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Investigation on the Crack Effect in the Cylinder and Matrix on the Backscattering Field Frequency Specifications using the Finite Element Method

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Abstract. A novel method to determine the health of the industrial parts is using the ultrasound scattering waves. Any changes in the structure of the scattering object or in the boundary conditions will cause a change in the scattering field. The frequency spectrum of the scattering time signal has valuable information, which is studied by resonant ultrasound spectroscopy (RUS). Since any defect, property changes, or changes in boundary conditions can affect the scattering field. Therefore, the possible defects in the piece are detected using the response of the scattering field. One possible defect in the fiber-reinforced composites is the existence of a crack in the matrix or fibers. In the present study, the effect of crack on the far-field backscattering amplitude spectrum is investigated using the finite element method (FEM). To this end, the effect of the crack's direction in the cylinder and matrix on the form function is scrutinized. The results show that the Rayleigh frequency modes are more sensitive to the cracks existing in the epoxy matrix than the Whispering-gallery frequency modes. Also, the existence of the crack in the aluminum cylinder has the most effect on the Whispering-gallery frequency modes. Besides, the existence of a horizontal crack in the aluminum cylinder leads to a significant reduction in these frequency modes. The validation of the research is determined by comparing the steel cylinder form function obtained from the finite element method's information and the analytical and experimental form functions in addition to the comparison of the aluminum cylinder form function and reference form function.

Keywords: Ultrasonic, Finite element method, Scattering, Form function, Crack.

1. Introduction

Form function and resonance spectrum are two important graphs obtained from the scattering wave field. any change in the geometrical or boundary conditions of the reflective object may change these graphs. The resonance frequencies and modes of the reflective object are among the important information of these graphs. The graphs are studied using Resonance Acoustic Spectroscopy (RAS) technique.

Faran [1] is one of the earliest researchers in the field of acoustic waves scattering. He investigated the scattering of the Isotropic sphere or cylinder floating in the water analytically. White [2] investigated the waves scattering from the elastic reflectors or cavities embedded in the solid medium. In1975, the scattering field resulting from the collision of a plane acoustic wave with two rigid cylinders was calculated by Young and Bertrand [3], using the direct matrix inversion and iterative procedure. The obtained results were in good agreement with experimental results. In 1992, Addison and Sinclair [4], based on the Faran[1], White [2] method, calculated the diffraction frequency response of Silicon Carbide (Sic) fiber embedded inside a titanium matrix in the frequency range $0 \leq f \leq 100$ MHz. A few years later, the mathematical relations of the Far Field Backscattering Amplitude Spectrum of an elastic cylinder embedded inside an epoxy medium were calculated by Betty et al., [5]. They used experimental and analytical methods to investigate further the form functions of the steel and copper cylinders, where a good agreement was observed. Flex et al. [6] and Garan et al. [7] studied the phenomenon of the constructive interference of the surface waves around an elastic body and the creation of the standing waves. By using normal mode expansion, Honarvar and Sinclair [8] proposed a mathematical model to calculate the far-field backscattering amplitude spectrum (form function) of the acoustic waves, produced by transverse isotropic cylinders immersed in fluid. Taheri et al. [9] proposed and investigated the analytical relations of the multiple scattering of elastic sound waves produced by isotropic and transverse isotropic cylinders embedded in a viscoelastic medium. In recent years, experimental methods have also been used to study the acoustic scattering field. Ripouche et al. [10] invented the first experimental method obtained based on the resonance identification and isolation method.

Sodagar et al. [11] proposed the application of the wave reflection echo as a reference frequency spectrum in the resonance identification and isolation method by using the short-pulse waves. The mismatch, observed in the scattering signal frequency spectrum and reference frequency spectrum in the rods having large diameters, which caused the lack of accuracy and reduction in the effective frequency range of the form function has been fixed using this reference spectrum. Sodagar and Pourshab [12] investigated the effect of surface disbonding of an elastic cylinder embedded in a solid medium on the Backscattering spectrum using the modified short pulse MIIR.

