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Research Paper

Influence of Temperature Pulse on a Nickel Microbeams under Couple Stress Theory

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Abstract. In this paper, the vibration of microbeams due to a temperature pulse has been investigated. The thermoelastic coupled equations for microbeam resonator have been derived via the modified theory of couple stress in connection with the generalized thermoelasticity with relaxation time. The analytical expressions for studied fields due to modified couple stress for the microbeam have been obtained by applying the Laplace transform method. In addition, some comparisons have been displayed in graphs to estimate the effects of different parameters such as the couple stress parameter and pulse of temperature on the considered fields. Numerical conclusions demonstrate that the estimation of deflection expected by the new theory is lower than that of the classical one. Comparisons are made with the results of different models in the absence and presence of couple stress theory. Particular cases of interest are also derived.

Keywords: Thermoelasticity, Microbeam, Temperature pulse, Couple stress theory.

1. Introduction

Micro- size mechanical resonators have great affect ability and also quick the reaction and are broadly utilized as modulators and sensors. Miniaturized scale and nano-mechanical materials have pulled in impressive consideration as of late because of their numerous imperative technological applications. The investigation of numerous influences on the attributes of resonators, for example, resounding quality and frequencies influences, is essential for designing great-performance mechanisms. Several authors have concentrated the vibration and temperature exchange procedure of beams [1-6]. Advanced apparatuses that have arisen on the foundation of micro-scale switches, modern science, the resonators, telephones, mirrors and pumps are cases of this approach [7, 8]. Many researchers investigated the Size-dependent, dynamics and stability analysis of micro-scale systems in the absence and presence of such dispersion forces. [9-13].

The traditional elasticity theory is not equipped for catching the size influence of microstructure and is suitable to study material conduct in macro-scale as it were. At the point when the size of the material under study diminishes, the exactness of the established classical theory diminishes and therefore its expectation of the material conduct in small and nano-scale doesn't concur with the results of experiments. The explanation behind such deviation is observed to be the major influence of microstructure [14, 15]. Consequently, the non-classical theories, for example, the gradient of the strain or the theories of couple stress are utilized to study the conduct of materials in these scales. Nontraditional theories contain length scale parameters which show the influence of microstructure in the conduct of the material. The theory of couple stress is observed to be the simplest sort of such models. This theory considers the rotation gradient notwithstanding the inclination of displacement, yet as found in the established model the rotation is supposed to be reliant to the displacement. The constitutive relations created for an isotropic material on the foundation of the classical theory of couple stress contain four flexible constants, i.e. two-length scale parameters and two Lame's constants [16, 17]. Kumar [18] investigated the reaction of a thermoelastic beam using the theory of modified couple stress subjected to a thermal source. Park and Gao [19] discussed the problem of a beam based on the Bernoulli-Euler model using the theory of modified couple stress.

Some researchers have studied thermoelastic interactions in micro-beams and micro-plates in the framework of non-classical continuum theory but still with the classical Fourier heat conduction law. The modified couple stress (MCS) theory is one of the main non-classical theories utilized in this regard. Analysis of TED in micro-structure based on non-classical or generalized heat conduction models but still with utilization of the classical continuum theory has been carried out by several researchers.

The current work aims to study the influence of modified couple stress theory on temperature and deflection of a microbeam in the context of Euler–Bernoulli model. The microbeam subjected to temperature pulse has been investigated based on generalized thermoelasticity theory. The solution of the basic equation has been obtained by utilizing the Laplace transform method. The analytical solutions for deflection are captured numerically for a Nickel micro-beam using MATHEMATICA software. The present model may be utilized as a part of micro-electro-mechanical applications such as transfer switches, recurrence channels, mass stream sensors, accelerometers and resonators.

2. Governing Equations

The basic equations based on couple stress isotropic thermoelastic medium developed by Hadjesfandiari and Dargush [15] without heat sources and body forces are given by:

Constitutive relations

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma\theta\delta_{ij} \tag{1}$$

The constitutive equation of the couple stress tensor will be in the form [19, 21]:

$$m_{ij} = 2\alpha\chi_{ij} \tag{2}$$

The symmetric curvature tensor in terms of rotations gradient can be written as [22]:

$$\chi_{kl} = \frac{1}{2}\left(\frac{\partial\omega_k}{\partial x_l} + \frac{\partial\omega_l}{\partial x_k}\right) \tag{3}$$

The rotation vector ω_i is related to the infinitesimal displacement vector as following [22]:

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j} \tag{4}$$

The small strain-displacement relations are,

$$e_{kl} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k}\right) \tag{5}$$

The generalized heat conduction equation proposed by Lord and Shulman (LS) will be in the form [23]:

$$\nabla(K\nabla\theta) = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left(\frac{K}{k}\frac{\partial\theta}{\partial t} + \gamma T_0\frac{\partial e_{kk}}{\partial t}\right) \tag{6}$$

where τ_0 is the thermal relaxation time. Lame's constant and shear modulus that can be defined as [22]:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)} \tag{7}$$

