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# 3D Optimization of Gear Train Layout Using Particle Swarm Optimization Algorithm

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**Abstract.** Optimization of the volume/weight in the gear train is of great importance for industries and researchers. In this paper, using the particle swarm optimization algorithm, a general gear train is optimized. The main idea is to optimize the volume/weight of the gearbox in 3 directions. To this end, the optimization process based on the PSO algorithm occurs along the height, length, and width of the gearbox to achieve the smallest possible gearbox. The constraints are divided into three types named geometrical, design and control constraints. The optimization process is presented for two and three-stage gear trains and by choosing different values for the gear ratio, input power and hardness of gears. The practical graphs for the optimum value of the weight/volume and all necessary design parameters of gearbox such as the number of stages, position, modulus of gears, face width of gears, and diameter of shafts are also presented. The results are validated by comparing with the results reported in the previous publications.

**Keywords**: Optimal gearbox layout, Weight/volume optimization, Particle swarm optimization (PSO), Gear train.

## 1. Introduction

The discipline of engineering challenges is boundless and has different variables consisting of linear and nonlinear constraints and equations each of which numerous researchers studied Heuristic methods to solve them. Heuristic methods don't necessitate derivatives of the functions and are powerful methods compared with the conventional methods. Heuristic methods find the best global solution, thus Gear train optimization has become more attractive for many researchers in recent years. The volume of gear trains depends on the configuration of affected parameters such as the location of gears, the number of gears, the number of teeth, etc. To attain the best parameters for the gearbox, many researchers used various methods for optimization. Gologlu and Zeyveli [1] implemented a differential evolution algorithm to optimize the weight of a single-stage gearbox with nonlinear constraint formulation. Patwal et al. [2] proposed TVAC-PSO-MS and used particle swarm optimization (PSO) for the production programing of pumped storage hydrothermal. Panda et al. [3] researched weight optimization for a single-stage gearbox consisting of spur gears. They used different evolution algorithms to achieve the optimum weight of spur gear set in a single-stage gearbox. They compared the results with other modern algorithms. Chenge and Jin. [4] worked on a new way to incorporate particle swarm optimization and social learning mechanisms for scalable optimization. Chong and Lee [5] utilized a Genetic Algorithm method to design the gear train. They applied GA to automate preliminary design. Zolfaghari et al. [6] worked on volume optimization of straight bevel gears by employing the evolutionary algorithm. To achieve this purpose, they used two optimization techniques including a Genetic Algorithm and a simulated annealing algorithm (SA). Miler et al. [7] utilized a Genetic algorithm to optimize the weight of a gear pair and studied on design of spur gear considering profile shift. Garg and Sharma. [8] used a fuzzy multi-objective optimization problem (FMOOP) and particle swarm optimization to solve multi-objective reliability-redundancy and nonlinear problems. Alexandru et al. [9] studied the steering gearbox design and simulation with a variable transmission ratio. They focused on important objectives consisted of mathematical models in theoretical bases, geometrical parameters and simulating the ability of the gearbox. Garg et. al. [10-12] worked on optimization of the constrained problems with hybrid PSO-GA and used heuristic methods (artificial bee colony algorithm) to approach better solutions for structural engineering design problems and proposed novel hybrid GA-GSA algorithm to improve the performance of the mechanism. Tudose et al. [13] considered the weight of gearbox as an objective function and used a two-phase evolutionary algorithm in formulation to optimize a two-stage helical gearbox. Rui et al. [14] presented how to used Genetic Algorithm to solve the multi-objective gear reducer design problems in optimization process. Kang et al. [15] presented optimization method for obtaining the optimum helix angle of gears. They presented a relation between the transmission error and contact ratio. Mendi et al. [16] depicted genetic algorithm to optimize the rolling bearing, shaft diameter and module. GA compared with The domain analytic method (AM) and the results showed, genetic algorithm is better than AM method to achieve optimal gear volume. Abderazek et al. [17] worked on spur gears and introduced a method for achieving the optimal tooth profile for gears. Fesharaki and Golabi.[18] used particle swarm optimization algorithm to find best place for piezoelectric actuator to reduce stress concentration around hole in plate. Yokota and Gen [19] studied on weight of gears and used genetic algorithm to achieve a solution method for optimum weight design. Savsani et al. [20] used simulated annealing and particle swarm optimization algorithms to achieve the optimum weight for a multi-stage gear train. Swantner and Campbell [21] worked on optimization of a gear train with a method that automates the design of gear trains which consists of various types of gear such as bevel, worm, spur and compound gears. Marjanovic et al. [22] studied on optimization of spur gear trains. They studied on position of shaft axes in gear train for reducing the volume of gearbox. Their strategy for selecting the optimal parameters has three stages: optimal materials, gear ratios, and position of shaft axes. They presented gear trains with 22% reduction in volume. Chong et al. [23] proposed an optimization algorithm with four important stages. At the first stage, the user selects the number of reduction stages. At the second stage, gear ratios are specified for each stage by using the random search method. Thirdly, basic parameters for gear design are generated by using the test methods. Finally, simulated annealing algorithm specified shafts position and other design parameters for minimizing the gearbox volume. Pomrehn and Papalambros [24], worked on discrete optimum design model for gear train that used spur gear pairs. Thompson et al. [25] Studied on optimal volume design for spur gear reduction units. They presented optimal design formulation for two and three-stage gear trains. Zarefar and Muthukrishnan[26] used random-search methodology for helical gear optimization. Salomon et. al. [27] worked on Optimization of gearbox design using active robust considered requirements with uncertain load**.** Ciavarella and Demelio [28] worked on optimization of fatigue life of gears, specific sliding and stress concentration by using numerical methods. Wang et al. [29,30] studied on optimum design of tooth profile of spur gears. Golabi et al. [31] worked on design optimization of multi-stage gear train based on minimum volume/weight. They used F-mincon method to optimize the different parameters of gear train such as gear ratio, input power, and strength of material. They presented the design parameters with some graphs such as number of stages, modules, shafts diameter and face width of gears. But in their research, the location of gears is considered to change in two directions (height and length).

In this paper, the optimum volume/weight of a gear train is investigated that the location of gears is varied in 3-dimensional direction (height, length, and width). In this point of view, the presented gearbox in this paper has the lowest possible weight for gear trains. The effective constraints are divided into 3 kinds of variables: geometrical, design and control constraints. To optimize the problem, particle swarm optimization (PSO) algorithm is used. By using fitness function and constraints a code is developed that named 3DGO_PSO (3D Global Optimization_ Particle Swarm Optimization) to solve and optimize the problem. The algorithm optimized (minimize) the weight of the gearbox and presented the location of gears in gear train, number of teeth, module, the width of gear and helical angle for each gear. The optimum parameters for the gearbox are presented as practical graphs for use. Finally, an example is presented to show how to use the graph and obtain the best parameters for each gearbox. The demonstrated results are validated by comparing the results with those reported in previous works.

## 2. Mathematical Model: Objective Function

Working on the optimization design of gearbox has been more attractive for researchers. In Gear trains, the location of gears can affect the minimum volume of the gearbox. The volume of a gearbox is the outcome of multiplying the length (L), width (W) and height (H) of the gearbox. In this paper, the location of gears changes along with the three directions in the gearbox and so, after implementation of the optimization algorithm the lowest possible volume/weight for the gearbox is identified.

It should be considered that the volume of gearbox depends on the layout of the gears, therefore a suitable layout provides a compact gearbox. Figure 1 illustrates the optimum layout of gears in a gear train.

To find the optimum volume/weight for the gearbox, the objective function must be specified first. But because all components in gearbox have almost identical density, the volume/weight function for the optimization process can be changed to the volume of all components of the gearbox. This volume of gearbox material is the sum of the volume of shell, shafts, and gears:

$$\text{Volume of Material} = V_{shell} + V_{gears} + V_{shafts} \tag{1}$$

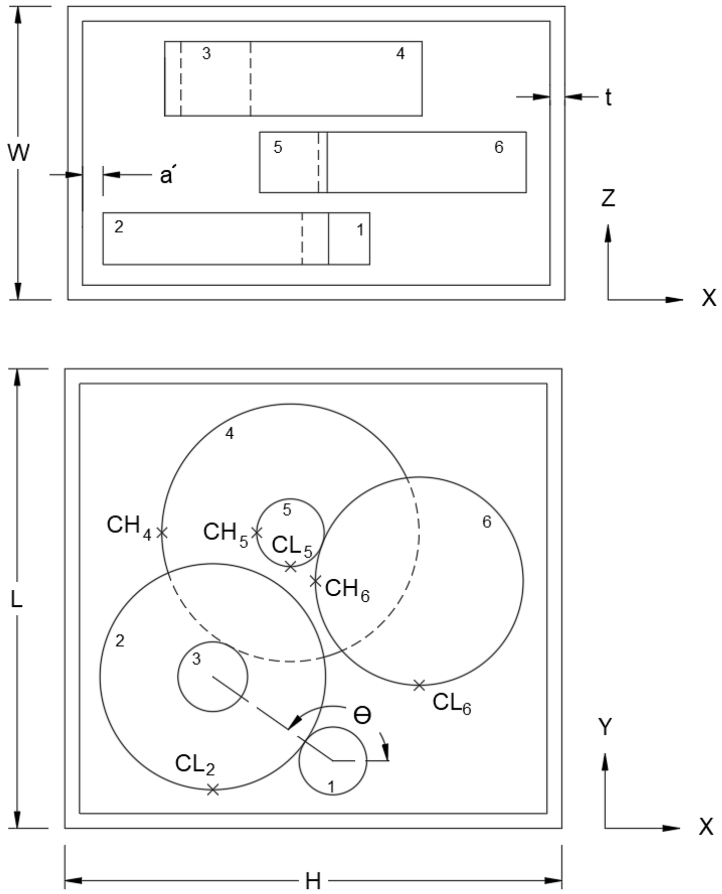


**Fig. 1.** The layout of gears in gear train

and the considering volumes are presented as:

$$\text{Volume of shell} = (W \times H \times L) - [(W-2t)\times(H-2t)\times(L-2t)] \tag{2}$$

$$\text{Volume of shafts} = \frac{\pi.d_1^2}{4}\times(L_{in}+W)+\sum_{i=2}^{n-1}\left(W\times\frac{\pi.d_i^2}{4}\right)+\frac{\pi.d_n^2}{4}\times(L_{out}+W) \tag{3}$$

$$\text{Volume of gears} = \sum_{i=1}^{2s} b_i \times \frac{\pi.\left(d_i^g\right)^2}{4} - \sum_{i=2}^{n-1}(b_{2i-1}+b_{2i-2})\times\frac{\pi.d_i^2}{4} - b_{2s}\times\frac{\pi.d_n^2}{4} - b_1\times\frac{\pi.d_1^2}{4} \tag{4}$$

