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Research Paper

The Method of Lines Analysis of Heat Transfer of Ostwald-de Waele Fluid Generated by a Non-uniform Rotating Disk with a Variable Thickness

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Abstract. In this article, it is aimed to address one of Ostwald-de Waele fluid problems that either, has not been addressed or very little focused on. Considering the impacts of heat involving in the Non-Newtonian flow, a variant thickness of the disk is additionally considered which is governed by the relation $z = a\,(r/R_O+1)^{-m}$. The rotating Non-Newtonian flow dynamics are represented by the system of highly nonlinear coupled partial differential equations. To seek a formidable solution of this nonlinear phenomenon, the application of the method of lines using von Kármán's transformation is implemented to reduce the given PDEs into a system of nonlinear coupled ordinary differential equations. A numerical solution is considered as the ultimate option, for such nonlinear flow problems, both closed-form solution and an analytical solution are hard to come by. The method of lines scheme is preferred to obtain the desired solution which is found to be more reliable and in accordance with the required physical expectation. Eventually, some new marvels are found. Results indicate that, unlike the flat rotating disk, the local radial skin friction coefficients and tangential decrease with the fluid physical power-law exponent increases, the peak in the radial velocity rises which is significantly distinct from the results of a power-law fluid over a flat rotating disk. The local radial skin friction coefficient increases as the disk thickness index m increases, while local tangential skin friction coefficient decreases, the local Nusselt number decrease, both the thickness of the velocity and temperature boundary layer increase.

Keywords: Ostwald-de Waele fluid; Rotating disk; Method of lines; Non-uniform thickness; Heat transfer.

1. Introduction

Steady flow of different kinds of fluids and warmth move produced by the pivot of the holder or any roundabout plate stays a significant point for the exploration because of its broad current innovative applications. The writing overview shows that the Newtonian flows caused by the rotation of a circular disk were first concentrated by Karman [1]. For example, warmed progression of Newtonian fluid close to the outside of the turning disk, was examined by Ram and Kumar [2] at the point when the consistency of the fluid is variation with the impacts of temperature. Rashidi et al. [3] examined the impacts of slip and attractive fields on the pivoting flow other than considering. The numerical arrangement has been looked for in the nanofluid flow by Sheikholeslami et al. [4]. Bachok et al. [5] have additionally chipped away at the rotating flow. They have utilized a finite difference scheme to obtain a suitable numerical solution a steady flow of viscous incompressible nanofluid. Analytical solution is discussion by Kendoush [6] to investigate the influence of heat transfer of a non-Newtonian flow over a round plate containing pores in it. Then again, the same kind of flow issue is contemplated with the exemption, to consider nanofluid of water as base fluid suspended with five unique sorts of molecule by Turkyilmazoglu [7]. MHD flow of water containing nano-sized copper particles is examined by Hayat et al. [8-14]. Their main focus is to get an analytical solution of nanofluid, which further, undergoes the influences of Brownian motion,

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thermophoresis and besides, the effects of slip applied at the boundary of the disk. Mustafa et al. [15] use the Keller-Box technique to accomplish a numerical arrangement while researching the progression of ferrofluid affected by the outside magnetic fields on an extending rotating disk. Soret-Dufour impacts along with the effect of slip and magnetic fields have been tested by Rahidi et al. [16]. They have taken the convective flow to expatiate around by the rotation of the disk. Lie group analysis is carried out by Asgher et al. [17] by applying Lie symmetry for acquiring an alluring solution when heated fluid flow over a plate that will, in general, extend due to the impacts of heat. Hayat et al. [18] put their efforts to get an asymptotic solution for the ferrofluid. The fluid is experiencing the homogenous-heterogeneous reactions over the surface of a round disk that is rotating with some speed in angle. The non-Newtonian effects in the rotating disk system are incorporated by several researchers. Mitschka et al. [19] used power-law constitutive equation to discuss non-Newtonian effects on the heated fluid in a rotating system. Anderssen et al. [20] validated the analysis of Mitschka and generalized their results for highly shear-thinning fluids. Attia [21] used the constitutive equation of Reiner-Rivilin fluid to investigate the non-Newtonian effects in addition to the porosity and suction/injection on thermally charged fluid due to rotating disk. Griffiths [22] discussed salient characteristics of rotating disk flow using the Carreau constitutive equation. The motion of generalized thermal conductivity in mixed convective boundary layer flows of GNFs over rotating disk is introduced by Ming et al. [23]. The flow along the boundary of a stretchable sheet is further extended by Fang et al. [24] reshaping the structure of sheet having a non-uniform surface. Later several successful attempts have been made by different researchers to discuss the flows over the surface of sheets having variant thickness [25-33]. In any case, less consideration is given to examine flow and heat transfer in turning variable thickness disk framework. The investigation of Zheng et al. [34] is spearheading toward this path. In any case, in another endeavor, Zheng et al. [35] have applied a multi-shooting strategy, on moderately confused stream issue of thermally led Power-law. the fluid of intensity law liquid over a plate moving with a certain speed having a variable thickness.

