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Finite Element Analysis of Functionally Graded Skew Plates in Thermal Environment based on the New Third-order Shear Deformation Theory

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Abstract. Functionally graded materials are commonly used in thermal environment to change the properties of constituent materials. The new numerical procedure of functionally graded skew plates in thermal environment is presented in this study based on the C0-form of the novel third-order shear deformation theory. Without the shear correction factor, this theory is also taking the desirable properties and advantages of the third-order shear deformation theory. We assume that the uniform distribution of temperature is embedded across the thickness of this structure. Both the rule of mixture and the micromechanics approaches are considered to describe the variation of material compositions across the thickness. Numerical solutions and comparison with other available solutions suggest that this procedure based on novel third-order shear deformation theory is accuracy and efficiency.

Keywords: Skew plates; Functionally graded materials; Finite element analysis; Third-order shear deformation theory; Thermal environment.

1. Introduction

Functionally graded materials have been successfully applied in numerous fields of engineering. The material is normally made from a mixture of ceramic and metal and provided the continuous variation of material properties from the bottom surface to the top surface of the plate. The functionally graded materials have obtained more attention in thermal environment applications, such as spacecraft, nuclear tank and so on. The analytical solutions [1-4] are valuable in certain cases, but in general cases with complicated geometries or complex conditions like a high temperature in the thermal environment, etc., are often limited. A slew of plate theories has been introduced in the past few decades [5, 6]. However, one may easily recognize that the third-order shear deformation plate theories are effective and accurate theories due to the quadratic variation of the transverse shear strains and stresses along the thickness of the plate as well as the shear locking free. Recently, Shi [7] gave a simple third-order shear deformation plate theory based on rigorous kinematics of displacements, initially applied to static analysis of isotropic and orthotropic beams and plates. The solutions obtained by Shi's theory have indicated to be more reliable and highly accurate than others [8-10]. Besides the analytical approaches, numerical methods are used in the structural analyses [11-26]. The Carrera unified formulation which allows finite element matrices/vectors to be derived in terms of fundamental nuclei can be also considered as a powerful higher-order technique to detect the accurate structural behavior of multilayered plates and shells [27-32]. A few studies related to functionally graded materials are presented in [33, 34], etc. In this study, the finite element analysis

of a functionally graded skew plate in a thermal environment based on Shi's theory with C0-form is the main objective.

The next sections of this paper are as follows. The concept of functionally graded skew plates, including the change of material properties under thermal conditions and the finite element formulation for static and free vibration analyses, are presented in Sect. 2. Numerical solutions of static bending deflections and natural frequencies for this structure not only with different skew angles but also with circular cutouts are shown in Sect.3. Some concluding remarks are given in the last section.

2. Functionally Graded Skew Plate and Finite Element Formulation

2.1 Functionally graded skew plate

Let us consider a functionally graded skew plate with geometry as plotted in Fig. 1a, 1b, and 1c. The bottom and top faces of the plate are to be fully metallic and ceramic and skew angle, namely Ψ , respectively. The mid-plane of the plate is the x-y plane. The z-axis is chosen perpendicular to the x-y plane.

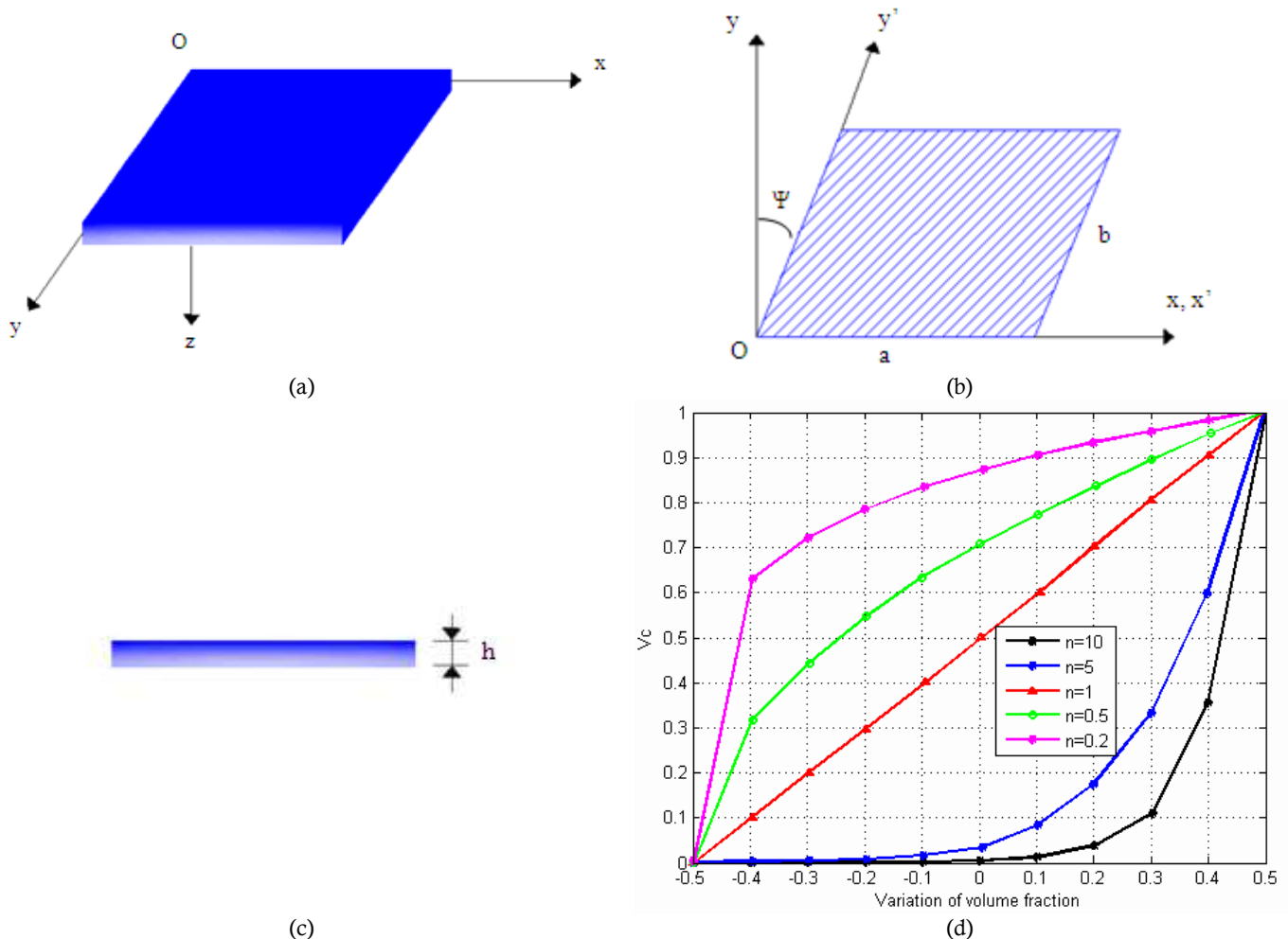


Fig. 1. The functionally graded skew plate a) in three-dimensional space, b) in the x-y plane, c) with thickness h , d) with the variation of volume fraction.

