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Dufour and Soret Effects on Unsteady Heat and Mass Transfer for Powell-Eyring Fluid Flow over an Expanding Permeable Sheet

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Abstract: In the present analysis, the Dufour and Soret effects on unsteady heat-mass transfer of a viscous incompressible Powell-Eyring fluids flow past an expanding/shrinking permeable sheet are reported. The fluid boundary layer develops over the variable sheet with suction/injection to the non-uniform free stream velocity. Under the symmetry group of transformations, the governing equations along with three independent variables, are converted into a system of PDEs with two independent variables. Finally, by employing the order-reduction technique the PDEs are transformed into ODEs, which are then solved numerically. The results are presented graphically and analyzed. The main advantage of this technique is that without any prior knowledge, one can easily find the scaling transformations, expanding velocity, suction/injection velocity, and free-stream velocity. From computed numerical results many important findings are obtained. Most importantly, thermal and concentration overshoots are found for larger values of Dufour and Soret numbers, respectively. Also, thermal and concentration crossing over found for different values of Soret and Dufour numbers, respectively.

Keywords: Unsteady heat and mass transfer; Soret and Dufour effects; Powell-Eyring fluid; Expanding sheet; Symmetry analysis.

1. Introduction

According to independent or dependent on the time at every point, flow can be categorized into two types, viz. steady and unsteady. Most of the flows that we encounter in our daily life are unsteady. The boundary layer theory of such kinds of flows is important in science and technology.

Using Lie group theoretic approach, Ma and Hui [1] studied two-dimensional unsteady boundary layer flows of Newtonian fluids over a flat plate. Later, Dholey and Gupta [2] extended this problem for moving the porous plate. Lie symmetry technique is a systematic mathematical procedure for determining symmetries of differential systems. The advantage of this technique is that without any prior knowledge of physical characters of the underlying variables one can easily determine all the symmetries under which a given system remains invariant (see Layek [3] (Chapter 8), Cantwell [4], Bluman and Kumei [5] and references therein). Jalil et al. [6], Jalil and Asghar [7] pioneered the application of Lie group method for determining the symmetries and invariant solutions of Powell-Eyring steady fluid flows. Using Lie symmetry analysis, Akgül and Pakdemirli [8] studied unsteady power-law fluid flow over a porous surface, and YÜRÜSOY [9] investigated unsteady boundary layer flow for Carreau fluid. Layek and Sunita [10] discovered the nature of the multitude of scalings in decaying isotropic turbulence. Using Lie Symmetry technique

Layek et al. [11] analyzed boundary layer Stagnation-Point flow and heat transfer of power-law fluid over a non-linear shrinking/stretching sheet.

In nature, most of the fluids are non-Newtonian. Among the available non-Newtonian fluid models, the Powell-Eyring fluid model is popular for vast applications in chemical engineering systems. In spite of the complicity of this rheological model, it has certain advantages over other non-Newtonian fluid models. It was obtained from the kinetic theory of gases and it suitably reduces to Newtonian behavior for small and large shear rates. Patel and Timol [12] illustrated MHD Powell-Eyring fluid flow by use of the method of satisfaction of asymptotic boundary conditions. The Powell-Eyring fluid flow of an expanding sheet was discussed by Javed et al. [13]. Whereas, Powell-Eyring fluid flow in a stretching cylinder with variable viscosity was described by Malik et al. [14]. In a stretching sheet, Hamid et al. [15] discovered dual solutions and stability analysis of flow and heat transfer of Casson fluid. Related to the flow of nanofluids some very recent work can be found in [16]-[21].

Heat transfer is the energy transfer due to a temperature difference in a medium or between two or more mediums. Heat transfer can take place in one of the three processes, conduction, convection, and radiation, whereas the mass transfer is the displacement of the mass from one position to another. It can be found in many mechanisms, such as absorption, drying, evaporation, etc. Prasad et al. [22] exposed Powell-Eyring fluid flow characteristics with heat transfer in a non-isothermal stretching sheet. The stagnation-point flow of Powell-Eyring fluid in the presence of melting heat transfer was examined by Hayat et al. [23]. In an unsteady expanding sheet, Khader and Megahed [24], Hayat et al. [25] and Reddy et al. [26] pioneered the flow and heat transfer of Powell-Eyring fluid. Hayat et al. [27] compared the series solution for Powell-Eyring fluid flow with a numerical solution. The thermal radiation effect in Powell-Eyring nano-fluid MHD flow caused due to a stretching cylinder was scrutinized by Hayat et al. [28]. In an exponentially shrinking sheet, Ara et al. [29] investigated the thermal radiation effect on the flow of Eyring-Powell fluid in the boundary layer. Nadeem and Saleem [30] analyzed the mixed convective flow of Eyring-Powell fluid along a rotating cone. Panigrahi et al. [31]-[32] and Malik et al. [33] derived mixed convective boundary layer MHD flow of Powell-Eyring fluid due to an expanding sheet. In an exponentially expanding sheet, Megahed [34] discovered flow and heat transfer with heat flux and changeable thermal conductivity of a Powell-Eyring fluid. Krishna et al. [35] derived dual solutions for unsteady Powell-Eyring fluid flow on an inclined stretching sheet. While, in an inclined expanding cylinder and taking heat absorption/generation effect, Rehman et al. [36] analyzed the dual stratified mixed convective flow of Eyring-Powell fluid. Zaib et al. [37] derived dual solutions of unsteady flow of a copper-water nano-fluid past an exponentially shrinking sheet with non-uniform suction. Ye [38] pioneered the melting process in a rectangular thermal storage cavity heated from vertical walls. Using a finite value method, the non-uniformity of electrode evaluation is conducted computationally based on the heat conduction equation by Ye [39]. By design method and modeling verification, Ye [40] derived uniform airflow distribution in duct ventilation.

For the simultaneous occurrence of heat and mass transfer in between the fluxes, the driving potential is very complex, as the energy flux can be created not only by temperature gradient but also by composition gradient. Energy flux caused by the concentration gradient is called Dufour or diffusion-thermo effect. The temperature gradient can also produce mass fluxes, and this is the Soret or thermo-diffusion effect. Soret and Dufour effects were neglected on the basis that they are in a smaller order of magnitude compared to the effects described by Fourier's and Fick's laws. There exists however exceptions. Eckert and Drake [41] prescribed lots of cases when this effect can't be neglected. The Soret effect is vital for isotope separation in a mixture of gases with very light molecular weight (H_2, He). The Dufour effect is considerable for medium molecular weight (N_2 air, etc.). Khan and Sultan [42] analyzed Soret and Dufour effects for the double-diffusive convective flow of non-Newtonian Eyring-Powell fluid for a cone in a porous medium. Hayat et al. [43] established Soret and Dufour effects on three-dimensional flow over exponentially expanding surface in a porous medium with chemical reaction and heat sink/source.

In the present analysis, we investigate unsteady heat and mass transfer for non-Newtonian Powell-Eyring fluid flow over an expanding/shrinking sheet with suction/injection taking into consideration both the Soret and Dufour effects with thermal radiation. By using the Lie symmetry technique, the governing partial differential equations are transformed into ordinary differential equations. Then, these equations are solved numerically using shooting method with the help of Mathematica software and the physical characteristics for the flow, heat and mass transfer are discussed. It is noted that thermal and concentration overshoots are found for larger values of Dufour and Soret numbers, respectively. Also, thermal and concentration crossing over found for different values of Soret and Dufour numbers, respectively.