A prevalent defect in the fiber-reinforced composite materials is the creation of the crack in the fiber or matrix. The ultrasonic waves are one of the most common methods used to inspect the composite materials. Under special conditions, when the wave hits the fiber, the wave scattering occurs. Scattering response from an elastic body is affected both by geometrical and mechanical properties. Since the existence of crack can cause a decrease in mechanical properties, therefore the scattering field can be used to detect the defection. In the present study, the effect of crack on the backscattered wave field of an aluminum cylinder is studied using the finite element method. By using the modified short pulse MIIR method on the results obtained from the modeling in the Abaqus software, the effect of the existence of the crack in an elastic cylinder and epoxy medium on the form function was studied.

2. Resonance Ultrasonic Spectroscopy Theory

When a wave collides with the interface of two objects having different mechanical properties, the phenomena such as refraction, reflection, and mode conversion occur. If the interface of two objects is rounded (but not necessarily circular), the scattering phenomenon will occur under the specific conditions.

Based on the frequency spectrum resonance scattering theory, the backscattering from an elastic body include both the smooth background and valleys and peaks, which correspond to the resonant frequencies of the body.

The available information on the scattering wave frequency spectrum was studied by Resonance Acoustic Spectroscopy. In this method, a broadband ultrasonic wave is radiated on the body. When an acoustic wave collides with the interface of two bodies, the surface waves are created around the interface of the two objects. Even if one of the natural frequencies of the reflective body exists within the frequency spectrum of the radiated wave, these waves have constructive interference, and as a result, the standing waves are created. In this case, the body acts like a wave generation resource and releases energy into the environment.

In the present study, the experimental conditions of the ultrasonic wave scattering of an aluminum cylinder embedded inside an epoxy environment are simulated by using FEM. Two quasi-harmonic and short-pulse methods facilitate the experimental analysis of resonate spectroscopy. In the quasi-harmonic method, a single-frequency wave is used, while in the short-pulse method, a broadband short-pulse wave is transmitted toward the target. In the present study, the short pulse method is used since it is less time- consuming. In this method, an elastic body is exposed to a short pulse wave with wide frequency bandwidth using an ultrasonic transducer. under these conditions, the transfer function of the measuring system affects the received frequency spectrum. As a result, the frequency spectrum of the backscattered wave is composed of the frequency spectrum resulted from the transducer (measuring system), and the frequency spectrum resulted from the backscattered waves, which are produced by the elastic body. By eliminating the frequency spectrum of the measuring system from the backscattered frequency spectrum, the resonances of the elastic body are identified. In a modified short pulse MIIR technique, instead of the reflected spectrum from a cylinder with high stiffness (rigid) and small diameter [13], the frequency spectrum reflection echo [11] is used as the reference frequency spectrum.

By eliminating the frequency effects of the measuring system from the received signal frequency spectrum and plotting the obtained spectrum in terms of the normalized frequency Ka (ω is the angular frequency, c is the wave speed in the epoxy matrix, $K = \omega / c$ is the wavenumber and a is the cylinder radius), the form function is obtained. The form function of an elastic cylinder can be calculated using relation (1) [11]:

$$\left|f_{\infty}\right| = \left(\frac{s_{(\omega)}}{s'_{(\omega)}}\right)\left|\frac{-2}{\sqrt{\pi i Ka}}\sum_{n=0}^{\infty}\frac{J'_n(Ka)}{H^{(1)'}_n(Ka)}\cos(n\varphi)\right| \tag{1}$$

where $s_{(\omega)}$ is the frequency spectrum of the received signal, $s'_{(\omega)}$ is the frequency spectrum of the reflection wave, φ is the receiving angle, J_n is the Bessel function, H_n is the Henkel function, and ε_n is the Neumann function. The relation of the Neumann function is as follows:

$$\varepsilon_n = \begin{cases} 1 & for \quad n = 0 \\ 2 & for \quad n \geq 0 \end{cases} \tag{2}$$