The volume fraction of the ceramic (V_c) and the metal (V_m) are described in (1) and the variation of volume fraction for several volume fraction coefficients of a functionally graded plate using the power-law distribution is plotted by Fig. 1d.

$$V_c = \left(\frac{z}{h} + \frac{1}{2} \right)^n; \quad V_m = 1 - V_c \quad \text{with } n \geq 0 \quad (1)$$

where z is the thickness coordinate variable with $-h/2 \leq z \leq h/2$ as well as c , m and n represent the ceramic, metal constituents and the non-negative volume fraction gradient index, respectively. In order to find the effective properties at a point of the functionally graded core, homogenization techniques such as the rule of mixture (RM) [5, 25] or the micro model (MM) [23, 24, 35] can be employed. Both homogenization techniques are used in the present study. And all values of E , ρ , ν and α that vary through the thickness of the plate are formulated as below.

$$E(z) = E_m + (E_c - E_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n \quad (2)$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n \quad (3)$$

$$\nu(z) = \nu_m + (\nu_c - \nu_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n \quad (4)$$

$$\alpha(z) = \alpha_m + (\alpha_c - \alpha_m) \left(\frac{1}{2} + \frac{z}{h} \right)^n \quad (5)$$

Alternatively, the micro model (MM) based on the Mori–Tanaka method is also given for the characterization of the material gradation. The effective bulk modulus K_{eff} and the shear modulus G_{eff} of a mixture of two constituents are determined

$$[K_{\text{eff}} - K_c] / [K_m - K_c] = V_m / \left\{ 1 + (1 - V_m) \left[\frac{K_m - K_c}{K_c + \frac{4}{3}G_c} \right] \right\} \quad (6)$$

$$[G_{\text{eff}} - G_c] / [G_m - G_c] = V_m / \left\{ 1 + (1 - V_m) \left[\frac{G_m - G_c}{G_c + \frac{G_c(9K_c + 8G_c)}{6(K_c + 2G_c)}} \right] \right\} \quad (7)$$

Here, (K_c, G_c) and (K_m, G_m) are the bulk and shear moduli of the ceramic and the metal constituents, respectively. The bulk and shear moduli of each material are obtained by the following (8). Finally, the effective Young's modulus E_{eff} and Poisson's ratio ν_{eff} are given by (9).

$$K_l = \frac{E_l}{3(1 - 2\nu_l)} \quad G_l = \frac{E_l}{2(1 + \nu_l)} \quad l \cong m, c \quad (8)$$

$$E_{\text{eff}} = \frac{9K_{\text{eff}}G_{\text{eff}}}{3K_{\text{eff}} + G_{\text{eff}}} \quad \nu_{\text{eff}} = \frac{3K_{\text{eff}} - 2G_{\text{eff}}}{2(3K_{\text{eff}} + G_{\text{eff}})} \quad (9)$$

The function of temperature $T(K)$ can be expressed [1]

$$P = P_0 (P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (10)$$

where $T = T_0 + \Delta T$ and $T_0 = 300^\circ\text{K}$ (ambient or free stress temperature), ΔT is the temperature change, and $P_0, P_{-1}, P_1, P_2, P_3$ are the coefficients of temperature $T(K)$, and are unique to each constituent.

2.2 Finite element formulation

According to the new theory of Shi [7], the three-dimensional displacement field (u, v, w) can be expressed in terms of C0-higher-order shear deformation theory and seven unknown variables as follows:

$$u(x, y, z) = u_0(x, y) + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3 \right) \phi_x^b + \frac{5}{4} \left(z - \frac{4}{3h^2}z^3 \right) \phi_x^s(x, y) \quad (11)$$

$$v(x, y, z) = v_0(x, y) + \left(\frac{1}{4}z - \frac{5}{3h^2}z^3 \right) \phi_y^b + \frac{5}{4} \left(z - \frac{4}{3h^2}z^3 \right) \phi_y^s(x, y) \quad (12)$$

$$w(x, y, z) = w_0(x, y) \quad (13)$$

It can be seen that the present theory is composed of seven unknowns including three axial and transverse displacements, four rotations due to the bending and shear effects. Under, the strain-displacement relations based on the small strain assumptions can be given as follows:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} u_{0,x} + z \frac{1}{4} (5\phi_{x,x}^s + \phi_{x,x}^b) + z^3 \left(\frac{-5}{3h^2} \right) (\phi_{x,x}^s + \phi_{x,x}^b) \\ v_{0,y} + z \frac{1}{4} (5\phi_{y,y}^s + \phi_{y,y}^b) + z^3 \left(\frac{-5}{3h^2} \right) (\phi_{y,y}^s + \phi_{y,y}^b) \\ u_{0,y} + v_{0,x} + z \frac{1}{4} (5\phi_{x,y}^s + 5\phi_{y,x}^s + \phi_{x,y}^b + \phi_{y,x}^b) + z^3 \left(\frac{-5}{3h^2} \right) (\phi_{x,y}^s + \phi_{y,x}^s + \phi_{x,y}^b + \phi_{y,x}^b) \\ \left(\frac{5}{4} \phi_y^s + \frac{1}{4} \phi_y^b + w_{,y} \right) + z^2 \left(\frac{-5}{h^2} \right) (\phi_y^s + \phi_y^b) \\ \left(\frac{5}{4} \phi_x^s + \frac{1}{4} \phi_x^b + w_{,x} \right) + z^2 \left(\frac{-5}{h^2} \right) (\phi_x^s + \phi_x^b) \end{Bmatrix} \quad (14)$$

in matrix form:

$$\begin{Bmatrix} \varepsilon \\ \gamma \end{Bmatrix} = \begin{Bmatrix} \varepsilon^{(0)} \\ \gamma^{(0)} \end{Bmatrix} + z \begin{Bmatrix} \varepsilon^{(1)} \\ \mathbf{0} \end{Bmatrix} + z^2 \begin{Bmatrix} \mathbf{0} \\ \gamma^{(2)} \end{Bmatrix} + z^3 \begin{Bmatrix} \varepsilon^{(3)} \\ \mathbf{0} \end{Bmatrix} \quad (15)$$

with

$$\varepsilon^{(0)} = \begin{Bmatrix} u_{0,x} \\ v_{0,y} \\ u_{0,y} + v_{0,x} \end{Bmatrix}; \quad \varepsilon^{(1)} = \frac{1}{4} \begin{Bmatrix} (5\phi_{x,x}^s + \phi_{x,x}^b) \\ (5\phi_{y,y}^s + \phi_{y,y}^b) \\ (5\phi_{x,y}^s + 5\phi_{y,x}^s + \phi_{x,y}^b + \phi_{y,x}^b) \end{Bmatrix}; \quad \varepsilon^{(3)} = \frac{-5}{3h^2} \begin{Bmatrix} \phi_{x,x}^s + \phi_{x,x}^b \\ \phi_{y,y}^s + \phi_{y,y}^b \\ \phi_{x,y}^s + \phi_{x,y}^b + \phi_{y,x}^s + \phi_{y,x}^b \end{Bmatrix} \quad (16)$$

$$\gamma^{(0)} = \begin{Bmatrix} \frac{5}{4} \phi_y^s + \frac{1}{4} \phi_y^b + w_{,y} \\ \frac{5}{4} \phi_x^s + \frac{1}{4} \phi_x^b + w_{,x} \end{Bmatrix}; \quad \gamma^{(2)} = \frac{-5}{h^2} \begin{Bmatrix} \phi_y^s + \phi_y^b \\ \phi_x^s + \phi_x^b \end{Bmatrix} \quad (17)$$