3. Mathematical Modeling

In order to study the effect of modified couple stress theory, we consider a thin microbeam has a length $L\ (0 \le x \le L)$, width $b\ (-b/2 \le y \le +b/2)$ and thickness $h\ (-h/2 \le z \le +h/2)$. Based on the Bernoulli-Euler beam theory, the displacements can be written as [18]:

$$u = -z\frac{\partial w}{\partial x}, \quad v = 0, \quad w = w(x,t), \tag{8}$$

where w is the transversal deflection. By substituting Eq. (8) into the Eq. (4), the rotations can be obtained as:

$$\omega_y = -\frac{\partial w}{\partial x}, \ \ \omega_x = \omega_z = 0 \tag{9}$$

Substituting Eq. (9) into Eq. (3), the components of curvature tensor can be expressed as:

$$\chi_{xy} = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2}, \quad \chi_{xz} = \chi_{zx} = \chi_{zy} = \chi_{yz} = 0 \tag{10}$$

With the help of Eq. (8), the axial stress is given by

$$\sigma_x = -E\left(z\frac{\partial^2 w}{\partial x^2} + \alpha_T\,\theta\right), \tag{11}$$

The bending moment M of the beam, can be calculated by using the relation [21]:

$$M = b\int_{-h/2}^{h/2} m_{xy}\,\mathrm{d}z + b\int_{-h/2}^{h/2} z\sigma_x\,\mathrm{d}z \tag{12}$$

By substituting Eqs. (2), (10) and (11) into Eq. (12), the bending moment of the cross-section is obtained as

$$M(x,t) = -EI\left(\frac{\partial^2 w}{\partial x^2} + \alpha_T M_T\right) - \alpha A\frac{\partial^2 w}{\partial x^2}, \tag{13}$$

where M_T is the thermal moment which is given by

$$M_T = \frac{12}{h^3}\int_{-h/2}^{h/2} \theta(x,z,t)\,z\,\mathrm{d}z. \tag{14}$$

The equation of transversely deflections of the beam can be expressed as [3]

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 w}{\partial t^2} \tag{15}$$

Substituting of the Eq. (13) into Eq. (15), the equation of motion of the beam takes the form

$$(EI + \alpha A)\frac{\partial^2 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} + \alpha_T EI\frac{\partial^2 M_T}{\partial x^2} = 0 \tag{16}$$

Introducing Eq. (8) into Eq. (6), the heat conduction equation can be expressed as:

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial z^2} = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[\frac{1}{k}\frac{\partial\theta}{\partial t} - \frac{\gamma T_0 z}{K_0}\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right] \tag{17}$$

We assume that the temperature increment is a sinusoidal variation along the thickness direction i.e.,

$$\theta(x,z,t) = \Theta(x,t)\sin(\pi z / h). \tag{18}$$

Inserting Eq. (18) into Eqs. (16) and (17), we get

$$(EI + \alpha A)\frac{\partial^2 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{24\alpha_T}{\pi^2 h}\frac{\partial^2\Theta}{\partial x^2} = 0. \tag{19}$$

$$M(x,t) = -(EI + \alpha A)\frac{\partial^2 w}{\partial x^2} - EI\frac{24\alpha_T}{\pi^2 h}\Theta, \tag{20}$$

After Multiplying Eq. (17) by zdz and integrating through the thickness of the beam from $-h/2$ to $h/2$, we get

$$\frac{\partial^2\Theta}{\partial x^2} - \frac{\pi^2}{h^2}\Theta = \left(1 + \tau_0\frac{\partial}{\partial t}\right)\left[\frac{1}{k}\frac{\partial\Theta}{\partial t} - \frac{\gamma T_0\pi^2 h}{24K_0}\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right], \tag{21}$$

We will use the following non-dimensional quantities:

$$(x',u',w',z') = \frac{1}{L}(x,u,w,z), \quad \Theta' = \frac{\gamma\Theta}{E}, \quad (t',t_0') = \frac{c}{L}(t,t_0), \quad \sigma_x' = \frac{\sigma_x}{E}, \quad M' = \frac{M}{bEh^2}, \ c = \sqrt{\frac{E}{\rho}} \tag{22}$$

