J. Appl. Comput. Mech., 8(2) (2022) 405-415
DOI: 10.22055/JACM.2019.31296.1858

ISSN: 2383-4536
jacm.scu.ac.ir

Shahid Chamran University of Ahvaz

Journal of Applied and Computational Mechanics

Research Paper

Topology Optimization of Laminated Composite Plates and Shells using Optimality Criteria

K.N.V. Chandrasekhar[1], V. Bhikshma[2], K.U. Bhaskara Reddy[3]

[1] Department of Civil Engineering, CVR College of Engineering, Hyderabad, Telangana, India, Email: biml.koralla1@gmail.com

[2] Department of Civil Engineering, Osmania University, Hyderabad, Telangana, India, Email: v.bhikshma@yahoo.co.in

[3] Department of Civil Engineering, CVR College of Engineering, Hyderabad, Telangana, India, Email: kalvauday@gmail.com

Received October 04 2019; Revised November 10 2019; Accepted for publication November 10 2019.
Corresponding author: K.N.V. Chandrasekhar (biml.koralla1@gmail.com)

Abstract. Laminated composite materials have the advantage of desired properties and are vastly replacing the existing traditional materials in Civil Engineering construction. In the present study, it is aimed to extend the study on the analysis of laminated composites plates and shells towards structural optimization. Topology optimization is performed using two different objective functions namely strain energy and fundamental frequency. The results of optimization have shown clearly that the distribution of material is dependent on the laminae. The optimal arrangement of material is obtained after using a cut-off relative density. It is confirmed to be a well-connected grid and is examined in detail. The results have shown that the optimal arrangement of material for a simply supported plate carrying a uniformly distributed load is at the centre of the edges and not towards the corners, Hence, the optimal arrangement of beams using strain energy is to align by joining the centre of the edges similar to plus (+) sign.

Keywords: Laminates; Composite; Plate; Strain energy; Fundamental frequency; Topology optimization.

1. Introduction

Laminated composites are one of the latest areas in the field of materials wherein the desired properties of the material can be obtained. over the last few years, laminated composites are finding their applications in Civil Engineering structures such as the deck of a bridge, composite steel to substitute the metallic steel which is normally utilized in several structures. This study is mainly concentrated on topology optimization of laminated composite plates.

After reviewing the literature, we found that some amount of work has been done in the field of laminated composite plates to determine non-dimensional fundamental frequencies. There is no existing literature related to topology optimization of laminated composite plates. The goal of this research work is to determine the optimal distribution of material with strain energy and fundamental frequency as an objective function. This study is performed to extend the field of analysis of laminated composite plates towards structural optimization.

Section 2 discusses the literature review on the free vibration analysis of laminated composite plates. Section 3 presents the methodology to perform this study. In section 4 the theoretical background is presented. Section 5 discussed the problem statement. In this study, we perform the topology optimization of laminated composite plates using two different types of objective functions. The first objective function is to optimize strain energy of the structure and the second objective function is to optimize the fundamental frequency of the structure. In section 6, analysis is performed with different types of lamina and one kind of geometry. The analysis is done using the finite element method and the meshing of the domain is performed using first-order four-node quadrilateral elements. The optimization is done using optimality criteria as the optimizer. Section 7 presents the conclusion along with the future study. In the end, the list of references used to conduct this study is provided.

1.1 Objectives of this study

To perform topology optimization of laminated composite plates and shells using strain energy as an objective function. To perform topology optimization of laminated composite plates and shells using fundamental frequency as the objective function.

1.2 Scope of the study

1. This study is limited to linear elastic analysis only 2. This study does not include a buckling analysis. 3. This study does not include hygrothermal effects.

Published online: March 20 2020

2. Literature Review

To study the free vibration characteristics J N Reddy [1] analysed doubly curved angle ply laminated Shell by using the finite element method based upon the yang-Norris-Stavsky theory. Two types of materials were used to obtain the fundamental frequencies by using a 2x2 mesh size. finite element solutions obtained by using various meshes and with and without in-plane displacement degrees of freedom (DOF). Results were validated with the existing journals.

Free vibration behaviour analysed by A.J.M. Ferreira, C.M.C. Roque, R.M.N. Jorge [2]for moderately thick plates and laminated composite plates by using FSDT Theory. The natural frequencies were found out for different laminated plates and for different skew angles. The plate is discretised into 11 divisions in the x-direction and 11 divisions in y directions, and for different mesh sizes, results were obtained. MQRBF method is very accurate which was used in this formulation. The results accurate for clamped skew laminated composite plates than simply supported plates. This method is a simple and very powerful method compared to other finite element methods.

Free vibration analysis was studied by G.R Liu, X Zhao,K.Y Dai [3] for a laminated composite shell by using a radial point interpolation method. In this analysis, FSDT was used to analyse the laminated composite plates. Shape functions were derived by using a radial basis and polynomial functions. To improve the convergence rate strain smoothing method was used. This code can also analyse both the thick and thin plates. Shear locking is also avoided in this analysis. Central deflections, Non-dimensional displacements, and nondimensional frequencies were found out for a laminated composite square plate with a square hole and clamped circular plate. By using nodal integration method eigenvalues were solved. Natural frequencies were also found out for a square plate with a complicated cutout.

The bending behaviour of a three-node triangular element was analysed by Jean-Louis Batoz [4] taking 9 degrees of freedom per each node. They have analysed 3 triangular elements namely the DKT element, SRI element, and HSM element. It was concluded that the most efficient triangular plate element among those three elements was DKT and HSM elements. For HSM and DKT triangular plate elements, natural frequencies were found out for cantilever triangular plate with 10 and 21 degrees of freedom. Central deflections have also been found out for simply supported plate and clamped type of boundary condition. The model with 21 DOF gives accurate frequencies for higher values than the model with 10 mass degree of freedom.

X. Zhang, K.S. Kim [5] obtained displacements for three-node triangular shell element and six degrees of freedom for each node. For a different type of boundary conditions, they have obtained displacements for laminated composite shells as well as for plates. Numerical results have shown that the present element is accurate and efficient for linear and geometrically nonlinear analysis of thin to moderately thick laminated composite plates.

2.1 Review on laminated composite shells

Formulation of shallow and thin shells was done by Chakravorthy[6] using first-order shear deformation theory. The effects of aspect ratio, fibre orientation geometrical parameters, and thickness to radius ratio were studied for different boundary conditions. They found that with the increase in the number of constraints and aspect ratio, the non-dimensional fundamental frequency increases in certain cases. They have achieved higher frequency when the fibre orientation is at 45 degrees.