Under Hooke's law, the constitutive equation is expressed as:

$$\boldsymbol{\sigma} = \mathbf{D}_m(\mathbf{z})(\boldsymbol{\varepsilon}^{(0)} + \mathbf{z}\boldsymbol{\varepsilon}^{(1)} + \mathbf{z}^3\boldsymbol{\varepsilon}^{(3)} - \boldsymbol{\varepsilon}^{(T)}) \quad (18)$$

$$\boldsymbol{\tau} = \mathbf{D}_s(\mathbf{z})(\boldsymbol{\gamma}^{(0)} + \mathbf{z}^2\boldsymbol{\gamma}^{(2)}) \quad (19)$$

in which:

$$\boldsymbol{\sigma} = [\sigma_x \quad \sigma_y \quad \sigma_{xy}]^T; \quad \boldsymbol{\tau} = [\tau_{yz} \quad \tau_{xz}]^T \quad (20)$$

$$\mathbf{D}_m(\mathbf{z}) = \frac{E(\mathbf{z})}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \quad (21)$$

$$\mathbf{D}_s(\mathbf{z}) = \frac{E(\mathbf{z})}{2(1+\nu)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (22)$$

$$\boldsymbol{\varepsilon}^{(T)} = [\varepsilon_x^{(T)} \quad \varepsilon_y^{(T)} \quad 0]^T = [\alpha(z)\Delta T \quad \alpha(z)\Delta T \quad 0]^T \quad (23)$$

The generalized displacements can hence be approximated as:

$$\mathbf{u}_0 = \mathbf{N}\mathbf{q}_e \quad (24)$$

with

$$\mathbf{u}_0 = [u_0 \quad v_0 \quad w \quad \phi_x^s \quad \phi_y^s \quad \phi_x^b \quad \phi_y^b]^T \quad (25)$$

$$\mathbf{N} = [N_1 \quad N_2 \quad N_3 \quad N_4] \quad (26)$$

$$\mathbf{q}_e = [\mathbf{q}_{1e} \quad \mathbf{q}_{2e} \quad \mathbf{q}_{3e} \quad \mathbf{q}_{4e}]^T \quad (27)$$

where $\mathbf{q}_e$ and N are the unknown displacement vector and the shape function vector. From Eqs. (16), (17) and (24), the strain can be rewritten as:

$$\boldsymbol{\varepsilon} = (\mathbf{B}_1 + \mathbf{B}_2 + \mathbf{B}_3) \mathbf{q}_e \quad (28)$$

$$\boldsymbol{\gamma} = (\mathbf{B}_4 + \mathbf{B}_5) \mathbf{q}_e \quad (29)$$

in which

$$\mathbf{B}_1 = \sum_{i=1}^4 \begin{bmatrix} \mathbf{N}_{i,x} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{N}_{i,y} & 0 & 0 & 0 & 0 & 0 \\ \mathbf{N}_{i,y} & \mathbf{N}_{i,x} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}; \quad \mathbf{B}_2 = \frac{1}{4} \sum_{i=1}^4 \begin{bmatrix} 0 & 0 & 0 & 5\mathbf{N}_{i,x} & 0 & \mathbf{N}_{i,x} & 0 \\ 0 & 0 & 0 & 0 & 5\mathbf{N}_{i,y} & 0 & \mathbf{N}_{i,y} \\ 0 & 0 & 0 & 5\mathbf{N}_{i,y} & 5\mathbf{N}_{i,x} & \mathbf{N}_{i,y} & \mathbf{N}_{i,x} \end{bmatrix} \quad (30)$$

$$\mathbf{B}_3 = -\frac{5}{3h^2} \sum_{i=1}^4 \begin{bmatrix} 0 & 0 & 0 & \mathbf{N}_{i,x} & 0 & \mathbf{N}_{i,x} & 0 \\ 0 & 0 & 0 & 0 & \mathbf{N}_{i,y} & 0 & \mathbf{N}_{i,y} \\ 0 & 0 & 0 & \mathbf{N}_{i,y} & \mathbf{N}_{i,x} & \mathbf{N}_{i,y} & \mathbf{N}_{i,x} \end{bmatrix} \quad (31)$$

$$; \mathbf{B}_5 = -\frac{5}{h^2} \sum_{i=1}^4 \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (32)$$

The normal forces, bending moments, higher-order moments and shear force can then be computed through the following relations.

$$\begin{aligned} \bar{\mathbf{N}} &= \{\tilde{N}_x \quad \tilde{N}_y \quad \tilde{N}_{xy}\}^T = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{xy}\}^T dz \\ &= \int_{-h/2}^{h/2} \mathbf{D}_m(z) (\boldsymbol{\varepsilon}^{(0)} + z\boldsymbol{\varepsilon}^{(1)} + z^3\boldsymbol{\varepsilon}^{(3)} - \boldsymbol{\varepsilon}^{(T)}) dz \mathbf{B}_4 = \sum_{i=1}^4 \begin{bmatrix} 0 & 0 & \mathbf{N}_{i,y} & 0 & \frac{5}{4} & 0 & \frac{1}{4} \\ 0 & 0 & \mathbf{N}_{i,x} & \frac{5}{4} & 0 & \frac{1}{4} & 0 \end{bmatrix} \end{aligned} \quad (33)$$

$$\bar{\mathbf{M}} = \{\tilde{M}_x \quad \tilde{M}_y \quad \tilde{M}_{xy}\}^T = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{xy}\}^T z dz = \int_{-h/2}^{h/2} \mathbf{D}_m(z) (\boldsymbol{\varepsilon}^{(0)} + z\boldsymbol{\varepsilon}^{(1)} + z^3\boldsymbol{\varepsilon}^{(3)} - \boldsymbol{\varepsilon}^{(T)}) z dz \quad (34)$$

$$\bar{\mathbf{P}} = \{\tilde{P}_x \quad \tilde{P}_y \quad \tilde{P}_{xy}\}^T = \int_{-h/2}^{h/2} \{\sigma_x \quad \sigma_y \quad \sigma_{xy}\}^T z^3 dz = \int_{-h/2}^{h/2} \mathbf{D}_m(z) (\boldsymbol{\varepsilon}^{(0)} + z\boldsymbol{\varepsilon}^{(1)} + z^3\boldsymbol{\varepsilon}^{(3)} - \boldsymbol{\varepsilon}^{(T)}) z^3 dz \quad (35)$$

$$\bar{\mathbf{Q}} = \{\tilde{Q}_y \quad \tilde{Q}_x\}^T = \int_{-h/2}^{h/2} \{\tau_{yz} \quad \tau_{xz}\}^T dz = \int_{-h/2}^{h/2} \mathbf{D}_s(z) (\boldsymbol{\gamma}^{(0)} + z^2\boldsymbol{\gamma}^{(2)}) dz \quad (36)$$

$$\bar{\mathbf{R}} = \{\tilde{R}_y \quad \tilde{R}_x\}^T = \int_{-h/2}^{h/2} \{\tau_{yz} \quad \tau_{xz}\}^T z^2 dz = \int_{-h/2}^{h/2} \mathbf{D}_s(z) (\boldsymbol{\gamma}^{(0)} + z^2\boldsymbol{\gamma}^{(2)}) z^2 dz \quad (37)$$