### 2.1. Calculating the width of the gear train (W)

According to figure 1, the width of the gear train can be expressed as:

$$W = 2a' + 2t + \frac{b_{firs\ gear}}{2} + \frac{b_{end\ gear}}{2} + z \tag{5}$$

where "z" is the distance between the center of first and end gear in the z-direction as shown in Fig. 1.

### 2.2. Calculating the height of gear train "H"

The height of the gear train as shown in figure 1 can be obtained as:

$$H = diff_H + 2a' + 2t \tag{6}$$

where, $diff_H$ is the difference between the top point of the gears and the lowest point of the gears in gear train as:

$$diff_H = \max(P_h) - \min(P_h) \tag{7}$$

and $P_h$ for all gears obtained from:

$$P_h(2i) = \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Cos}(\theta_i)+a_i \pm \left(C_h(2i)\right) = \\ \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Cos}(\theta_i)+\left(r_{2i-1}+r_{2i}\right)\mathrm{Cos}(\theta_i)\pm\left(C_h(2i)\right) \qquad (i=1,2,\ldots,s) \tag{8}$$

$$P_h(2i-1) = \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Cos}(\theta_i)+a_i \pm \left(C_h(2i-1)\right) = \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Cos}(\theta_i) \\ +\left(r_{2i-1}+r_{2i}\right)\mathrm{Cos}(\theta_i)\pm\left(C_h(2i-1)\right) \qquad (i=1,2,\ldots,s) \tag{9}$$

In equations 8 and 9, the term "$C_h$" is the edge of each gear in the “x” direction according to Fig. 1.

### 2.3. Calculating the length of gear train “L”

The length of the gear train obtained:

$$L = diff_L + 2a' + 2t \tag{10}$$

where "$diff_L$" is the difference between the first point of the first gear and endpoint of the end gear along the “y” direction and presented as:

$$diff_L = \max(P_l) - \min(P_l) \tag{11}$$

and, $P_l$ for all gears in gear train obtained from:

$$P_l(2i) = \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Sin}(\theta_i)+z_i \pm \left(C_l(2i)\right) = \\ \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Sin}(\theta_i)+\left(r_{2i-1}+r_{2i}\right)\mathrm{Sin}(\theta_i)\pm\left(C_l(2i)\right) \quad (i=1,2,\ldots,s) \tag{12}$$

$$P_l(2i-1) = \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Sin}(\theta_i)+z_i \pm \left(C_l(2i-1)\right) = \\ \sum_{i=2}^{s}\left(r_{2i-3}+r_{2i-2}\right)\mathrm{Sin}(\theta_i)+\left(r_{2i-1}+r_{2i}\right)\mathrm{Sin}(\theta_i)\pm\left(C_l(2i-1)\right) \quad (i=1,2,\ldots,s) \tag{13}$$

where in Eqs. 12 and 13, "$C_l$" is the edge of each gear in the “y” direction as shown in Fig. 1.

## 3. Mathematical Model: Constraints

To find the optimum volume/weight for the gear train, the necessary constraint must be considered. The constraints affect the optimum values of volume/weight of gear train, divided into three types of constraints as geometrical, design and control parameters constraints.

### 3.1. Geometrical Constraints

A geometrical constraint is defined to avoid the clashes. So, a geometrical constraint should control the area that there is the possibility of the clash between the gear and the next shaft in each stage (as shown in Fig. 2) or the clash between the gears as shown in Fig. 3. The minimum possible distance for all of the gears in each stage in separate planes can be written as:

$$r_{2i}\,\mathrm{Cos}(\theta) < r_{2i+2}\,\mathrm{Cos}(\theta) + r_{2i+1} + a' \tag{14}$$

Also, to optimize the location of gears, a constraint is considered to search if the gears can place along with the location of previous gears in gear train. This constraint formulated in equation 15 and show in Fig. 2.

$$C_{ij} > \frac{d^p_{i-1}}{2} + \frac{d^g_{j-1}}{2} \tag{15}$$

$$r_{2i+3} + r_{2i} \operatorname{Cos}(\theta) < r_{2i+2} \operatorname{Cos}(\theta) + r_{2i+1} + C \tag{16}$$

where, "$i$" and "$j$" are the paired gear.

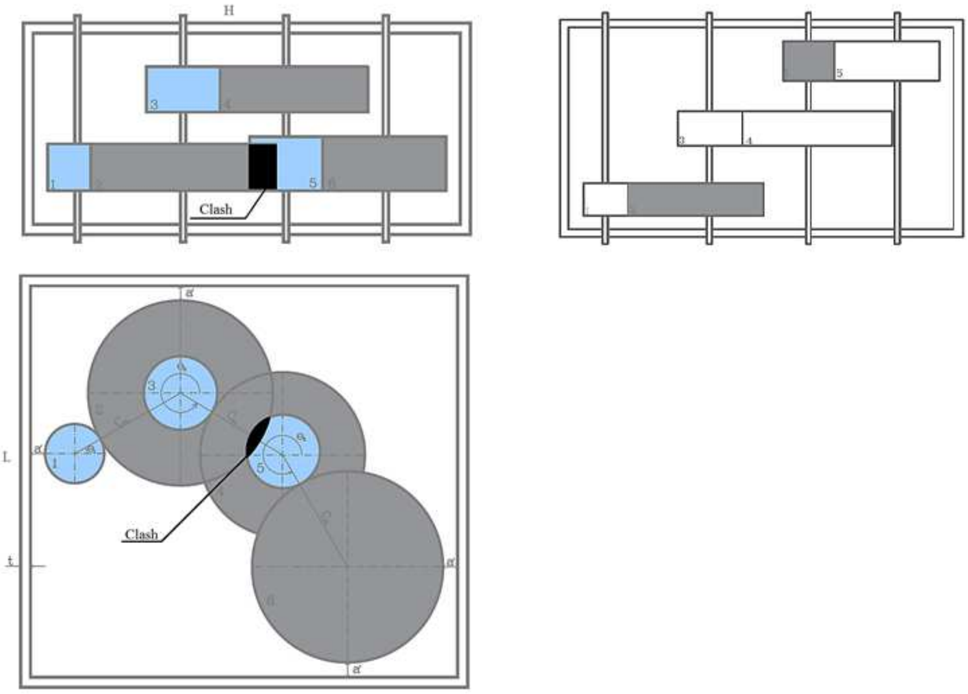


**Fig. 2.** Arranged gear position in gear train in the same or different plane

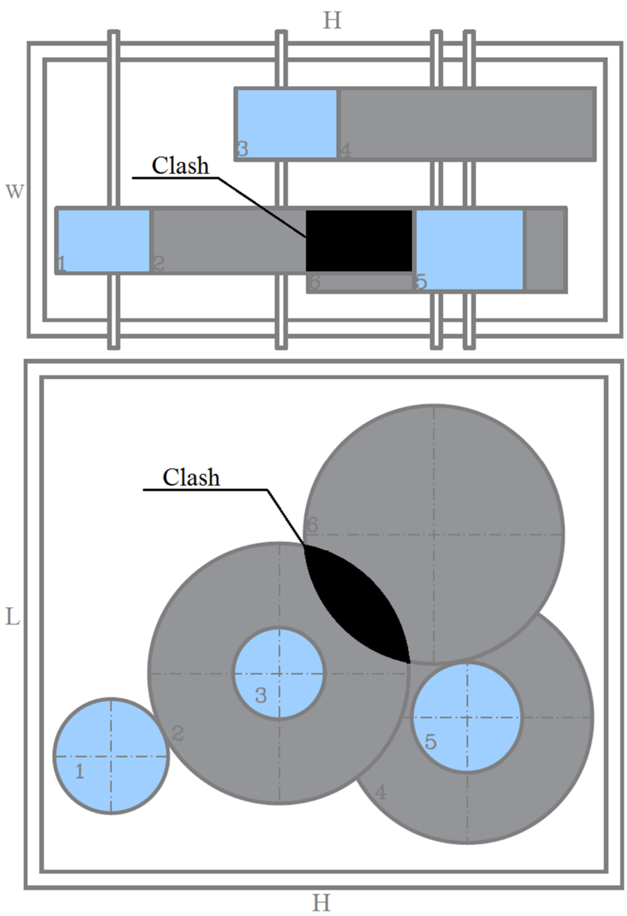


**Fig. 3.** Possible interface (clash) between non-paired in the separate plane (3D optimization)

### 3.2. Design constraints

The design constraints for the gearbox have been divided into three parts including bending strength and pitting resistance for each gear and the strength of shafts. All of the design constraints are indicated in Table 1 [32].