The paper by Won-hong[7] presents the study on forced vibration analysis on laminated composite plates and shells under different types of loadings. The natural strain method is used to overcome the shear locking and membrane effect. They used nine node quadrilateral elements and six degrees of freedom per each node to conduct this study. They found that the damping effect and rotary inertia effect have a significant impact on the response. R.K.Kapania [8] presented the finite element method of static, free vibration and thermal analysis of composite plates and shells using a three-node triangular element with six degrees of freedom per node. Their study can be used to perform buckling analysis. Izadi [9] studied on the free vibration analysis of joined conical shells. Using hamilton's principle and first-order shear deformation theory five equilibrium equations are derived. The effects of semi vertex angle, boundary conditions, number of layers and length of shells were investigated. Tiangui [10] presented the analysis of vibrations of composite shells with different shell curvatures. The effects of rotary inertia and shear deformation were studies using first-order shear deformation theory. Viswanathan et.al. [11] did a free vibration analysis of symmetric angle ply laminated cylindrical shells with various thicknesses. The thickness of shells varies linearly, sinusoidal and exponentially. Spline function is adopted and to determine the eigenvalues point collocation method is adopted. They found that the variation in thickness affects frequency. The frequency decreases with an increase in length for all thicknesses of layers of shells. Chate [12] performed dynamic analysis of shells. They observed that the results were higher when the ply angle is 60^0. Hosseini-Hashemi [13] studied transversely isotropic spherical shells. Fundamental frequencies were determined for different shell parameters such as shear modulus ratio and curvature ratio. They found that the fundamental frequencies for SCSC and SSSS boundary constraints decrease with an increase in curvature ratio. Wu and Wu [14]present a free vibration analysis of laminated conical shells using the asymptotic approach. Differential quadrature method is adopted for solving the problems and determine the fundamental frequencies of conical and cylindrical shells. Hakan [15] presented a static analysis of laminated cylindrical and conical shells. The solution for the governing differential equations is obtained from the discrete singular convolution method. They found that the method gives accurate results when the mesh is small. Shu [16] studied the behavior of composite laminated shells using generalised quadrature method. The displacements were expressed as Fourier series and they found that the method converges very fast with higher accuracy. Timarci [17] studied the effect of free vibrations on finite, circular and closed cylindrical shells made of one or more monoclinic shells unified shear deformable theory. They found that the numerical behavior of the solution is stable. Francesco [18] presented the general formulation for free vibrations of thick and thin doubly curved composite shells and plates with various curvatures using a two-dimensional higher-order equivalent single layer theory. The study assumed that transverse normal is extensible to include the stretching effect, higher-order shear deformations to formulate governing differential equations. Chakravorthy[19] formulated the shell by considering two principal radii of curvature and radius of cross curvature using first-order shear deformation theory. Numerical results were obtained by varying the geometrical and material properties. They found that fibre orientation 90 degrees gives the highest natural frequency with four-layered symmetric and antisymmetric lamina. Chakravarthy [20] studied the behavior of free vibration behavior of doubly curved shells like conoids, hyperbolic and elliptical paraboloids using eight-node quadrilateral elements having five degrees of freedom each. They found that the truncated conoid is stiffer than the full conoid. The fundamental frequency of hyperbolic paraboloids increases with the decrease of thickness to curvature ratio. The frequencies increase with the increase of angle of orientation of fibre up to 45 degrees and thereafter decreases until the orientation reaches 90 degrees.

Gokhan Serhat and Ipek Basdogan [21] presented a multi-objective methodology for laminated composite plates with the dynamic load. They optimized the design variables namely thickness of the plate, orientation of angles and lamination parameters. They conducted optimization technique on a flat plate with different dimensions and boundary conditions. Cameron et.al. [22] performed optimization of structures using different types of elements namely Quad4, Quad8, Quad9, triangular and hexagon 6 node elements with wachpress type interpolation functions. An example of a Messerschmitt Bolkow Blohm (MBB) beam is solved and the results were presented. Guilian [23] proposed the TIMP method for topology optimization of plate structures. They solved several complex problems with design variables and loading conditions. Liu [24] in his research paper aims at applying layerwise deformation theory and Isogeometric analysis to model multi-layered laminated composites. A few numerical examples are solved to verify the validity of the proposed model. Liu [25] in his paper presented a layer-based discretisation to model plasticity in structural frames to capture the gradual plastification of the section. The performance of the proposed model is applied to understand the yielding of beams and frames under both small and large deformations. Veedu et.al. [26] did their research on reinforcing lamina with carbon nanotubes to improve the mechanical properties of the composite lamina. The carbon nanotubes are provided in between the lamina to enhance the strength in the thickness direction. The carbon nanotubes are grown in between the multi-layered composites resulting in a 3D effect between plies and loading. The fabricated 3D composites are tested for fracture toughness, damping, thermoelastic behavior, thermal and electrical conductivities making these structures multifunctional.

3. Methodology

The flowchart which is shown in Fig.1a explains the procedure followed to determine the non-dimensional fundamental frequencies boundary conditions for different laminae. For a given geometry and material properties, the element stiffness matrix and the element mass matrix are formulated. The element matrices are then assembled to form the global stiffness matrix and global mass matrix respectively. The Eigen frequencies are then determined. Fig 1c shows the flowchart to perform topology optimization. First-order sensitivity analysis is performed in this study. The first derivative of the fundamental equation is found and the relative densities are determined using optimality criteria. The same procedure is repeated in every iteration and then the final optimal distribution is determined.

The flow chart in Fig.1b shows the steps involved to formulate the element stiffness matrix and force vector. The element matrices are assembled to form a global stiffness matrix and global force vector. The nodal displacements are determined. Topology optimization of the laminated composite plate and shells can be performed using first-order sensitivity analysis of the strain energy. The newer values of relative densities for each element are determined using optimality criteria as shown in Fig.1d. The steps are repeated until the optimal distribution of material is determined.