2. Formulation of the Problem

Consider the heat and mass transfer of two-dimensional unsteady boundary layer flow of a viscous incompressible non-Newtonian Powell-Eyring fluid along with an expanding/shrinking sheet with non-uniform suction, Dufour-Soret effects, and thermal radiation. The Geometrical and Physical sketch of the flow problem is given in Fig. 1.

The shear-stress tensor for the Powell-Eyring fluid is given by [44]

$$\tau_{xy} = \mu \frac{\partial \bar{u}}{\partial y} + \frac{1}{\beta_1} \sinh^{-1} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial y} \right) \quad (1)$$

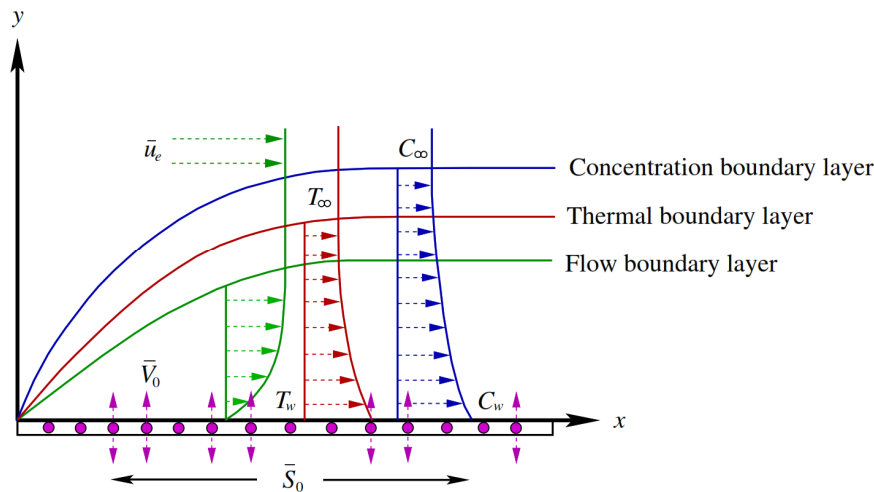


Fig.1. Physical sketch of the flow problem.

where μ is the coefficient of viscosity, β_1 and C_1 are material fluid parameters. The second order approximation of $\sinh^{-1}(\partial \bar{u} / C_1 \partial \bar{y})$ function is given by [34]

$$\sinh^{-1}\left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}}\right) \cong \frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{1}{6} \left(\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}}\right)^3, \left|\frac{1}{C_1} \frac{\partial \bar{u}}{\partial \bar{y}}\right| < 1. \quad (2)$$

Under the usual boundary layer approximations, the governing equations of the mass, momentum, energy and concentration for the unsteady flow can be written as [25], [26], [34], [35], [43]

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \quad (3)$$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial \bar{u}_e}{\partial \bar{t}} + \bar{u}_e \frac{\partial \bar{u}_e}{\partial \bar{t}} + \left(\nu + \frac{1}{\rho \beta_1 C_1}\right) \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{1}{2 \rho \beta_1 C_1^3} \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}, \quad (4)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho c_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho c_p} \frac{\partial \bar{q}_r}{\partial \bar{y}} + \frac{D_m k_t}{c_s c_p} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2}, \quad (5)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D_m \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + \frac{D_m k_t}{T_m} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2}, \quad (6)$$

with the boundary conditions

$$\bar{u}(\bar{x}, \bar{y}, \bar{t}) = \bar{S}_0(\bar{x}, \bar{t}), \bar{v}(\bar{x}, \bar{y}, \bar{t}) = \bar{V}_0(\bar{x}, \bar{t}), \bar{T} = T_w(\bar{x}, \bar{t}), \bar{C} = C_w(\bar{x}, \bar{t}) \quad \text{at } \bar{y} = 0, \quad (7)$$

$$\bar{u}(\bar{x}, \bar{y}, \bar{t}) \rightarrow \bar{u}_e(\bar{x}, \bar{t}), \bar{T} \rightarrow T_\infty, \bar{C} \rightarrow C_\infty \quad \text{at } \bar{y} \rightarrow \infty, \quad (8)$$

where $\bar{x}$ -direction is along with the plate, $\bar{y}$ perpendicular to the plate, $\bar{t}$ is the time, $\bar{u}$ and $\bar{v}$ are, respectively, the velocity components in the $\bar{x}$ and $\bar{y}$ directions, $\nu (= \mu/\rho)$ is the kinematic viscosity, $\bar{T}$ is the temperature, $\bar{C}$ is the concentration, κ is the thermal conductivity of the fluid, ρ is the constant fluid density, c_p the specific heat at constant pressure, $\bar{q}_r$ is the radiation heat-flux, D_m is the coefficient of mass diffusivity, k_t is the thermal diffusion ratio, c_s is the concentration susceptibility, T_m is the mean fluid pressure, $\bar{S}_0(\bar{x}, \bar{t})$ is the moving surface velocity, $\bar{V}_0(\bar{x}, \bar{t})$ is the suction or injection velocity of the surface, $T_w(\bar{x}, \bar{t})$ is the surface temperature, $C_w(\bar{x}, \bar{t})$ is the surface concentration, $\bar{u}_e(\bar{x}, \bar{t})$ is the free stream velocity outside the boundary layer, and T_∞ and C_∞ are uniform ambient temperature and concentration of the flow respectively.

Using Rosseland approximation for thermal radiation [45], one obtains $\bar{q}_r = -(4\sigma^*/3k_t)(\partial \bar{T}^4/\partial \bar{y})$, where σ^* is the Stefan-Boltzman constant, k_t is the absorption coefficient. We consider the temperature variation within the flow field in such way that $\bar{T}^4$ can be expanded in a Taylor's series. Expanding $\bar{T}^4$ about T_∞ and neglecting higher order

terms we get, $\bar{T}^4 \cong 4T_\infty^3 \bar{T} - 3T_\infty^4$.

In order to make Eq. (3), (8) non-dimensional, we introduce the following dimensionless variables

$$\left. \begin{aligned} x &= \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L} \sqrt{Re}, \quad u = \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{U_0} \sqrt{Re}, \quad T = \frac{\bar{T} - T_\infty}{T_w - T_\infty}, \\ C &= \frac{\bar{C} - C_\infty}{C_w - C_\infty}, \quad t = \frac{U_0}{L} \bar{t}, \quad u_e = \frac{\bar{u}_e}{U_0}, \quad S_0 = \frac{\bar{S}_0}{U_0}, \quad V_0 = \frac{\bar{V}_0}{U_0} \sqrt{Re}, \end{aligned} \right\} \quad (9)$$

where L , U_0 and Re are, respectively, the length of the horizontal surface, average free stream velocity and the Reynolds number. Then, the flow equations are transformed into

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (10)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + (1 + \epsilon) \frac{\partial^2 u}{\partial y^2} - \epsilon \delta \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2}, \quad (11)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Pr} \frac{\partial^2 T}{\partial y^2} + \frac{16}{3R} \frac{\partial^2 T}{\partial y^2} + Du \frac{\partial^2 C}{\partial y^2}, \quad (12)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 T}{\partial y^2}, \quad (13)$$

with

$$u(x, y, t) = S_0(x, t), \quad v(x, y, t) = V_0(x, t), \quad T = 1, \quad C = 1, \quad \text{at } y = 0 \quad (14)$$

$$u(x, y, t) \rightarrow u_e(x, t), \quad T \rightarrow 0, \quad C \rightarrow 0 \quad \text{at } y \rightarrow \infty \quad (15)$$

In Equations. (10)-(13), the dimensionless parameters are $Pr = \rho c_p \nu / \kappa$, the Prandtl number; $R = k_1 \rho C_p \nu / \sigma^* T_\infty^3$, the radiation parameter; $Du = D_m k_t (C_w - C_\infty) / c_s c_p \nu (T_w - T_\infty)$, the Dufour number; $Sc = \nu / D_m$, the Schmidt number; $Sr = D_m k_t (T_w - T_\infty) / T_m \nu (C_w - C_\infty)$, the Soret number; $\epsilon = 1 / \rho \nu \beta_1 C_1$ and $\beta = U_0^3 / 2 \nu L C_1^2$, the Powell-Eyring fluid parameters.

