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# A Note on the Hydromagnetic Blasius Flow with Variable Thermal Conductivity

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**Abstract**. In this paper, the influence of the transverse magnetic field is unraveled on the development of steady flow regime for an incompressible fluid in the boundary layer limit of a semi-infinite vertical plate. The sensitivity of real fluids to changes in temperature suggests a variable thermal conductivity modeling approach. Using appropriate similarity variables, solutions to the governing nonlinear partial differential equations are obtained by numerical integration. The approach used here is based on using the shooting method together with the Runge-Kutta-Fehlberg integration scheme. Representative velocity and temperature profiles are presented at various values of the governing parameters. The skin-friction coefficient and the rate of heat transfer are also calculated for different parameter values. Pertinent results are displayed graphically and discussed. It is found that the heat transfer rate improves with an upsurge in a magnetic field but lessens with an elevation in the fluid thermal conductivity.

**Keywords**: MHD, Blasius flow, Variable thermal conductivity, Heat transfer.

## 1. Introduction

Virtually all the fluid of industrial and engineering applications are useful in energy dissipation in a thermal system [1], for instance, the temperature is known to alter the fluid thermal conductivity significantly, and in control systems it is well known that interference with magnetic field alters the flow behavior of most electrically conducting fluids. This control strategy plays an important role in skin friction and the rate of heat transfer. Other important applications are witnessed in many geophysical situations where MHD problem arises from the origin of the Earth's magnetic field to the prediction of space weather, the damping of turbulent fluctuations in semiconductor melts during crystal growth and even in the measurement of the flow rates of beverages in the food industry. Other interesting application of MHD are not limited to spraying in metallurgical engineering, electrochemistry and electroplating processing, and other surface occurrences. In view of these wide applications, several researchers have worked on various aspects of magnetohydrodynamics in the boundary layer region. For instance, Soundalgekar and Takhar [2] pioneered a study on flow with heat transfer using the Blasius-type model. The result showed that the magnetic field does create stress in the fluid thereby causing the generation of heat with negligible induced magnetic field. A similar study by Rossow [3] on transverse magnetic field influence on the thermal structure of an electrically conducting viscous fluid past a semi-infinite flat plate was conducted with weak induced magnetic field induction assumption. Takhar *et al.* [4] examined the convective viscous dissipating flow over a semi-infinite vertical plate while Sakiadis [5] studied the boundary layers on a continuous semi-infinite sheet moving steadily through a quiescent fluid environment. Kumari *et al.* [6] analyzed the MHD flow and heat transfer over a stretching surface with prescribed wall temperature or heat flux. Non-similar, laminar, steady, electrically conducting forced convection liquid metal boundary flow with induced magnetic field effects have been studied by Beg *et al.* [7]. Takhar [8] has considered aligned magnetic field effects on laminar hydrodynamic boundary layer convection along an impulsively-started semi-infinite plate. Srinivasa and Eswara [9] studied unsteady laminar boundary layer flow due to an impulsively stretching surface. Devi and Nagraj [10] investigated heat and mass transfer in unsteady magneto hydrodynamic flow over a semi-infinite flat plate. Recently, Elbashbeshy [11] considered heat transfer over a stretching surface with variable and uniform surface heat flux subject to injection and suction. Chamkha [12] studied the problem of steady, laminar, free convection flow over a vertical porous surface in the presence of magnetic field. Watanabe [13] numerically analyzed the natural convection hydrodynamic wedge flow in the presence of a transverse magnetic field. Other relevant work can be found in Ref. [14-37]. The present paper extends the previous works to study the exponential law of velocity of fluid for the boundary layer problem in the presence of uniform magnetic field.