Eqs. (33), (34), (35), (36) and (37) can be presented in the matrix form:

$$\begin{bmatrix} \bar{\mathbf{N}} \\ \bar{\mathbf{M}} \\ \bar{\mathbf{P}} \\ \bar{\mathbf{Q}} \\ \bar{\mathbf{R}} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{A}} & \bar{\mathbf{B}} & \bar{\mathbf{E}} & \mathbf{0} & \mathbf{0} \\ \bar{\mathbf{B}} & \bar{\mathbf{D}} & \bar{\mathbf{F}} & \mathbf{0} & \mathbf{0} \\ \bar{\mathbf{E}} & \bar{\mathbf{F}} & \bar{\mathbf{H}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{A}} & \hat{\mathbf{B}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \hat{\mathbf{B}} & \hat{\mathbf{D}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}^{(0)} \\ \boldsymbol{\varepsilon}^{(1)} \\ \boldsymbol{\varepsilon}^{(3)} \\ \boldsymbol{\gamma}^{(0)} \\ \boldsymbol{\gamma}^{(2)} \end{bmatrix} - \begin{bmatrix} \bar{\mathbf{N}}^{(T)} \\ \bar{\mathbf{M}}^{(T)} \\ \bar{\mathbf{P}}^{(T)} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (38)$$

with

$$(\bar{\mathbf{A}}, \bar{\mathbf{B}}, \bar{\mathbf{D}}, \bar{\mathbf{E}}, \bar{\mathbf{F}}, \bar{\mathbf{H}}) = \int_{-h/2}^{h/2} (1, z, z^2, z^3, z^4, z^6) \mathbf{D}_m(z) dz \quad (39)$$

$$(\hat{\mathbf{A}}, \hat{\mathbf{B}}, \hat{\mathbf{D}}) = \int_{-h/2}^{h/2} (1, z^2, z^4) \mathbf{D}_s(z) dz \quad (40)$$

$$(\bar{\mathbf{N}}^{(T)}, \bar{\mathbf{M}}^{(T)}, \bar{\mathbf{P}}^{(T)}) = \int_{-h/2}^{h/2} \mathbf{D}_m(z) (1, z, z^3) \{1 \ 1 \ 0\}^T \alpha(z) \Delta T dz \quad (41)$$

The total strain energy of a plate due to the normal forces, shear force, bending moments and higher-order moments can be given by:

$$U = \frac{1}{2} \int_{V_e} \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV - \int_{S_e} \mathbf{u}^T \mathbf{f} dS = \frac{1}{2} \mathbf{q}_e^T \int_{S_e} \left(\begin{array}{l} \mathbf{B}_1^T \bar{\mathbf{A}} \mathbf{B}_1 + \mathbf{B}_1^T \bar{\mathbf{B}} \mathbf{B}_2 + \mathbf{B}_1^T \bar{\mathbf{E}} \mathbf{B}_3 + \\ + \mathbf{B}_2^T \bar{\mathbf{B}} \mathbf{B}_1 + \mathbf{B}_2^T \bar{\mathbf{D}} \mathbf{B}_2 + \mathbf{B}_2^T \bar{\mathbf{F}} \mathbf{B}_3 + \\ + \mathbf{B}_3^T \bar{\mathbf{E}} \mathbf{B}_1 + \mathbf{B}_3^T \bar{\mathbf{F}} \mathbf{B}_2 + \mathbf{B}_3^T \bar{\mathbf{H}} \mathbf{B}_3 + \\ + \mathbf{B}_4^T \hat{\mathbf{A}} \mathbf{B}_4 + \mathbf{B}_4^T \hat{\mathbf{B}} \mathbf{B}_5 + \mathbf{B}_5^T \hat{\mathbf{B}} \mathbf{B}_4 + \\ + \mathbf{B}_5^T \hat{\mathbf{D}} \mathbf{B}_5 \end{array} \right) dS \mathbf{q}_e - \quad (42)$$

$$- \mathbf{q}_e^T \int_{S_e} (\mathbf{B}_1^T \bar{\mathbf{N}}^{(T)} + \mathbf{B}_2^T \bar{\mathbf{M}}^{(T)} + \mathbf{B}_3^T \bar{\mathbf{P}}^{(T)}) dS - \mathbf{q}_e^T \int_{S_e} \mathbf{N}^T \mathbf{f} dS + \frac{1}{2} \int_{S_e} (\boldsymbol{\varepsilon}^{(T)})^T \bar{\mathbf{A}} \boldsymbol{\varepsilon}^{(T)} dS$$

$$U = \frac{1}{2} \mathbf{q}_e^T \mathbf{K}_e \mathbf{q}_e - \mathbf{q}_e^T \mathbf{F}_e^{(T)} - \mathbf{q}_e^T \mathbf{F}_e + \mathbf{C}^{(T)} = \mathbf{q}_e^T \left(\frac{1}{2} \mathbf{K}_e \mathbf{q}_e - \mathbf{F}_e^{(T)} - \mathbf{F}_e \right) + \mathbf{C}^{(T)} \quad (43)$$

The kinetic energy is shown as:

$$T = \frac{1}{2} \int_{V_e} \dot{\mathbf{u}}^T \boldsymbol{\rho}(z) \dot{\mathbf{u}} dV = \frac{1}{2} \dot{\mathbf{q}}_e^T \left\{ \int_{V_e} \mathbf{N}^T \mathbf{L}^T \boldsymbol{\rho}(z) \mathbf{L} \mathbf{N} dV \right\} \dot{\mathbf{q}}_e = \frac{1}{2} \dot{\mathbf{q}}_e^T \mathbf{M}_e \dot{\mathbf{q}}_e \quad (44)$$

in which $\mathbf{L}$ is clearly described as:

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & \left(\frac{1}{4} z - \frac{5}{3h^2} z^3 \right) \frac{\partial}{\partial x} & \frac{5}{4} \left(z - \frac{4}{3h^2} z^3 \right) & 0 \\ 0 & 1 & \left(\frac{1}{4} z - \frac{5}{3h^2} z^3 \right) \frac{\partial}{\partial y} & 0 & \frac{5}{4} \left(z - \frac{4}{3h^2} z^3 \right) \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (45)$$

and the mass matrix of the element is presented:

$$\mathbf{M}_e = \int_{V_e} \mathbf{N}^T \mathbf{L}^T \boldsymbol{\rho}(z) \mathbf{L} \mathbf{N} dV = \int_{S_e} \mathbf{N}^T \left(\int_{-h/2}^{h/2} \boldsymbol{\rho}(z) \mathbf{L}^T \mathbf{L} dz \right) \mathbf{N} dS \quad (46)$$

For bending analysis, the bending solutions can be obtained by solving the following equation:

$$\mathbf{K} \mathbf{d} = \mathbf{F} + \mathbf{F}^{(T)} \quad (47)$$

The dynamic equations for solving eigenvalue can be given as:

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{d} = 0 \quad (48)$$

For skew plates, the edges of elements are not parallel to the global axes of the plate. Hence it is necessary to transform the element matrices from global to local axes by using nodal-transformation matrix $\mathbf{T}$ so that the degrees of freedom of the nodes can be conveniently expressed, respectively

$$\mathbf{T} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 & 0 & 0 \\ \sin \psi & \cos \psi & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \psi & -\sin \psi \\ 0 & 0 & 0 & \sin \psi & \cos \psi \end{bmatrix} \quad (49)$$