Applying non-dimensional variables (22), the basic equations can be rewritten as following (dropping the primes for convenience)

$$\frac{\partial^4 w}{\partial x^4}+A_1\frac{\partial^2 w}{\partial t^2}+A_2\frac{\partial^2 \Theta}{\partial x^2}=0, \tag{23}$$

$$\frac{\partial^2 \Theta}{\partial x^2}-A_3\Theta=\left(1+\tau_0\frac{\partial}{\partial t}\right)\left[A_4\frac{\partial \Theta}{\partial t}-A_5\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial x^2}\right)\right], \tag{24}$$

$$\sigma_x=-z\frac{\partial^2 w}{\partial x^2}-\theta, \tag{25}$$

$$M(x,t)=-A_6\frac{\partial^2 w}{\partial x^2}-A_7\Theta, \tag{26}$$

where

$$\begin{gathered} A_1=\frac{\rho A c^2}{EI+\alpha A},\quad A_2=\frac{24E^2 I\alpha_T}{\pi^2 h\gamma\left(EI+\alpha A\right)},\quad A_3=\frac{\pi^2 L^2}{h^2}\ ,A_4=\frac{cL}{k}, \\ A_5=\frac{\gamma^2 T_0\pi^2 h}{24KE},A_6=\frac{\left(EI+\alpha A\right)}{bEh^2 L},\ A_7=\frac{24I\alpha_T}{\pi^2 hb}. \end{gathered} \tag{27}$$

4. Initial and Boundary Conditions

The initial conditions of the considered problem are taken as

$$w(x,t)\big|_{t=0}=\left.\frac{\partial w(x,t)}{\partial t}\right|_{t=0}=0,\quad \Theta(x,t)\big|_{t=0}=\left.\frac{\partial \Theta(x,t)}{\partial t}\right|_{t=0}=0, \tag{28}$$

To solve the considered problem, the following boundary conditions are considered. Firstly, we consider the two ends of the microbeam are clamped, i.e.

$$w(x,t)\big|_{x=0,L}=0,\qquad \left.\frac{\partial w(x,t)}{\partial x}\right|_{x=0,L}=0. \tag{29}$$

Secondary, we assume that the surface $x=0$ of the microbeam is exposed to temperature pulse in the form

$$\Theta(0,t)=\Theta_0\begin{cases}\sin(\omega t), & 0\le t\le\dfrac{\pi}{\omega}, \\ 0, & t>\dfrac{\pi}{\omega},\end{cases} \tag{30}$$

where Θ_0 is the amplitude and ω is the temperature pulse. Also, the temperature at $x=L$ satisfy the following relation

$$\frac{\partial \Theta}{\partial x}=0\ ,\ x=L. \tag{31}$$

5. Solution in the Transformed Domain

Applying the Laplace transform which defined by

$$\bar{f}(x,s)=\int_0^\infty e^{-st} f(x,t)dt, \tag{32}$$

on Eqs. (23) to (26) and using the initial conditions (28), one acquires

$$\frac{d^4\bar{w}}{dx^4}+A_1 s^2\bar{w}+A_2\frac{d^2\bar{\Theta}}{dx^2}=0, \tag{33}$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2}-B_1\right)\bar{\Theta}=-B_2\frac{\mathrm{d}^2\bar{w}}{\mathrm{d}x^2}, \tag{34}$$

$$\bar{\sigma}_x = -z\frac{d^2\bar{w}}{dx^2} - \bar{\theta}, \tag{35}$$

$$\bar{M} = -A_6\frac{d^2\bar{w}}{dx^2} - A_7\bar{\Theta}, \tag{36}$$

where

$$B_1 = A_3 + s(1+\tau_0 s)A_4,\ \ B_2 = s(1+\tau_0 s)A_5 \tag{37}$$

From Eqs. (33) and (34), we get the following differential equation for $\bar{w}$ or $\bar{\Theta}$:

$$\left[\frac{d^6}{dx^6} - A\frac{d^4}{dx^4} + B\frac{d^2}{dx^2} - C\right]\left\{\bar{w},\bar{\Theta}\right\} = 0, \tag{38}$$

where the coefficients A, B and C are given by

$$A = B_1 + A_2B_2, \quad B = s^2A_1, \quad C = A_1B_1s^2 \tag{39}$$

Introducing m_i $(i=1,2,3)$ Eq. (38) can take the form

$$[(D^2-m_1^2)(D^2-m_2^2)(D^2-m_3^2)]\left\{\bar{w},\bar{\Theta}\right\} = 0, \tag{40}$$

where $D = d/dx$ and m_1^2, m_2^2 and m_3^2 are the characteristic roots of the equation

$$m^6 - Am^4 + Bm^2 + C = 0. \tag{41}$$

The roots of Eq. (41) satisfy the well-known relations:

$$m_1^2 + m_2^2 + m_2^2 = A,\ \ m_1^2\ m_2^2 + m_2^2m_3^2 + m_3^2m_1^2 = B,\ \ m_1^2\ m_2^2\ m_2^2 = C \tag{42}$$