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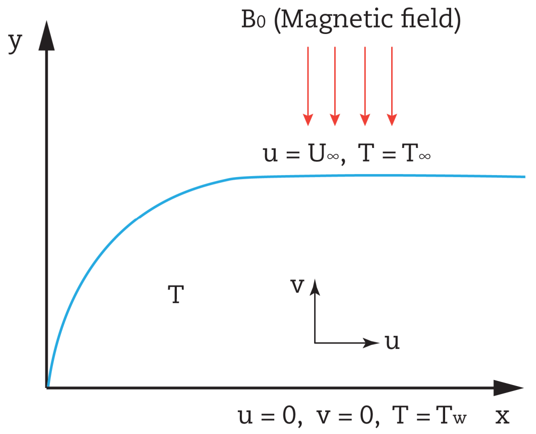


**Fig. 1.** Problem geometry

All the studies above neglects the influence of temperature on the fluid thermal conductivity, however, the assumption that the thermal conductivity of a fluid is constant is not true in practical purposes since thermal conductivity is either a linear or nonlinear function of temperature. Therefore, a more accurate computation must admit these changes. The present study therefore focuses on the inclusion of combined effects of constant magnetic field and variable thermal conductivity in the study of the electrically conducting fluid flow of Blasius type in the boundary layer of the semi-infinite heated plate [1]. In the following section, the mathematical analysis is presented with the flow assumptions.

## 2. Problem Formulation

Consider the convective boundary layer flow adjacent to a heated vertical plate, the wall is maintained at a constant wall temperature $T_w$ whereas the uniform ambient temperature is $T_\infty$. Using the Cartesian coordinates (x, y), where the y-axis is taken along a semi-infinite plate in the direction of flow, and the x-axis is taken perpendicular to it. A transverse magnetic field of uniform strength (0, $B_0$, 0) is assumed to be applied to the plate (see figure 1). In addition, the combined effects Ohmic heating and viscous dissipation is assumed to be negligible.
Without the induced magnetic field, the appropriate conservation equations are from the mass conservation, the momentum and the energy equation [1-4, 16-21]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u-U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right), \tag{3}$$

subject to the boundary conditions

$$\left.\begin{array}{c} u(x,0)=0,\;\; v(x,0)=0,\;\; T(x,0)=T_w,\\ u(x,\infty)\to U_\infty(x),\;\; T(x,\infty)\to T_\infty, \end{array}\right\} \tag{4}$$

In order to facilitate the analysis of full set of governing equations, along with the boundary conditions the equations (1)-(3) are dimensionless and the following similarity variables are introduced [3],

$$\eta=y\sqrt{\frac{mU_0}{4\upsilon}}e^{\frac{mx}{4}},\psi=U_0e^{\frac{mx}{4}}\sqrt{\frac{4\upsilon}{mU_0}}f(\eta),\theta=\frac{T-T_\infty}{T_w-T_\infty},U_\infty(x)=U_0e^{\frac{mx}{2}},\alpha=\frac{k}{\rho c_p}, \tag{5}$$

The thermal conductivity is taken as usual as in [5]

$$k=k_0[1+\beta\theta], \tag{6}$$

Substituting the similarity variables into the governing equations (1) to (3), we obtain the equations:

$$f'''-2f'^2+ff''-4M(f'-1)+2=0 \tag{7}$$

$$(1+\beta\theta)\theta''+\beta\theta'^2+\Pr f\theta'=0 \tag{8}$$

where the prime symbols indicates the differentiation with respect to $\eta$. The boundary conditions (4) then become

$$\left.\begin{array}{c} f(0)=f'(0)=0,\;\; \theta(0)=1,\\ f'\to 1,\;\; \theta\to 0,\;\; as\;\; \eta\to\infty. \end{array}\right\} \tag{9}$$

Other quantities of engineering interest are the skin friction $C_f$ and the Nusselt number $Nu$ which are given as:

$$\mathrm{Re}_x^{1/2} C_f = f''(0), \quad \mathrm{Re}_x^{-1/2} Nu = -\theta'(0), \tag{10}$$

where $Re_x$ is the local Reynolds number.

