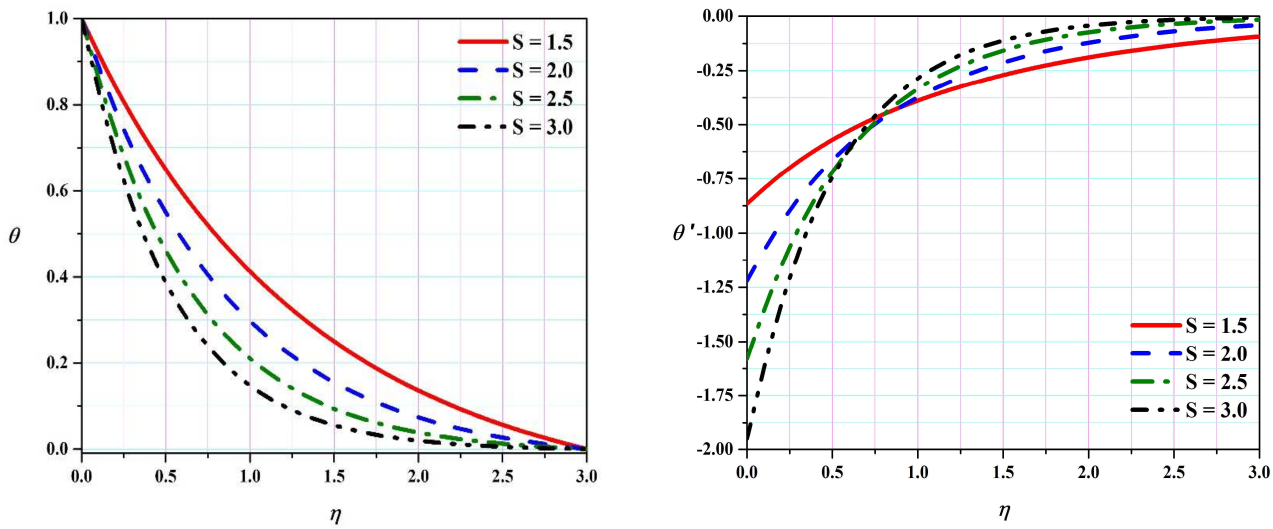
(a) (b)

Fig. 6. Effect of the magnetic parameter on $\theta(\eta)$ and $\theta'(\eta)$ (S=2, Ec=0.2, Pr=0.71)



(a) (b)

Fig. 7. Effect of the suction parameter on $f(\eta)$ and $f'(\eta)$ (M=2)



(a) (b)

Fig. 8. Effect of the suction parameter on $\theta(\eta)$ and $\theta'(\eta)$ (M=2, Ec=0.1, Pr=0.7)

Also for validity, Tables 1 and 2 show the values obtained from CM and numerical method. In these tables, the error is defined as:

$$Error = \left| f(\eta)_{Numerical} - f(\eta)_{Analytical} \right|$$
$$Error = \left| \theta(\eta)_{Numerical} - \theta(\eta)_{Analytical} \right| \tag{30}$$

According to Figs. 3 and 4 and Tables 1 and 2, it is concluded that the analytical method used for solving the governing equations is useful to predict the behavior of flow and heat transfer of Newtonian fluid over a permeable and shrinking sheet in the presence of the magnetic field.

In the following, the effect of physical parameters of the velocity distribution, temperature and the heat transfer rate are investigated. Figs. 5 (a and b) and Figs. 6 (a and b) represent the effect of the magnetic parameter on the fluid field. Figs. 5 (a and b) show that for a given position of η, by increasing of M, $f(\eta)$ increases and $|f'(\eta)|$ decreases. In the present study, according to Eq. 6, $f(\eta)$ and $f'(\eta)$ are related to velocity components v and u, respectively. Therefore, the augmentation of the magnetic parameter increases v but decreases u.

According to Fig. 6 (a and b), for a given position of η, by increasing of M, $\theta(\eta)$ decreases and also the temperature gradient [$-\theta'(0)$] reduces. Thus, the heat transfer rate on the wall decreases. Figs. 7 (a and b) and Figs. 8 (a and b) depict the effect of suction parameter on the fluid field. Figs. 7 (a and b) show that for a given position of η, by increasing of S, $f(\eta)$increases and $|f'(\eta)|$ decreases. Indeed, v increases and u reduces.