**Table 1.** Design Constraints

| | | |
|---|---|---|
| Diameter constraint | $d \geq \left\{ \left[ \left( \frac{M_a}{S_e} \right)^2 + \left( \frac{T_m}{S_y} \right)^2 \right]^{\frac{1}{2}} \cdot \frac{32 S_{FS}}{\pi} \right\}^{\frac{1}{3}}$ | (17) |
| $\sigma_{Bending} \leq \sigma_{Allowable(Bending)}$ | $K_V . F_t . K_s . K_o . \frac{K_B . K_H}{Y_J . m_t . b} < \frac{Y_N}{Y_z . Y_\theta} . \frac{\sigma_{FP}}{S_F}$ | (18) |
| $\sigma_{Contact} \leq \sigma_{Allowable(Contact)}$ | $\left( K_V . F_t . K_s . K_o . \frac{Z_R}{Z_I} . \frac{K_H}{b . d_{wl}} \right)^{\frac{1}{2}} . Z_E < \frac{Z_W . Z_N . \sigma_{HP}}{Y_z . Y_\theta . S_H}$ | (19) |

### 3.3. Control parameter constraints

Control parameter constraints are presented in table 2. These constraints affect the optimization process to achieve the initial value consideration for the gear train and satisfy the initial assumptions for gear train such as considering the ratio or modulus selecting [33]:

**Table 2.** Control Parameter Constraints

| | | |
|---|---|---|
| Minimum teeth | $N^p \geq \frac{2 \mathrm{Cos}(\delta_{2i-1})}{\left( 2 \left( \frac{N^g}{N^p} \right) + 1 \right) \mathrm{Sin}^2(\varphi_t)} \left( \sqrt{\left( 2 \left( \frac{N^g}{N^p} \right) + 1 \right) \mathrm{Sin}^2(\varphi_t) + \left( \frac{N^g}{N^p} \right)^2} + \frac{N^g}{N^p} \right)$ | (20) |
| Overall ratio | $R_e = \Pi_{i=1}^{s-1} \frac{M_{2i-1}}{M_{2i}}$ | (21) |
| Reduction the ratio of the gear train | $N^P \leq N^g$ | (22) |
| Gear face width | $3.\pi.m_t \leq F \leq 5.\pi.m_t$ | (23) |
| Modulus constant number | $1 \leq m^p \leq 50$ | (24) |
| The constraint for modulus o | $m^g = m^p$ | (25) |

## 4. Optimization process

To optimize the gear train volume/weight with previous consideration for objective function and constraints, the particle swarm optimization algorithm (PSO) is used. PSO method is the pivotal entry into a computation technique that used meta-heuristic according to stochastic optimization that used the behavior of the population. The PSO algorithm was introduced by Eberhart and Kenney [34] and this algorithm used the social behavior of birds or fishes to find the optimum point of the problem like food.

In the PSO algorithm, each bird (particle) wanders in the problem space randomly. They are potential solutions and assumed the position of particles, Velocity, and final fitness function. Each particle randomly searching to find a piece of food in problem space and they have the same question that where the target or food is but in each iteration the particles just know how far the targets or foods are in space. One particle that is nearest to the target or food is effective to follow so for converging to optimum target each particle updating generations. At the initial step, all particle has a random position and zero velocity. Then, based on the best location for particles the velocity update and the particles move to a new location. The new velocity and new location for each particle updates in each iteration so that all particles arrive and converge to one point (optimum point). During the search process in problem space, the best location indeed best fitness value in each iteration for each particle ($P_{best}$) and the best location for all particles is historical best value that is maximum food source or value of fitness function obtained ($G_{best}$) save and use for next iteration [34]. Two best values ($P_{best}$ and $G_{best}$) used for updating velocity and position vectors for any of the N particles in population. Particle velocity obtained from the way each N particles move all over in problem space. That consists of three terms: in the first described the inertia or momentum prohibits the particle from an extremely changing direction. The second called the self (individual) intelligence that is tendency of particles toward their own best locations in each particle's memory. At the last, named the social (group) intelligence, denotes the particle steers to move towards the general (global) best situation (location) of the whole population. In this paper to optimize the considering problem, a code based on PSO algorithm was developed called 3DGO-PSO. The fitness function and all constraints are considered in the 3DGO-PSO to solve and find the best values for gear train parameters. The implementation steps of 3DGO-PSO are presented in Fig. 4.

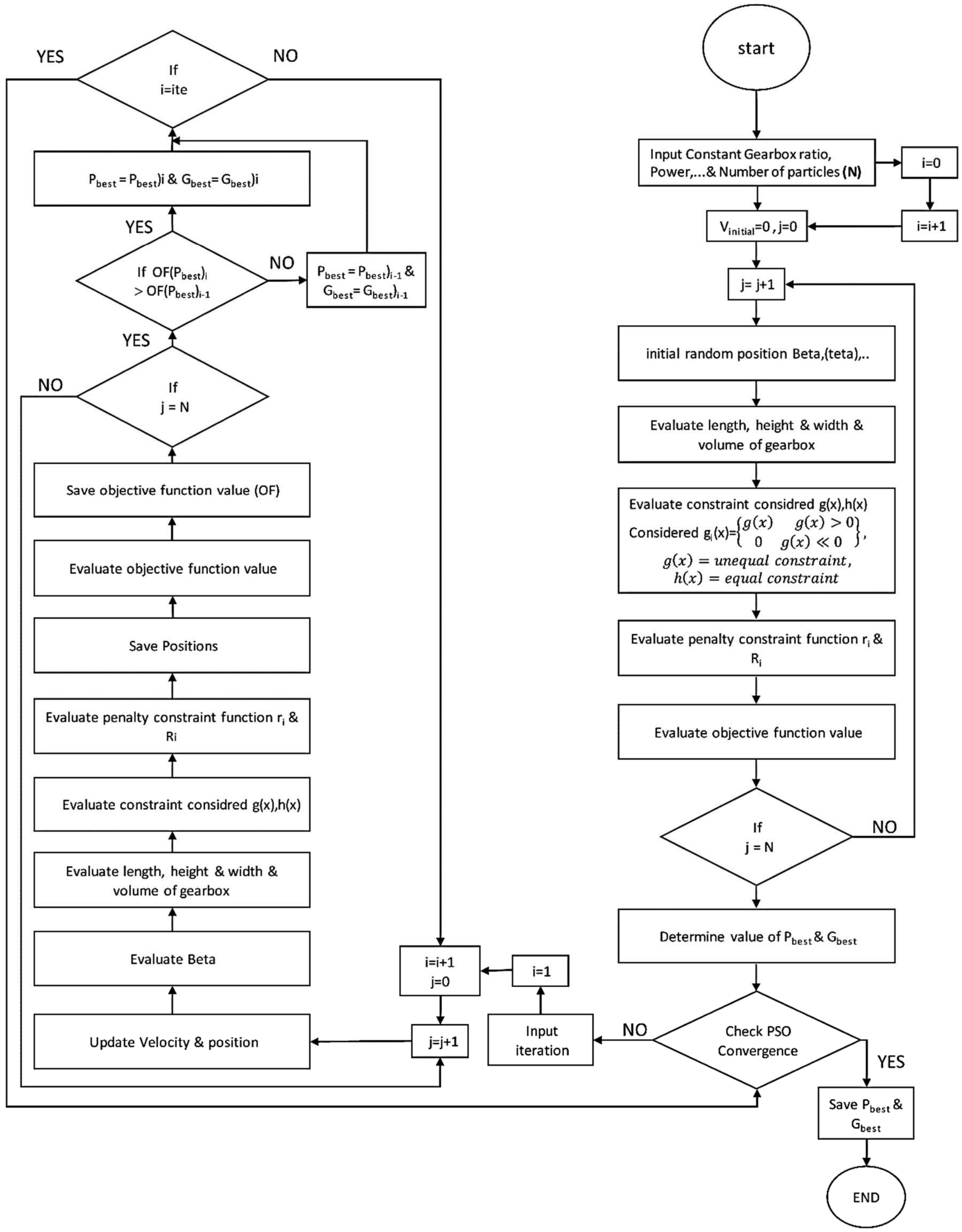


**Fig. 4.** Implementation PSO flowchart

For using the objective function equation (1) and the constrains together, the constraints added to the objective function as penalty functions. So if considering the objective function "V" and the equality constraints as $H_i = 0,\ \ i = 1,2,...n$ and the inequality constraints as $G_j \le 0,\ j = 1,2,...m$ and considering that the PSO algorithm tries to maximize the function, the main objective function in the program presented as:

$$\text{main objective function} = \frac{1}{1+\varphi} \tag{26}$$

where,

$$\varphi = V + \sum_{i=1}^{n} H_i + \sum_{j=1}^{m} g_j \tag{27}$$

and

$$g_j = \max\left\{0,\ G_j\right\}, \quad j = 1,2,\ldots,m \tag{28}$$

## 5. Results and Example

In this paper, for presenting the results, various input parameters for the gear train are considered and explained in 5 steps.

Step 1: The input parameters are presented in Table 3. Using these parameters for the gearbox, the 3DGO-PSO solve the problems and find the best values for all components of the gear train and all values are presented as a pra ctical graph for gear train.

Step 2: To use the practical graph, Fig. 6 presented a flowchart for selecting the best values for gear train parameters. According to Fig. 6, at the first, the number of stages, comparison between second and third stages parameters (50 hp ≤ Power ≤ 200 hp) is extracted for considering the transmission power and overall ratio for the gearbox as shown in Figs. 7. Then the reduction ratio for each stage of the three-stage gear train obtains from Figs. 8. Next Figs. 9 to 33, show the optimum parameters for each stage of two and three-stage gear train.