3. Problem Simulation using FEM

The piece studied here is an aluminum or steel cylinder with infinite length and circular cross-section located in an epoxy medium. By applying the force, pressure, or displacement, the transducer effect can be simulated in the FE method as a wave generator. In the research, the pressure has been used to simulate the transducer. In order to calculate the applied pressure resulted from locating the normal transducer on the piece, the pulse-form of the Eq. (3) has been used [14]:

$$\begin{aligned} &(1 - \cos(\frac{2\pi f t}{N}))\cos(2\pi f t) \quad for \quad 0 < t < \frac{N}{f} \\ &0 \quad otherwise \end{aligned} \tag{3}$$

where f is the excitation frequency in terms of Hz, N is the number of the excitation signal cycles and t is the excitation time. In this paper, the radiation wave has two cycles, and its central frequency is 1MHz. Figs. (2) and (3) show the time signal of the sent wave and its frequency spectrum.
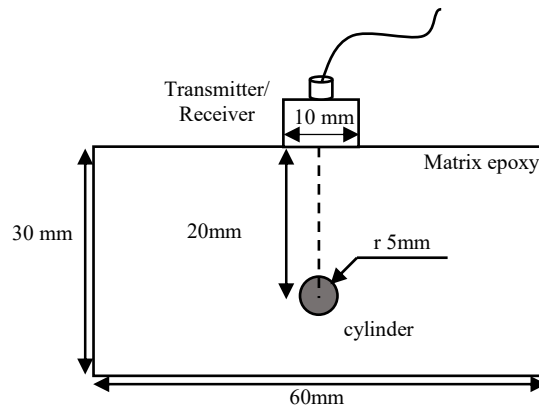
Fig. 1. A schematic of the geometrical structure of the cylinder embedded in the epoxy medium and how the measuring system located.
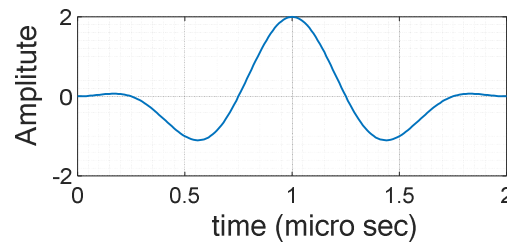

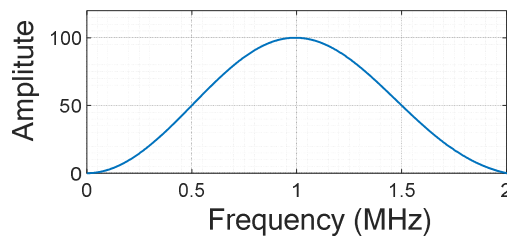**Fig. 2.** Radiative wave with frequency 1MHz and the number of cycle N=2.

Fig. 3. Frequency spectrum of the sent wave.

A schematic of the dimensions of the piece and how the ultrasonic transducer located are shown in Fig. (1). Full connection is established between cylinder and epoxy matrix, using tie constraint, and the cylinder is embedded in the matrix environment. By assigning the four-node plane strain element CPE4R, the infinite length condition is satisfied. Also, it is a common assumption to consider that the transducer is 2-dimensional; the same assumption has been considered in articles [12] and [14]. The existence of the reflected waves of the walls within the received signals makes it difficult to analyze the results. In an experimental test, by allocating large length and width to the test piece, the echo of the reflected waves from surfaces is separated from the backscattered signal. Increasing the dimensions of the piece in the finite element model increases the runtime. For this reason, in this research, infinite elements are used on the test piece surface, such as ref [14]. In order to prevent the interference of the reflected waves of the walls with the scattering waves, the 4-node linear plane strain one-way infinite quadrilateral CINPE4 was applied on the matrix's walls. Fig. (4) shows how the finite and infinite elements were used. Also, Table 1 listed the mechanical properties of the materials. Given that the explicit solver of Abaqus software has been used, the minimum time step of integrating to ensure the convergence of the analysis results is obtained from the following relation:

$$\Delta t \le \Delta t_{cr} = \frac{l_{min}}{c_l} \tag{4}$$

where l_{min} denotes the minimum element size and c_l denotes the value of longitudinal wave speed.