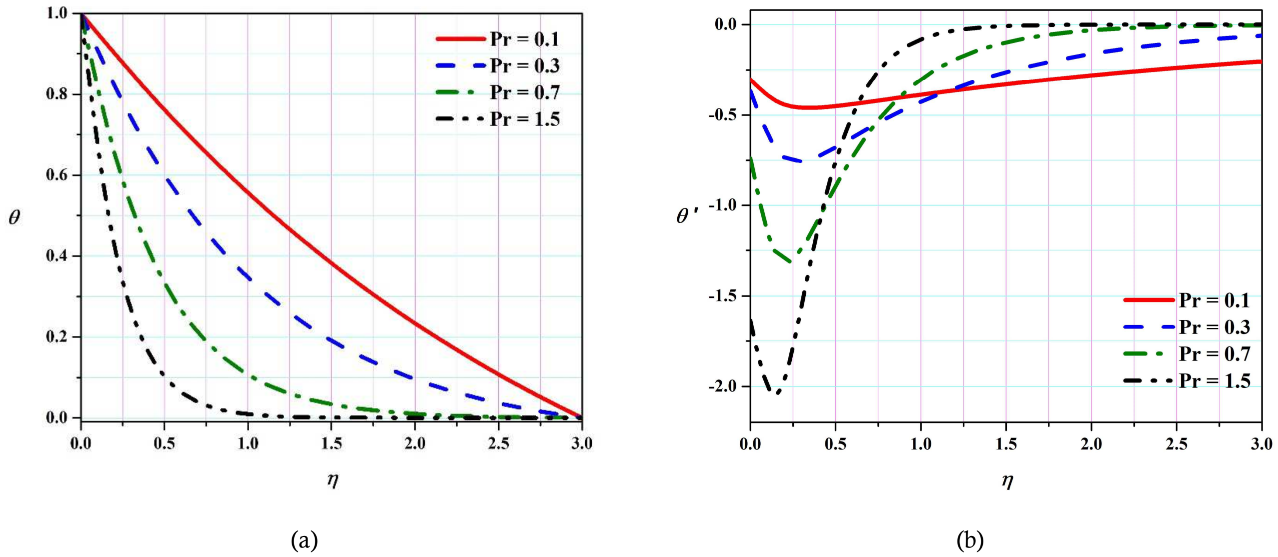


Fig. 9. Effect of Prandtl number on $\theta(\eta)$ and $\theta'(\eta)$ (M=2.5, S=3.5, Ec=1)