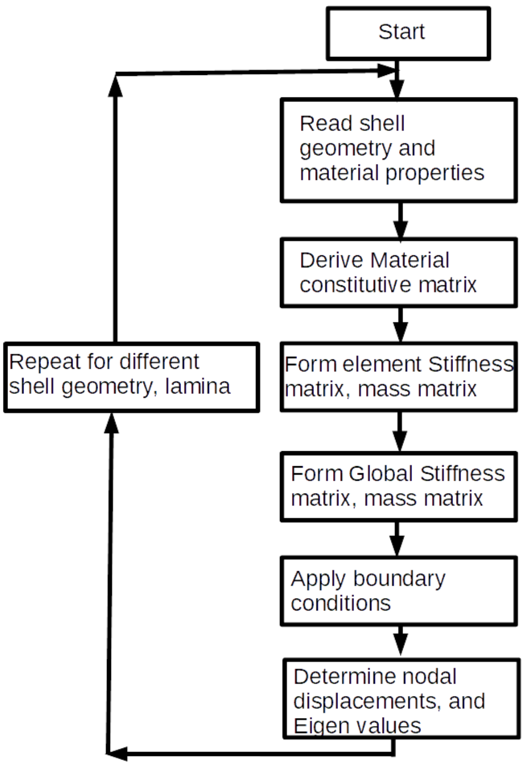

Fig. 1a. Flowchart showing the steps involved to perform this study

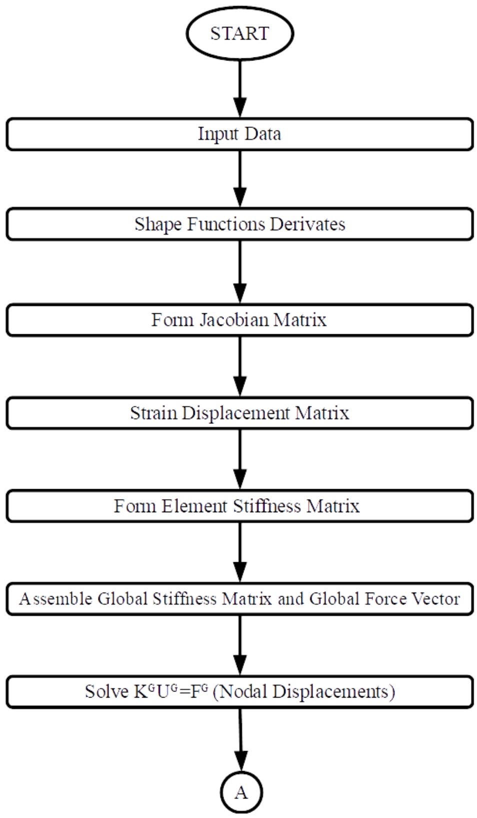

Fig. 1b. Steps to perform static analysis and determine the nodal displacements

The steps required to form the global stiffness matrix and global force vector and perform the static analysis are shown in Fig.1b. Fig.1c shows the flow chart to perform first-order sensitivity analysis with strain energy as an objective function. Fig.1d shows the flow chart to perform the first-order sensitivity analysis of fundamental equations and determine the newer values of relative density using optimality criteria.

4. Theoretical Background

The constitutive equations for the shell are given by

$$\{F\} = [D]\{\varepsilon\}, \qquad \{F\} = \{N_x,\ N_y,\ N_{xy},\ M_x,\ M_y,\ M_{xy},\ Q_x,\ Q_y\}^T \tag{1}$$

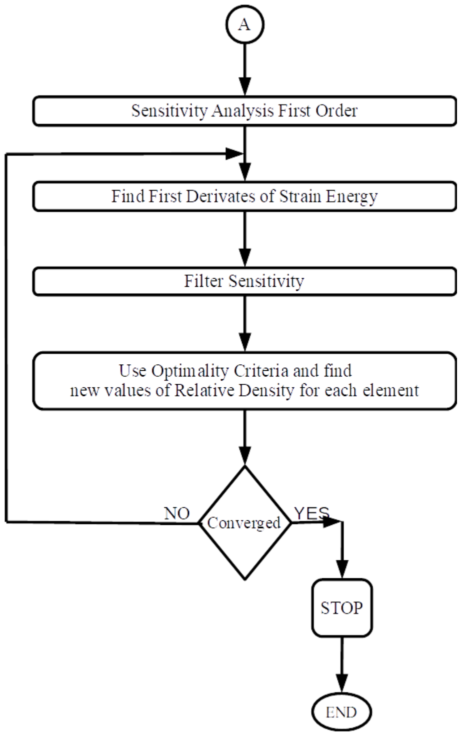

Fig. 1c. The sensitivity analysis using strain energy of the structure

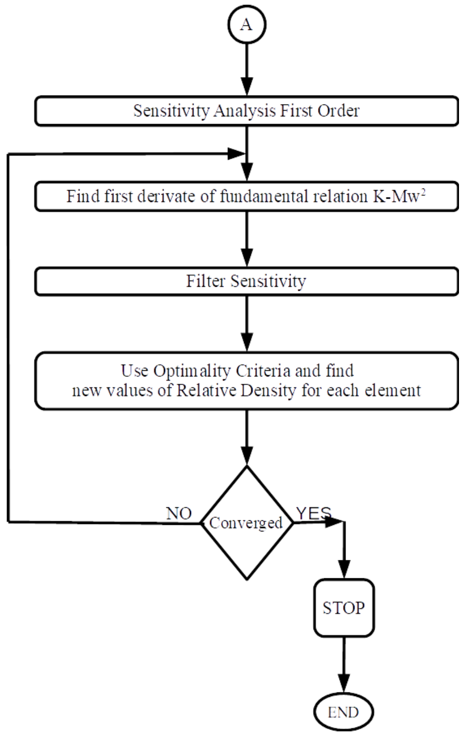

Fig. 1d. The sensitivity analysis to perform frequency-based topology optimization

where N_x, N_y, N_{xy} are the in-plane force resultants, M_x, M_y, M_{xy} are the moment resultants, Q_x, Q_y are the transverse shear resultants. The force and moment resultants are expressed as

$$\{N_x,\ N_y,\ N_{xy},\ M_x,\ M_y,\ M_{xy},\ Q_x,\ Q_y\}^T = \int_{-h/2}^{h/2} \{\sigma_x,\ \sigma_y,\ \tau_{xy},\ \sigma_z z,\ \sigma_y z,\ \tau_{xy} z,\ \tau_{xz},\ \tau_{yz}\}^T dz$$
$$\{\varepsilon\} = \{\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0,\ k_x,\ k_y,\ k_{xy},\ Q_x,\ Q_y\}^T \tag{2}$$

The elements of stiffness matrix [D] are

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix}$$
$$A_{ij} = \sum_{k=1}^{n} \left(Q_{ij}\right)_k (z_k - z_{k-1}),\ B_{ij} = \frac{1}{2}\sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right),\ D_{ij} = \frac{1}{3}\sum_{k=1}^{n} \left(Q_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right),\ i, j = 1,2,6$$
$$S_{ij} = \sum_{k=1}^{n} \left(F_i F_j\right)\left(G_{ij}\right)_k (z_k - z_{k-1}) \tag{3}$$

where $(Q_{ij})_k$ are the elements of elastic constant matrix and

$$\left(\overline{Q_{ij}}\right)_k = [T_1]^{-1} \left(Q_{ij}\right)_k [T_1] \text{ for i, j = 1,2,6}$$
$$\left(\overline{Q_{ij}}\right)_k = [T_2]^{-1} \left(Q_{ij}\right)_k [T_2] \text{ for i, j = 4,5} \tag{4}$$

in which

$$[T_1] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix},\quad [T_2] = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \tag{5}$$