To use the proper shape function for calculating the precise corresponded variables and also to prevent the divergence of finite element results, the size of elements in terms of the minimum emitted wavelength (λ_{min}) in a material is recommended as Eq. (5) [15]:
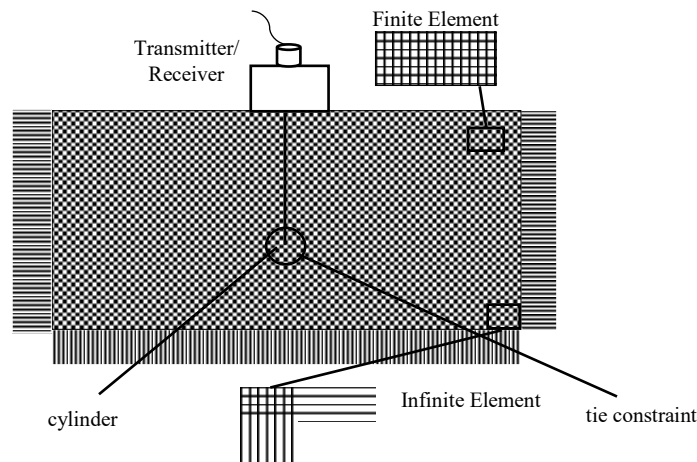
$$l_{min} \le \frac{\lambda_{min}}{20} \tag{5}$$

In this research, for epoxy, steel, and aluminum material, the element size 39μm, 84μm, and 86μm were used, respectively.

Table 1. Elastic and acoustic properties

Material	Density (Kg/m³)	Young modulus (GPa)	Poisson ratio	Compression Velocity (m/s)	Transverse Velocity (m/s)
Steel	7800	208	0.28	5890	3230
Aluminum	2690	68.95	0.33	6420	3040
Epoxy	1129	3.5	0.4	2645	1331

Fig. 4. The structure of the elements in the finite element model.
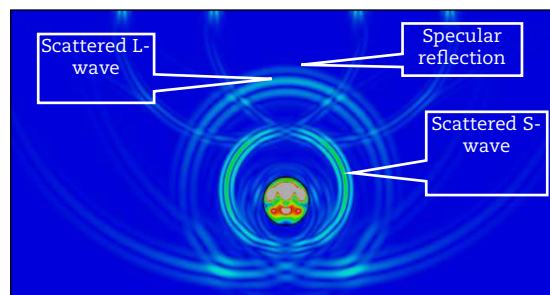
Fig. 5. Ultrasonic wave scattering after a collision of the radiative wave with the steel cylinder embedded in the epoxy matrix.
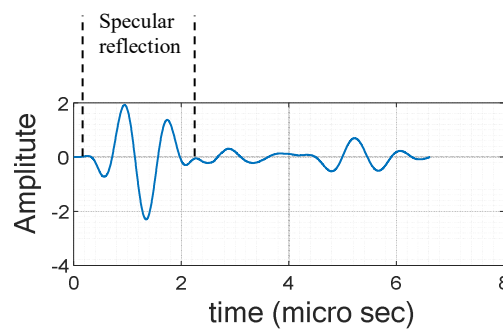
Fig. 6. Backscattering time signal in a case of coaxial steel cylinder and ultrasonic transducer.
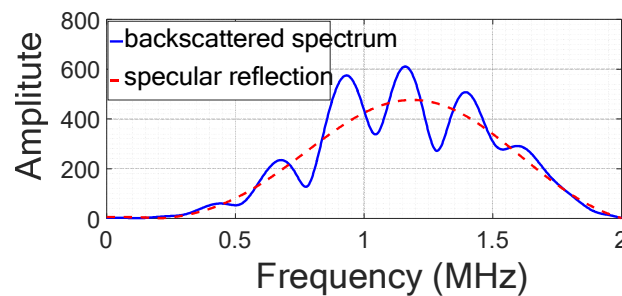
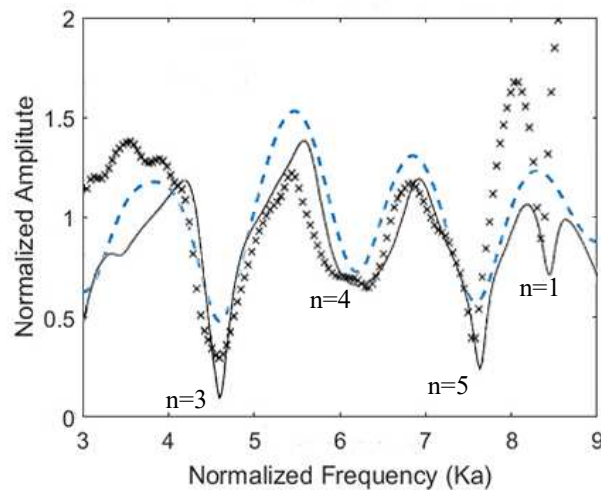**Fig. 7.** Frequency spectrum of the backscattered signal and reflected eco.