**Table 3.** Elected Specific input data

| Input parameters | Transmission power (hp) | Hardness of material (BHN) | Gear train ratio |
|---|---|---|---|
| Elected Specific values | 2, 5, 10, 20, 30, 50, 80, 100, 150, 200 | 200, 300, 400 | 1.5, 2, 3, 5, 8, 10, 15, 20, 40, 50 |

**Table 4.** Input data for applicable example consideration with Golabi et al. [31]

| Input parameters Example | Transmission power (hp) | Hardness of material (BHN) | Gear train ratio |
|---|---|---|---|
| Example input data | 150 | 400 | 15 |

**Table 5.** Comparison between results obtained from the presented paper and previous publication by Golabi et al. [31]

| Total Ratio=15, Power=150 hp, hardness of material=400 BHN | | | |
|---|---|---|---|
| Description | | Ref [24] | Presented Research |
| u1 | First Stage | 2.6 | 2.7 |
| u2 | Second Stage | 2.6 | 2.6 |
| u3 | Third stage | 2.2 | 2.2 |
| Module | First Stage | 5 | 6 |
| | Second Stage | 8 | 9 |
| | Third stage | 10 | 8 |
| Face Width (mm) | First Stage | 51 | 77 |
| | Second Stage | 82 | 81 |
| | Third stage | 170 | 112 |
| Shaft Diameter (mm) | First Stage | 35 | 33 |
| | Second Stage | 43 | 41 |
| | Third stage | 125, 72 | 87, 68 |
| Gear Position Angle | First Stage | - | 130 |
| | Second Stage | - | 240 |
| | Third stage | - | 210 |
| Volume (mm^3) | | 2.6 e7 | 2.2 e7 |
| Difference | | | -15 % |

Step 3: In order to explain how to use the practical graph, an example of selecting the best parameters for a special gearbox is presented.

Step 4: But the considering example is the same as considered in reference [31] to compare and validate the results of this paper and those reported in reference [31].

The input data for considering the example are presented in Table 4. As mentioned above, by using a flowchart from Fig.

5, the best number for reduction stages are extracted from Figs. 9 as 3 stages gearbox. Next, the partial ratio for gear train achieved from Fig. 13. Finally, the optimal values for the considering gear train are obtained from Figs. 39 to 44 (see the Appendix).

The final step (validate): The results obtained from the practical graph in this paper and those obtained from reference [31] are presented in Table 5. Golabi et al. [31] worked on optimization weight/volume of the multistage gear train in 2 directions (length and width). But in this paper the optimization process is implemented in 3 directions (length, with and height of gear train). The results presented in this paper show that the optimum volume has about 15% less volume than the volume reported by Golabi et al. [31]. Figure 5 illustrated the trend of the fitness function during the optimization process. The computational process has been done with the Intel Core i7-3216QM CPU@ 2.10GHZ 2.10 GHz, 8.00 Gigabytes of RAM, and the solving process has been taken about 61000 seconds for optimization. The convergence for the PSO algorithm is that the changes in volume do not exceed 1 $cm^3$ for 1000 iteration.

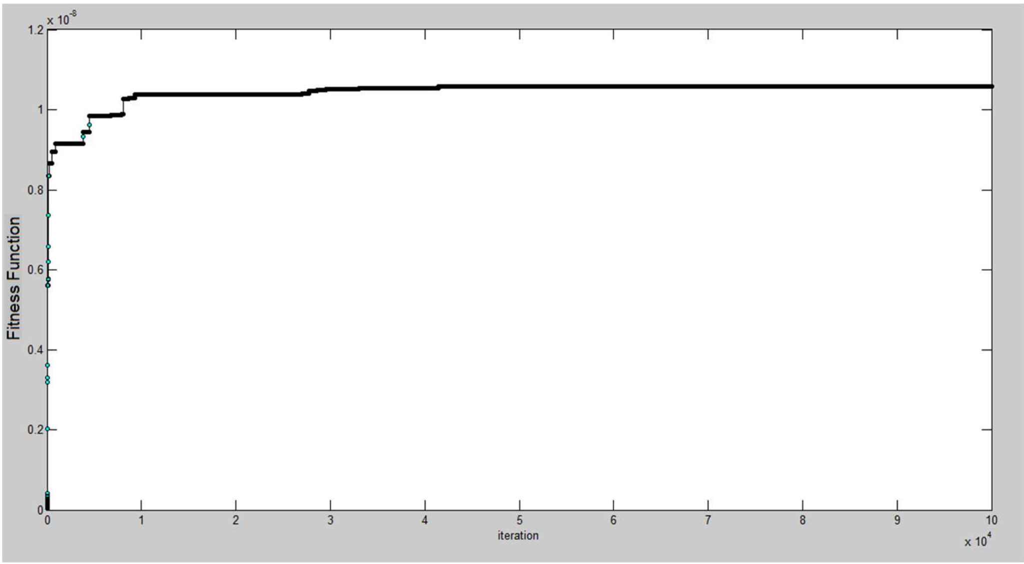


**Fig. 5.** Trend of fitness during the optimization

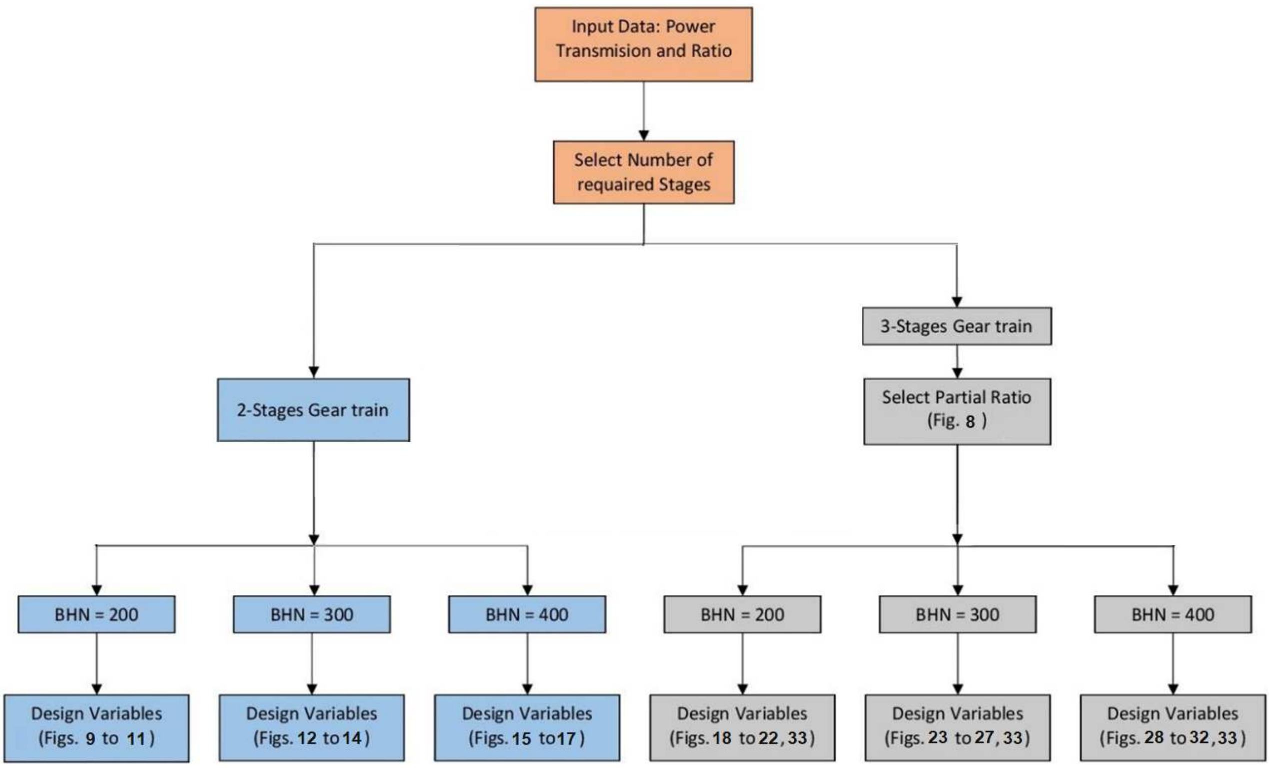


**Fig. 6.** Flowchart to define design parameters in practical curves

## 6. Conclusion

The domain of engineering challenges is broad and has different variables consisted of linear and nonlinear constraints and equations each of which researchers studied heuristic methods to solve them. heuristic methods don't need derivatives of the functions and are substantial methods compared with the traditional methods proceeding to the best global solution. Garg, H.[35,36] used heuristic methods with soft computing to analyze the Performance of industrial system. In this paper, Particle swarm optimization method is employed for the 3D dimensional and layout optimization process of a multi-stage gearbox to achieve minimum weight/volume. To attain the optimal weight/volume of the gearbox, the volume of material of gearbox components is considered as the objective function and the necessary constraints are considered for the optimization process. Highlighted features in this paper consist of, Firstly, minimum volume/weight objective function in the 3D general form for gear train is offered. Secondly, Suitable Practical curves are appropriate for one until three-stages gear trains for obtaining the best gearbox parameters are introduced. Thirdly, Optimization processes implemented on two and three-stage gears train by selecting different input data include gear ratio, power and hardness, Number of best stages, gear position angle, modulus, shaft diameter and face width for gears are offered for gearbox. The considering constraints divided into three types as a geometrical constraint, design constraints and control parameter constraints. The optimization process implemented on one, two and three stages for gearbox and all gear trains optimized in 3 direction length, width, and height. The input parameters for gear train optimization are included as gear ratio, power, and strength of materials. Then the practical graphs are extracted from optimization results to achieve the minimum weight/volume of the gearbox. Value of optimum weight/volume and all the necessary design parameters of gearbox such as the position of gears, face width of gears, number of stages, shaft diameter and module of gears are presented as a practical graph. In the final, an example is presented to show how to use the graphs and the results are validating with those reported in the previous publication. The results show that optimization the gearbox in three directions can reduce the volume of material of gearbox components and lead to a smaller gearbox.