All the above audit give a very clear proof that the fluid flows joined by the transmission of heat over a sheet having a non-uniform is a charming point of ongoing research, for the researchers and specialists because of its applications in basic and mechanical engineering. The inspiration driving the present paper is to examine the rotational progression of Ostwald-de Waele fluid, because of the round development of a disk. The setup of the flow is round and with non-uniform thickness. Also, the stream is additionally affected by heat. The streaming issue is scientifically detailed as far as the arrangement of nonlinear and coupled fractional differential conditions. Von Karman's transformation changes the development of PDEs into ODEs. A numerical methodology is received, to conquer the issue of commonly related and nonlinear flow marvel. In this specific circumstance, the method of lines plan is wanted to arrive at a solid and imposing arrangement. The remarkable flow and warmth move highlights are examined all together, and some very critical discoveries have been enrolled in the end, also.

2. Mathematical Model

Considering consistent and warmed flow of Ostwald-de Waele fluid on a round disk. The flow is created due to the turn of the disk, about Z-pivot, having the rakish velocity Ω, as appeared in the graph. The velocity of the fluid alongside the warm appropriation is given by:

$$V = Q + e_r + R e_\theta + S e_r \,, \tag{1}$$

$$T = T(r, z). \,, \tag{2}$$

Here Q, R and S are separate, parts of velocity in , θ and z-direction, while T speaks to the temperature of the warmed and turning Ostwald-de Waele fluid. The mathematical model of Ostwald-de Waele fluid with power-law index "n".

$$\mu = \mu_o \left\{ \left(\frac{\partial Q}{\partial z} \right)^2 + \left(\frac{\partial R}{\partial z} \right)^2 \right\}^{\frac{n-1}{2}} , \tag{3}$$

The fundamental administering equations depicting the rotational warmed flow wonder are given as:

$$\frac{Q}{r} + \frac{\partial Q}{\partial r} + \frac{\partial S}{\partial z} = 0, \,, \tag{4}$$

$$\rho \left(\frac{\partial Q}{\partial r} + S \frac{\partial Q}{\partial z} + \frac{R^2}{r} \right) = \left(\frac{\partial}{\partial z} \mu \frac{\partial Q}{\partial z} \right), \,, \tag{5}$$

$$\rho \left(\frac{\partial R}{\partial r} + S \frac{\partial R}{\partial z} + \frac{Q R}{r} \right) = \left(\frac{\partial}{\partial z} \mu \frac{\partial R}{\partial z} \right), \,, \tag{6}$$

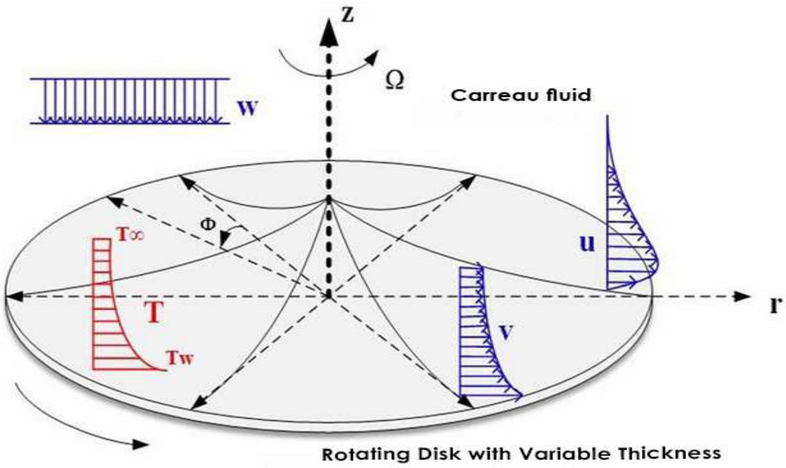

Fig. 1. Configuration of the non-uniform rotating disk

$$\rho c_p\left(Q\frac{\partial T}{\partial r}+S\frac{\partial T}{\partial z}\right)=\left(\frac{\partial}{\partial z}\lambda\frac{\partial T}{\partial z}\right),, \tag{7}$$