3. Numerical Solutions

In this section, the numerical solutions for static bending and free vibration analyses of functionally graded skew plates in the thermal environment are presented. Some representative numerical examples of this structure having different skew angles are considered and analyzed. On the other hand, the functionally graded skew plate with a cutout is also considered. Not only the simply supported but also the fully clamped boundary conditions are used in this paper. The simply supported boundary conditions for this procedure is as follows:

$$v_0 = w = \phi_y^s = \phi_y^b = 0, \text{ at } x = 0, x = a \text{ and } u_0 = w = \phi_x^s = \phi_x^b = 0, \text{ at } y = 0, y = b \quad (50)$$

and the fully clamped boundary conditions are as below:

$$u_0 = v_0 = w = \phi_x^s = \phi_x^b = \phi_y^s = \phi_y^b = 0, \text{ at } x = 0, x = a \text{ and } y = 0, y = b \quad (51)$$

Table 1 gives the different material properties of functionally graded skew plates made of the ceramic (Al_2O_3 , Si_3N_4 , ZrO_2) and the metal (SUS304) [1, 3].

Table 1. Temperature-dependent coefficient of Young's modulus E (Pa), thermal expansion coefficient α (1/K), Poisson's ratio ν , mass density ρ (kg/m³) for various materials.

Ceramic/Metal	P ₀	P ₋₁	P ₁	P ₂	P ₃	P (300°K)
Al₂O₃						
E (Pa)	349.55e9	0	-3.853e-4	4.027e-7	-1.673e-10	320.24e9
α (1/K)	6.8269e-6	0	1.838e-4	0	0	7.203e-6
ν	0.26	0	0	0	0	0.260
ρ (kg/m ³)	3800	0	0	0	0	3800
Si₃N₄						
E (Pa)	348.43e9	0	-3.070e-4	2.160e-7	-8.946e-11	322.27e9
α (1/K)	5.8723e-6	0	9.095e-4	0	0	7.475e-6
ν	0.24	0	0	0	0	0.240
ρ (kg/m ³)	2370	0	0	0	0	2370
ZrO₂						
E (Pa)	244.27e9	0	-1.371e-3	1.214e-6	-3.681e-10	168.06e9
α (1/K)	12.766e-6	0	-1.491e-3	1.006e-5	-6.778e-11	18.591e-6
ν	0.288	0	1.133e-4		0	0.298
ρ (kg/m ³)	3657	0	0		0	3657
SUS304						
E (Pa)	201.04e9	0	3.079e-4	-6.534e-7	0	207.79e9
α (1/K)	12.330e-6	0	8.086e-4	0	0	15.321e-6
ν	0.326	0	-2.002e-4	3.797e-7	0	0.318
ρ (kg/m ³)	8166	0	0	0	0	8166

Firstly, a fully simply supported functionally graded skew plate made of Al/ZrO₂ subjected to a uniform load P is studied. Some geometrical properties of plate are $a/b = 1$ and $h = 0.1$. The material properties of Al and ZrO₂ are also given by $\nu_m = 0.3$, $E_m = 70\text{GPa}$ and $\nu_c = 0.3$, $E_c = 151\text{GPa}$. The maximum central deflection of this structure is respectively normalized by $\bar{w} = [100w_c E_m h^3] / [12(1 - \nu_m^2) P a^4]$. Table.2 presents a comparison of the normalized deflections of example gained by this method based on two homogenization techniques RM & MM and other solutions in [19]. The solutions reported in Table.2 are calculated for various values of the skew angle and the volume fraction coefficient. In order to confirm the correctness of this procedure to the functionally graded skew plates in the thermal environment, the temperature under consideration is set to be $T = 300^\circ\text{K}$ ($\Delta T = 0$). The same previous structure is given but it is now made of Si₃N₄/SUS304 instead. The material properties of Si₃N₄/SUS304 for this case can be found in Table 1. Under the analysis, the central deflections can be obtained by using finite element formulation based on Shi's theory with C0-form and normalized by $\bar{w} = [100w_c E_m h^3] / [12(1 - \nu_m^2) P a^4]$. They are compared with the analytical solutions of Wattanasakulpong et al. [1] for skew angle equal 0, as reported in Table 3. Noted that E_m and ν_m are Young's modulus

and Poisson's ratio of metal at $T = 300^\circ\text{K}$ detailed in Table 1. Several solutions from the current study for each value of Ψ and n are also found in Table 3.

Table 2. Comparison of the normalized deflections for various values of the volume fraction exponent n and skew angle ψ .

$\Psi (^\circ)$	$\bar{w}$							
	$n=0$		$n=0.5$		$n=1$		$n=2$	
	[19]	Present (RM)	[19]	Present (RM)	[19]	Present (RM)	[19]	Present (RM)
0	0.3461	0.3522	0.4621	0.4464	0.5134	0.5022	0.5562	0.5549
15	0.3106	0.3146	0.4147	0.3974	0.4608	0.4469	0.4993	0.4949
30	0.2200	0.2193	0.2936	0.2742	0.3263	0.3081	0.3538	0.3430
45	0.1137	0.1050	0.1515	0.1298	0.1686	0.1458	0.1831	0.1633
60	0.0356	0.0263	0.0473	0.0325	0.0527	0.0366	0.0576	0.0414

$\Psi (^\circ)$	$\bar{w}$							
	$n=0$		$n=0.5$		$n=1$		$n=2$	
	[19]	Present (MM)	[19]	Present (MM)	[19]	Present (MM)	[19]	Present (MM)
0	0.3461	0.3522	0.4621	0.4611	0.5134	0.5164	0.5562	0.5666
15	0.3106	0.3146	0.4147	0.4112	0.4608	0.4605	0.4993	0.5059
30	0.2200	0.2193	0.2936	0.2841	0.3263	0.3183	0.3538	0.3519
45	0.1137	0.1050	0.1515	0.1349	0.1686	0.1513	0.1831	0.1683
60	0.0356	0.0269	0.0473	0.0342	0.0527	0.0385	0.0576	0.0433

Table 3. The normalized deflections for various values of Ψ and n .

$\Psi (^\circ)$	$\bar{w}$							
	$n=0.5$		$n=1$		$n=5$		$n=10$	
	[1]	Present (RM)	[1]	Present (RM)	[1]	Present (RM)	[1]	Present (RM)
0	0.325	0.3248	0.343	0.3464	0.380	0.3847	0.396	0.3998
15	-	0.2835	-	0.3025	-	0.3382	-	0.3516
30	-	0.1906	-	0.2036	-	0.2302	-	0.2394
45	-	0.0903	-	0.0965	-	0.1103	-	0.1147
60	-	0.0247	-	0.0265	-	0.0306	-	0.0319

$\Psi (^\circ)$	$\bar{w}$							
	$n=0.5$		$n=1$		$n=5$		$n=10$	
	[1]	Present (MM)	[1]	Present (MM)	[1]	Present (MM)	[1]	Present (MM)
0	0.325	0.3280	0.343	0.3493	0.380	0.3870	0.396	0.4019
15	-	0.2863	-	0.3051	-	0.3403	-	0.3534
30	-	0.1925	-	0.2055	-	0.2316	-	0.2405
45	-	0.0912	-	0.0975	-	0.1110	-	0.1152
60	-	0.0250	-	0.0267	-	0.0308	-	0.0320

Next, the influence of the ratio a/b of functionally graded skew plates on the mechanical deflection is also studied. By completing it, a functionally graded skew plate made of $\text{Al}_2\text{O}_3/\text{SUS304}$ with some geometrical properties such as skew angle $\Psi=15^\circ$ and $a/h=10$ are taken, sustaining in high-temperature conditions from $T = 300^\circ\text{K}$ up to 800°K . Various values of the ratio $a/b = 0.2; 0.5; 1; 2$ and 5 are studied. The volume fraction exponent of this structure $n = 1$ is also taken. The numerical solutions of the normalized deflections of a fully clamped functionally graded skew plate are thus presented in Table 4 and also depicted in Fig. 2a.