Fig. 8. Form function of a steel cylinder embedded in epoxy medium using FEM (dashed line), theoretical method [5] (continuous line), and experimental method [5] (star).
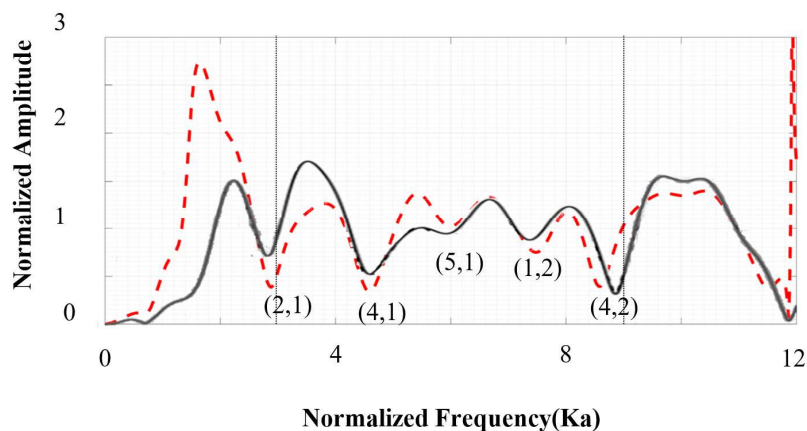

Fig. 9. Form function of an aluminum cylinder embedded in epoxy medium obtained from the proposed method (dashed lines) and the form function of the aluminum cylinder of reference [12] (continuous line).

4. Numerical Rresults

4.1. Validation

One way to validate the presented models is the comparison of the form function of the steel cylinder resulted from the FEM with the values obtained from the analytical and experimental methods. To this end, by exciting the steel cylinder embedded in the epoxy medium using a typical transceiver, the form function is plotted. Fig. (2) shows the sent pulse and Fig. (3) shows this pulse in the frequency-domain. The sent pulse has the central frequency 1MHz, and bandwidth ranges from 0.5-1.5MHz. Therefore, the results are valid in the normalized frequency (Ka) range 3-9. At the moment of colliding the wave with the cylinder, the mode conversion phenomenon occurs, which can be seen in Fig. (5). The received signal and its corresponding reflection echo are shown in Fig. (6). Fig (7) shows the frequency spectrum, which corresponds with the backscattered signal and Specular Reflection.

Form function is one of the crucial data in scattering response of the body being radiated by ultrasonic waves. The correct evaluation of form function is of high importance due to valuable supplied information. Form function of steel cylinder will be obtained by inserting the information of Figure 7 into Equation 1 (substitute backscattered spectrum data for $s_{(\omega)}$ and specular reflection data for $s'_{(\omega)}$) and plotting the $|f_{\infty}|$ (normalized amplitude) in terms of Ka (normalized frequency). By comparing this form function with the form function of the steel cylinder calculated by analytical and experimental methods, the validation of the studied model is obtained (Fig. (8)). As can be seen, there is much similarity and agreement for the general form function and the location of their resonance frequencies. Incompatibility between form functions near normalized frequencies of 3 and 9, were formerly observed in article [12].