The governing equations are partial and very difficult to solve. So, we will transform them to self-similar ordinary differential equations. For that, here we adopt Lie symmetry group theory [4] to the equations to obtain the symmetry transformations of the variables under which the equations remain form invariant. Note that the equations may remain form invariant under many transformations, but we will consider only those transformations under which the boundary conditions are also form invariant. Since the equations contain three independent variables, we have adopted the method twice to obtain the desired similar ordinary differential equations with boundary conditions.

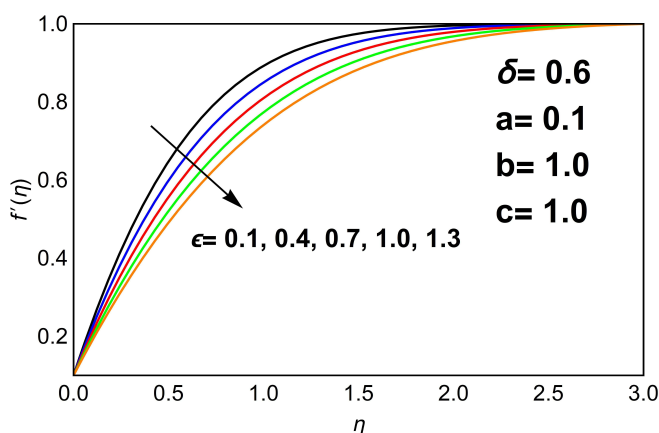


Fig. 2. Velocity profiles for different values Powell-Eyring fluid parameter ϵ .

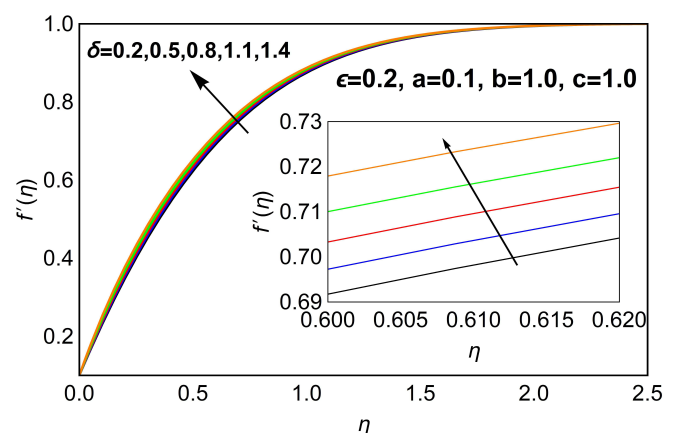


Fig. 3. Velocity profiles for different values Powell-Eyring fluid parameter δ .

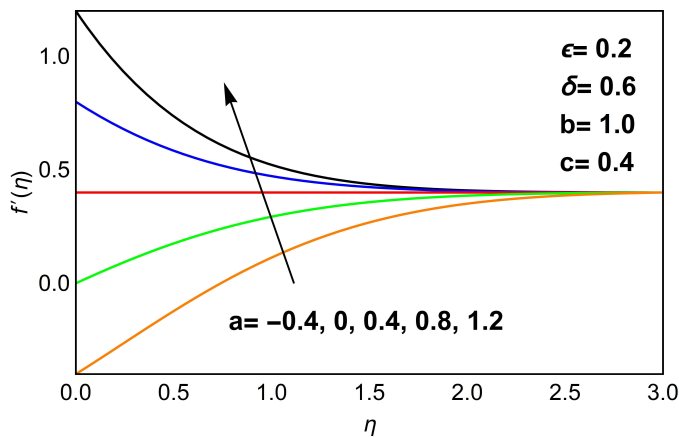


Fig. 4. Velocity profiles for different values of stretching/shrinking parameter a .

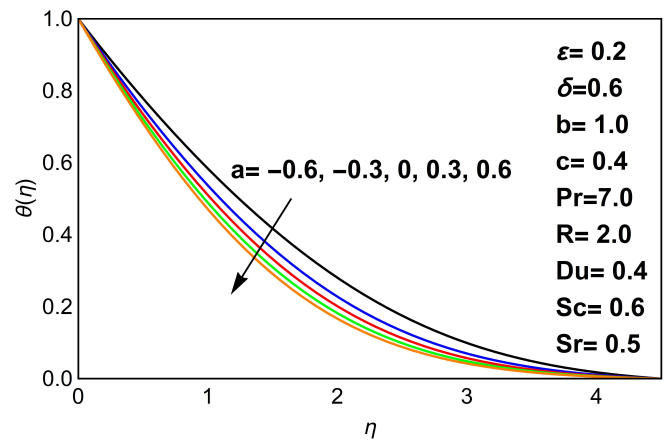


Fig. 5. Temperature profiles for different values of stretching/shrinking parameter a .

3. Symmetry Analysis of the Governing Equations

There exist many standard books (see Cantwell [4], Layek [3] (Chapter 8), Bluman and Kumei [5] and references therein) on Lie Group theory. Also, the Lie Group theory is now extensively used in many Laminar and Turbulent flow to obtain similarity variables and self-preserving functions. Here we described the method very briefly as follows: Consider the following partial differential equation

$$F(x, U, U^{(1)}, U^{(2)}, \dots, U^{(n)}) = 0 \quad (16)$$

where $x = (x_1, x_2, \dots, x_k)$ is the independent variable, $U = (U_1(x), U_2(x), \dots, U_l(x))$ is the dependent variables, $U^{(n)}$ is the n -th order derivative of U . The main idea of the Lie is to find transformations of variables that do not change the functional form of differential equation (16). A transformation $(X = (\phi, \psi): D \times \mathbb{R} \rightarrow D, D \subseteq \mathbb{R}^n)$ such that $T_\epsilon \equiv x^* = \phi(x, U, \epsilon)$ and $U^* = \psi(x, U, \epsilon)$, $\epsilon \in S \subset \mathbb{R}$ is a continuous parameter and forming a group, with $x^* = x$ and $U^* = U$ when $\epsilon = 0$ is called a symmetry group of transformation of Equation (16) iff equivalence $F(x, U, U^{(1)}, U^{(2)}, \dots, U^{(n)}) = 0 \Leftrightarrow F(x^*, U^*, U^{*(1)}, U^{*(2)}, \dots, U^{*(n)}) = 0$ holds. The infinitesimal form of the transformations T_ϵ can be written as

$$x^* = x + \epsilon \xi(x, U) + o(\epsilon^2) \text{ and } U^* = U + \epsilon \eta(x, U) + o(\epsilon^2), \quad (17)$$