We finally show the influence of the skew angle Ψ of functionally graded skew plates on the mechanical deflection. This plate made of $\text{Al}_2\text{O}_3/\text{SUS304}$ with $a/b=0.2$ and $a/h=10$ is given, upholding the high-temperature conditions from $T = 300^\circ\text{K}$ up to 800°K for the same case, respectively. Various values of the skew angle such as $0^\circ; 15^\circ; 30^\circ; 45^\circ$ and 60° are listed below. The volume fraction exponent $n = 1$ is taken again for this analysis. The normalized deflections

of a fully clamped functionally graded skew plate are given in Table 5 and also depicted again in Fig. 2b. Noted that the rule of mixture (RM) is only used in the above two parts.

Table 4. Influence of the ratio a/b on the normalized deflections with the rule of mixture (RM).

a/b	$T=300^0(K)$	400	500	600	700	800
0.2	0.1153	0.1173	0.1197	0.1227	0.1264	0.1313
0.5	0.0195	0.0198	0.0202	0.0207	0.0213	0.0221
1	0.0056	0.0057	0.0058	0.0060	0.0061	0.0063
2	0.0026	0.0026	0.0027	0.0027	0.0028	0.0029
5	0.0015	0.0015	0.0016	0.0016	0.0016	0.0017

Table 5. Influence of the skew angle Ψ on the normalized deflections with the rule of mixture (RM).

$\Psi (^{\circ})$	$T=300^0(K)$	400	500	600	700	800
0°	0.1213	0.1233	0.1258	0.1290	0.1330	0.1381
15°	0.1153	0.1173	0.1197	0.1227	0.1264	0.1313
30°	0.0957	0.0973	0.0993	0.1017	0.1048	0.1088
45°	0.0598	0.0608	0.0620	0.0635	0.0654	0.0679
60°	0.0201	0.0204	0.0208	0.0213	0.0219	0.0227

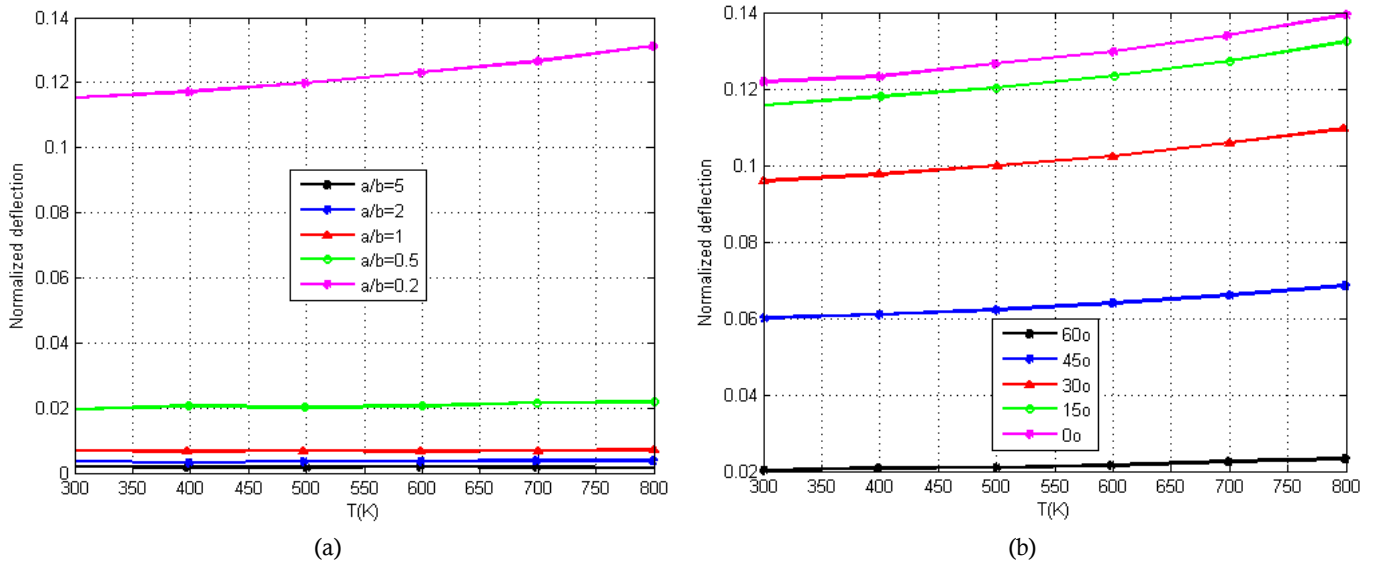


Fig. 2. a) Variation of the ratio a/b and its influence on the normalized deflections of a fully clamped functionally graded skew plate with $a/h = 10$, $\Psi = 15^{\circ}$ made of $Al_2O_3/SUS304$, b) Variation of skew angle Ψ and its influence on the normalized deflections of a fully clamped functionally graded skew plate with $a/b=0.2$, $a/h = 10$ made of $Al_2O_3/SUS304$.

Furthermore, the natural frequency of functionally graded skew plates in the thermal environment is investigated. The influence of skew angle on the frequency of the $Si_3N_4/SUS304$ skew plate with $a/h=10$ and two homogenization techniques RM & MM is considered. Two kinds of boundary conditions are used and the solutions are presented in Tables 6 and 7. On the other hand, several values of skew angles (0° , 15° , and 30°) are taken. The temperatures $T_c = 400^{\circ}K$ and $T_m = 300^{\circ}K$ are applied on the top and the bottom of this structure. The first six modes are calculated for various values of skew angle and the normalized frequencies are built by $\Omega = (\omega a^2 / h) [\rho_0 (1 - \nu^2) / E_0]^{1/2} / 2\pi$ as given in [20], where E_0 and ρ_0 are the reference values of E_m and ρ_m at $T = 300^{\circ}K$ as given in Table 1. It can be seen that an increase in the skew angle of the plate increases the frequency for all modes and this progression is not dependent on the value of n . It is also shown that an increase in the parameter n from ceramic to metal portion reduces the frequency because of the less stiffness of the plate. It can be concluded that the volume fraction exponent n is one of the essential parameters for understanding the vibration features of functionally graded skew plates. Among the two boundary conditions introduced, the highest frequency is obtained for a fully clamped skew plate and the lowest frequency is given for other conditions. This is due to the restraints of vibration on the boundaries of the structure. Furthermore, the first six modes of a simply supported functionally graded skew plate for $a/b = 1$ are also depicted in Fig. 3.

Table 6. Comparison of the normalized frequencies of the simply supported functionally graded skew plates with $a/h = 10$.