Like the previous model, the form function of the aluminum cylinder is calculated. Fig. (1) shows how the measuring system and the cylinder are located. Also, the sent echo, its corresponding frequency spectrum, and the type of the element used are shown in Figs. (2),(3) and (4), respectively.

By comparing the form function with the form function of an aluminum calculated in reference [12], the proposed model re-evaluated (Fig. (9)). The same behavior of the form function in the normalized frequency (Ka) range 3-9 indicated the validity of the proposed model. The resonance frequencies in the form function are indicated by subscribes (n, l). Where n indicates the model number (the half of the number of all nodes in the standing wave shape around the cylinder in a constant frequency) and l indicates the type of the circumferential wave around the cylinder. Where l=1 shows the pseudo-Rayleigh and l=2,3, ... indicates the whispering gallery wave.
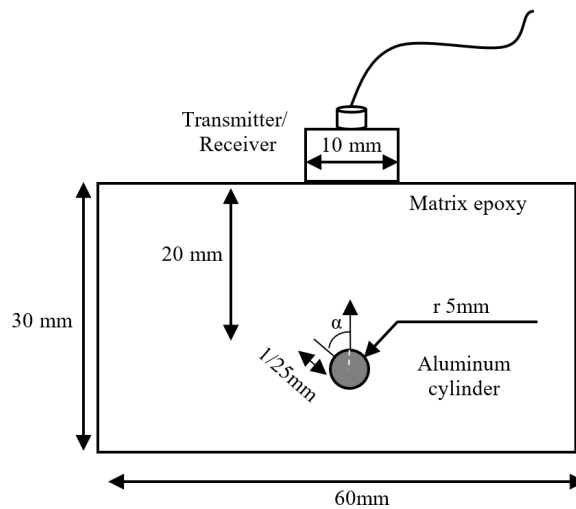
Fig. 10. A schematic of the geometrical structure of an aluminum cylinder embedded in an epoxy medium and how the measuring system located in a case of crack existence in the epoxy matrix.
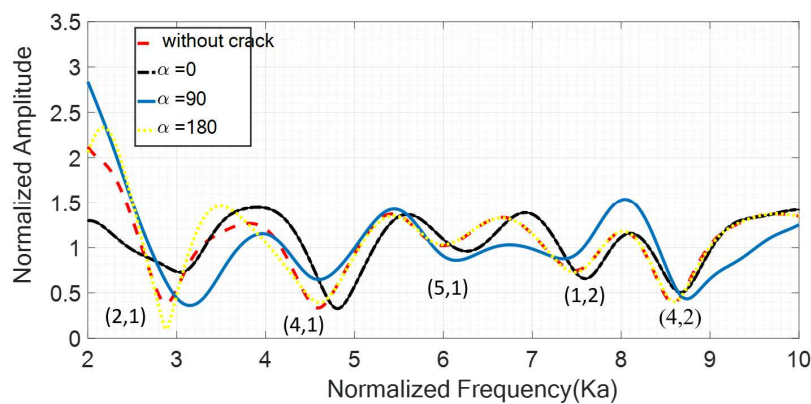