The solution of Eq. (40) can be written as

$$\bar{w} = \sum_{i=1}^{3}\left(L_i\cosh(m_ix) + M_i\sinh(m_ix)\right), \tag{43}$$

$$\bar{\Theta} = \sum_{i=1}^{3}\beta_i\left(L_i\cosh(m_ix) + M_i\sinh(m_ix)\right), \tag{44}$$

where L_i and M_i are some parameters depending on the parameter s and

$$\beta_i = -\frac{B_2m_i^2}{m_i^2 - B_1} \tag{45}$$

With the aid of Eqs. (43) and (44), the bending moment $\bar{M}$ is given by

$$\bar{M} = \sum_{i=1}^{3}\gamma_i\left(L_i\cosh(m_ix) + M_i\sinh(m_ix)\right), \tag{46}$$

where $\gamma_i = A_6m_i^2 + A_7\beta_i$. Also, displacement is given by

$$\bar{u} = -z\frac{\mathrm{d}\bar{w}}{\mathrm{d}x} = z\sum_{i=1}^{3}m_i\left(L_i\sinh(m_ix) + M_i\cosh(m_ix)\right). \tag{47}$$

Additionally, the strain and the stress are given by

$$\bar{e} = \frac{\mathrm{d}\bar{u}}{\mathrm{d}x} = -z\sum_{i=1}^{3}m_i^2\left(L_i\cosh(m_ix) + M_i\sinh(m_ix)\right). \tag{48}$$

$$\bar{\sigma}_x = -\sum_{i=1}^{3}\left(zm_i^2 + \beta_i\sin(\pi z/h)\right)\left(L_i\cosh(m_ix) + M_i\sinh(m_ix)\right). \tag{49}$$

In the Laplace transform domain, the boundary conditions (29) -(31) take the forms

$$\bar{w}(x,s)\big|_{x=0,L} = 0, \qquad \left.\frac{\mathrm{d}\,\bar{w}(x,s)}{\mathrm{d}\,x}\right|_{x=0,L} = 0, \tag{50}$$

$$\bar{\Theta}(x,s)\Big|_{x=0} = \frac{\omega \Theta_0}{s^2+\omega^2} = \bar{G}(s). \tag{51}$$

$$\frac{\partial \bar{\Theta}}{\partial x} = 0, \quad x = L. \tag{52}$$

Solving the above system of linear equations, we obtain the parameters C_i and C_{i+3}.

6. Inversion of the Laplace Transforms

To get the numerical results of the present application in the time domain, we apply the Riemann-sum approximation procedure. In this method, any function $\bar{f}(x,z,s)$ in the Laplace transform domain can be reversed to the physical domain $f(x,z,t)$ by using the following formula [22]

$$f(x,z,t) = \frac{e^{\upsilon t}}{t}\left[\tfrac{1}{2}\mathrm{Re}[\bar{f}(x,z,\upsilon)] + \mathrm{Re}\sum_{n=0}^{N}\left(\bar{f}\left(x,z,\upsilon+\frac{in\pi}{t}\right)(-1)^n\right)\right], \tag{53}$$

where Re is the real part of function, $i=\sqrt{-1}$ and $\upsilon \approx 4.7/t$ for faster convergence [23].

7. Numerical results

The numerical analysis of the analytical results obtained in the previous sections with respect to thermodynamic temperature change, deflection, vibration displacement, bending moment and stress in the beam are discussed. The effect of temperature pulse, the presence and absence of couple stress and thermal relaxation time on the field quantities are investigated. It is assumed that the beam is made of Nickel with the following mechanical and thermal properties [24]

Table1. Geometrical and material properties of the Nickel micro-beam

Parameters	Physical data
Young's modulus (E)	210 GPa
Poisson's ratio (ν)	0.31
Density (ρ)	8900 kg/m
Thermal conductivity (K)	92 W/mK
Specific heat at constant volume (C_E)	438 J/kgK
Coefficient of linear thermal expansion (α_t)	13×10^{-6} K^{-1}
Couple stress parameter (α)	2.5 Kg m s-2
Ambient Temperature (T_0)	300 K

The aspect ratios of the beam are fixed as $b/h = 0.5$, $L/h = 10$, $t = 0.12$ and $h = 10\,\mu$m. The numerical values of temperature change θ, distributions of the deflection w, stress σ_x, thermal moment M, displacement u have been computed using the approximation method defined in Eq. (53). Numerical calculations have been carried out with the help of MATHEMATICA software programming. The numerical results have been presented graphically in Figs. 1–15.

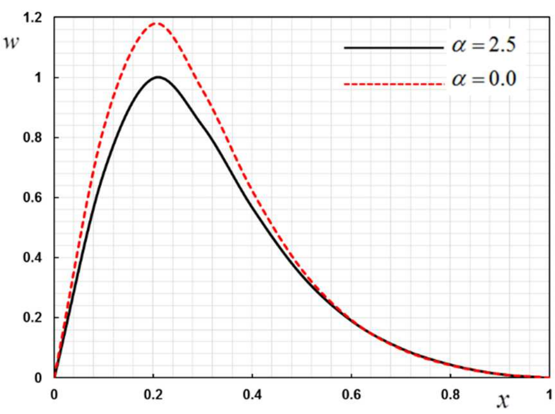

Fig. 1. The transverse deflection distributions versus the axial direction with different modified couple stress parameter α $(\tau_0 = 0.1, \omega = 0.2, z = h/3)$.