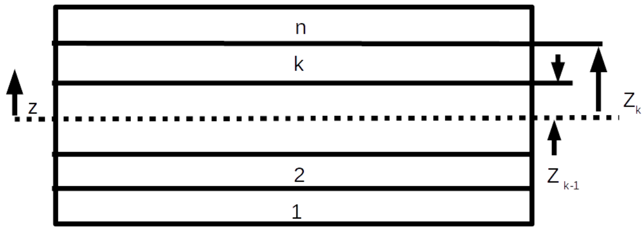

Fig. 2. Layers of lamina [27]

where m = $\cos\theta$ and n = $\sin\theta$

$$[Q_{ij}] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}, i,j = 1,2,6 \;,\; [Q_{ij}] = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix}, i,j = 4,5 \tag{6}$$

in which

$$\begin{gathered} Q_{11} = \frac{E_{11}}{1-\upsilon_{12}\upsilon_{21}} \\ Q_{12} = \frac{\upsilon_{12}E_{11}}{1-\upsilon_{12}\upsilon_{21}}, \\ Q_{22} = \frac{E_{22}}{1-\upsilon_{12}\upsilon_{21}} \\ Q_{44} = G_{13} \\ Q_{55} = G_{23} \\ Q_{66} = G_{12} \end{gathered} \tag{7}$$

F_i and F_j are the two factors which is considered as one when the shell is thin. When the shell is moderately thick the product is considered as 5/6 as this is the shear correction factor from the theory of elasticity.
The strain displacement relations of first order theory of shells,

$$\{\varepsilon_x,\ \varepsilon_y,\ \gamma_{xy},\ \gamma_{xz},\ \gamma_{yz}\}^T = \{\varepsilon_x^0,\ \varepsilon_y^0,\ \gamma_{xy}^0,\ \gamma_{xz}^0,\ \gamma_{yz}^0\}^T + z\{k_x,\ k_y,\ k_{xy},\ k_{xz},\ k_{yz}\}^T \tag{8}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x} - \dfrac{w}{R_{xx}} \\ \dfrac{\partial \upsilon}{\partial y} - \dfrac{w}{R_{yy}} \\ \dfrac{\partial u}{\partial y} + \dfrac{\partial \upsilon}{\partial x} - 2\dfrac{w}{R_{xy}} \\ \alpha + \dfrac{\partial w}{\partial x} \\ \beta + \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \\ k_{xz} \\ k_{yz} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial \alpha}{\partial x} \\ \dfrac{\partial \beta}{\partial y} \\ \dfrac{\partial \alpha}{\partial y} + \dfrac{\partial \beta}{\partial x} \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

4.1 Finite element formulation

A four-node curved orthotropic quadrilateral finite element [28] is used for shell analysis. Five degrees of freedom are defined at every node (u, v, w, α, β). The relations between the displacement w.r.t ξ and η co-ordinates and degrees of freedom.

$$u = \sum_{i=1}^{4} N_i u_i,\ \ \upsilon = \sum_{i=1}^{4} N_i \upsilon_i,\ \ w = \sum_{i=1}^{4} N_i w_i,\ \ \alpha_x = \sum_{i=1}^{4} N_i \alpha_{xi},\ \ \beta_y = \sum_{i=1}^{4} N_i \beta_{yi} \tag{11}$$

The displacements of the elements can be expressed in terms of shape functions and nodal degrees of freedom

$$U = [N]\{d\} \tag{12a}$$

$$U = \begin{Bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{Bmatrix} \tag{12b}$$

4.2 Stiffness matrix for an element [29]

The strain of the element is $\varepsilon = [B]\{\delta\}$. The strain displacement matrix is given by

$$[B] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & -\dfrac{N_i}{R_{xx}} & 0 & 0 \\ 0 & N_{i,y} & -\dfrac{N_i}{R_{yy}} & 0 & 0 \\ N_{i,y} & N_{i,x} & -\dfrac{2N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & N_{i,y} & N_{i,x} \\ 0 & 0 & N_{i,x} & N_i & 0 \\ 0 & 0 & N_{i,y} & 0 & N_i \end{bmatrix} \tag{13}$$

$$[k] = \iint [B]^T [E][B]\, dx\, dy$$

The stiffness matrix of the element is solved by the numerical integration method.

4.3 Mass matrix of an element

The mass matrix of the element is given by

$$[M] = \iint [N]^T [P][N]\, dx\, dy$$

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix},\ [P] = \sum_{i=1}^{8} \begin{bmatrix} I_1 & 0 & 0 & 0 & I_2 \\ 0 & I_1 & 0 & -I_2 & 0 \\ 0 & 0 & I_1 & 0 & 0 \\ 0 & -I_2 & 0 & I_3 & 0 \\ I_2 & 0 & 0 & 0 & I_3 \end{bmatrix} \tag{14}$$

$$I_1 = \int \rho dz,\ I_2 = \int \rho z\, dz,\ I_3 = \int \rho z^2\, dz$$

The element mass matrix is assembled to form a global mass matrix.

4.4 Load vector due to the static force

The force vector for each element is given by

$$\{P_e\} = \iint N_i \{q \quad 0 \quad 0\}^T dx\, dy \tag{15}$$

where q is the uniformly distributed load due to static force. The element force vector is assembled to form a global force vector.

4.5 Non-dimensional fundamental frequency [30]

The condition for free vibration analysis

$$[K] - \omega^2 [M] = 0 \tag{16}$$

where K is Global stiffness matrix, M is Global mass matrix and ω is fundamental frequency. Nondimensional frequencies are determined by using the formula given below [31]

$$\lambda = \omega a^2 \left(\frac{\rho}{E_{22} h^2} \right)^{1/2} \tag{17}$$

where a is the length of the side, h denotes the total thickness, E_{22} is the modulus of elasticity in Y direction and ρ is the mass density.

5. Problem Statement

Two types of problems are solved here in this paper.