where $\xi = \partial \phi / \partial \epsilon|_{\epsilon=0}$ and $\eta = \partial \psi / \partial \epsilon|_{\epsilon=0}$ are called the infinitesimals of the transformations. Again, global form T_ϵ of the transformations (17) can hence be determined by integrating the first order system $\partial x^* / \partial \epsilon = \xi(x^*, U^*)$, $\partial y^* / \partial \epsilon = \eta(x^*, U^*)$, with the initial conditions $x^* = x$, $y^* = y$ at $\epsilon = 0$. The infinitesimal transformation (17) can be represented by the infinitesimal generator given by $X = \xi_i \partial / \partial x_i + \eta_j \partial / \partial U_j$, $i = 1, 2, \dots, k$; $j = 1, 2, \dots, l$. The infinitesimals ξ, η of the differential equation $F = 0$ can be determined using the defining equation $X^{[n]} F|_{F=0} = 0$, where $X^{[n]}$ is the prolongation of X to all the derivatives of the dependent variables U w.r.t x up to order n of the equation (16). The n -th prolongation $X^{[n]}$ is defined as

$$X^{[n]} = \xi_i \frac{\partial}{\partial x_i} + \eta_j \frac{\partial}{\partial U_j} + \zeta_i \frac{\partial}{\partial U_{j_i}} + \zeta_{i_1 i_2} \frac{\partial}{\partial U_{j_{i_1 i_2}}} + \dots + \zeta_{i_1 i_2 \dots i_n} \frac{\partial}{\partial U_{j_{i_1 i_2 \dots i_n}}} \quad (18)$$

where U_{j_i} is the partial derivative of the variable U_j with respect to the variable x_i , $\zeta_{i_1 i_2 \dots i_k} = D_{i_1} \dots D_{i_k} (\eta_j - \xi_i U_{j_i}) + \xi_i U_{j_{i_1 \dots i_k}}$; $i, i_1, \dots, i_k = 1, 2, \dots, k$ and D_{i_k} denotes the total differential operator with respect to the variable x_k .

We shall now focus on the invariant solutions of Eq. (10), (15) by employing the Lie symmetry analysis (see Layek [3] (Chapter 8), Cantwell [4], Bluman and Kumei [5] and references therein). The equations has three independent variables t, x, y and five dependent variables $u(t, x, y)$, $v(t, x, y)$, $u_e(t, x)$, $T(t, x, y)$, $C(t, x, y)$. So, the infinitesimal generator of these equations is of the following form

$$X_1 = \xi_1 \frac{\partial}{\partial t} + \xi_2 \frac{\partial}{\partial x} + \xi_3 \frac{\partial}{\partial y} + \eta_1 \frac{\partial}{\partial u} + \eta_2 \frac{\partial}{\partial v} + \eta_3 \frac{\partial}{\partial u_e} + \eta_4 \frac{\partial}{\partial T} + \eta_5 \frac{\partial}{\partial C}. \quad (19)$$

By using the Mathematica package **IntroSymmetry.m** (Cantwell [4]), the infinitesimals are found as

$$\left. \begin{aligned} \xi_1 &= a_1 + a_2 t, \xi_2 = h(t) + \frac{3}{2} a_2 x, \xi_3 = g(t, x) + \frac{1}{2} a_2 y, \\ \eta_1 &= h'(t) + \frac{1}{2} a_2 u, \eta_2 = \frac{\partial g}{\partial t} + \frac{\partial g}{\partial x} u - \frac{1}{2} a_2 v, \eta_3 = h'(t) + \frac{1}{2} a_2 u_e, \\ \eta_4 &= a_3 + a_4 C + a_5 T, \eta_5 = a_6 + \left[a_5 - \frac{a_4 (3R \cdot Pr + 16Pr \cdot Sc + 3R \cdot Sc)}{3R \cdot Pr \cdot Du \cdot Sc} \right] C + a_4 \frac{Sr}{Du} T, \end{aligned} \right\} \quad (20)$$

where the group parameters $a_1, a_2, a_3, a_4, a_5, a_6 \in \mathbb{R}$ are arbitrary constants and $h(t), g(t, x)$ are arbitrary functions. Now, the boundary conditions (14)-(15), will remain form invariant under the transformation for this infinitesimal if $g(x, t) = a_3 = a_4 = a_5 = a_6 = 0$ and the functions $S_0(x, t), V_0(x, t), u_e(x, t)$ satisfy the following equations

$$\left. \begin{aligned} h'(t) + \frac{1}{2} a_2 S_0 &= (a_1 + a_2 t) \frac{\partial S_0}{\partial t} + \left[h(t) + \frac{3}{2} a_2 x \right] \frac{\partial S_0}{\partial x}, \\ \frac{1}{2} a_2 V_0 + (a_1 + a_2 t) \frac{\partial V_0}{\partial t} + \left[h(t) + \frac{3}{2} a_2 x \right] \frac{\partial V_0}{\partial x} &= 0, \\ h'(t) + \frac{1}{2} a_2 u_e &= (a_1 + a_2 t) \frac{\partial u_e}{\partial t} + \left[h(t) + \frac{3}{2} a_2 x \right] \frac{\partial u_e}{\partial x}. \end{aligned} \right\} \quad (21)$$

without loss of generality, we assume that $h(t) = 0$. So, there are left one translational parameter a_1 and one scaling parameter a_2 . We can write infinitesimal generators for each of the parameter as

$$X_{a_1} = \frac{\partial}{\partial t}, \quad X_{a_2} = t \frac{\partial}{\partial t} + \frac{3}{2} x \frac{\partial}{\partial x} + \frac{1}{2} y \frac{\partial}{\partial y} + \frac{1}{2} u \frac{\partial}{\partial u} - \frac{1}{2} v \frac{\partial}{\partial v} + \frac{1}{2} u_e \frac{\partial}{\partial u_e}. \quad (22)$$

and the Eq. (10), (15) read the same under the following global transformation groups

$$\left. \begin{aligned} G_{a_1} : t^* &= t + a_1, x^* = x, y^* = y, u^* = u, v^* = v, u_e^* = u_e, T^* = T, C^* = C, \\ G_{a_2} : t^* &= t e^{a_2}, x^* = x e^{\frac{3}{2} a_2}, y^* = y e^{\frac{1}{2} a_2}, u^* = u e^{\frac{1}{2} a_2}, v^* = v e^{-\frac{1}{2} a_2}, u_e^* = u_e e^{\frac{1}{2} a_2}, T^* = T, C^* = C. \end{aligned} \right\} \quad (23)$$

Using invariant surface condition of the curve $F(x, U) = 0$ according to which the solution is to be invariant if $XF = 0$. Using the characteristic method [5], we have found the characteristic system as

$$\frac{dt}{a_1 + a_2 t} = \frac{dx}{\frac{3}{2} a_2 x} = \frac{dy}{\frac{1}{2} a_2 y} = \frac{du}{\frac{1}{2} a_2 u} = \frac{dv}{-\frac{1}{2} a_2 v} = \frac{du_e}{\frac{1}{2} a_2 u_e} = \frac{dT}{0} = \frac{dC}{0}. \quad (24)$$

Solving the above equations, we found the following invariants (self-similar variables and self-preserving functions) in dimensionless form as

$$\left. \begin{aligned} X &= \frac{x}{(a_{12} + t)^{\frac{3}{2}}}, \quad Y = \frac{y}{(a_{12} + t)^{\frac{1}{2}}}, \quad U(X, Y) = \frac{u}{(a_{12} + t)^{\frac{1}{2}}}, \quad V(X, Y) = v (a_{12} + t)^{\frac{1}{2}}, \\ U_e(X) &= \frac{u_e(x, t)}{(a_{12} + t)^{\frac{1}{2}}}, \quad T = P(X, Y), \quad C = Q(X, Y), \end{aligned} \right\} \quad (25)$$

where $a_{12} = a_1 / a_2$ is arbitrary and may be called as virtual origin. Using (25), Eqs. (10) - (15) are transformed into the following PDEs