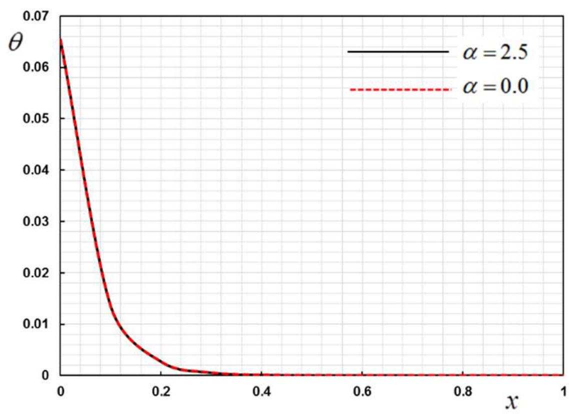

Fig. 2. The temperature distributions versus the axial direction with different modified couple stress parameter α $(\tau_0 = 0.1, \omega = 0.2, z = h/3)$.

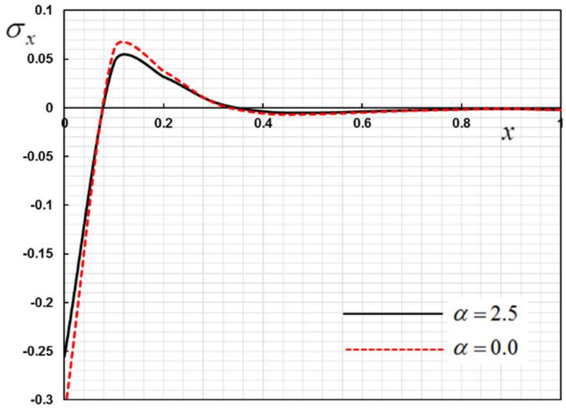

Fig. 3. The stress distributions versus the axial direction with different modified couple stress parameter α $(\tau_0 = 0.1, \omega = 0.2, z = h/3)$.

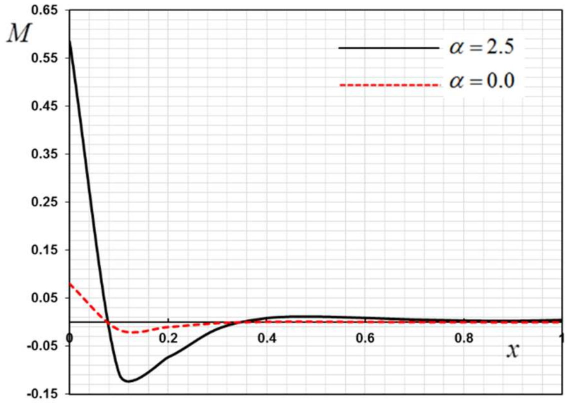

Fig. 4. The bending moment versus the axial direction with different modified couple stress parameter α $(\tau_0 = 0.1, \omega = 0.2, z = h/3)$.

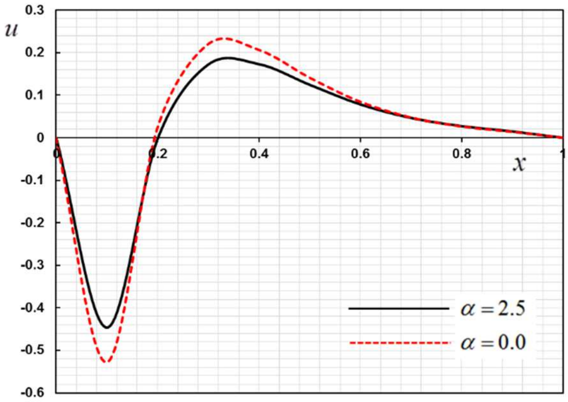

Fig. 5. The displacement versus the axial direction with different modified couple stress parameter α $(\tau_0 = 0.1, \omega = 0.2, z = h/3)$.

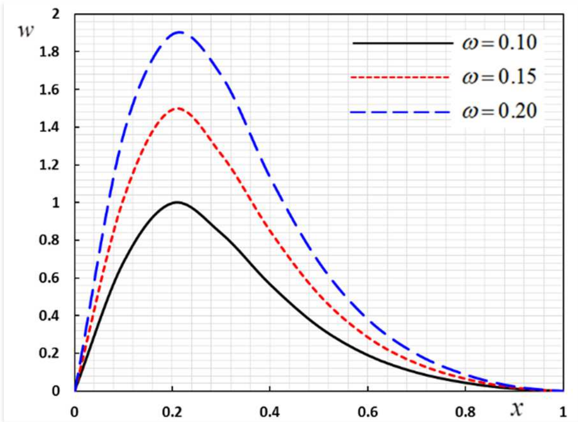

Fig. 6. The transverse deflection distributions versus the axial direction with different temperature pulse ω $(\tau_0 = 0.1, \alpha = 2.5, z = h/3)$.