## Author Contributions

Mehrdad Hoseiniasl conducted the experiments and analyzed the empirical results and Javad Jafari Fesharaki planned the scheme, initiated the project and suggested the experiments. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

## Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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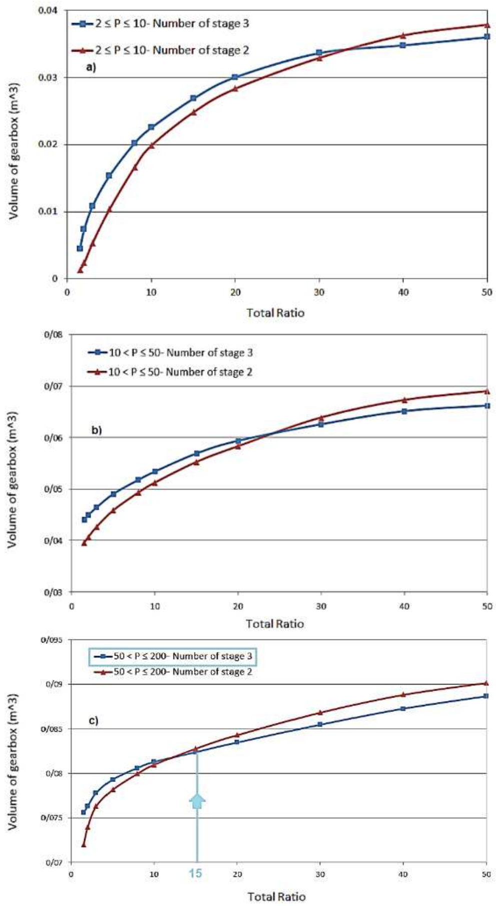


**Fig. 7.** The optimum number of stages, the comparison between second and third stages parameters, a) 2 hp ≤ Power ≤ 10 hp, b) 10 hp ≤ Power ≤ 50 hp, c) 50 hp ≤ Power ≤ 200 hp

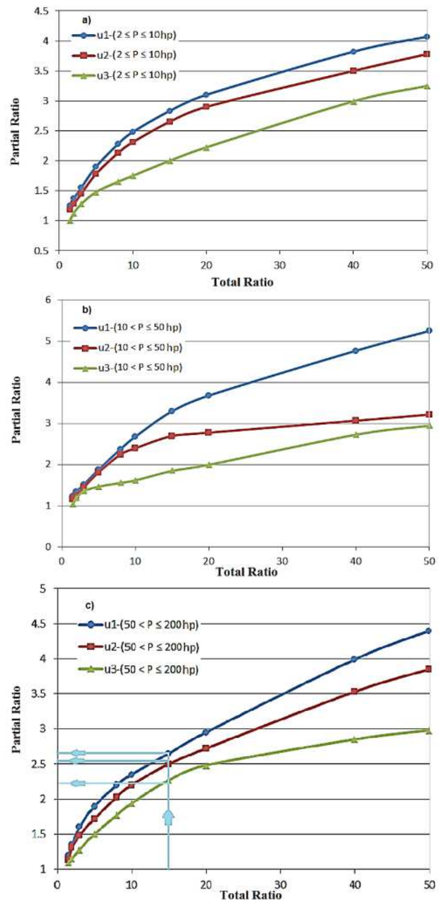


**Fig. 8.** The optimum partial ratio of third-stage, a) 2 hp ≤ Power ≤ 10 hp, b) 10 hp ≤ Power ≤ 50 hp, c) 50 hp ≤ Power ≤ 200 hp