## 3. Results and Discussion

In this section, graphical results from the modelling and simulation of the nonlinear problem is resented for Blasius -type model experiencing transverse magnetic field is presented noting the effect of temperature dependent thermal conductivity. Surprisingly, as reported in figures 2 and 3, when the effect of magnetic field on the fluid flow (Air at Pr = 1) is taken into consideration, both the velocity and temperature boundary layer thicknesses diminish. Consequently, the fluid moves closer to the plate surface while the fluid temperature declines due to exchange of heat between the fluid and the plate surface. Figure 4 explains the impact of the Prandtl number on the thermal distribution, as presented in the graph, increasing the Prandtl number lessens the thermal boundary thickness and the fluid temperature distribution. The simple reason for this behavior is due a drop in the thermal diffusivity as the Prandtl number increases. In figure 5, the effect of increasing thermal conductivity parameter on the fluid temperature is shown. Interestingly, thermal boundary layer thickness is enhanced and the fluid temperature rises with a boost in thermal conductivity. This is expected, since more heat flows into the fluid from the plate as fluid thermal conductivity rises. Finally, results in figures 6 & 7 confirm that the skin friction at the plate surface increases with increasing values of the magnetic field intensity parameter while the rate of heat transfer is seen to decrease with a rise in thermal conductivity of the fluid. Meanwhile, a boost in Prandtl number enhances the Nuselt number. These observations also agree with the numerical results displayed in tables 1 and 2. Clearly both the skin friction and Nusselt number exhibit very strong positive correlation with the magnetic field while an increase in the thermal conductivity parameter exhibits a strong negative correction with the Nusselt number as shown in table 2. Figure 8 shows a comparison between the results for Nusselt number obtained for a special case of this present study when (M=0, β=0) to that of ref. [31] and a perfect agreement is observed. This validates the accuracy of our numerical results displayed in this paper.