ψ (°)	n	Mode					
		1	2	3	4	5	6
0°	$n=0.0$, [20]	2.0061	4.8528	4.8528	7.4805	9.2543	9.2543
	Present (RM)	1.9828	4.8293	4.8293	7.4199	9.3638	9.3638
	Present (MM)	1.9828	4.8293	4.8293	7.4199	9.3638	9.3638
	$n=0.5$, [20]	1.3756	3.3344	3.3644	5.1483	6.3693	6.3723
	Present (RM)	1.3682	3.3338	3.3338	5.1222	6.4683	6.4683
	Present (MM)	1.3619	3.3180	3.3180	5.0977	6.4366	6.4367
	$n=1.0$, [20]	1.2022	2.9125	2.9125	4.4991	5.5643	5.5686
	Present (RM)	1.1981	2.9181	2.9181	4.4852	5.6599	5.6600
	Present (MM)	1.1934	2.9060	2.9060	4.4661	5.6345	5.6347
15°	$n=2.0$, [20]	1.0776	2.6100	2.6100	4.0304	4.9825	4.9865
	Present (RM)	1.0737	2.6119	2.6119	4.0142	5.0586	5.0587
	Present (MM)	1.0704	2.6033	2.6033	4.0005	5.0402	5.0403
	$n=0.0$, [20]	2.1240	4.7543	5.4808	7.5202	9.7136	10.0960
	Present (RM)	2.1075	4.6388	5.3726	7.3342	9.4495	10.0839
	Present (MM)	2.1075	4.6388	5.3726	7.3342	9.4495	10.0839
	$n=0.5$, [20]	1.4581	3.2678	3.7680	5.1773	6.6868	6.9537
	Present (RM)	1.4556	3.2037	3.7104	5.0654	6.5280	6.9682
	Present (MM)	1.4487	3.1884	3.6926	5.0410	6.4960	6.9340
30°	$n=1$, [20]	1.2749	2.8547	3.2916	4.5249	5.8417	6.0765
	Present (RM)	1.2743	2.8039	3.2476	4.4346	5.7122	6.0972
	Present (MM)	1.2691	2.7921	3.2338	4.4155	5.6866	6.0695
	$n=2$, [20]	1.1431	2.5585	2.9496	4.0537	5.2303	5.4405
	Present (RM)	1.1409	2.5086	2.9053	3.9669	5.1050	5.473
	Present (MM)	1.1372	2.5002	2.8954	3.9531	5.0864	5.4271
	$n=0.0$, [20]	2.5542	5.1870	6.9332	8.0161	9.9596	11.208
	Present (RM)	2.5455	4.6786	6.6033	7.3323	9.8536	10.3429
	Present (MM)	2.5455	4.6786	6.6033	7.3323	9.8536	10.3429
	$n=0.5$, [20]	1.7585	3.5693	4.7719	5.5237	6.9207	7.7743
	Present (RM)	1.7612	3.2343	4.5665	5.0704	6.8103	7.1539
	Present (MM)	1.7528	3.2186	4.5441	5.0456	6.7768	7.1185
	$n=1$, [20]	1.5392	3.1192	4.1695	4.8291	6.0239	6.7724
	Present (RM)	1.5412	2.8302	3.9955	4.4374	5.9594	6.2613
	Present (MM)	1.5345	2.8180	3.9775	4.4174	5.9322	6.2323
	$n=2$, [20]	1.3806	2.7958	3.7355	4.3260	5.3316	6.0061
	Present (RM)	1.3772	2.5301	3.5689	3.9646	5.3240	5.5916
	Present (MM)	1.3723	2.5213	3.5559	3.9501	5.3042	5.5704

Finally, the functionally graded skew plate with circular cutout at location $x/a=0.45$, $y/a=0.75$ and $a/b=1$, $R/a=0.1$ under fully clamped boundary condition as in Fig. 4a is considered. Based on the change of high-temperature conditions that are followed by [21], the first ten natural frequencies of Al/Al₂O₃ skew plate with $\psi=0^\circ$ are presented in Table 8. It can be seen that these values have a good agreement with Janghorban's results based on ANSYS software [21]. Some frequencies from the micro model (MM) have better accuracy than others from the rule of mixture (RM), respectively.



Table 7. Comparison of the dimensionless frequencies of fully clamped functionally graded skew plates ($a/h = 10$).

ψ (°)	n	Mode					
		1	2	3	4	5	6
0°	$n=0.0$, [20]	3.4690	6.6929	6.6929	9.4163	11.3369	11.4450
	Present (RM)	3.4276	6.7007	6.7007	9.3752	11.4878	11.5998
	Present (MM)	3.4276	6.7007	6.7007	9.3752	11.4878	11.5998
	$n=0.5$, [20]	2.3888	4.6153	4.6153	6.4762	7.8247	7.9011
	Present (RM)	2.3710	4.6393	4.6393	6.4920	7.9627	8.0405
	Present (MM)	2.3595	4.6163	4.6163	6.4597	7.9223	7.9996
	$n=1.0$, [20]	2.0860	4.0300	4.0300	5.6718	6.8302	6.8973
	Present (RM)	2.0741	4.0577	4.0577	5.6796	6.9626	7.0306
	Present (MM)	2.0651	4.0309	4.0309	5.6528	6.9279	6.9957
	$n=2.0$, [20]	1.8668	3.6025	3.6025	5.0671	6.0975	6.1575
	Present (RM)	1.8532	3.6204	3.6204	5.0663	6.2013	6.2622
	Present (MM)	1.8467	3.6067	3.6067	5.0465	6.1756	6.2363
15°	$n=0.0$, [20]	3.6667	6.6507	7.4396	9.5553	11.8853	12.3530
	Present (RM)	3.5130	6.4580	7.2380	9.2835	11.5846	12.2428
	Present (MM)	3.5130	6.4580	7.2380	9.2835	11.5846	12.2428
	$n=0.5$, [20]	2.5255	4.5868	5.1316	6.5934	8.2046	8.5294
	Present (RM)	2.4303	4.4718	5.0118	6.4302	8.0301	8.4873
	Present (MM)	2.4185	4.4497	4.9870	6.3981	7.9893	8.4441
	$n=1$, [20]	2.2054	4.0050	4.4806	5.7564	7.1613	7.4451
	Present (RM)	2.1260	3.9111	4.3836	5.6250	7.0215	7.4213
	Present (MM)	2.1167	3.8929	4.3632	5.5982	6.9866	7.3842
	$n=2$, [20]	1.9734	3.5798	4.0043	5.1419	6.3917	6.6448
	Present (RM)	1.8993	3.4892	3.9105	5.0162	6.2537	6.6091
	Present (MM)	1.8927	3.4759	3.8956	4.9964	6.2277	6.5815
30°	$n=0.0$, [20]	4.3621	7.3539	9.1829	10.3505	13.6765	13.7720
	Present (RM)	3.8450	6.4179	8.3784	9.2433	11.9329	12.3930
	Present (MM)	3.8450	6.4179	8.3784	9.2433	11.9329	12.3930
	$n=0.5$, [20]	3.0068	5.0746	6.3381	7.1463	9.4779	9.5115
	Present (RM)	2.6610	4.4457	5.8048	6.4072	8.2730	8.5961
	Present (MM)	2.6481	4.4236	5.7758	6.3749	8.2309	8.5522
	$n=1$, [20]	2.6257	4.4305	5.5331	6.2382	8.2556	8.3003
	Present (RM)	2.3277	3.8880	5.0765	5.6039	7.2341	7.5184
	Present (MM)	2.3174	3.8699	5.0523	5.5767	7.1979	7.4805
	$n=2$, [20]	2.3484	3.9581	4.9412	5.5689	7.3231	7.4028
	Present (RM)	2.0787	3.4677	4.5255	4.9940	6.4423	6.6947
	Present (MM)	2.0713	3.4544	4.5077	4.9739	6.4153	6.6665