The speed and warm limit conditions for this pivoting stream issue are enrolled as:

$$Q=0,\ R=\Omega r,\ S=0,\ T=T_s\ \ at\ \ z=a\left(\frac{r}{R_0}+1\right)^{-m}, \tag{8}$$

Furthermore, also flow at the outrageous is characterized as:

$$Q=0,\ R=0,\ T=T_\infty\ \ at\ \ z\to\infty., \tag{9}$$

Thermal conductivity for the present examination is characterized as:

$$\lambda=\lambda_o\left\{\left(\frac{\partial Q}{\partial z}\right)^2+\left(\frac{\partial R}{\partial z}\right)^2\right\}^{\frac{n-1}{2}}, \tag{11}$$

Here λ_o be the thermal consistency coefficient. In addition, the numerical articulation for outspread and just as, distracting shear stress, is individually given as:

$$\tau_{s_r}=\mu\frac{\partial Q}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}=\mu_0\left\{\left(\frac{\partial Q}{\partial z}\right)^2+\left(\frac{\partial R}{\partial z}\right)^2\right\}^{\frac{n-1}{2}}\frac{\partial Q}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}, \tag{11}$$

$$\tau_{s_\phi}=\mu\frac{\partial R}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}=\mu_0\left\{\left(\frac{\partial Q}{\partial z}\right)^2+\left(\frac{\partial R}{\partial z}\right)^2\right\}^{\frac{n-1}{2}}\frac{\partial R}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}, \tag{12}$$

One can utilize the accompanying numerical relation to calculate heat flux density of this thermally directed nonlinear fluid flow:

$$q_s=\mu\frac{\partial T}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}=-\lambda_0\left\{\left(\frac{\partial Q}{\partial z}\right)^2+\left(\frac{\partial R}{\partial z}\right)^2\right\}^{\frac{n-1}{2}}\frac{\partial T}{\partial z}\bigg|_{z=A(\frac{r}{R_0}+1)^{-m}}, \tag{13}$$

3. Method of Lines

Method of lines is a semi-discrete technique [36–41] which includes reducing an IBVP to a system of (ODE) in time using a discretization in space. The subsequent -ODE system can be explained utilizing the standard initial value software, which may utilize a variable time-step/variable request approach with time local error control. The most significant bit of leeway of the MOL approach is that it has not just the straightforwardness of the unequivocal strategies [42] but also the superiority (stability advantage) of the implicit ones unless a poor numerical method for the solution of ODEs is utilized. It is conceivable to accomplish higher-request approximations in the discretization of spatial subordinates without noteworthy increments in the computational unpredictability. This method has wide appropriateness to physical and synthetic frameworks displayed by PDEs. The models that incorporate the arrangement of blended frameworks of logarithmic conditions, ODEs and PDEs, the goals of soak moving fronts, parameter estimation and ideal control and numerous other physical issues.

So as to utilize this methodology for fathoming (5)–(7), we discretize the arrange x with M (M even) consistently spaced grid points $x_i = x_{i-1} + h$, $x_0 = 0$, $x_M = l$, $i = 1, 2, \ldots, M$. Remark that $h = 1/M$ and we can likewise compose$x_i = ih$. We utilize a second-order difference estimate for the second derivative in x in grid points x_i , $i = 1, 2, \ldots, M - 1$.

The fundamental thought of the method of lines comprises a discretization of the space variable and in substitution of partial derivatives by finite-difference expressions; the time variable keeps a consistent structure and arrangements over the lines parallel with the pivot of this variable are processed. Give us a chance to subdivide the arrangement space of our application into uniform rectangular work by the lines. The derivative u_x in equations (5-7) is computed by finite differences scheme in two ways:

1. three-point centered approximations $Q_x = \dfrac{Q_{i+1} - Q_{i-1}}{2h} + o(h)^2$

2. seven-point centered approximations $Q_x = \dfrac{Q_{i-2} - 8Q_{i-1} - 8Q_{i+1} - Q_{i+2}}{12h} + o(h)^4$