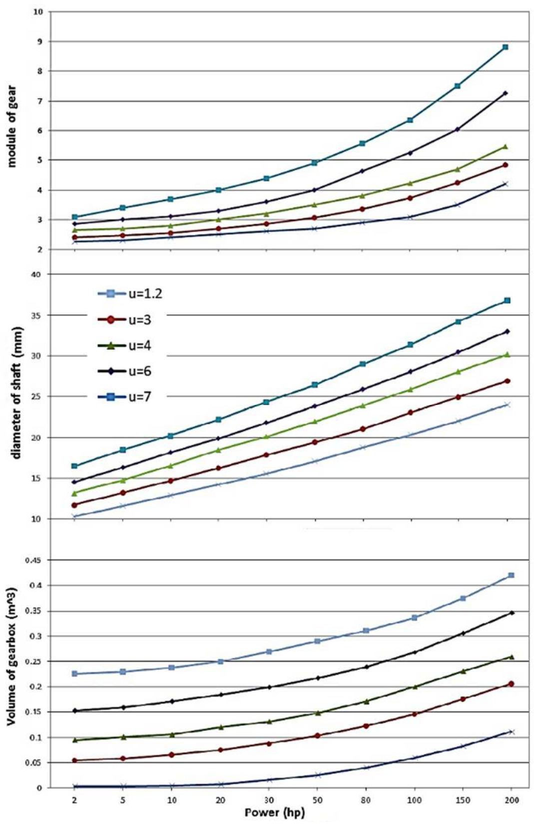


**Fig. 9.** Parameters of optimum gear train for 2- stage BHN = 200 - (stage-one)

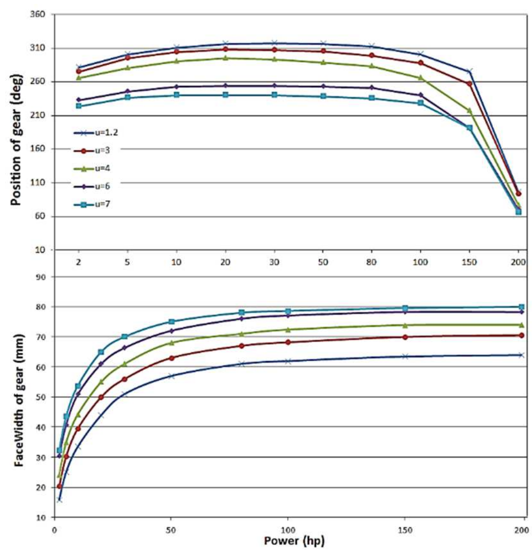


**Fig. 10.** Face width and position of gear for 2-stage gear train BHN = 200- (stage one)

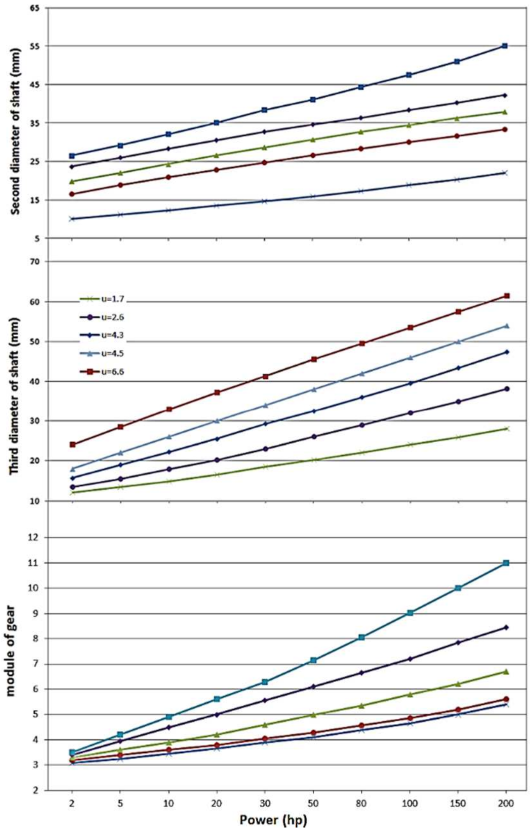


**Fig. 11.** Module and shaft diameters for 2-stage gear train BHN = 200 - (stage two)

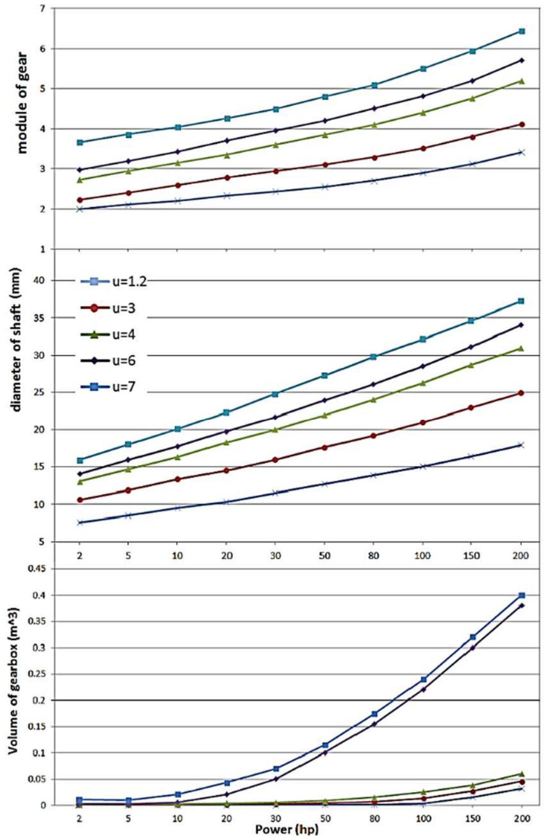


**Fig. 12.** Parameters of optimum gear train for 2- stage BHN = 300 - (stage-one)

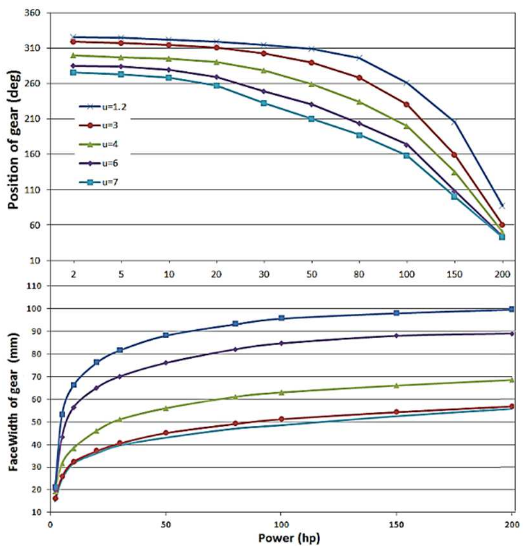


**Fig. 13.** Face width and position of gear for 2-stage gear train BHN = 300 - (stage one)

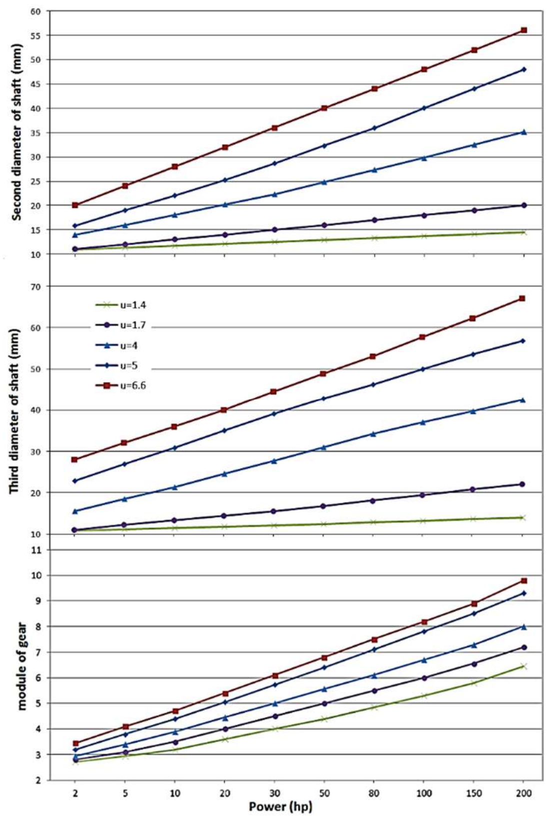


**Fig. 14.** Module and shaft diameters for 2-stage gear train BHN = 300 – (stage two)

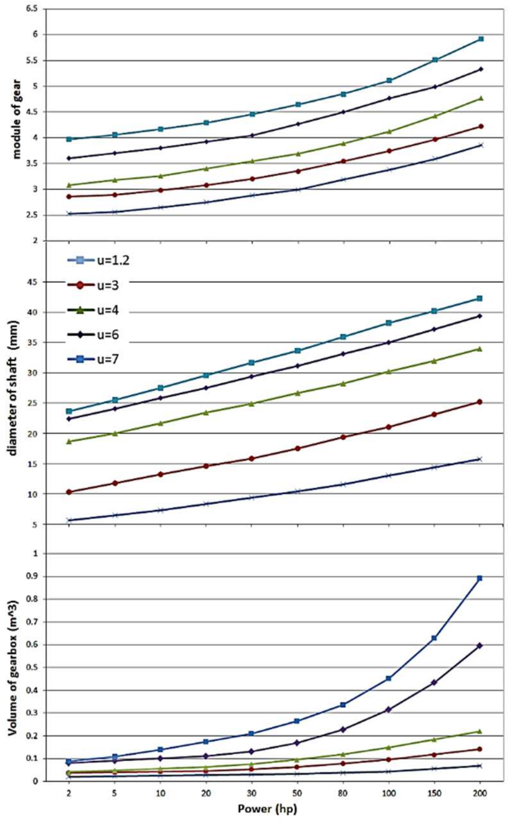


Fig. 15. Parameters of optimum gear train for 2- stage BHN = 400 - (stage one)

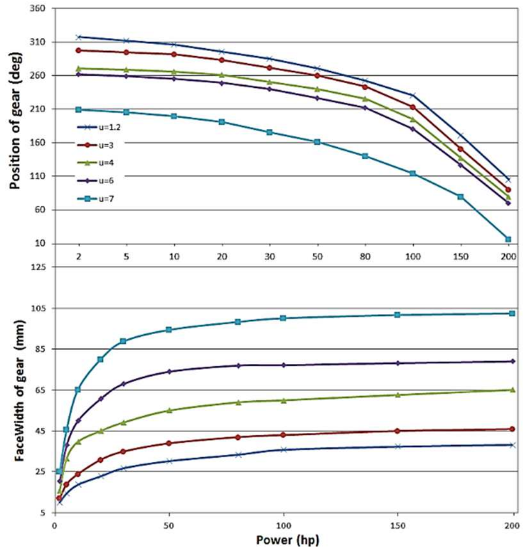


Fig. 16. Face width and position of gear for 2-stage gear train BHN= 400- (stage one)

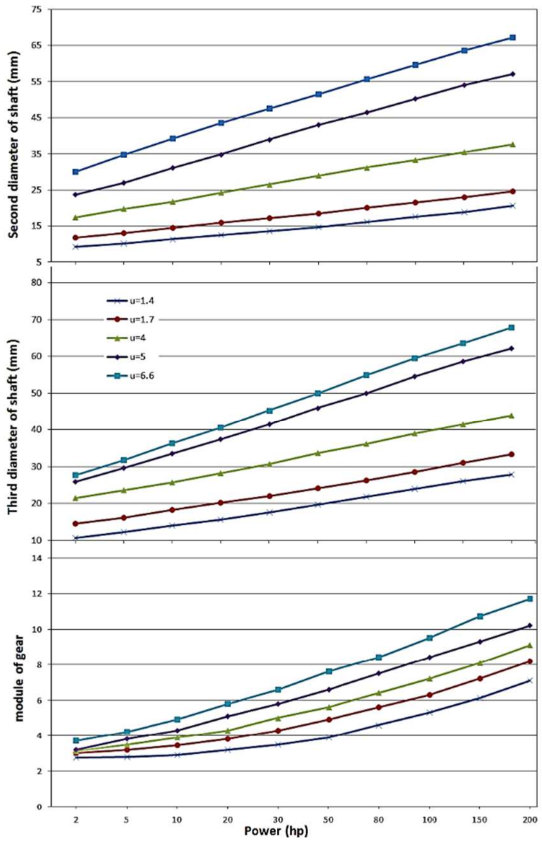


Fig. 17. Module and shaft diameters for 2-stage gear train BHN= 400- (stage two)

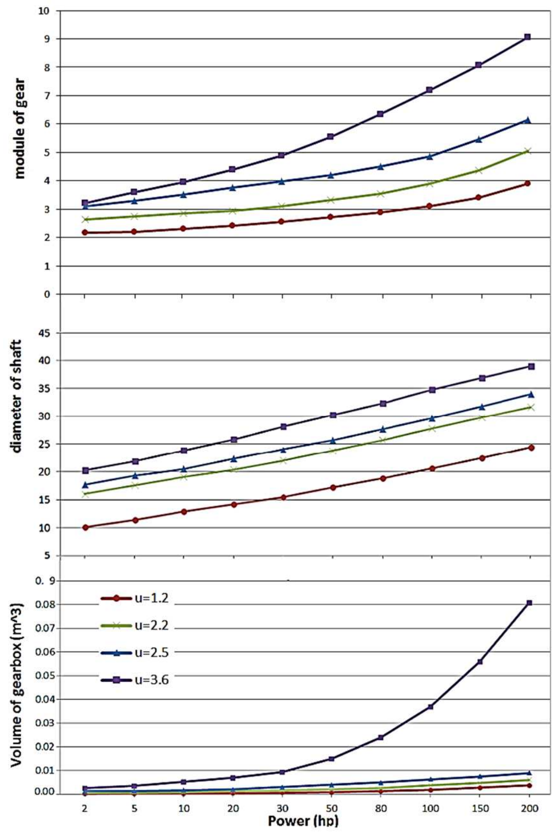


Fig. 18. Volume, shaft diameter and module for 3-stage gear train BHN = 200 – (stage one)