Case I: Couple Stress Effect

The effect of couple stress on numerical calculations of the field quantities is illustrated in Figs. 1-5 in the presence ($\alpha \neq 0$) and the absence of couple stress ($\alpha = 0$). Figure 1 displays the variations of deflection w in the microbeam for $\alpha = 2.5, 1.5, 0.0$ when the parameters ω and τ_0 are fixed. It can be seen from Fig. 1 that the variation of lateral vibration for modified couple stress model is greater in comparison to for the classical model. Also, it is observed that the deflection increases from zero and becomes maximum near $x = 0.2$, then it decreases gradually as x increases and becomes again zero near $x = 0$ and satisfies the boundary condition.

Fig. 2 demonstrates the temperature change θ of the micro-beam versus distance x for two the various values of the modified couple stress parameter α. It can be clearly concluded from the figure that parameter α has small effect on temperature values. Also, the numerical values of temperature decrease monotonically in the microbeam moving in the wave propagation direction.

The influence of different values of the parameter α on the axial stress σ_x along the axial direction is presented in Fig. 3. As seen in Fig. 3, difference behavior based on the presence and absence of couple stress. It is observed in Fig. 3that, the new couple stress theory predicts lower values of natural axial stress in comparison with the classical theory.

Figure 4 displays the variant of bending moment M with axial distance x in the presence and absence of couple stress in the beam. From Figure 4, it is observed that couple stress parameter α has a significant effect on bending moment distribution.

Fig. 5 presents the distribution of the displacement u of the microbeam for various values of α. We observe that the parameter α has a significant effects on the displacement distribution u. Moreover, the values of u start decreasing with distance x in the range $0 \le x \le 0.1$, thereafter increasing to maximum amplitudes in the wide range $0.1 \le x \le 0.3$.

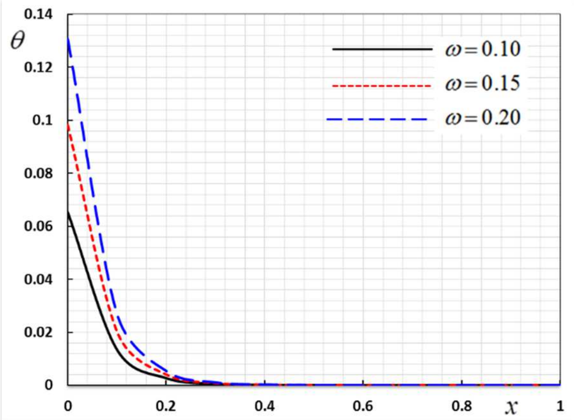

Fig. 7. The temperature distributions versus the axial direction with different temperature pulse ω $(\tau_0 = 0.1,\ \alpha = 2.5,\ z = h/3)$.

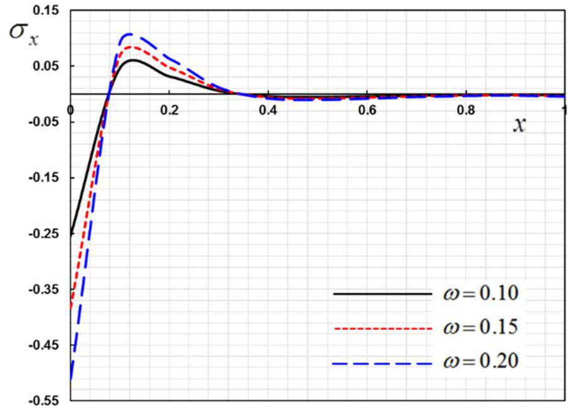

Fig. 8. The stress distributions versus the axial direction with different temperature pulse ω $(\tau_0 = 0.1,\ \alpha = 2.5,\ z = h/3)$.

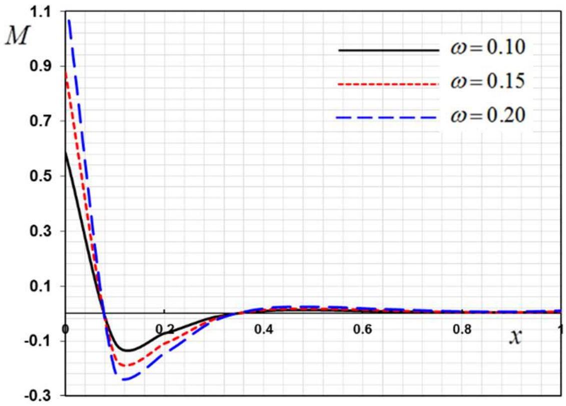

Fig. 9. The bending moment versus the axial direction with different temperature pulse ω $(\tau_0 = 0.1,\ \alpha = 2.5,\ z = h/3)$.