5.1 Optimize strain energy

The objective function is to optimize Strain Energy subject to,

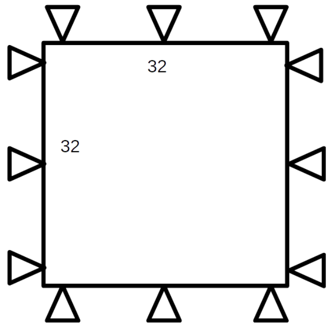

Fig. 3. The initial design domain of a simply supported plate (SSSS)

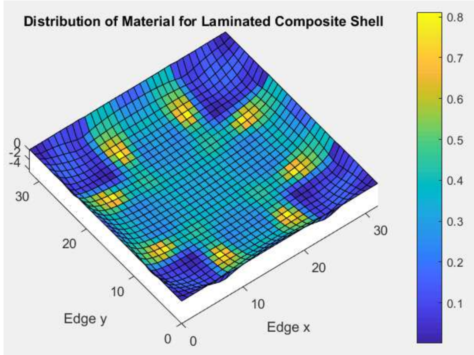

Fig. 4a. Magnified deformed shape of the laminated composite plate -45/45 laminae

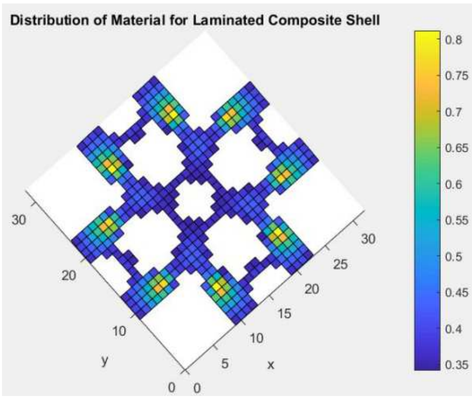

Fig. 4b. Optimal distribution of material -45/45 laminae SSSS plate

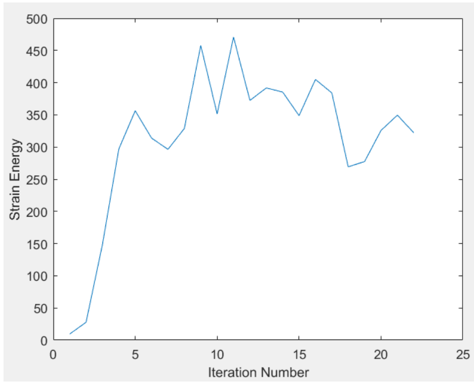

Fig. 4c. Strain energy on Y-axis v/s iteration on X-axis for the laminated composite plate with -45/45 laminae

Volume >= Min. Volume
Stress <= Permissible stress
Displacement <= Permissible displacement

Given,
Young's Modulus of elasticity
Poisson's ratio
Weight density

5.2 Optimize fundamental frequency

The objective function is to optimize. fundamental frequency
subject to,
Volume >= Min volume

Given,
Young's modulus of elasticity
Poisson's ratio
Weight density

6. Analysis

6.1 Problem definition

Topology optimization of plates is performed in this section. Two objective functions are used. In the first part of the section strain energy is used as the objective function. In the second part of this section fundamental frequency is used as the objective function.

A simply supported SSSS laminated composite plate of size 32 units x 32 units is meshed using four-node first order 32 x 32 quadrilateral elements as shown in Fig.3. The thickness of the plate is 0.1 and Poisson's ratio is taken as 0.25. The Young's modulus of elasticity is taken as E_{11} is equal to 25 x 10^6. The Young's modulus of elasticity in the second direction is equal to E_{22} = 25E_{11}. The shear modulus of elasticity is taken as 0.5E_{22}. A uniformly distributed load having a magnitude of 0.1 units acting vertically downwards is applied on the plate. The penalisation factor for the stiffness matrix is taken as 3. The input data is given here. $E_{11} = E_{xx} = 25 \times 10^6$, $E_{22} = E_{yy} = 1 \times 10^6$, $G_{xy} = 0.5E_{22}$, $G_{xz} = 0.5 \times 10^6$, $G_{yz} = 0.2 \times 10^6$. Fig.3 shows the initial design domain. The edges are simply supported. The boundary condition for nodes along the x-axis is 10110 and nodes along y-axis 01101. 1 stands for zero displacement and 0 stands for when the displacement is allowed. For nodes along the x-axis, the y-displacement 'v' and rotation β are allowed. For nodes along the degrees y-axis, the x-displacement 'u' and rotation α are allowed. Optimality criteria method is applied to perform topology optimization.

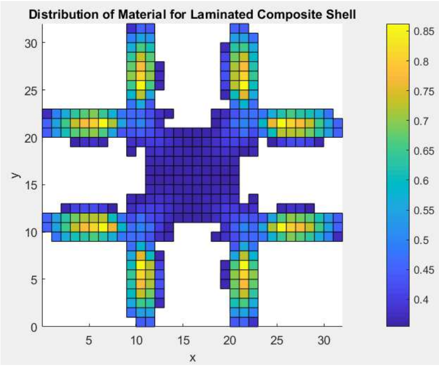

Fig. 5. Optimal distribution of material with the lamina 0/90/0/90

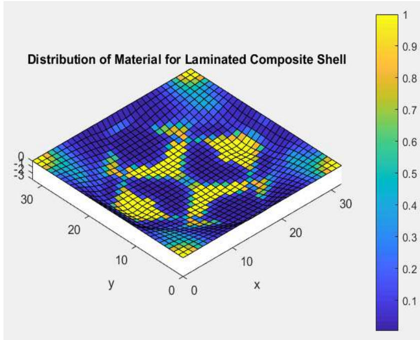

Fig. 6a. 3D view of the first mode shape

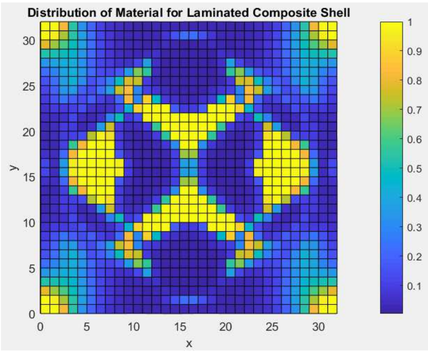

Fig. 6b. Top view of the optimal distribution

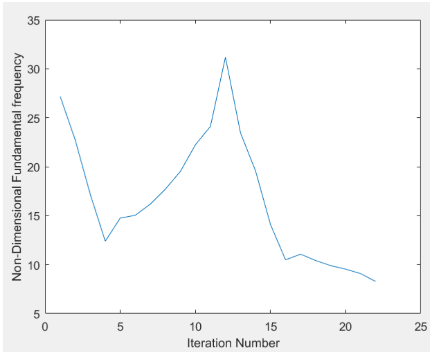

Fig. 6c. Non-dimensional fundamental frequency on Y-axis v/s iteration on X-axis for frequency-based optimization of laminated composite plate with -45/45 laminae

6.1.1 Topology optimization of laminated composite plated with Strain energy as the objective function

6.1.1.1 The total number of laminae is equal to 2 and they are -45/45.

The strain energy of the structure at every iteration is evaluated. The penalisation factor for the stiffness matrix is taken as 3. Graph 1 in Fig.4c shows the variation of the strain energy on the Y-axis and iteration number on the X-axis. The optimal set of parameters for optimality criteria are given here. The total number of iterations is 22. The filter radius is equal to 2 and the moves limit is equal to 0.3. The stabilisation factor is taken as equal to 0.50. The minimum volume of the material is 0.30 or 30%. Fig.4a shows the deformed shape of the laminated composite plate after 22 iterations. Fig.4b shows the optimal distribution of material with a cutoff for relative density taken equal to 0.34.