Fig. 11. Form function of an aluminum cylinder embedded in an epoxy medium using FEM in healthy case and existence of a crack in the epoxy matrix.
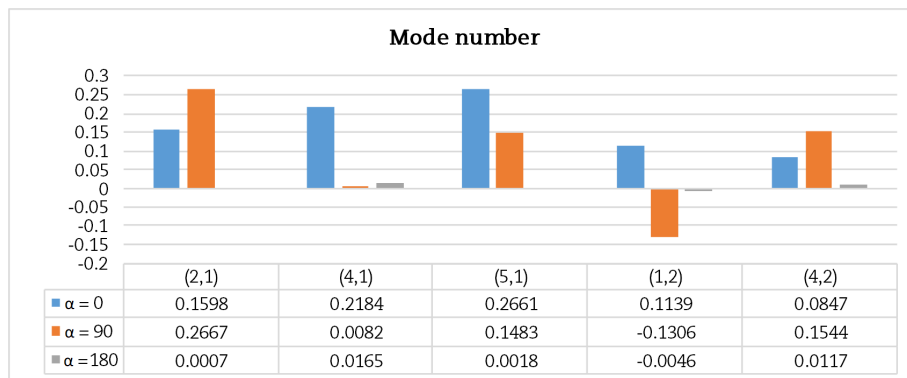

Mode number					
	(2,1)	(4,1)	(5,1)	(1,2)	(4,2)
$\alpha = 0$	0.1598	0.2184	0.2661	0.1139	0.0847
$\alpha = 90$	0.2667	0.0082	0.1483	-0.1306	0.1544
$\alpha = 180$	0.0007	0.0165	0.0018	-0.0046	0.0117

Fig. 12. Variations of normalized frequency due to the existence of a crack in the epoxy matrix

4.2. Crack effect on the form function

In this section, the effect of the crack on the backscattered wave field is investigated. For this purpose, the received signal is analyzed using a short pulse method of isolation and identification of resonances. The effect of these defects on the far-field backscattered spectrum is further investigated by creating a crack in cylinder or epoxy (at the border contact with cylinder). In the finite element model, cracks are defined in such a way that there is no bond between the two edges of the crack, while all other surfaces, except for crack, are confined with tie constraint.

4.2.1. Crack in epoxy

In this section, by creating a 1.25mm –long crack on the matrix-cylinder contact boundary and with three different orientations, the effect of the crack on the form function is studied. The schematic of how these cracks located is shown in Fig. (10).

The received signal was analyzed using a short pulse method of isolation and identification of resonances, and later the form function was plotted. In Fig. (11)The form function, where there is a crack at angles 0, 90, and 180 degrees inside the matrix, is compared with the healthy matrix form function.

As can be seen from Fig. (12), the existence of crack at angle zero degree caused an increase in all frequency modes. Moreover, the minimum variations were observed in a case of crack with an angle of 180 degrees.
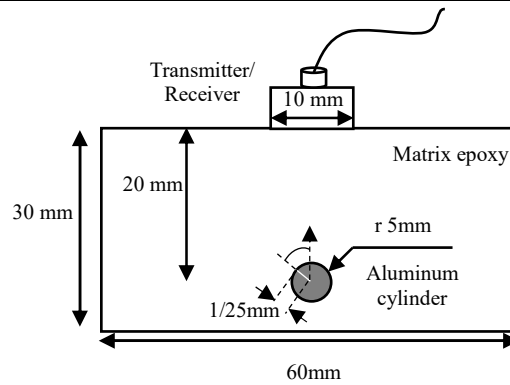
Fig. 13. A schematic of the geometrical structure of an aluminum cylinder embedded in an epoxy medium and how the measuring system located in a case of crack existence in an aluminum cylinder.
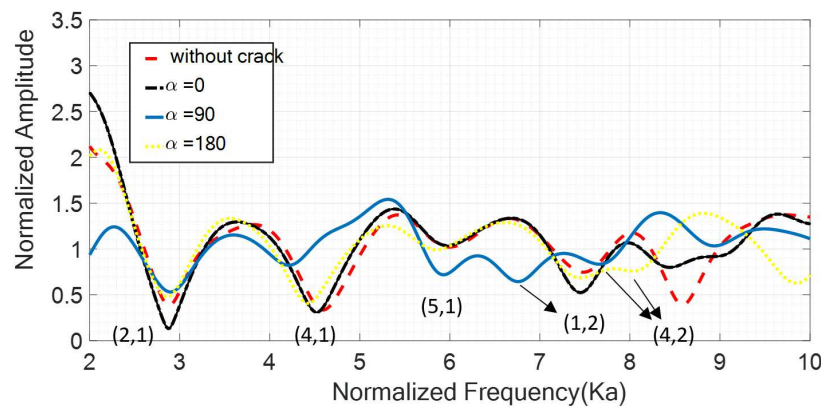