Applying the above finite difference schemes to Eqs. (7-9) yields a system of ordinary differential equations for the unknown Q_i as functions in t as follows:

$$\frac{Q_i}{r} + \frac{dQ_i}{dr} + \frac{S_{i+1} - S_{i-1}}{2h} = 0,\ i = 1,2,3,\ldots\ldots\ldots\ldots N-1.$$

$$\rho\frac{Q_i}{1}.\frac{dQ_i}{dr} + W_i\frac{Q_{i+1} - Q_{i-1}}{2h} = \mu\frac{Q_{i+1} - 2Q_i + Q_{i-1}}{h^2},\ i = 1,2,3,\ldots\ldots\ldots\ldots N-1.$$

$$\rho\frac{Q_i}{1}.\frac{dQ_i}{dr} + W_i\frac{Q_{i+1} - Q_{i-1}}{2h} = \mu\frac{Q_{i+1} - 2Q_i + Q_{i-1}}{h^2},\ i = 1,2,3,\ldots\ldots\ldots\ldots N-1. \tag{14}$$

$$\rho(\frac{Q_i}{1}.\frac{dR_i}{dr} + S_i\frac{R_{i+1} - R_{i-1}}{2h} + \frac{Q_i R_i}{r}) = \mu\frac{R_{i+1} - 2R_i + R_{i-1}}{h^2},\ i = 1,2,3,\ldots\ldots\ldots\ldots N-1.$$

$$\rho c_p(\frac{Q_i}{1}.\frac{dT_i}{dr} + S_i\frac{T_{i+1} - T_{i-1}}{2h}) = \lambda\frac{T_{i+1} - 2T_i + T_{i-1}}{h^2},\ i = 1,2,3,\ldots\ldots\ldots\ldots N-1.\ ,$$

Then, any popular ODE solvers should be used for solving the above equations, we will use classical four order Runge–Kutta scheme (RK4):

$$U^{n+1} = U^n + \frac{\Delta t(k_1 + 2k_2 + 2k_3 + k_4)}{6},$$

$$K_1 = F(U^n), \quad K_2 = F(U^n + \frac{\Delta t}{2}K_1), \tag{15}$$

$$K_3 = F(U^n + \frac{\Delta t}{2}K_2), \qquad K_4 = F(U^n + \Delta t \cdot K_3),$$

As the flow problem is mathematically formulated in terms of a nonlinear coupled system of partial differential equations. In order to reduce the system of PDEs into ODEs. It is approximate to define the following transformation:

$$\begin{gathered} Q = r^* R_0 \Omega F(\eta), \quad R = r^* R_0 \Omega G(\eta), \quad T = (T_S - T_\infty)\Theta(\eta) + T_\infty \\ S = R_0\Omega\left(1 + r^*\right)^{-m}\left(\frac{\Omega^{2-n}R_0^2\rho}{\mu_o}\right)^{-1/(n+1)} H(\eta) \end{gathered}, \tag{16}$$

where η is defined as:

$$\eta = \frac{z}{R_0}\left(\frac{\Omega^{2-n}R_0^2\rho}{\mu_o}\right)^{1/(n+1)}\left(1 + r^*\right)^m, \tag{17}$$

where $r^* = r / R_0$ is the dimensionless radius.

$$\begin{aligned} F &= f(\eta - \alpha) = f(\zeta), \ \ G = g(\eta - \alpha) = g(\zeta), \\ H &= h(\eta - \alpha) = h(\zeta), \ \ \Theta = \theta(\eta - \alpha) = \theta(\zeta), \end{aligned} \tag{18}$$

here $\alpha = a(\Omega^{2-n} R_0^2 \rho / \mu_o)^{-1/(n+1)} / R_0$ denotes thickness coefficient, in non-dimensional form of the disk and $Re = \Omega^{2-n} R_0^2 \rho / \mu_o$ is called Reynold's number. Eqs. (4)–(7) are reduced in the following form:

$$2f + m(\xi + \alpha)\varepsilon f' + h' = 0, , \tag{19}$$

$$f^2 + m(\zeta + \alpha)\varepsilon f f' - g^2 + f' h = r^{(n-1)}(1+r)^{m(n+1)}((f'^2 + g'^2)^{\frac{n-1}{2}} f')', , \tag{20}$$

$$2f g + m(\zeta + \alpha)\varepsilon f g' - g' h = r^{(n-1)}(1+r)^{m(n+1)}((f'^2 + g'^2)^{\frac{n-1}{2}} g')', , \tag{21}$$

$$2f g + m(\zeta + \alpha)\varepsilon f g' - g' h = r^{(n-1)}(1+r)^{m(n+1)}((f'^2 + g'^2)^{\frac{n-1}{2}} g')', , \tag{22}$$

$$m(\zeta + \alpha)\varepsilon f \theta' + \theta' h = \frac{1}{p_r}(1+r)^{m(n+1)}((f'^2 + g'^2)^{\frac{n-1}{2}} \theta')', , \tag{23}$$