The cutoff relative density as 0.34. Maximum downward deflection = 4.5257 units. The Strain energy is equal to 322.1664 units. The deflected shape of the plate is uniform and regular. The distribution in Fig.4b shows that the optimal arrangement of material is at the centre of the edges at not at the corners. If we consider the plate element as a footing, then the optimal arrangement of beams underneath the plate element carrying a uniformly distributed load is near the centre of edges similar to a '+' sign.

The graph shows the variation of strain energy on the y-axis and the iteration number on the x-axis for the optimization of the laminated composite plate with -45/45 laminae. A few more examples are shown in Fig.6

6.1.1.2 The total number of laminae is equal to 4 and they are 0/90/0/90 laminae

Fig. 5 shows the optimal distribution of material for 0/90/0/90 lamina. The cutoff relative density is 0.35. The number of iterations is 22. The maximum downward deflection is 4.5186 units. The Strain energy is equal to 206.7188 units. The deflected curve is regular and uniform. The elements in yellow color carry the maximum value of relative density and participate in carrying the load acting on the structure. The elements in blue color carry lower value for relative density and do not participate in carrying the load acting on the structure. This is the optimal distribution of material having higher bending stiffness and can carry loading under simply supported conditions. The optimal distribution is in the form of plus '+' sign. With strain energy as the objective function, the optimal distribution of material is near the centre of the edges and the lamina is best oriented along 0 degrees and 90 degrees in alternating layers.

6.1.2 Topology optimization of laminated composite plates with fundamental frequency as the objective function

The fundamental frequency at every iteration is evaluated. The first-order sensitivity analysis is performed by considering the first derivative of the fundamental equation $[K]-\omega^2[M]=0$. The optimizer used in this study is optimality criteria to calculate the newer values of relative density at every iteration. The penalisation factor for the stiffness matrix is taken as 3 and the penalisation factor for the mass matrix is taken as 2. The graph shown in Fig.6c shows the non-dimensional frequency on Y-axis and iteration on X-axis. The graph shows a volatile nature as the stabilisation factor is low. The optimal set of parameters for optimality criteria are given here. The total number of iterations is 22. The filter radius is equal to 2 and the move limit is equal to 0.3. The stabilisation factor is taken as equal to 0.50. The minimum volume of the material is 0.30 or 30%. Fig.6a shows the first mode shape of vibration of the laminated composite plate after 22 iterations. Fig.6b shows the optimal distribution of material.

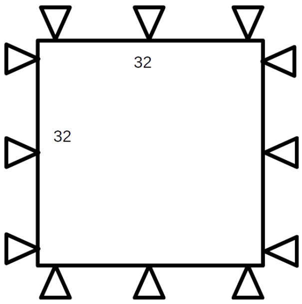

Fig. 7. The initial design domain of a simply supported elliptical shell

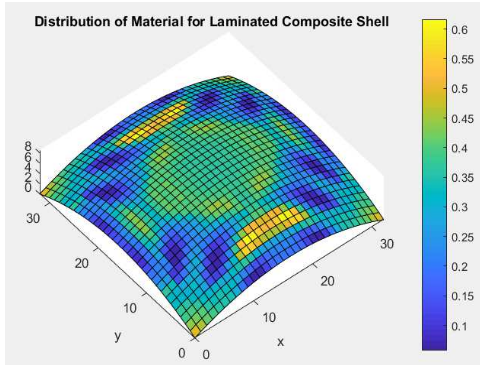

Fig. 8a. Strain energy-based optimal distribution of material for SSSS Elliptical paraboloid for 0/90/0 laminae after 30 iterations

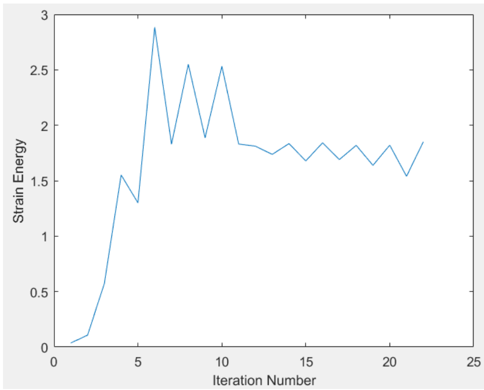

Fig. 8b. Strain energy on Y-axis v/s iteration number on the x-axis for strain energy-based topology optimization of laminated composite elliptical paraboloid shell with lamina 0/90/0.

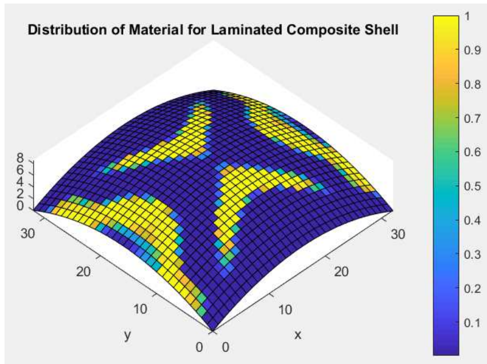

Fig. 9a. The fundamental frequency based optimal distribution of material after 13 iterations for laminated composite SSSS Elliptical shell with -45/45 laminae

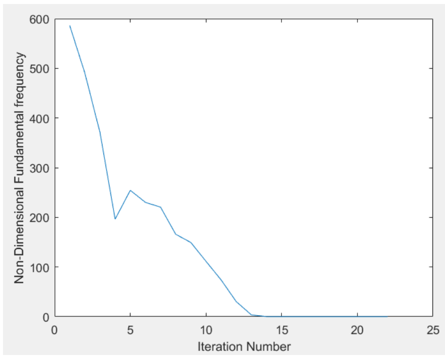

Fig. 9b. Non-dimensional fundamental frequency v/s iteration number curve for laminated composite SSSS elliptical paraboloid shell with -45/45 laminae

Fig.6a, 6b showing the distribution of material for Eigen frequency-based optimization of laminated composite SSSS plate with -45/45 laminae after 22 iterations. The distribution near the centre of the edges has a higher value of relative density as shown with the yellow color. These elements participate with higher weightage towards the objective function. The terms corresponding to these elements in the stiffness matrix and mass matrix will be multiplied by the penalised relative density for the element. The elements in blue color do not participate in the stiffness of the matrix as their value for the relative density is low.