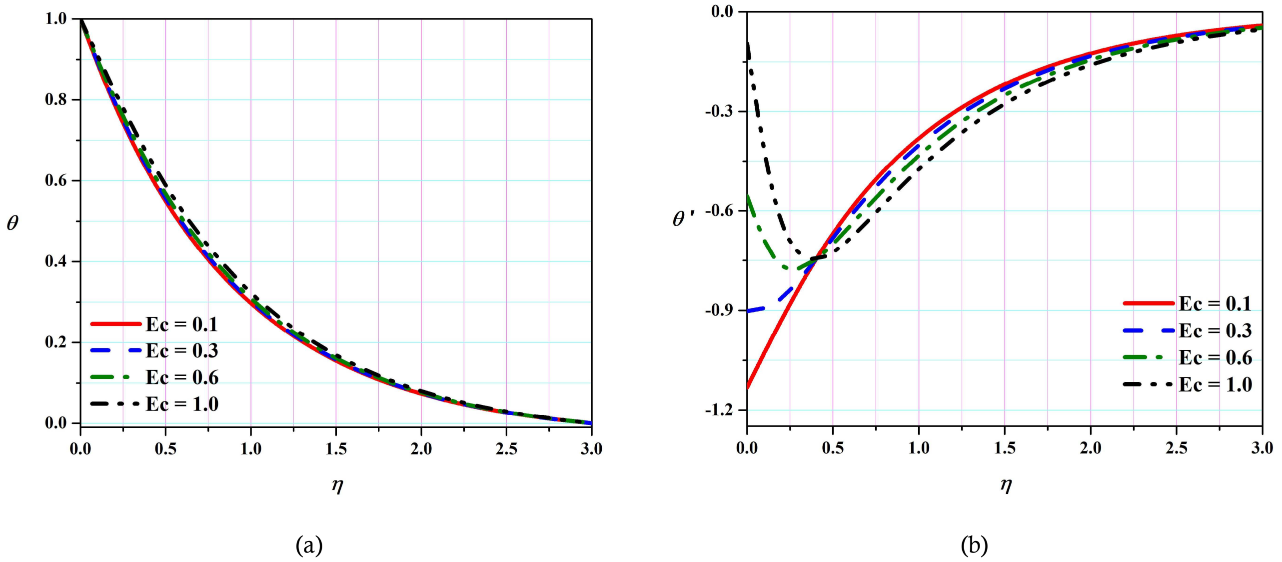


Fig. 10. Effect of Eckert number on $\theta(\eta)$ and $\theta'(\eta)$ (M=2, S=2, Pr=0.7)

According to Fig. 8 (a and b), for a given position of η by increasing of S, $\theta(\eta)$ decreases but the temperature gradient [-$\theta'(0)$] increases. Thus, the heat transfer rate on the wall increases. Figs. 9 (a and b) and Figs. 10 (a and b) indicate the effect of Prandtl number and Eckert number on the heat transfer of the fluid. Figs. 9 (a and b) show that for a given position of η, by increasing of Prandtl number, $\theta(\eta)$ reduces but the temperature gradient [-$\theta'(0)$] increases. Thus, the heat transfer rate on the wall increases, whereas the Eckert number has an opposite effect.

6. Conclusion

An analytical study on the flow and heat transfer of a Newtonian fluid over a permeable shrinking sheet under the influence of a uniform magnetic field considering joule dissipation was investigated. First, the nonlinear partial differential equations were converted to nonlinear ordinary differential equations by an appropriate similarity transformation. Then the equations were solved by an analytical Collocation Method (CM). For the validity, the results of the analytical method were compared with numerical results. It was shown that there is good agreement between them. The effect of the physical parameters such as the magnetic parameter, suction parameter, Prandtl number, and Eckert number were investigated and depicted. The following conclusions can be drawn out:

- By increasing the magnetic and suction parameters, $f(\eta)$ increases and $|f'(\eta)|$ decreases. Therefore, v is increased whereas u is decreased over the sheet.
- The augmentation of the suction parameter and Prandtl number can lead to the reduction of the temperature and the temperature gradient [-$\theta'(0)$] growth. Thus the heat transfer rate on the wall increases.
- By increasing the magnetic parameter, the temperature, the temperature gradient [-$\theta'(0)$] and thus the heat transfer rate on the wall decreases.
- Increasing the Eckert number has an opposite effect on temperature and heat transfer rate of the sheet, hence, they decline.

Author Contributions

M. Javanmard initiated the project, suggested the problem, developed the mathematical modeling and conducted the analytical solution; M.H. Taheri conducted the numerical solution; N. Askari examined the validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

B_0	External magnetic field (T)	u	Tangential velocity component (m/s)
c	Sheet shrinking rate ($1/s$)	u_w	Sheet velocity (m/s)
c_p	Fluid specific heat ($j/kg.k$)	v	Normal velocity component (m/s)
Ec	Eckert number	v_w	Distribution of wall mass suction (m/s)
k	Thermal conductivity ($w/m.k$)	x	Coordinate in the direction of the sheet (m)
M	Magnetic parameter	y	Coordinate in the direction normal to the sheet (m)
Pr	Prandtl number	ρ	Density (kg/m^3)
S	Mass suction parameter	σ	Electrical conductivity $(\Omega.m)^{-1}$
T	Temperature (k)	υ	Kinematic viscosity (m^2/s)
T_w	Sheet temperature (k)	η	Similarity parameter
T_∞	Free flow temperature (k)	θ	Dimensionless temperature

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