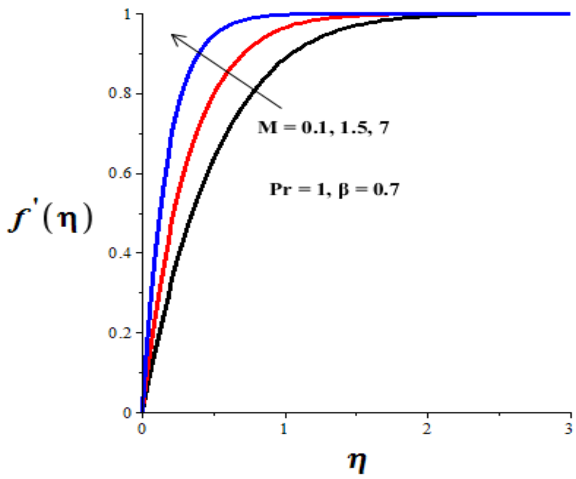


**Fig. 2.** Velocity profiles with increasing M.

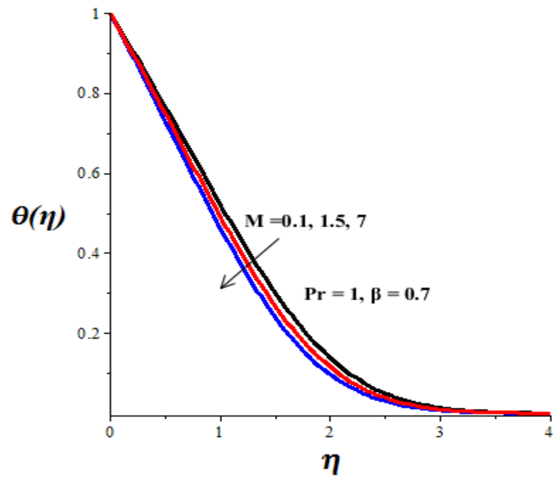


**Fig. 3.** Temperature profiles with increasing M.

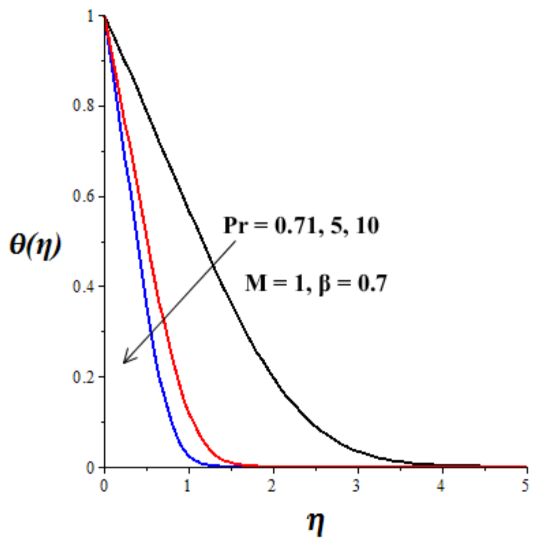


**Fig. 4.** Temperature profiles with increasing Pr.

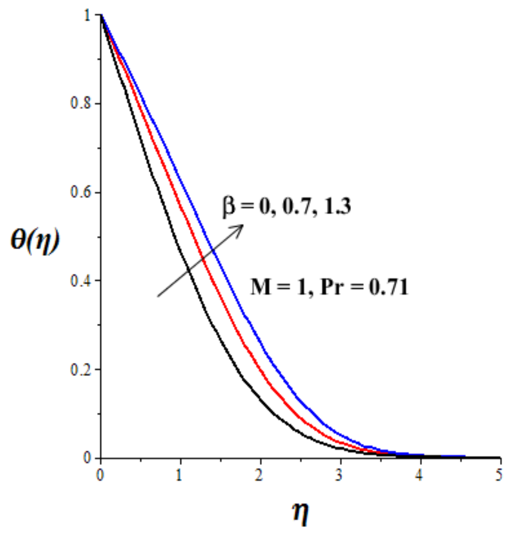


**Fig. 5.** Temperature profiles with increasing β.

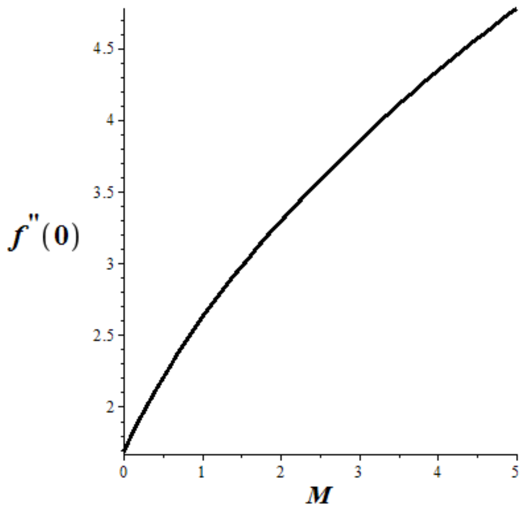


Fig. 6. Skin friction with increasing M.

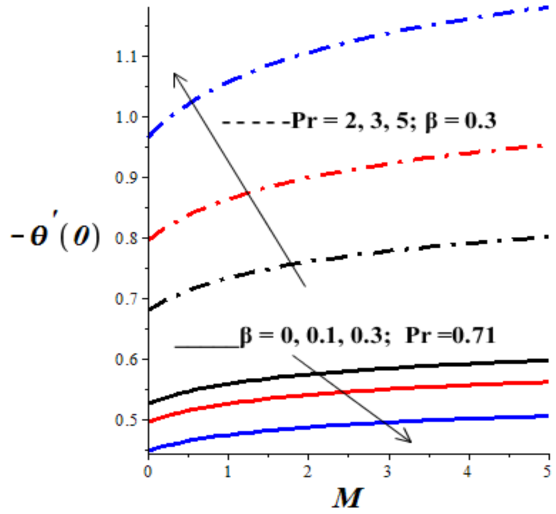


Fig. 7. Nusselt number with increasing M, β and Pr.