6.2 Problem definition

Topology optimization of laminated composite shells is performed in this section. Two objective functions are used. In the first part of the section strain energy is used as the objective function. In the second part of this section fundamental frequency is used as the objective function.

A simply supported SSSS laminated composite shell of size 32 units x 32 units is meshed using four-node first order 32 x 32 quadrilateral elements as shown in Fig.7. The thickness of the plate is 0.1 and Poisson's ratio is taken as 0.25. The Young's modulus of elasticity is taken as E_{11} is equal to 25 x 10^6. The total number of laminae is equal to 3 and they are 0/90/0. The Young's modulus of elasticity in the second direction is equal to $E_{22} = 25E_{11}$. The shear modulus of elasticity is taken as $0.5E_{22}$. A uniformly distributed load having a magnitude of 2 units acting vertically downwards is applied on the plate. The penalisation factor for the stiffness matrix is taken as 3. The input data is given here. $E_{11} = E_{xx} = 25 \times 10^6$, $E_{22} = E_{yy} = 1 \times 10^6$, $G_{xy} = 0.5E_{22}$, $G_{xz} = 0.5 \times 10^6$, $G_{yz} = 0.2 \times 10^6$. Fig.3 shows the initial design domain. For the elliptical laminated composite shell, the radius of curvature along x and y directions is equal a taken as R_{xx} = 30 and R_{yy} = 30. For the hyperbolic paraboloid composite shell, the radius of curvature along x and y directions is taken as equal to R_{xx} = 30 and R_{yy} = -30. For the conoidal shell, the radius of curvature along the x-axis is taken as R_{xx} = 30 and the ratio of hl/hh is taken as equal to 0.60/4 = 0.15. For the cylindrical shell, the radius of curvature along the x-direction is taken as an r equal to R_{xx} = 30. Optimality criteria method is applied to perform topology optimization.

6.2.1 Topology optimization of laminated elliptical paraboloid shell with Strain energy as the objective function

The strain energy of the structure at every iteration is evaluated. The penalisation factor for the stiffness matrix is taken as 3. Graph 1 in Fig.8b shows the variation of the strain energy on the Y-axis and iteration number on the X-axis. The optimal set of parameters for optimality criteria are given here. The total number of iterations is 22. There are three laminae 0/90/0. The filter radius is equal to 2 and the move limit is equal to 0.3. The stabilisation factor is taken as equal to 0.50. The minimum volume of the material is 0.30 or 30%. Fig.8a shows the optimal distribution of material for the laminated composite shell after 22 iterations. The optimal distribution in Fig.8a shows that the material near the centre of the edges carries a maximum value of the relative density as shown in the yellow color. This material participates in carrying the load acting on the structure. The material in blue whose value of relative density is low does not participate in carrying the load.

6.2.2 Topology optimization of laminated elliptical paraboloid shells with fundamental frequency as the objective function

The number of laminae is taken as two. The lamina is -45/45. The lamina oriented at 45 degrees gives the highest value for the fundamental frequency. The optimal distribution of material as shown in Fig.9a shows that the elements in yellow color carrying higher value of relative density and participate in higher stiffness of the structures. The elements in blue color carry the lower value of relative density and contribute less towards the objective function. The material at the centre of the edges and near the centre of the shell is yellow in color. As shown in Fig. 9b the variation of the non-dimensional fundamental frequency on the Y-axis and the iterations on the X-axis. The initial value of the frequency is high for the lamina -45/45. The total number of iterations is 22.

7. Conclusion

Topology optimization of SSSS composite laminated plates was performed in this study. Two types of problems were solved here in this research paper. Topology optimization with strain energy as the objective function and fundamental frequency as the objective function for several laminae were considered. There was no existing literature in this area of study. The optimal alignment of beams for the simply supported plate carrying uniformly distributed load was at the centre of the edges in the form of (+) sign and not at the corners. A few examples with different laminae were solved and the results were presented. The measures of the deflection and the strain energy were presented here. Finally, the lamina when arranged in the form of the plus (+) was showing higher bending stiffness compared to all other laminae for an SSSS plate. The lamina should be oriented along this direction which is 0 degrees and 90 degrees. The topology optimization of simply supported plate with fundamental frequency as the objective function has shown that the material near the centre has higher values of relative density and correspondingly these elements participate with the higher weightage of relative density raised to the penalisation factor and contribute more towards the objective function. The material in blue color carried lower values for relative density and did not participate in stiffness of the structure. Topology optimization of the laminated composite elliptical paraboloid shell has shown that the elements near the centre of the edge carried higher values of relative density and were shown in yellow color. The elements in blue color did not participate in the stiffness of the structure.

8. Future Study

This present research study opens a new door and presents a method to perform topology optimization of laminated structures.

1. Topology optimization of laminated composite structures with different geometry and boundary conditions subjected to hygrothermal loading can be a research topic.
2. Topology optimization of laminated composite structures using swarm intelligence algorithms is a new area of study.
3. Performance-based topology optimization of composite structures can be a good research topic for further study.

Author Contributions

K.N.V. Chandrasekhar planned the scheme, initiated the project and developed the mathematical modelling and code; V. Bhikshma suggested the methodology and timeline schema for conducting the project; K.U. Bhaskara Reddy developed the code and conducted the analysis and validated the results. The manuscript was written through the contribution of all the authors. All authors discussed the results, reviewed and approved the final version of the manuscript.

Acknowledgments

Not applicable.

Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship and publication of this article.

Funding

The authors received no financial support for the research, authorship and publication of this article.

Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

References

[1] J.N. Reddy, Free vibration of antisymmetric, angle -ply laminated plates including transverse shear deformation by the finite element method, *Journal of Sound and Vibration*, j(4), 1979, M-576.
[2] A.J.M. Ferreira, C.M.C. Roque, R.M.N. Jorge, Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions, *Computer Methods in Applied Mechanics and Engineering*, 194, 2005, 4265–4278.
[3] G.R. Liu, X. Zhao, K.Y. Dai, Z.H. Zhong, G.Y. Li, X. Han, Static and free vibration analysis of laminated composite plates using the conforming radial point interpolation method, *Journal of Composites Science and Technology*, 68, 2008, 354–366.
[4] J.-L. Batoz, A study of three-node triangular plate bending elements, *International Journal for Numerical Methods in Engineering*, 15, 1980, 1771-1812.
[5] Y.X. Zhang, K.S. Kim, A simple displacement-based 3-node triangular element for linear and geometrically nonlinear analysis of laminated composite plates, *Computer Methods in Applied Mechanics and Engineering*, 194, 2005, 4607–4632.
[6] D. Chakravorthy, J.N. Bandyopadhyay, FINITE ELEMENT FREE VIBRATION ANALYSIS OF DOUBLY CURVED LAMINATED COMPOSITE SHELLS, *Journal of Sound and Vibration*, 191(4), 1996, 491-504.
[7] W.-H. Lee, S.-C. Han, Free and forced vibration analysis of laminated composite plates and shells using a 9-node assumed strain shell element, *Computational Mechanics*, 39, 2006, 41-58.
[8] R.K. Kapania, P. Mohan, Static, free vibration and thermal analysis of composite plates and shells using a flat triangular shell element, *Computational Mechanics,* 17, 1966, 343-357.
[9] M.H. Izadi, S. Hosseini-Hashemi, M. Habibnejad Korayem, Analytical and fem solutions for free vibration of joined cross-ply laminated thick conical shells using shear deformation theory, *Archive of Applied Mechanics*, 88, 2018, 2231–2246.
[10] T. Ye, G. Jin, Z. Su, X. Jia, A unified Chebyshev-Ritz formulation of vibration analysis of composite laminated deep open shells with arbitrary boundary conditions, *Archive of Applied Mechanics*, 84, 2014, 441-471.
[11] K.K. Viswanathan, J.H. Lee, Z. Abdul Aziz, I. Hossain, Free vibration of symmetric angle-ply laminated cylindrical shells of variable thickness, *Acta Mechanica,* 221, 2011, 309-319.
[12] A. Chate, R. Rikards, A. Korjakin, Analysis of free damped vibrations of laminated composite cylindrical shells, *Mechanics of Composite Materials*, 31(5), 1995, 39-47.
[13] S. Hosseini-Hashemi, R. Atashipour, M. Fadace, U.A. Girhammar, An exact closed-form procedure for free vibration analysis of laminated spherical shell panels based on Sanders theory, *Archive of Applied Mechanics*, 82, 2012, 985-1002.
[14] C.P. Wu, C.H. Wu, Asymptotic differential quadrature solutions for the free vibration of laminated conical shells, *Computational Mechanics*, 25, 2000, 346-357.
[15] H. Ersoy, B. Akgoz, O. Civalek, Static analysis of laminated conical shells by discrete singular convolution (dsc) approach, *KSCE Journal of Civil Engineering*, 18, 2014, 1455–1463.
[16] C. Shu, Free vibration analysis of composite laminated conical shells by generalised differential quadrature, *Journal of Sound and Vibration*, 194(4), 1996, 587-604.
[17] T. Timarci, K.P. Soldatos, Vibrations of angle-ply laminated circular cylindrical shells subjected to different sets of edge boundary conditions, *Journal of Engineering Mechanics*, 37, 2000, 211-230.
[18] F. Tornabene et. al., General higher-order equivalent single layer theory for free vibrations of doubly-curved laminated composite shells and panels, *Composite Structures*, 104, 2013, 94-117.
[19] D. Chakravarthy, J.N. Bandyopadhyay and P.K.Sinha, Free vibration analysis of point-supported laminated composite doubly curved shells-a finite element approach, *Computers and Structures*, 54(2), 1995, 191-198.
[20] D. Chakravarthy, J.N. Bandyopadhyay, P.K.Sinha, Finite element free vibration analysis of doubly curved laminated composite shells, *Journal of Sound and Vibration*, 191(4), 1996, 491-504.
[21] G. Serhat, I. Basdogan, Multi-objective optimization of composite plates using lamination parameters, *Material and Design*, 180, 2019, 107904.
[22] C. Talischi, G.H. Paulino, C.H. Le, Topology optimization using wachspress type interpolation with hexagonal elements, *AIP Conference proceeding* , 973, 2008, 309.
[23] G. Yi, Y. Sui, TIMP method for topology optimization of plate structures with displacement constraints under multiple loading cases, *Structural and Multidisciplinary Optimization*, 53(6), 2016, 1185-1196.
[24] N. Liu, A.E. Jeffers, Isogeometric analysis of laminated composite and functionally graded sandwich plates based on a layerwise displacement theory, *Composite Structures*, 176, 2017, 143-153.
[25] N. Liu, A.E. Jeffers, Adaptive isogeometric analysis in structural frames using a layer-based discretization to model spread of plasticity, *Computers and Structures*, 196, 2018, 1–11.
[26] V.P. Veedu et.al., Multifunctional composites using reinforced lamina with carbon nanotubes forests, *Nature Materials*, 5, 2006, 457-462.
[27] K.S. Sai Ram, P.K. Sinha, Hygrothermal effects on the bending characteristics of laminated composite plates, *International Journal of Computers and Structures*, 40, 1991, 1009-1015.
[28] C.S. Krishnamoorthy, *Finite Element Analysis, Theory and Programming*, Tata McGraw Hill Publishers, 1999.
[29] K. Bakshi1, D. Chakravorty, Relative performances of composite conoidal shell roofs with parametric variations in terms of static, free and forced vibration behaviour, *International Journal of Civil Engineering*, 12(2), 2014, 201-213.
[30] H.S. Das, D. Chakravorty, Natural frequencies and mode shapes of composite conoids with complicated boundary conditions, *Journal of Reinforced Plastics and Composites*, 27(13), 2008, 1397-1415.
[31] A.J.M. Ferreira, C.M.C. Roque, R.M.N. Jorge, Free vibration analysis of symmetric laminated composite plates by FSDT and radial basis functions, *Computer Methods in Applied Mechanics and Engineering,* 194, 2005, 4265–4278.

ORCID iD

K.N.V. Chandrasekhar https://orcid.org/0000-0003-3930-1196
K.U. Bhaskara Reddy https://orcid.org/0000-0003-2174-3404

How to cite this article: Chandrasekhar K.N.V., Bhikshma V., Bhaskara Reddy K.U. Topology Optimization of Laminated Composite Plates and Shells using Optimality Criteria, *J. Appl. Comput. Mech.*, 8(2), 2022, 405–415. https://doi.org/10.22055/JACM.2019.31296.1858