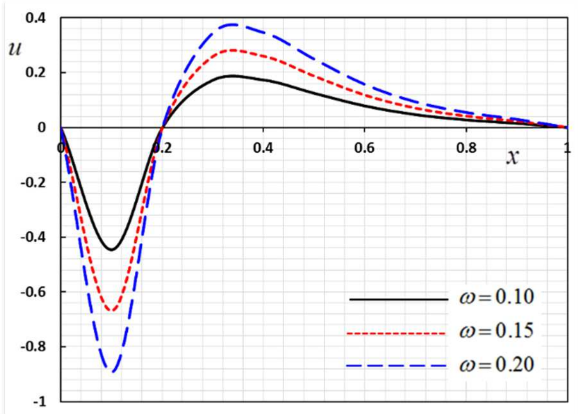

Fig. 10. The displacement versus the axial direction with different temperature pulse ω $(\tau_0 = 0.1,\ \alpha = 2.5,\ z = h/3)$.

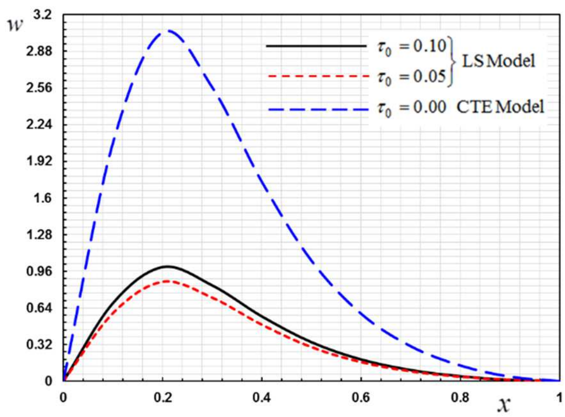

Fig. 11. The transverse deflection distributions versus the axial direction with different theories of thermoelasticity $(\alpha = 2.5,\ \omega = 0.2,\ z = h/3)$.

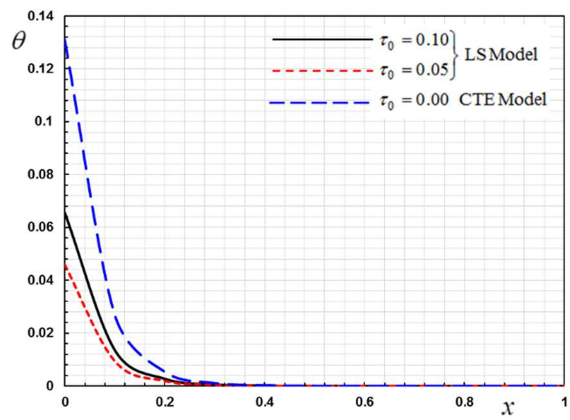

Fig. 12. The temperature distributions versus the axial direction with different theories of thermoelasticity $(\alpha = 2.5,\ \omega = 0.2,\ z = h/3)$.

Case II: The Effect of Temperature Pulse

The influence of the temperature pulse ω on all the fields is displayed in Figs. (6-10). Based on the obtained results, at higher pulses there are many differences between the results of couple stress theory and classical one. The values of deflection of the microbeam are observed to be maximum at $x = 0.2$ and decreases from this point on both sides of the beam. Increasing the value of the temperature pulse ω causes increasing in the values of the magnitude of all the studied fields which is very clear in the peak points of the illustrated curves.

Case III: The effect of the thermal relaxation time

In this case, we study the effect different values of thermal relaxation time τ_0 on the different fields when temperature pulse and the modified couple stress parameter α remain constants. The graphs in Figures 11-15 represent the figures predicted by the two theories of thermoelasticity which are obtained as a special case of the current LS model.
To obtain the coupled theory of thermoelasticity (CTE), we put $\tau_0 = 0$ and to get the Lord-Shulman (LS) theory, we take $\tau_0 > 0$. The magnitudes of the field variables in the LS model is larger compared to the CTE model. We also observe that the relaxation time τ_0 have a large influence on the different field quantities.
It is detected that the micro-beam displays more deflection in case of the classical theory (CT) than that in the case of generalized one (LS). The behavior of the curves in the two theories of thermoelasticity are usually quite similar. The mechanical distributions show that the thermal wave propagates with a finite speed in the medium. Also, the values of the field variables in classical theory (CTE model) are unlike compared to the generalized theory.