Table 8. Comparison of the frequencies of fully clamped functionally graded skew plates ($\psi = 0^\circ$) with circular cutout at location $x/a=0.45$, $y/a=0.75$, $a/b=1$, $R/a=0.1$

Mode	Frequency			Mode	Frequency		
	[21]	Present (RM)	Present (MM)		[21]	Present (RM)	Present (MM)
1	581.24	580.71	579.94	6	2481.3	2480.9	2479.44
2	920.03	921.13	920.06	7	2485.7	2484.3	2483.67
3	920.69	921.35	920.57	8	2984.2	2985.0	2984.33
4	1414.7	1413.64	1412.91	9	3697.4	3698.8	3697.61
5	1815.8	1816.47	1815.75	10	3704.7	3706.7	3705.18

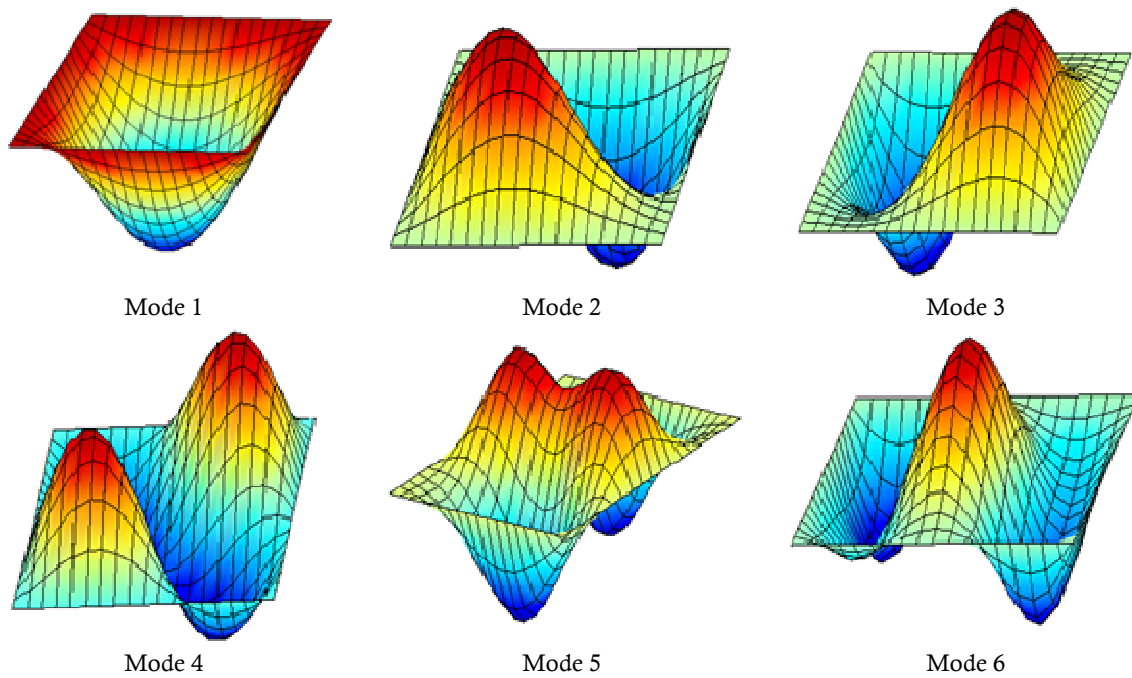


Fig. 3. Visualization of the first six modes of a simply supported functionally graded skew plate ($a/h = 10$) with the rule of mixture (RM).

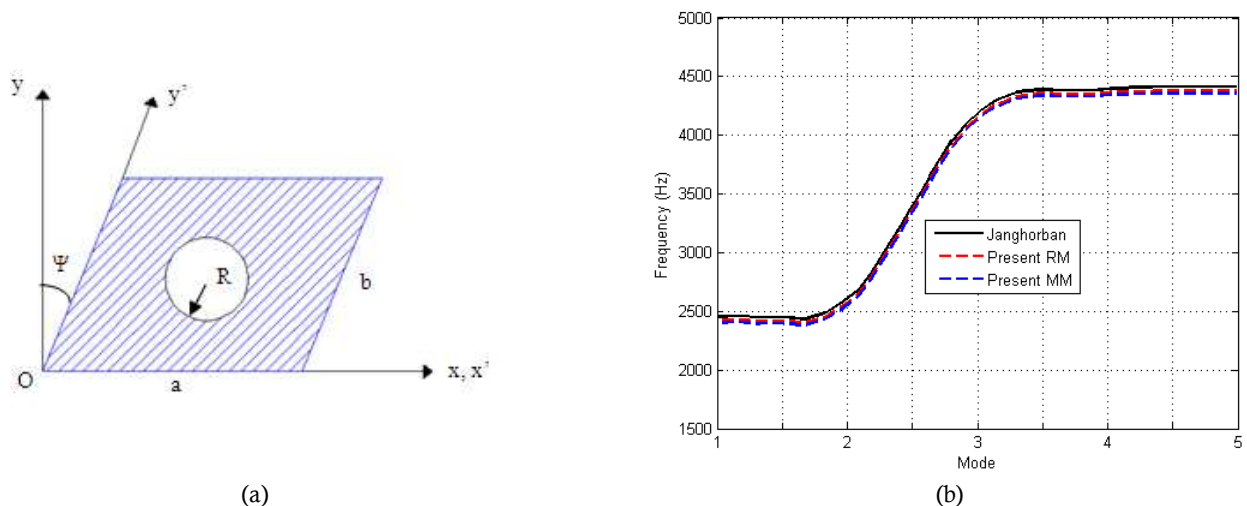


Fig. 4. a) The functionally graded skew plate with circular cutout, b) Comparison of the frequencies of fully clamped functionally graded skew plates ($\Psi = 30^\circ$) with circular cutout at location $x/a = 0.45$, $y/a = 0.75$ and $a/b = 1$, $R/a = 0.25$

By changing the skew angle Ψ from 0° up to 30° and the ratio R/a from 0.1 up to 0.25, the comparison of the natural frequencies of fully clamped functionally graded skew plate with a circular cutout in the thermal environment is shown in Fig. 4b. It is interesting to note that the obtained numerical solutions based on Shi's theory and two homogenization techniques RM & MM match very well with those from [21].

4. Conclusions

The present paper deals with the finite element analysis of functionally graded skew plates in thermal environment based on the new third-order shear deformation theory under C0-form. Both homogenization techniques were used and the effective properties were considered to be temperature-dependent. The computational accuracy of the developed finite element model was established by comparing the obtained results with the results in other literature. The effects of various parameters such as aspect ratio, thickness ratio, volume fraction gradient index, boundary conditions and temperature on deflections as well as natural frequencies were discussed. The paper also helps to supplement the knowledge for engineers in design.

Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.



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