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad (26)$$

$$\frac{U}{2} - \frac{3}{2} X \frac{\partial U}{\partial X} - \frac{Y}{2} \frac{\partial U}{\partial Y} + U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{U_e}{2} - \frac{3}{2} X \frac{\partial U_e}{\partial X} + U_e \frac{\partial U_e}{\partial X} + (1 + \epsilon) \frac{\partial^2 U}{\partial Y^2} - \epsilon \delta \left(\frac{\partial U}{\partial Y} \right)^2 \frac{\partial^2 U}{\partial Y^2}, \quad (27)$$

$$-\frac{3}{2} X \frac{\partial P}{\partial X} - \frac{Y}{2} \frac{\partial P}{\partial Y} + U \frac{\partial P}{\partial X} + V \frac{\partial P}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 P}{\partial Y^2} + \frac{16}{3R} \frac{\partial^2 P}{\partial Y^2} + Du \frac{\partial^2 Q}{\partial Y^2}, \quad (28)$$

$$-\frac{3}{2}X \frac{\partial Q}{\partial X} - \frac{Y}{2} \frac{\partial Q}{\partial Y} + U \frac{\partial Q}{\partial X} + V \frac{\partial Q}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 Q}{\partial Y^2} + Sr \frac{\partial^2 P}{\partial Y^2}, \quad (29)$$

The boundary condition (14) and (15) remain form invariant and hold for the above solution form (25) only if $S_0(x, t)$, $V_0(x, t)$ are of the form

$$S_0(x, t) = (a_{12} + t)^{1/2} A(X), \quad V_0(x, t) = (a_{12} + t)^{-1/2} B(X). \quad (30)$$

Hence, the boundary conditions (14), (15) under the above symmetry group of transformations Equation (25) become

$$U(X, Y) = A(X), \quad V(X, Y) = B(X), \quad P = 1, \quad Q = 1, \quad \text{at } Y = 0, \quad (31)$$

$$U(X, Y) \rightarrow U_e(X), \quad P \rightarrow 0, \quad Q \rightarrow 0 \quad \text{at } Y \rightarrow \infty. \quad (32)$$

Till now, we obtain Symmetry transformations that reduce the number of independent variables from three to two.

We now apply the stream function $U = \partial \psi / \partial Y$, $V = -\partial \psi / \partial X$ to reduce the number of dependent variable. So the equation (26) is automatically satisfied. The Eqs. (27) - (32) are transformed into

$$\frac{1}{2} \frac{\partial \psi}{\partial Y} - \frac{3}{2} X \frac{\partial^2 \psi}{\partial X \partial Y} - \frac{Y}{2} \frac{\partial^2 \psi}{\partial Y^2} + \frac{\partial \psi}{\partial Y} \frac{\partial^2 \psi}{\partial X \partial Y} - \frac{\partial \psi}{\partial X} \frac{\partial^2 \psi}{\partial Y^2} = \frac{U_e}{2} - \frac{3}{2} X \frac{\partial U_e}{\partial X} + U_e \frac{\partial U_e}{\partial X} + (1 + \epsilon) \frac{\partial^3 \psi}{\partial Y^3} - \epsilon \delta \left(\frac{\partial^2 \psi}{\partial Y^2} \right)^2 \frac{\partial^3 \psi}{\partial Y^3}, \quad (33)$$

$$-\frac{3}{2} X \frac{\partial P}{\partial X} - \frac{Y}{2} \frac{\partial P}{\partial Y} + \frac{\partial \psi}{\partial Y} \frac{\partial P}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial P}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 P}{\partial Y^2} + \frac{16}{3R} \frac{\partial^2 P}{\partial Y^2} + Du \frac{\partial^2 Q}{\partial Y^2}, \quad (34)$$

$$-\frac{3}{2} X \frac{\partial Q}{\partial X} - \frac{Y}{2} \frac{\partial Q}{\partial Y} + \frac{\partial \psi}{\partial Y} \frac{\partial Q}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial Q}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 Q}{\partial Y^2} + Sr \frac{\partial^2 P}{\partial Y^2}, \quad (35)$$

with the boundary conditions

$$\frac{\partial \psi}{\partial Y} = A(X), \quad \frac{\partial \psi}{\partial X} = -B(X), \quad P = 1, \quad Q = 1, \quad \text{at } Y = 0, \quad (36)$$

$$\frac{\partial \psi}{\partial Y} \rightarrow U_e(X), \quad P \rightarrow 0, \quad Q \rightarrow 0 \quad \text{at } Y \rightarrow \infty. \quad (37)$$

Now, our aim is to transform the PDES (33), (35) with boundary conditions (36), (37) to an ODEs by using the same Lie Group.

4. Second Reduction of Order

The equations (33), (35) contains two independent variables (X, Y) and four dependent variables (ψ, U_e, P, Q) . So infinitesimal generator of the eqns. (33), (35) has the following form

$$X_2 = \xi_1 \frac{\partial}{\partial X} + \xi_2 \frac{\partial}{\partial Y} + \eta_1 \frac{\partial}{\partial \psi} + \eta_2 \frac{\partial}{\partial U_e} + \eta_3 \frac{\partial}{\partial P} + \eta_4 \frac{\partial}{\partial Q}. \quad (38)$$

The infinitesimals $(\xi_1, \xi_2, \eta_1, \eta_2, \eta_3, \eta_4)$ are obtained by solving the defining equations with the help of Mathematica package **IntroToSymmetry.m** (Cantwell [4]) as

$$\left. \begin{aligned} \xi_1 &= b_{10} + b_{11}X + b_{12}Y + b_{13}\psi + b_{14}U_e + b_{15}P + b_{16}Q, \\ \xi_2 &= b_{20} + b_{21}X + b_{22}Y + b_{23}\psi + b_{24}U_e + b_{25}P + b_{26}Q, \\ \eta_1 &= b_{30} + b_{31}X + b_{32}Y + b_{33}\psi + b_{34}U_e + b_{35}P + b_{36}Q, \\ \eta_2 &= b_{40} + b_{41}X + b_{42}Y + b_{43}\psi + b_{44}U_e + b_{45}P + b_{46}Q, \\ \eta_3 &= b_{50} + b_{51}X + b_{52}Y + b_{53}\psi + b_{54}U_e + b_{55}P + b_{56}Q, \\ \eta_4 &= b_{60} + b_{61}X + b_{62}Y + b_{63}\psi + b_{64}U_e + b_{65}P + b_{66}Q. \end{aligned} \right\} \quad (39)$$

Using invariant surface condition, the characteristic equations analogous to the scaling variables $b_{11}, b_{22}, b_{33}, b_{44}, b_{55}, b_{66}$ can be written as

$$\frac{dX}{b_{11}X} = \frac{dY}{b_{22}Y} = \frac{d\psi}{b_{33}\psi} = \frac{dU_e}{b_{44}U_e} = \frac{dP}{b_{55}P} = \frac{dQ}{b_{66}Q}. \quad (40)$$

Solving the above-mentioned characteristic equations, we have the invariants in dimensionless form as

$$\eta = \frac{Y}{X^{b_2}}, \psi = X^{b_3} f(\eta), U_e = cX^{b_4}, \theta = X^{b_5} P(\eta), \phi = X^{b_6} Q(\eta). \quad (41)$$

Here $b_2 = b_{22}/b_{11}$, $b_3 = b_{33}/b_{11}$, $b_4 = b_{44}/b_{11}$, $b_5 = b_{55}/b_{11}$, $b_6 = b_{66}/b_{11}$, $c = \text{const.}$

Using the transformations in Eq. (41), the non-linear partial differential Equations. (33)-(35) are transformed into following ordinary differential equations if $b_2 = b_5 = b_6 = 0$, $b_3 = b_4 = 1$:

$$(1+\epsilon)f''' - f'^2 + ff'' - \epsilon\delta f''^2 f''' + f' + \frac{\eta}{2}f'' + c^2 - c = 0, \quad (42)$$

$$-\frac{\eta}{2}\theta' - f\theta' = \frac{1}{Pr}\theta'' + \frac{16}{3R}\theta'' + Du\phi'', \quad (43)$$

$$-\frac{\eta}{2}\phi' - f\phi' = \frac{1}{Sc}\phi'' + Sr\theta'', \quad (44)$$

where $\delta = \beta X^2$ is the dimensionless local material fluid parameter.