The corresponding boundary conditions can be written as:

$$\begin{aligned} &f(0) = 0, \ \ g(0) = 1, \ \ h(0) = 0, \ \ \theta(0) = 1, \ \ at \ \xi \to 0, \\ &f(\xi) = 0, \ \ g(\xi) = 0, \ \ \theta(\xi) = 0, \ \ at \ \xi \to \infty, \end{aligned} \tag{24}$$

where the prime represents the derivative with respect to ζ, f, g, h, θ are dimensionless radial, tangential, axial velocity and temperature respectively, the local radial skin friction coefficient and tangential skin friction coefficient are given by:

$$\mathrm{Re}^{\frac{1}{n+1}} C_{f_r} = r^n (1+r)^{mn} \left\{ f'^2(0) + g'^2(0) \right\}^{\frac{n-1}{2}} f'(0), \tag{25}$$

$$\mathrm{Re}^{\frac{1}{n+1}} C_{f_\phi} = r^n (1+r)^{mn} \left\{ f'^2(0) + g'^2(0) \right\}^{\frac{n-1}{2}} g'(0), \tag{26}$$

$$\mathrm{Re}^{\frac{1}{n+1}} N Q_r = -\mathrm{Re}^{\frac{1}{n+1}} \frac{q_s r}{\lambda_0 \left(\left(\frac{\partial Q}{\partial z} \right)^2 + \left(\frac{\partial R}{\partial z} \right)^2 \right)^{\frac{n-1}{2}} (T_s - T_\infty)} = -r(1+r)^m \theta'(0), \tag{27}$$

To obtain a reliable solution to Eqs. (19)-(22) subject to corresponding boundary conditions given in Eqs. (23)-(24), the method of lines is utilized. This method is much faster and more flexible to use as compared to other methods. It has been extensively used and tested on boundary layer flows. By means of said method, one the solution can be attained by using the four steps, namely (i) first reduce the system of equations to a first-order system, (ii) then write the difference equations by means of central differences, (iii) now linearize the resulting nonlinear equation by Newton's Raphson method if any and finally elimination technique is used to solve the linear system.

4. Results and Discussion

In this section of the article, a careful talk is completed with respect to the results of the different variables and parameters associated with the introduced examination. A visual parametric examination shows how the flow is being influenced over the rotating disk. Uncommon accentuation is given to see the impacts of primary parameters m, n, Pr and We on this concerned fluid flow and heat transfer qualities. Figs. (2)-(5) are plotted to inspect the velocity parts and temperature for different estimations of n. Figs. (6)- (9) are plotted to watch, how the thickness of the rotating disk m, can modify the conduct of the velocity and temperature of this spinning fluid. One has no trouble to locate that every radial, tangential and axial velocity hampered, comparing to variety in the size of the rotating disk. The Nusselt number $\theta(\zeta)$ different values of P_r is plotted in Figs. (10). The behavior of the Nusselt number is found to be an increase one, for the variation of disk's thickness, Prandtl number, and Weissenberg number. Figs. (11)-(12) examine the radial skin friction coefficients.