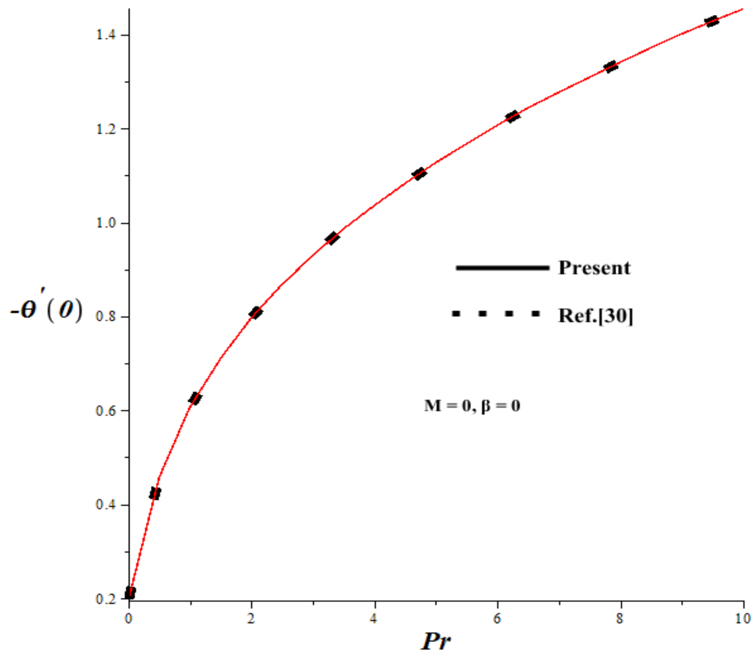


Fig. 8. Numerical validation of present study with ref. [31].

Table 1. Computations showing the skin friction $f''(0)$ and Nusselt number $-\theta'(0)$.

| M | β | Pr | $f''(0)$ | $-\theta'(0)$ |
|---|---|---|---|---|
| 0.1 | 0.7 | 0.71 | 1.801510747 | 0.386981748 |
| 1 | 0.7 | 0.71 | 2.615314412 | 0.405034450 |
| 3 | 0.7 | 0.71 | 3.851825639 | 0.421969183 |
| 5 | 0.7 | 0.71 | 4.778647860 | 0.430212129 |
| 1 | 1 | 0.71 | 2.615314412 | 0.369050103 |
| 1 | 1.5 | 0.71 | 2.615314412 | 0.325931415 |
| 1 | 2 | 0.71 | 2.615314412 | 0.295236160 |
| 1 | 0.7 | 1 | 2.615314412 | 0.469394185 |
| 1 | 0.7 | 5 | 2.615314412 | 0.907408760 |
| 1 | 0.7 | 10 | 2.615314412 | 1.186445909 |

Table 2. Computations showing the $f''(0)$ and $-\theta'(0)$ correlation coefficients with respect to parameters.

| Data Points | Correlation Coefficient(r) | Data Points | Correlation Coefficient(r) |
|---|---|---|---|
| $(M, f''(0))$ | 0.992653 | $(M, -\theta'(0))$ | 0.956928 |
| $(\beta, f''(0))$ | 0 | $(\beta, -\theta'(0))$ | -0.995832 |
| $(Pr, f''(0))$ | 0 | $(Pr, -\theta'(0))$ | 0.983645 |

## 4. Conclusions

Inspired by the zeal to document the combined impact of a constant magnetic field on the steady Blasius-type problem, a numerical integration approach based on Runge-Kutta-shooting-method was performed with similarity transformation to change the nonlinear partial differential equations to the corresponding boundary-value problem. Summarily, the major contributions to existing knowledge are as follows:

i- wall skin friction is an increasing function of the magnetic field parameter.
ii- heat transfer rate increases with Prandtl number and magnetic field but drops with increasing values of the variable thermal conductivity parameter.
iii- Both thermal and velocity boundary layer thickness lessen with a rise in magnetic field intensity.
iv- Fluid temperature increases with a rise in thermal conductivity but diminishes with a rise in Prandtl number.

## Conflict of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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## Nomenclature

| | |
|---|---|
| $u$ ,$v$ | Components of velocity in the $x$ and $y$ directions |
| T | Temperature |
| $\sigma, \rho$ , $\upsilon$ | Electrical conductivity, density and kinematic viscosity respectively |
| $B_0$ | Magnetic induction |
| $\alpha$ | Thermal diffusivity |
| $U_\infty$ | Free stream velocity |
| $\eta$ | Dimensionless similarity variable |
| $m$ | Positive constant |
| $k_0$ | Free stream thermal conductivity |
| $\beta = a(T_W , T_\infty)$ | Thermal conductivity parameter |
| $M = \sigma B_0^2 / m\rho U_\infty$ | Magnetic parameter |
| $\Pr = \upsilon / \alpha$ | Prandtl number |
| $\beta$ | Variable thermal conductivity parameter |

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