Also, we assume $A(X), B(X)$ are of the form $A(X) = aX$, $B(X) = b$ where a is the dimensionless expanding/shrinking parameter with $a > 0$ for expanding surface and $a < 0$ for shrinking surface, and b is the dimensionless mass flux parameter with $b > 0$ for suction and $b < 0$ for injection respectively.

So, the boundary conditions (36), (37) become

$$f' = a, f = b, \theta = 1, \phi = 1 \text{ at } \eta = 0, \quad (45)$$

$$f' \rightarrow c, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty. \quad (46)$$

5. Numerical Procedure

The above non-linear differential Equations (42), (44) along with boundary conditions (45), (46) form a two-point boundary value problem and are solved using shooting method, by converting it into an initial value problem. In this method, we have chosen a suitable finite value of $\eta \rightarrow \infty$, say η_∞ . We set the following first order systems

$$f' = z, z' = p, p' = \frac{z^2 - fp - z - \frac{\eta}{2}p - c^2 + c}{1 + \epsilon - \epsilon\delta p^2}; \quad (47)$$

$$\theta' = r, r' = \frac{\left[\left(-\frac{\eta}{2}r - fr \right) \frac{1}{Sc} - Du \left(-\frac{\eta}{2}s - fs \right) \right]}{\left[\left(\frac{1}{Pr} + \frac{16}{3R} \right) \frac{1}{Sc} - Du \cdot Sr \right]}; \quad (48)$$

$$\phi' = s, s' = \frac{\left[\left(-\frac{\eta}{2}r - fr \right) Sr - \left(\frac{1}{Pr} + \frac{16}{3R} \right) \left(-\frac{\eta}{2}s - fs \right) \right]}{\left[Du \cdot Sr - \left(\frac{1}{Pr} + \frac{16}{3R} \right) \frac{1}{Sc} \right]}; \quad (49)$$

with

$$f(0) = b; z(0) = a; \theta(0) = 1; \phi(0) = 1 \} \quad (50)$$

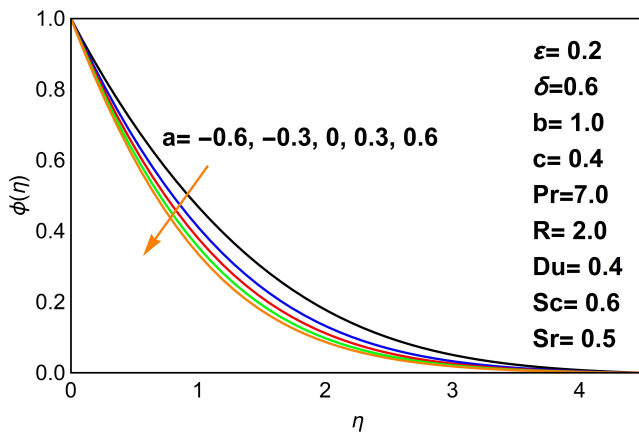


Fig. 6. Concentration profiles for different values of stretching/shrinking parameter a .

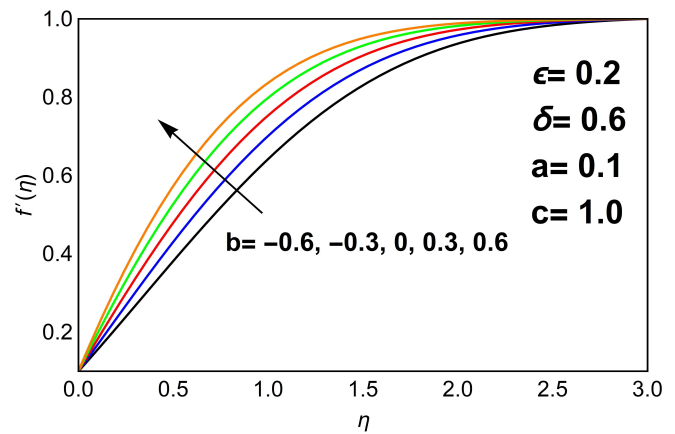


Fig. 7. Velocity profiles for different values of suction/injection parameter b .

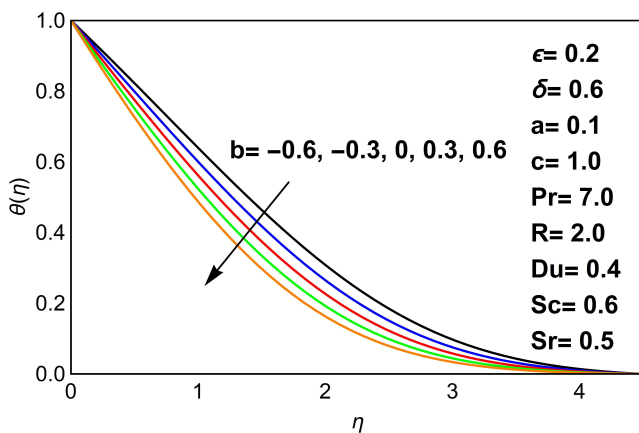


Fig. 8. Temperature profiles for different values of suction/injection parameter b .

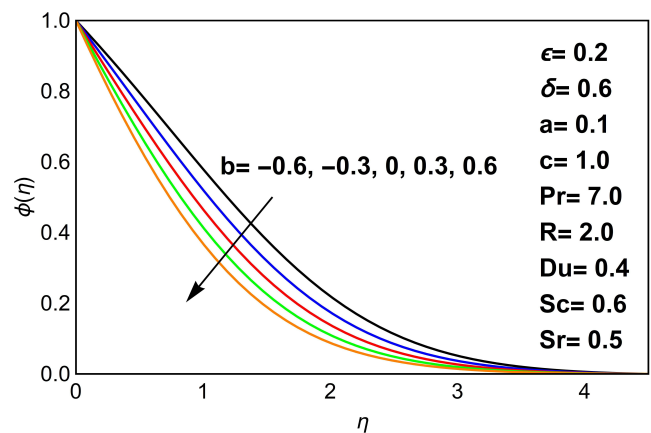


Fig. 9. Concentration profiles for different values of suction/injection parameter b .

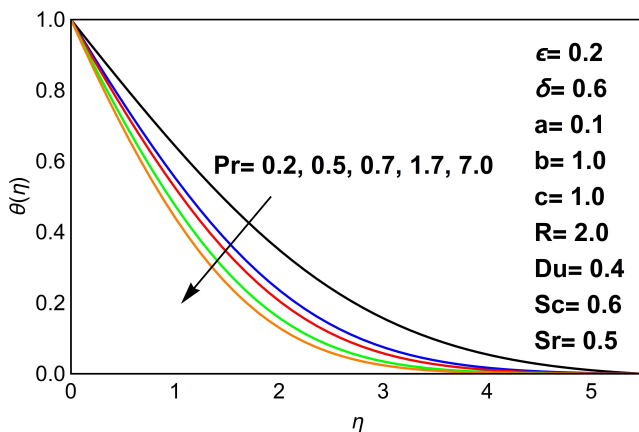


Fig. 10. Temperature profiles for different values of Prandtl number Pr .