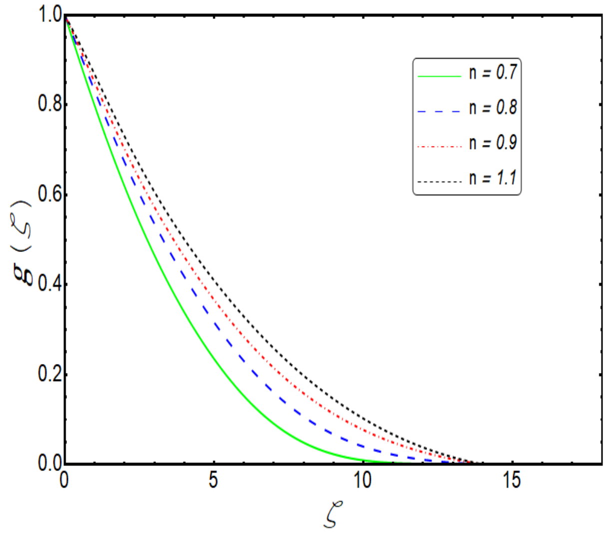

Fig. 2. Radial-velocity $f(\zeta)$ for different n

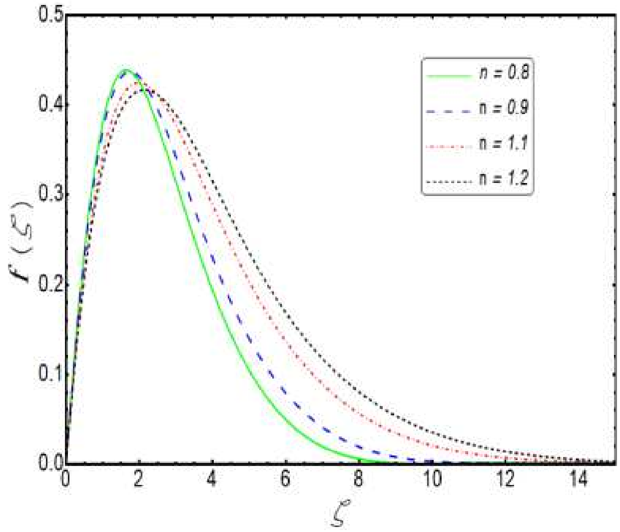

Fig. 3. Tangential-velocity $g(\zeta)$ for different n

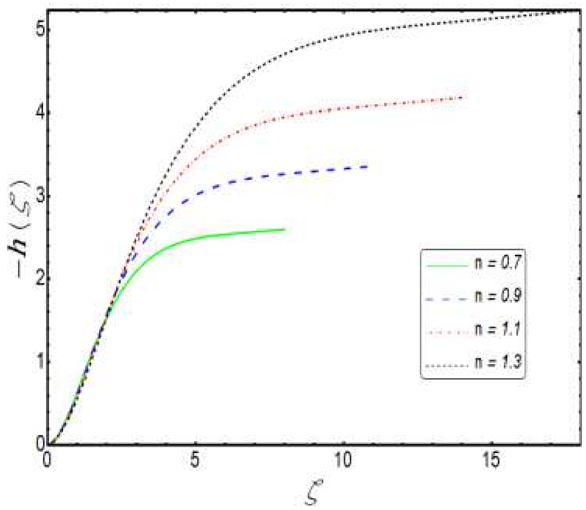

Fig. 4. Axial-velocity $-h(\zeta)$ for different n

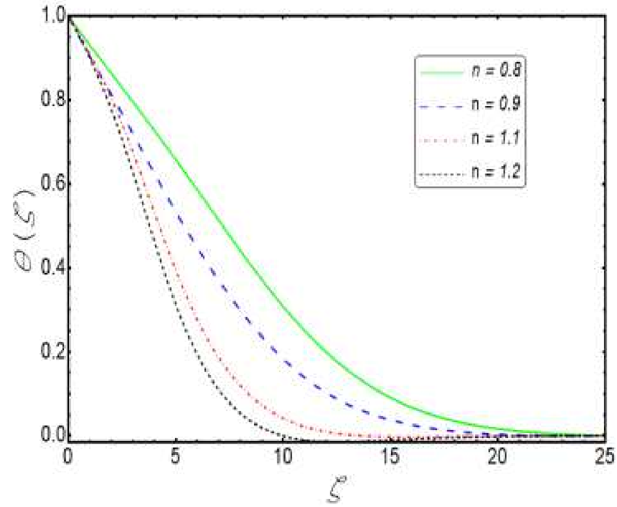

Fig. 5. Temperature $\theta(\zeta)$ for n

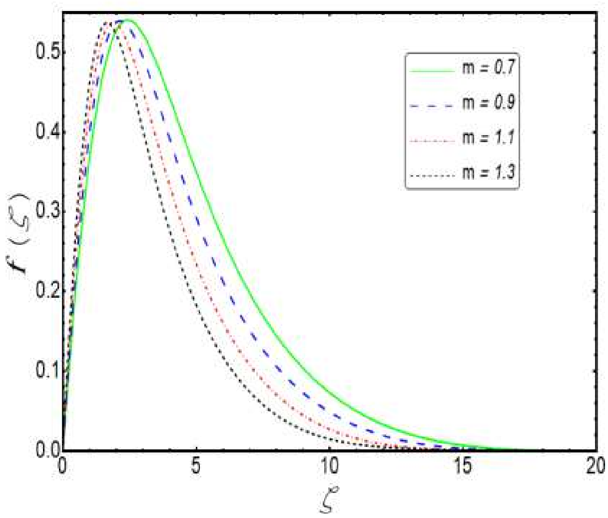

Fig. 6. Radial-velocity $f(\zeta)$ for m

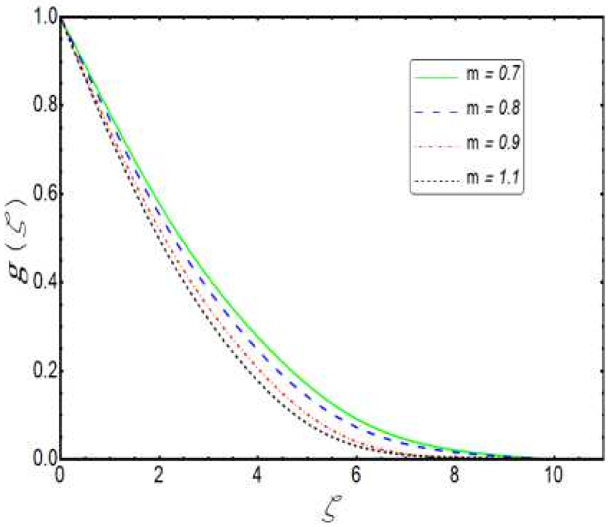

Fig. 7. Tangential-velocity $g(\zeta)$ for m

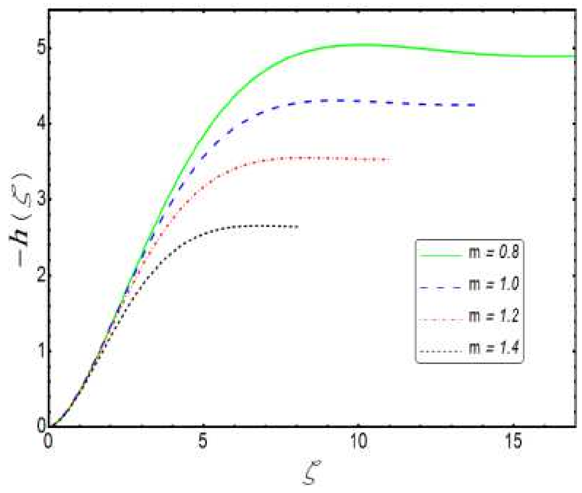

Fig. 8. Axial velocity $-h(\zeta)$ for m

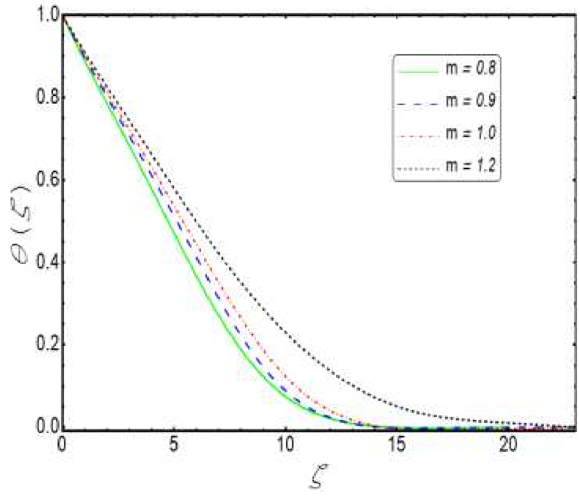

Fig. 9. Temperature $\theta(\zeta)$ for different m

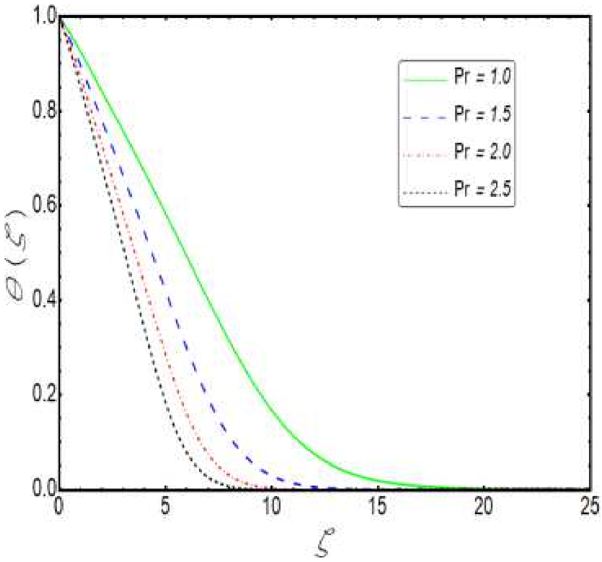

Fig. 10. Temperature $\theta(\zeta)$ for different Pr

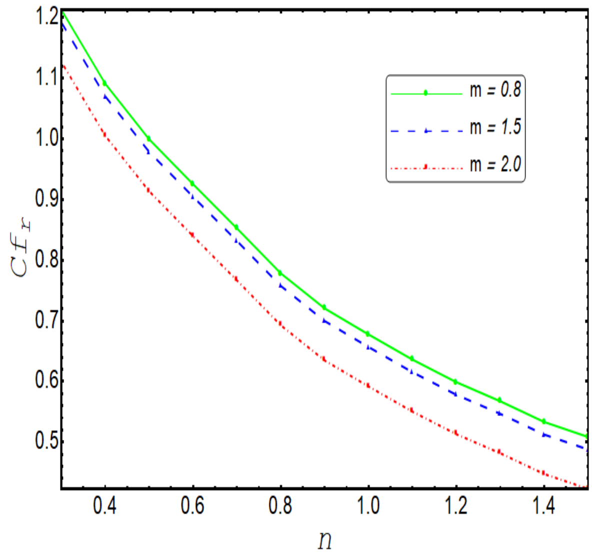

Fig. 11. Local radial skin friction Cf_r for m

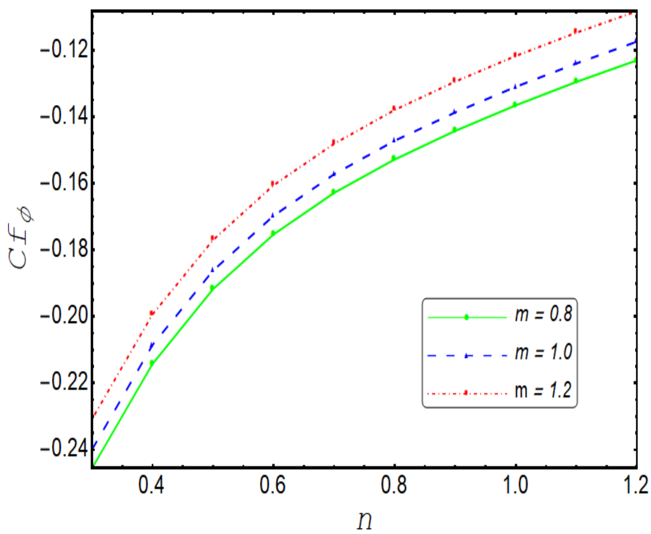

Fig. 12. Local tangential skin friction $Cf_\emptyset$ for m

5. Conclusion

A steady boundary layer flow and heat transfer of a generalized Newtonian fluid were analyzed over the rotating disk. From the configuration of the disk, it is to keep in mind that the thickness