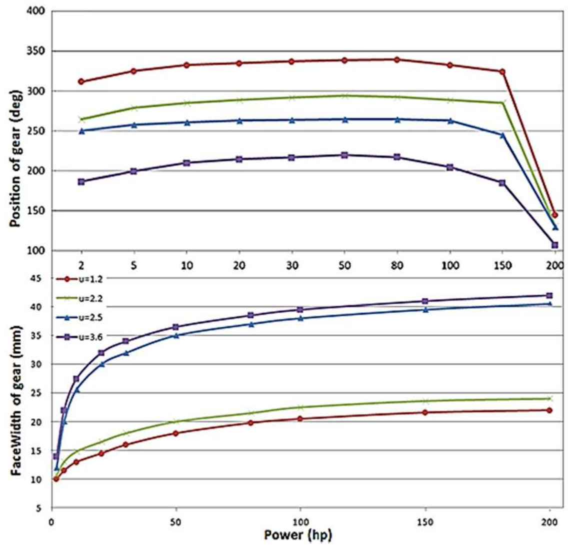


**Fig. 19.** Face width and position of gear for 3-stage gear train BHN = 200 – (stage one)

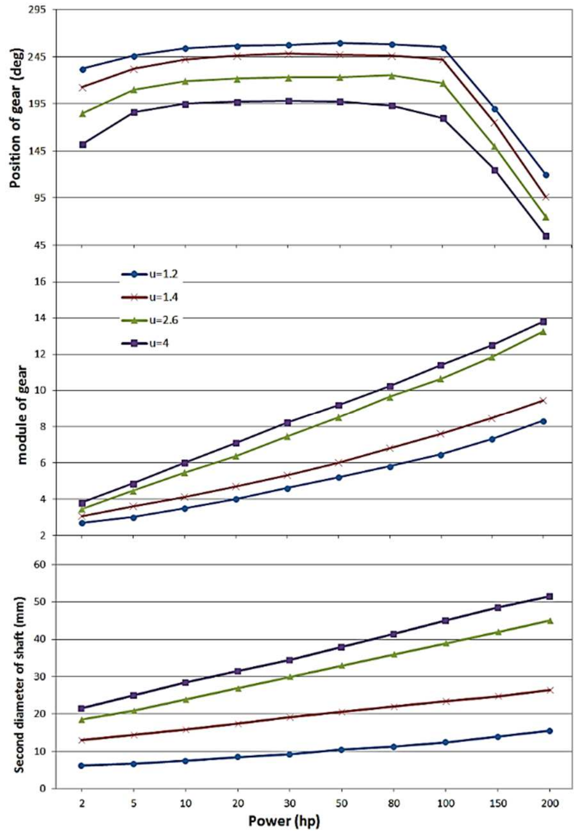


**Fig. 20.** Shaft diameter, module and position of gear for 3-stage gear train BHN = 200 – (stage two)

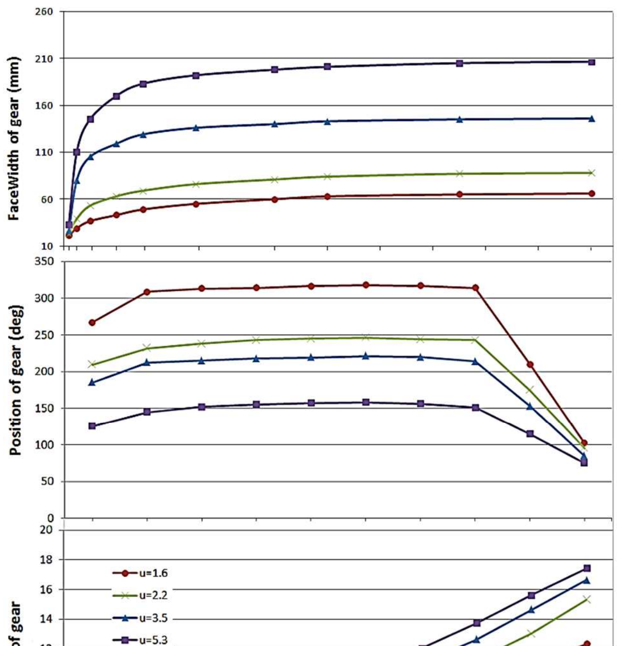


**Fig. 21.** Module, position and face width of gear for 3-stage gear train BHN = 200 – (stage three)

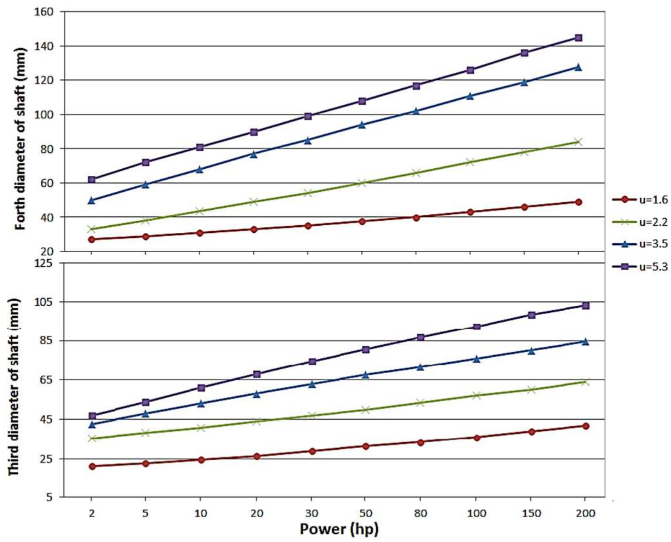


**Fig. 22.** Shaft diameters for 3-stage gear train BHN =200

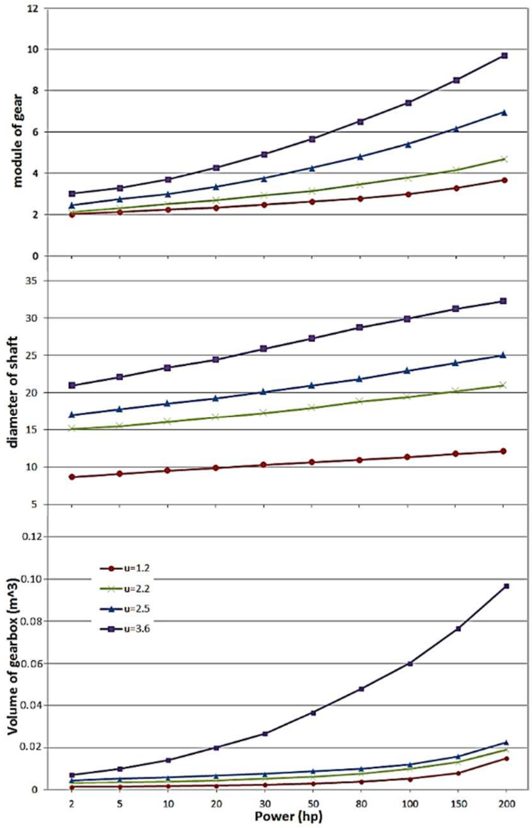


**Fig. 23.** Volume, shaft diameter and module for 3-stage gear train BHN = 300 – (stage one)

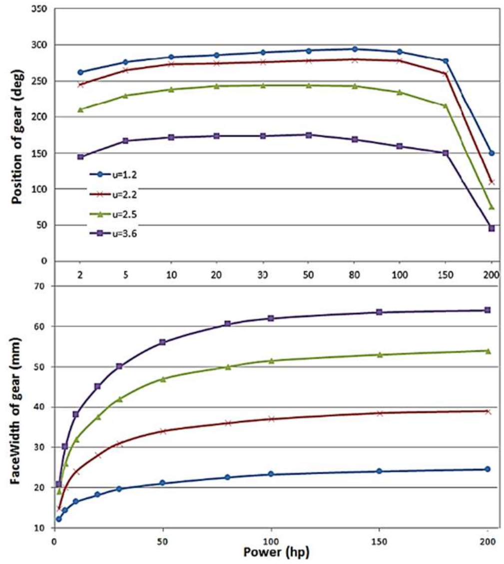


**Fig. 24.** Face width and position of gear for 3-stage gear train BHN = 300 – (stage one)

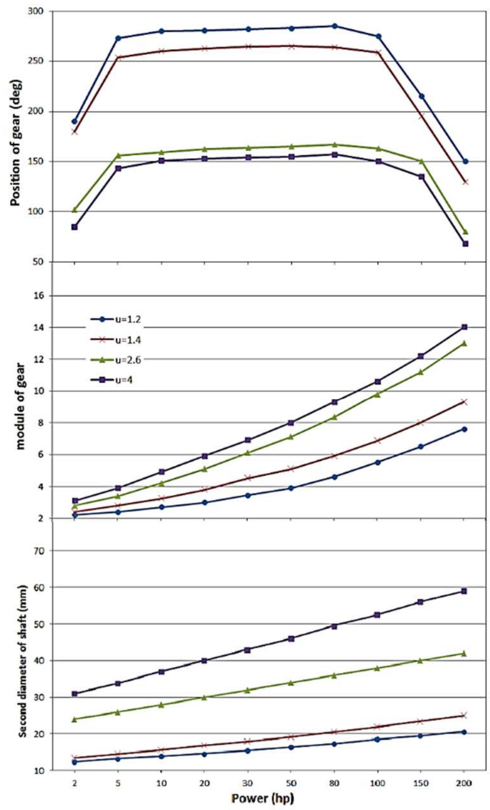


**Fig. 25.** Shaft diameter, module and position of gear for 3-stage gear train BHN = 300 – (stage two)

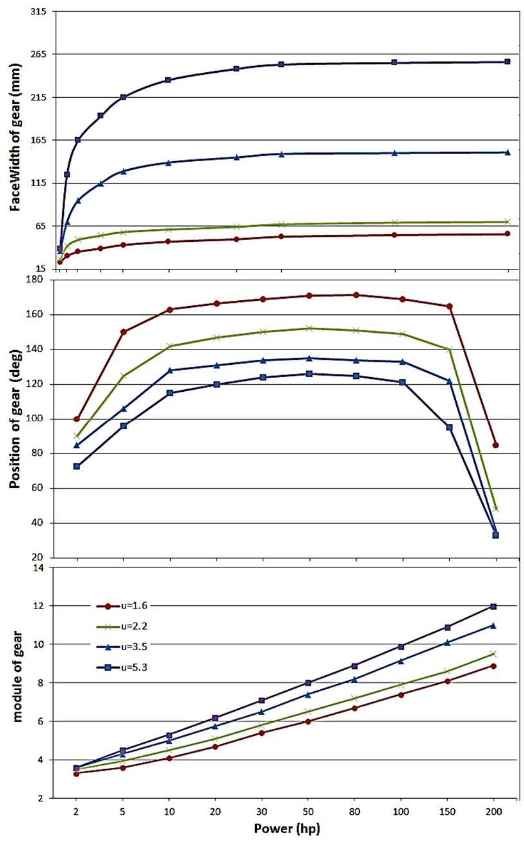