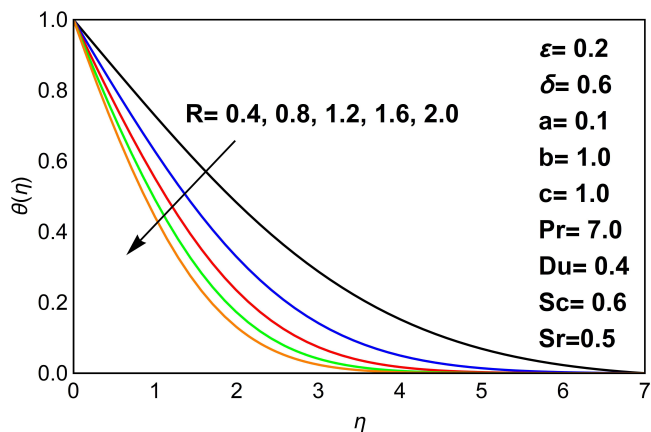


Fig. 11. Temperature profiles for different values of radiation parameter R .

To solve (47)-(49) with (50) as an initial value problem we must need values for $p(0)$, i.e., $f''(0)$, $r(0)$ i.e., $\theta'(0)$ and $s(0)$, i.e., $\phi'(0)$, but no such values are given in the problem. The initial guess values for $f''(0)$, $\theta'(0)$ and $\phi'(0)$ are chosen and fourth order Runge-Kutta method is applied to obtain the solution. We compare the computed values of $f'(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ at η_∞ with the given boundary conditions $f'(\eta_\infty) = c$, $\theta(\eta_\infty) = 0$ and $\phi(\eta_\infty) = 0$ and adjust values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ using Newton-Raphson method to get better approximation for the solution. The step size is taken as $\Delta\eta = 0.01$. The process is repeated until we get the result correct up to desired accuracy of 10^{-5} level.

6. Results and Discussion

The numerical results are illustrated graphically in the form of a non-dimensional velocity, temperature, concentration profiles. The parameters are chosen arbitrary, where $Pr = 0.71$ corresponds physically to the air at $20^\circ C$, $Pr = 1$ corresponds to electrolyte solution such as salt-water and $Pr = 7.0$ corresponds to pure water. Throughout the study of results, we consider the value of the governing parameters as $\epsilon = 0.2$, $\delta = 0.6$, $Pr = 7.0$, $R = 2.0$, $Du = 0.4$, $Sc = 0.6$, $Sr = 0.5$. These values are taken as default unless otherwise specified.

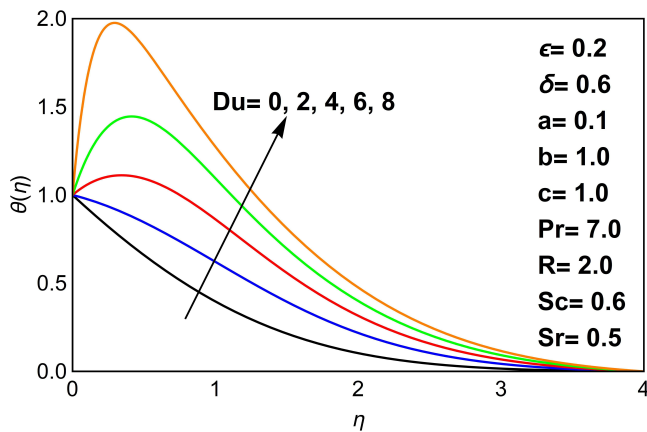


Fig. 12. Temperature profiles for different values of Dufour number Du .

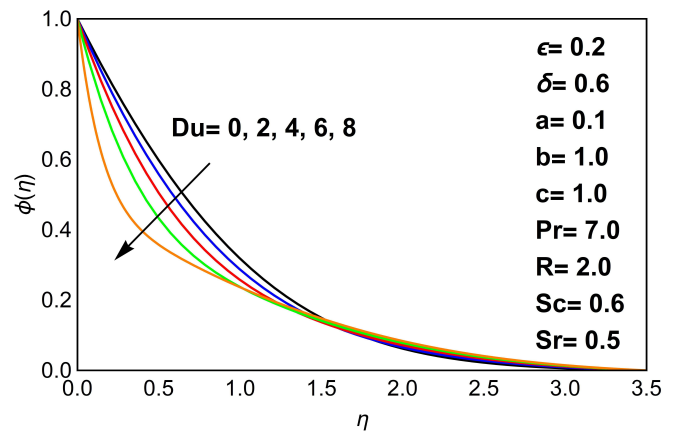


Fig. 13. Concentration profiles for different values of Dufour number Du .

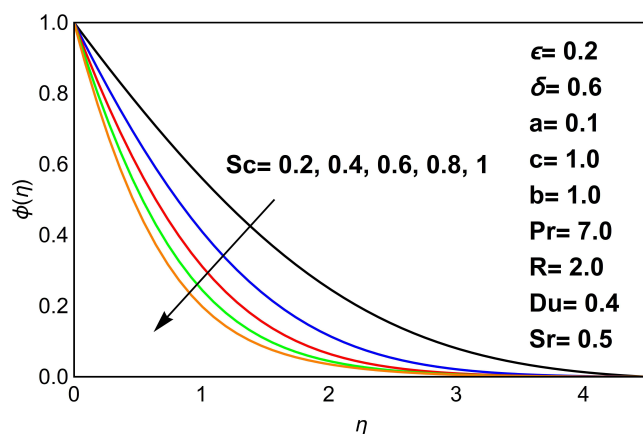


Fig. 14. Concentration profiles for different values of Schmidt number Sc .

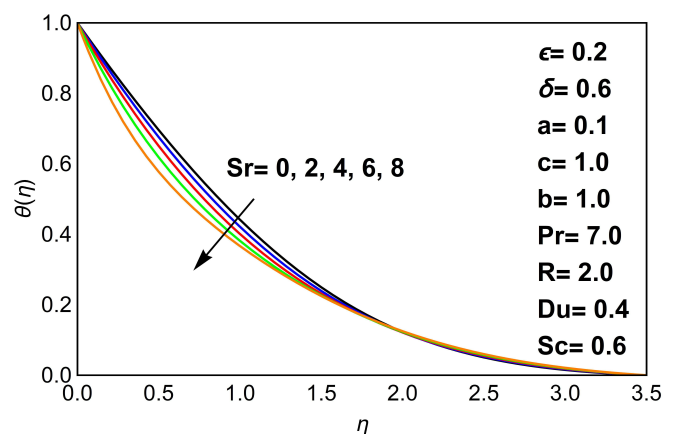


Fig. 15. Temperature profiles for different values of Soret number Sr .

To confirm the validity of numerical scheme and the obtained results, our obtained results of skin-friction for steady case and different Powell-Eyring fluid parameters δ with $\epsilon = 0.2$, $c = 0$ are compared with published work Khan et al. [46] in Table 1 and those data are established excellent agreement.

Figs. 2 - 3 depict the effect of Powell-Eyring fluid parameters ϵ and δ on self-similar velocity profiles. It is found that the velocity decreases for increasing ϵ and so the flow boundary layer thickness increases for rising ϵ . Whereas, the other Powell-Eyring fluid parameter δ shows the opposite behavior, but its effect is less prominent. It is noted that the impact of ϵ and δ on temperature and concentration profiles is very less, so we have not given these graphs.