Fig. 14. Form function of an aluminum cylinder embedded in epoxy medium and how the measuring system located in healthy case and existence of a crack in the aluminum cylinder.
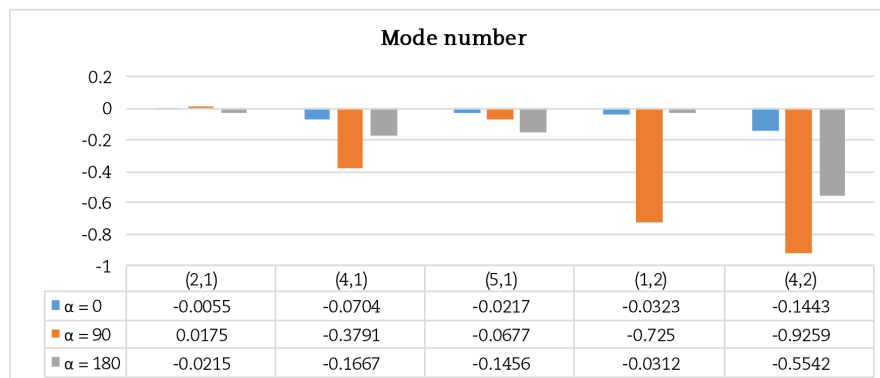

Mode number	(2,1)	(4,1)	(5,1)	(1,2)	(4,2)
$\alpha = 0$	-0.0055	-0.0704	-0.0217	-0.0323	-0.1443
$\alpha = 90$	0.0175	-0.3791	-0.0677	-0.725	-0.9259
$\alpha = 180$	-0.0215	-0.1667	-0.1456	-0.0312	-0.5542

Fig. 15. Variations of Normalized Frequency due to the existence of a crack in the aluminum cylinder.

4.2.2. Crack in cylinder

In this section, by creating a 1.25mm-long crack on the matrix-cylinder contact boundary and with three different orientations, the effect of the crack on the form function is studied. The schematic of how these cracks located is shown in Fig. (13).

By deconvoluting the received signal where these defects exist, the form function is plotted. In Fig. (14). The form function, where there are three types of crack located at angles 0, 90, and 180 degrees inside the matrix, has been compared with healthy matrix form function. As can be seen from Fig. (15), the existence of crack at angle 90 degree, in contrast to lack of the crack, caused severe variations at the point where the normalized frequencies are located. Moreover, the existence of crack at angle zero degree does not cause many variations in the form function of an aluminum cylinder.

5. Conclusion

The form function depends on the configuration and the geometry of the scatterer. Any physical change in the scatterer or in the boundary conditions led to a change in the form function. In the present research, the effect of the crack on the frequency spectrum of the far-field backscattered was investigated. To this end, the effect of crack orientation in the epoxy matrix and aluminum cylinder on the form function was scrutinized. It was found that the Rayleigh frequency modes are more sensitive to the existence of the crack in the epoxy matrix. Also, the whispering gallery frequency modes are more sensitive to the existence of a crack in the aluminum cylinder, and the existence of the crack in the cylinder caused more displacement in these frequency modes than the other modes. The existence of crack in the right-hand side of the cylinder caused more displacement in frequency modes. The validation of the proposed method was performed by comparing the form function of the steel cylinder obtained from the FEM with experimental and analytical results as well as comparing the form function of aluminum cylinder with form function of the reference [12].

Author Contributions

O. Noormohammadi Arani initiated the project and developed the mathematical and finite element modeling and conducted the solutions. A. Yaghootian and S. Sodagar identified the problem under discussion and suggested the solutions; The manuscript was written through the contribution of all authors. All authors discussed the results, reviewed, and approved the final version of the manuscript.

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Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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