**Fig. 26.** Module, position and face width of gear for 3-stage gear train BHN = 300 – (stage three)

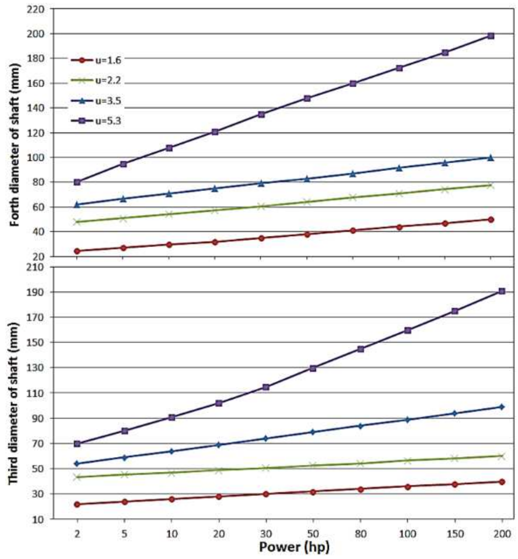


**Fig. 27.** Shaft diameters for 3-stage gear train BHN =300

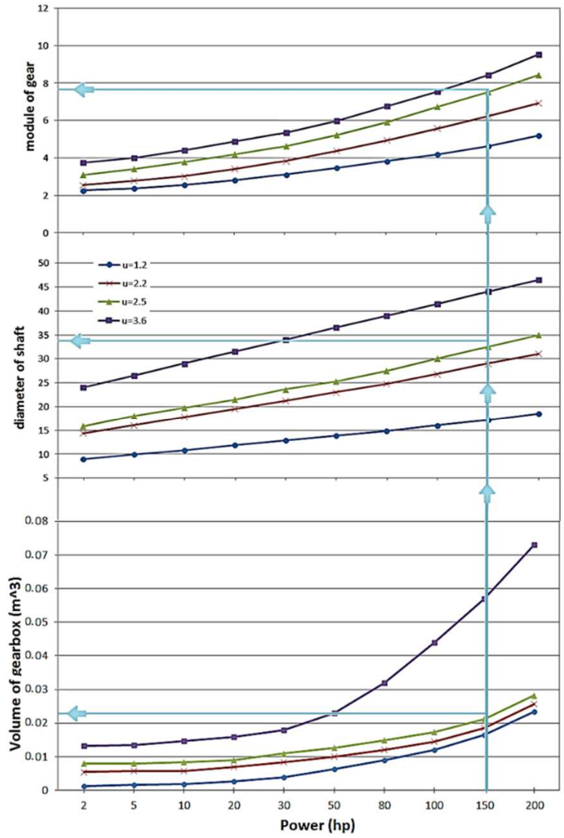


**Fig. 28.** Volume, shaft diameter and module for 3-stage gear train - BHN = 400- (stage one)

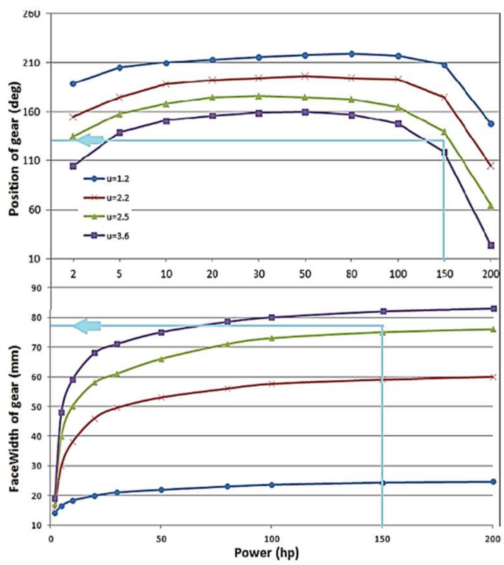


**Fig. 29.** Face width and position of gear for 3-stage gear train BHN = 400 – (stage one)

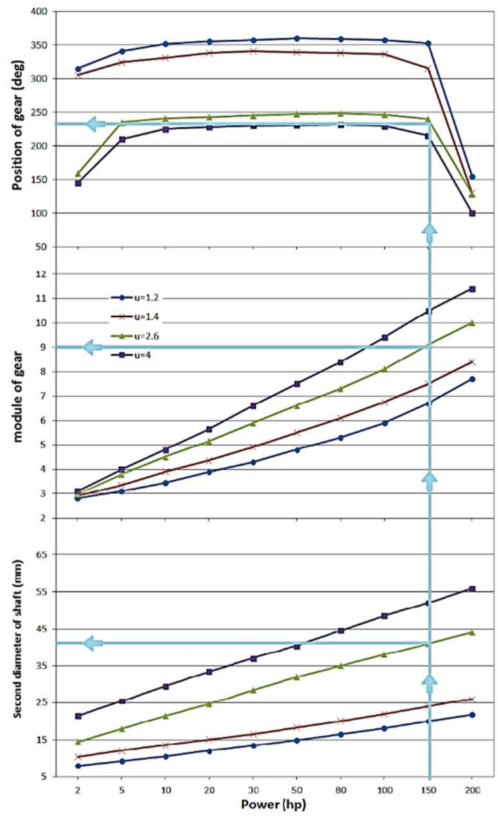


**Fig. 30.** Shaft diameter, module and position of gear for 3-stage gear train BHN = 400 – (stage two)

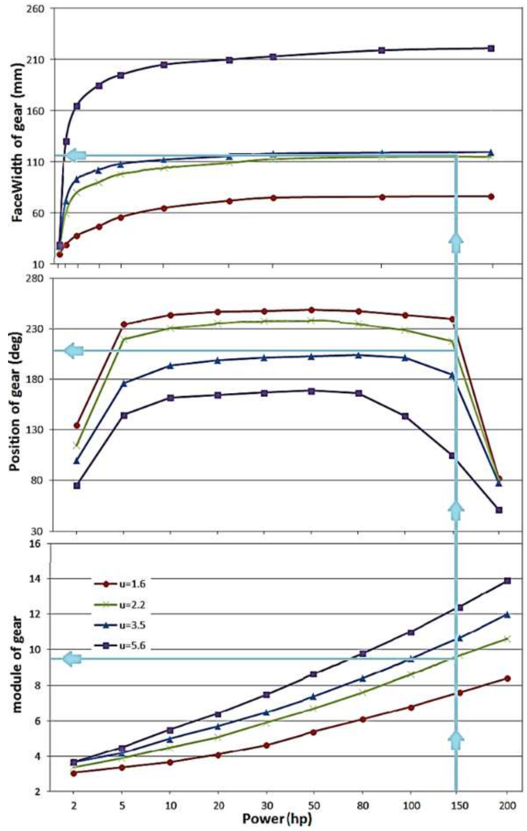


**Fig. 31.** Module, position and face width of gear for 3-stage gear train BHN = 400 – (stage three)

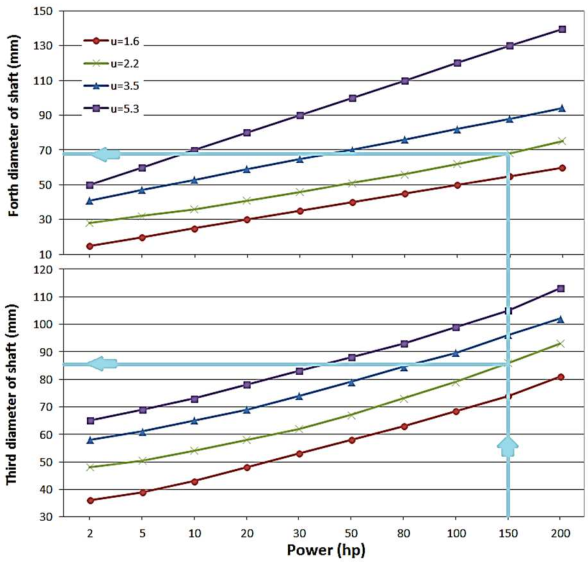


**Fig. 32.** Shaft diameters for 3-stage gear train BHN = 400

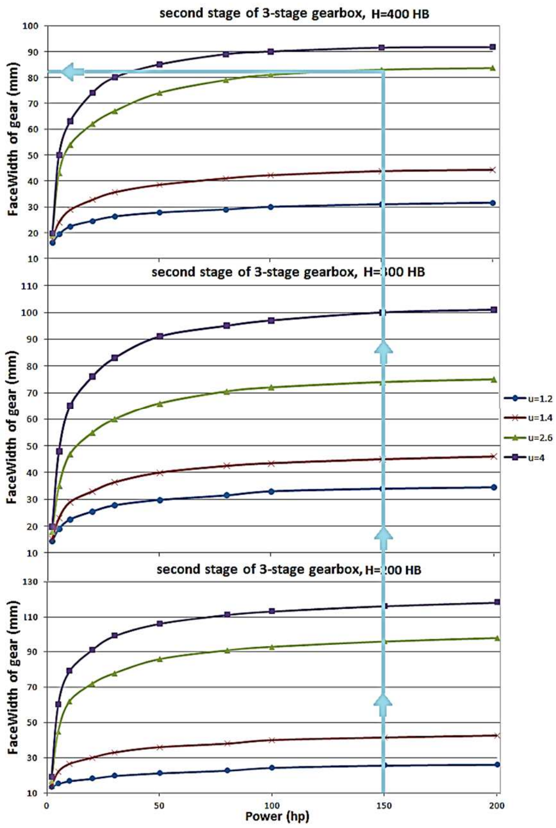


**Fig. 33.** The face width of gears for 3-stage gear train BHN =200, 300 and 400