Figs. 4 - 6 present effects of expanding/shrinking parameter a on the velocity, temperature and concentration profiles. It is obvious that $a > 0$ corresponds to expanding sheet, $a < 0$ corresponds to shrinking sheet and $a = 0$ corresponds to plane sheet. We can spot that for an increase of expanding parameter a (assuming velocity ratio of the expanding sheet to the free stream velocity is greater than unity i.e. $a > c$), the velocity profile $f'(\eta)$ along the plate increases, and accordingly the boundary layer thickness enlarges. For increase of shrinking parameter $a (< 0)$, the velocity profile $f'(\eta)$ along the plate increases, but the flow boundary layer thickness decreases. Also, for increase of expanding/shrinking parameter a , the temperature profile $\theta(\eta)$ and the concentration profile $\phi(\eta)$ (at a fixed η) along the plate decrease with constant other parameters. As a result, the concentration and thermal boundary layer thickness lessen for increasing a .

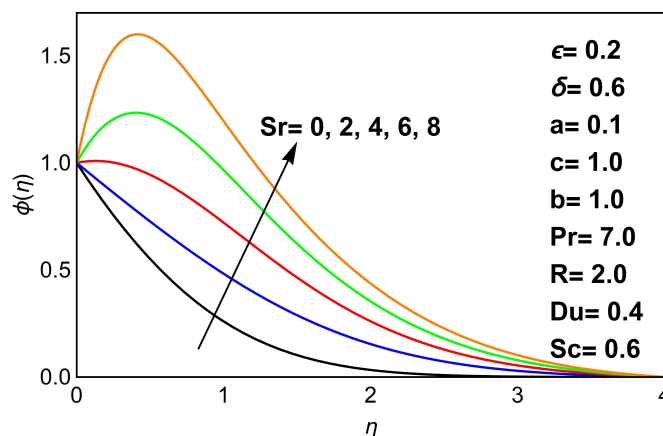


Fig. 16. Concentration profiles for different values of Soret number Sr .

Table 1. Comparison of skin-friction coefficient with Khan et al. [46] for steady case and for different values of δ with $\epsilon = 0.2$, $c = 0$.

δ	Khan et al. [46]	Present study
0.1	1.0940	1.09402
0.2	1.0924	1.09239
0.3	1.0909	1.09091
0.4	1.0894	1.08942
0.5	1.0878	1.08784
0.6	1.0862	1.08618
0.7	1.0847	1.08467
0.8	1.0832	1.08323
0.9	1.0816	1.08162
1.0	1.0797	1.07968

The suction/injection of fluid mass has a major impact on velocity, concentration and temperature and those are illustrated Figs. 7 - 9. We see that the value of velocity in the boundary layer enhances with larger suction/injection parameter b , but the temperature and the concentration profile decrease with b . So, the suction/injection decreases the flow, concentration and thermal boundary layer thicknesses.

The prediction of the influence of Prandtl number Pr and radiation parameter R on the temperature profiles are depicted in Figs 10 - 11. It is obvious that for increasing Pr the temperature profiles decrease (at a fixed η). So, the thermal boundary layer thickness reduces for increasing Pr . Similar to the Prandtl number, we notice that the temperature profiles decrease for increasing radiation parameter R . Thus, the thermal boundary layer thickness decreases for increasing R .

Figs. 12 - 13 reveal the temperature and concentration profiles for dissimilar values of the Dufour number Du . We see that for the rise of Dufour number Du causes an increment of temperature profiles. The diffusion-thermo effect results in the enlargement of thermal boundary layer thickness with thermal overshoot for larger values of Du . In the case of concentration profiles, initially it decreases with increasing Du , but after a certain stage, it increases with increasing Du , i.e., concentration crossing over is found. So, the concentration boundary layer thickness also enlarges with Du .

While the concentration profiles for unlike values of Schmidt number Sc is demonstrated in Fig. 14. It is monitored that when Sc increases, the concentration profile decreases. As it is known that mass diffusivity causes enhancement in concentration boundary layer thickness and Sc is inversely proportional to mass diffusivity, so the outcome is justified.

Finally, the effects of Soret number Sr on the temperature and concentration profiles are portrayed through Figs. 15 - 16. Similar to concentration for Du variation, it is seen that initially, temperature profile decreases for increasing Sr , but after some distance from the sheet, it increases with increasing Sr . Hence, though it is small but there is a growth in thermal boundary layer thickness increases with Sr showing thermal crossing over. Also, the concentration profiles increase with increasing Sr . Thus, the thermo-diffusion effect, i.e., larger values of Sr makes the concentration boundary layer thickness thick. Also, concentration overshoot found for higher values of Sr .

7. Concluding Remarks

In this work, the combined heat and mass transfer in an unsteady, incompressible, viscous, non-Newtonian Powell-Eyring fluid along with expanding/shrinking sheet with suction/injection, Dufour and Soret effects are investigated. The Lie-group transformations are applied to obtain the invariants of the system. The equations are then solved using numerical technique and the acquired results are plotted graphically. From the present study, we may conclude the following points:

- The flow boundary layer thickness increases with increasing Powell-Eyring fluid parameter ϵ and expanding parameter $a(>c)$ but it decreases with increasing Powell-Eyring fluid parameter δ , shrinking parameter and suction/injection parameter b .



- ii. The thermal boundary layer thickness reduces with increasing expanding/shrinking parameter a , suction/injection parameter b , Prandtl number Pr and radiation parameter R , but it increases with larger Dufour number Du and Soret number Sr .
- iii. The thickness of the Concentration boundary layer lessens with increasing expanding/shrinking parameter a , suction/injection parameter b and Schmidt number Sc , but it increases with increasing Dufour number Du and Soret number Sr .
- iv. Thermal and concentration overshoots are found for many values of Dufour and Soret numbers respectively and it grows with their increments.
- v. While for different values of Soret and Dufour numbers the thermal and concentration crossing over found.

Author Contributions

The manuscript was planned, formulated, written through the equal contribution of all authors. All authors discussed the results, reviewed and approved the final form of the manuscript.

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Conflict of Interest

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Nomenclature

a, b, c	Constants	R	Radiation parameter
C	Concentration	Re	Reynold number
C_1	Material fluid parameter	Sc	Schmidt number
C_w	Concentration at the wall	Sr	Soret number
C_∞	Free stream concentration	S_0	Moving surface velocity
c_p	Specific heat at constant pressure	t	Time
c_s	Concentration susceptibility	T	Temperature
D_m	Coefficient of mass diffusivity	T_m	Mean fluid pressure
Du	Dufour number	T_w	Temperature at the wall
f	Dimensionless stream function	T_∞	Free stream temperature
f'	Dimensionless velocity	u, v	Velocities in x, y -directions
k_T	Thermal diffusion ratio	U_0	Characteristic velocity
k_1	Absorption coefficient	u_e	Free stream velocity
L	Characteristic length	V_0	Suction/injection
Pr	Prandtl number	x, y	Cartesian coordinates

Greek symbols

τ_{xy}	Shear stress tensor	ϵ, δ	Powell-Eyring fluid parameter
β_1	Material fluid parameter	μ	Coefficient of viscosity
κ	Thermal conductivity of the fluid	η	Similarity variable
ρ	Density of the fluid	ψ	Stream function
ν	Kinematic viscosity	θ	Dimensionless temperature
σ^*	Stefan-Boltzman constant	ϕ	Dimensionless concentration

Subscript

w	Condition at the surface	∞	Condition at infinity
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