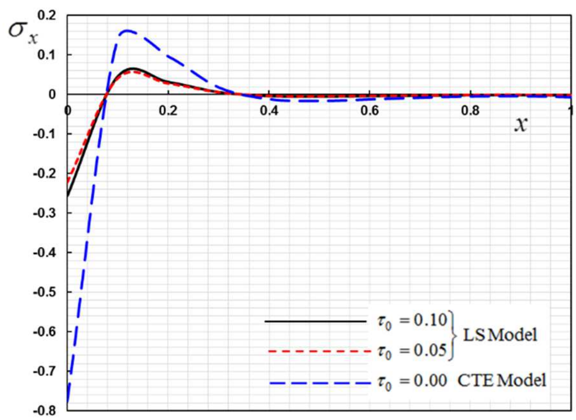

Fig. 13. The stress distributions versus the axial direction with different theories of thermoelasticity ($\alpha = 2.5,\ \omega = 0.2,\ z = h/3$).

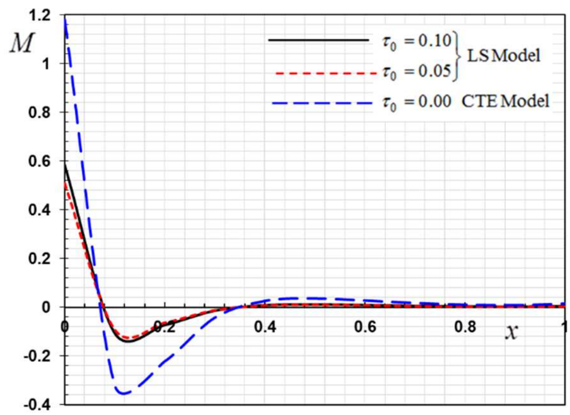

Fig. 14. The moment of bending versus axial direction with different theories of thermoelasticity ($\alpha = 2.5,\ \omega = 0.2,\ z = h/3$).

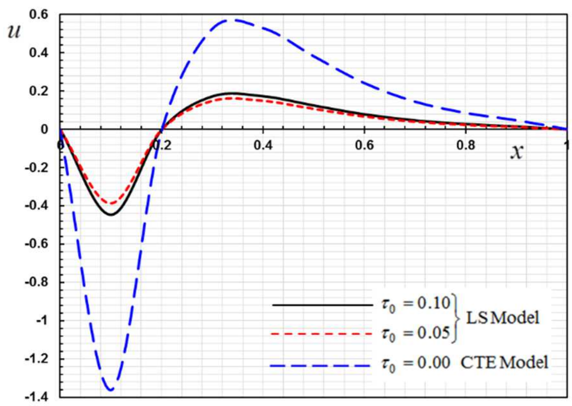

Fig. 15. The displacement versus the axial direction with different theories of thermoelasticity ($\alpha = 2.5,\ \omega = 0.2,\ z = h/3$).

8. Conclusion and Perspectives

The current paper was an investigation of the elastic vibrations in a thermoelastic Euler–Bernoulli micro-beam exposed to temperature pulse heating. The modified couple stretch theory and Lord-Shulman (LS) model were utilized and the impact of the temperature pulse parameter was displayed. A mathematical model that governs the studied problem were introduced and derived. To get the systematic expressions for deflection, temperature, displacement and bending moment in the small scale beam, the technique Laplace transform was utilized. The effects of the parameters of couple stress and the temperature pulse on the studied fields were studied and introduced graphically. As indicated by the outcomes appeared in all illustrated figures, it was observed that the parameter of couple stress has manifest impacts on all field variables. In addition, the thermoelastic deflections, displacement and temperature have a strong dependency on the temperature pulse parameter. This study was very essential for micro-scale problems because in these cases the material parameters are temperature dependent. Also, the obtained results can be valuable for mechanical engineers in the designing of small scale resonators for MEMS applications. The study of lateral deflection, thermal moment and axial stress average is a significant problem of continuum mechanics.

Author Contributions

Author 1 planned the scheme, initiated the project and suggested the experiments; Author 2 conducted the experiments and analyzed the empirical results; Author 3 developed the mathematical modeling and examined the theory validation. The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

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Nomenclature

T_0	environment temperature
(x, y, z)	Cartesian coordinates system
u, w	displacement components
σ_{xx}	axial mechanical stress component
α	the couple stress parameter
e_{ij}	linear strain tensor
t_0	mechanical relaxation time due to viscosity
$\theta = T - T_0$	temperature change
ρ	material density
$k = K / \rho C_E$	Thermal diffusivity
K	thermal conductivity
ω_i	components of rotation vector
e_{kij}	alternate tensor
χ_{kl}	symmetric curvature
C_E	specific heat at uniform strain
E	Young modulus
ν	Poisson's ratio
α_t	thermal expansion coefficient
δ_{ij}	Kronecker delta function
$\gamma = \alpha_t E / (1 - 2\nu)$	coupling parameter
λ, μ	Lame’s constant
L	Length of the beam
b	width of the beam
h	thickness of the beam
$w(x, t)$	transversal deflection
$A = bh$	beam cross-section area
$I = bh^3 / 12$	inertia moment of the cross-section
EI	flexural rigidity of the beam
M_T	thermal moment

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