of the disk was not uniform, for it changed from the center to its edge. The rotational flow of the non-Newtonian fluid was mathematically modeled by nonlinear and coupled partial differential equations. A very famous Von Karmon's transformation was implemented to convert PDEs into the set of nonlinear and coupled ordinary differential equations. A numerical solution was sought for this swirling flow problem by using a well-known numerical method *"method of lines"*. This method does not require large computer memory and avoids linearization and physically unrealistic assumptions. It can be applied to the nonlinear systems of partial differential equations as well. MOL was provided very accurate numerical solutions for linear or nonlinear PDE's in comparison with other existing methods. Finally, the obtained mathematical results were analyzed through graphs and some of the most significant findings related to present investigation, are summarized as:

- An increase in the power-law exponent of fluid, supports velocity and temperature limit layers become slenderer.
- The expansion of the plate thickness index offers to ascend to the thickness of the velocity and temperature limit layer together with local radial skin friction coefficient, while the local tangential skin friction coefficient and Nusselt number reduction.
- As Prandtl number accretion, the thickness of the temperature limit layer decreases and the nearby Nusselt number expands.
- The size of the disk thickness results to rise the local Nusselt number.
- The presence of elasticity in fluid reduces both radial and tangential skin friction coefficients.

Nomenclature

R_o	Include radius, m	η	Dimensionless similarity variable
q_w	Heat flux density, $\frac{W}{m^2}$	λ	Thermal conductivity, $\frac{W}{m\,K}$
P_r	Prandtl number	θ	Dimensionless temperature
Cf_r	Local radial skin friction coefficient on the disk	τ_{s_r}	Radial shear stress on the disk, N
$Cf_\emptyset$	Local tangential skin friction coefficient on the disk	τ_{s_ϕ}	Tangential shear stress on the disk, N
(f,g,h)	Dimensionless velocity components	ρ	Fluid density, $\frac{\text{kg}}{m^3}$
P	Fluid pressure, Pa	Ω	Angular velocity of the disk, S^{-1}
T	Fluid temperature in the boundary layer, K	ζ	Dimensionless similarity variable
α	Dimensionless thickness coefficient of the disk		

Conflict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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